PL

Παρασκευή, 21 Ιανουαρίου 2022 3:44 μμ

Πχ) Από κοινή σ.π. $f(x,y)$ των τ.μ. X κ' Y δίνονται:

$$f(1,1) = f(2,1) = \frac{1}{8}, \quad f(1,2) = \frac{1}{4}, \quad f(2,2) = \frac{1}{2}$$

α) Από κοινή σ.π.

$x \backslash y$	1	2	$f_X(x)$
1	1/8	1/4	3/8
2	1/8	1/2	5/8
$f_Y(y)$	2/8	6/8	1

→ Περιθώρια του X

→ Περιθώρια του Y.

ΔΕΣΜΕΥΜΕΝΕΣ σ.π.

$f_{X|Y}(x|Y=y)$ κ' $f_{Y|X}(y|X=x)$.

Γνωρίζουμε: $f_{X|Y}(x|Y=y) = \dfrac{f_{X,Y}(x,y)}{\;}$

$f_Y(y)$

και: $f_{Y|X}(y|X=x) = \dfrac{f_{X,Y}(x,y)}{f_X(x)}$

Οπότε:

$f_{X|Y}(x|y)$

x \ y	1	2
1	1/2	1/3
2	1/2	2/3

$f_{Y|X}(y|x)$

y \ x	1	2
1	1/3	1/5
2	2/3	4/5

$$f_{X|Y}(1|Y=1) = \frac{f_{X,Y}(1,1)}{f_Y(1)} = \frac{1/8}{2/8} = \frac{1}{2}$$

⋮

Τέλος, οι X κ' Y τ.μ. δεν είναι ανεξάρτητες

Άρα: $f_{X,Y}(x,y) \neq f_X(x) \cdot f_Y(y)$

β) $E(X) = ;$ κ' $E(Y) = ;$

$$E(X) = \sum_{x=1}^{2} x \cdot f_X(x) = 1 \cdot \frac{3}{8} + 2 \cdot \frac{5}{8} = \frac{13}{8}$$

$$E(Y) = \sum_{y=1}^{2} y\, f_Y(y) = 1 \cdot \frac{1}{4} + 2\,\frac{3}{4} = \frac{7}{4}.$$

Επίσης, ζητείται:

$E[X \mid Y=y] = ;$ κ' $E[Y \mid X=x] = ;$

Έχουμε:

$$E[X \mid Y=1] = \sum_{x=1}^{2} x \cdot f_{X|Y}(x \mid y=1) =$$

$$= 1 \cdot \frac{1}{2} + 2\,\frac{1}{2} = \frac{3}{2}$$

$$E[X \mid Y=2] = \sum_{x=1}^{2} x\, f_{X|Y}(x \mid y=2) =$$

$$= 1 \cdot \frac{1}{3} + 2\frac{2}{3} = \frac{5}{3}$$

$$E[Y|X=1] = \sum_{y=1}^{2} y \cdot f_{Y|X}(y|x=1) =$$

$$= 1 \cdot \frac{1}{3} + 2\frac{2}{3} = \frac{5}{3}$$

$$E[Y|X=2] = \sum_{y=1}^{2} y \cdot f_{Y|X}(y|x=2) =$$

$$= 1 \cdot \frac{1}{5} + 2\frac{4}{5} = \frac{9}{5}$$

$$E(X+Y) = E(X) + E(Y) = \frac{13}{8} + \frac{7}{4} = \frac{27}{8}.$$

$$E(XY) = \sum_{x}\sum_{y} xy \cdot f_{X,Y}(x,y) =$$

$$= 1 \cdot \frac{1}{8} + 2\frac{1}{4} + 2\frac{1}{8} + 4\frac{1}{2} = \frac{23}{8}.$$

$$E\left(\frac{X}{Y}\right) = \sum_{x}\sum_{y} \frac{x}{y} f_{X,Y}(x,y) =$$

$$= 1\frac{1}{8} + \frac{1}{2} \cdot \frac{1}{4} + \frac{2}{1}\frac{1}{8} + 1\frac{1}{2} = 1$$

$$E\left(\frac{Y}{X}\right) = \sum_x \sum_y \frac{y}{x} f_{X,Y}(x,y) =$$

$$= 1\frac{1}{8} + \frac{2}{1}\frac{1}{4} + \frac{1}{2}\frac{1}{8} + 1\frac{1}{2} = \frac{19}{16}$$

$$V(X) = E(X^2) - E(X)^2$$

$$E(X^2) = \sum_{x=1}^{2} x^2 f_X(x) =$$

$$= 1^2\frac{3}{8} + 2^2\frac{5}{8} = \frac{3}{8} + \frac{20}{8} = \frac{23}{8}$$

$$V(X) = \frac{23}{8} - \left(\frac{13}{8}\right)^2 = \_\_\_\_\_\_$$

$$C(X,Y) = E(XY) - E(X)\cdot E(Y)$$

$$= \frac{23}{8} - \frac{13}{8}\cdot\frac{14}{8} = \_\_\_\_\_$$

Τέλος

$$\rho = \frac{C(X,Y)}{\sqrt{V(X)}\sqrt{V(Y)}} = \frac{?}{?\ \ ?}$$

Πχ] Η από κοινού σ.π.π. των X κ' Y είναι

$$f(x,y) = \begin{cases} 2e^{-x-2y}, & \text{αν } x>0,\ y>0 \\ 0, & \text{αλλού.} \end{cases}$$

α) $$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_0^{\infty} 2e^{-x}e^{-2y}\,dy =$$

$$= 2e^{-x}\int_0^{\infty} e^{-2y}\,dy = 2e^{-x}\cdot\left[-\frac{e^{-2y}}{2}\right]_0^{\infty} = e^{-x}$$

Ομοίως: $$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \int_0^{\infty} 2e^{-x-2y}\,dx = 2e^{-2y}$$

Ισχύει: $$f_{X,Y}(x,y) = 2e^{-x-2y} = e^{-x}\cdot 2e^{-2y} = f_X(x)\,f_Y(y)$$

άρα είναι ανεξάρτητες.

β) $$P(X>1) = \int_1^{\infty} f_X(x)\,dx = 1 - \int_0^1 f_X(x)\,dx$$

$$= 1 - F_X(1) = 1 - [1 - e^{-1}] = e^{-1}$$

$$= 1 - F_x(1) = 1 - [1 - e] = e$$

$$P(Y<3) = \int_0^3 f_Y(y)\,dy = F_Y(3) = 1 - e^{-2\cdot 3} = 1 - e^{-6}$$

$$P(X>2 \mid Y<1) = P(X>2) = 1 - P(X \leq 2)$$

$$= 1 - F_X(2) = - - -$$

$$P(X<Y) = \int_0^\infty \int_x^\infty f_{X,Y}(x,y)\,dy\,dx$$

$$= \int_0^\infty \int_0^y f_{X,Y}(x,y)\,dx\,dy.$$