REC3D: An Accumulative Reconstruction Algorithm based on Volume Intersectional Information for PET

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Abstract

A new PET image reconstruction algorithm from raw data, based on analytical geometry relations and without *a priori* image information, is presented in the current study. The REC3D algorithm transforms the difficult mathematical problem of the PET image reconstruction in a simple geometrical one. The developed technique utilizes the accumulated ray density distribution in a predefined voxelized volume with appropriate dimensions which covers a given field of interest. The density distribution is the accumulation product of a geometrically weighted intersection of the annihilation line, as it is defined by the detected position of the two anti-diametrically emitted annihilation photons, with all the affected voxels. The final 3D tomographic image is created by properly interpreting the voxel slices of the predefined volume. The algorithm is computationally optimized using an acceleration method, which scans only the affected voxels along the Line of Interest (LoR) in a continuous way. Following this technique for each pair of the detected annihilation photons, the computational time is reduced significantly. The efficiency of this method is evaluated with several phantoms simulated inside the GEANT4/GATE environment and the reconstruction results are compared with other widely used analytical and iterative algorithms.

Keywords: `PET`, `Image Reconstruction`, `GEANT4/GATE`

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1. Introduction

The Positron Emission Tomography (PET) imaging technique is used in diagnosis and biomedical research. It has proved to be particularly useful for studying brain and heart functions and certain biochemical processes involving these organs (e.g., glucose metabolism and oxygen uptake). A chemical compound in PET, which is called radiotracer, labeled with a short-lived positron-emitting radionuclide of Carbon, Oxygen, Nitrogen or Fluorine is injected into the patients body. PET scanners collect data from the radiotracer distribution. Iterative PET image reconstruction algorithms can achieve superior image quality when compared with conventional analytic reconstruction methods, which usually assume very simple models and present streak artifacts [1],[2]. However, iterative algorithms require optimization techniques and computational times for fully 3D reconstruction which can limit the practical use of these algorithms. Consequently, in recent years many techniques have been proposed to reduce the complexity and number of iterations required for effective convergence [3],[4].

The motivation behind the present work is the development of a new PET reconstruction algorithm. The REC3D uses raw data information and unlike other commonly used algorithms does not require an *a priori* image assumption or a tremendous number of data; REC3D is also not bound or restricted by any PET scanner geometry. This novel algorithm transforms the difficult mathematical reconstruction problem to a pure geometrical one. The developed technique utilizes the accumulated density distribution in a predefined voxelized volume with the appropriate dimensions which covers the field of interest.

2. The REC3D Reconstruction Algorithm

All PET scanners can provide raw data for the position of two anti-diametrically and simultaneously detected photons. With this information, the interior of the detector Field of View (FoV) is a big volume divided in N_x, N_y, N_z cubic voxels along the X, Y and Z axis respectively. The Line of Interest (LoR) of every event will intersect some of these cubic voxels. The reconstruction problem basically becomes an intersection procedure of a given line with a voxel. This problem can be further simplified by the definition that every voxel consists of six planes. Immediately the geometrical problem is converted to an even simpler one, the intersection of a line with a plane.

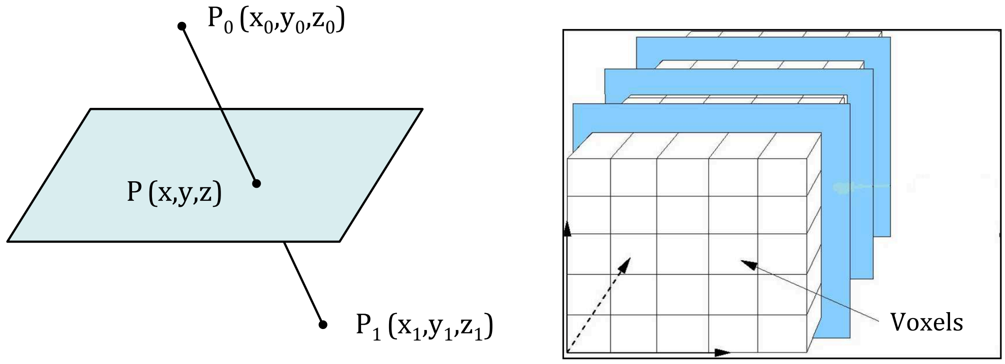

Figure 1: Intersection of the annihilation line with a plane and the definition of the volume of interest

The annihilation line is primarily defined by the two opposite 511 keV photons, detected at the points $P_0(x_0, y_0, z_0)$ and $P_1(x_1, y_1, z_1)$ respectively. The line intersects a given plane at the point P(x,y,z), which is given by the analytical equation:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0} \tag{1}$$

For a given plane vertical to one of the system axis X, Y, Z, one of the fractions is arithmetically defined and therefore the full coordinates of the intersection point P(x,y,z) are calculable. For example, for the vertical plane to axis Z at the point $\mathcal{Z}$, its intersection point with the line P_1P_2 is given by:

$$P = \left[\frac{\mathcal{Z} - z_0}{z_1 - z_0}(x_1 - x_0) + x_0 \ , \ \frac{\mathcal{Z} - z_0}{z_1 - z_0}(y_1 - y_0) + y_0 \ , \ \mathcal{Z} \right] \tag{2}$$

The accumulative weight factor, which is used to assign the luminosity distribution in each voxel is the Euclidean distance of the line inside the voxel. The line passes through the voxel from two points $\mathcal{C}_1(u_1, v_1, w_1)$ and $\mathcal{C}_2(u_2, v_2, w_2)$. The Euclidean distance $\mathcal{D}is$ is calculated:

$$\mathcal{D}is = |\mathcal{C}_1 - \mathcal{C}_2| = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2 + (w_1 - w_2)^2} \tag{3}$$

The REC3D algorithm utilizes this technique to scan the whole volume, voxel by voxel, creating the final 3D image.

2.1. Accelerated REC3D

The REC3D algorithm shows the great advantage of minimizing the computation time compared to the other commercially used algorithms. In order to make this advantage even greater an acceleration technique is used. Instead of scanning the whole voxelized volume in order to save computation time and having $\mathcal{D}is$ already known, we take a step further and calculate the first derivatives. This calculation will present a predicted path for the LoR inside the volume. Following the direction of the LoR only the voxels that contribute to the image are scanned.

$$DivX = \frac{C_{1x} - C_{2x}}{\mathcal{D}is} \tag{4}$$

$$DivY = \frac{C_{1y} - C_{2y}}{\mathcal{D}is} \tag{5}$$

$$DivZ = \frac{C_{1z} - C_{2z}}{\mathcal{D}is} \tag{6}$$

In order to ensure high accuracy a specific step to the calculation of the derivative is introduced. Starting from the intersection points C_{1x}, C_{1y}, C_{1z} and with the given voxel dimensions $X_{step}, Y_{step}, Z_{step}$ an increasing active voxel search along the LoR is computed with the QQ_i points:

$$QQ_{ix} = C_{1x} + i \cdot 0.001 \cdot X_{step} \cdot DivX \tag{7}$$

$$QQ_{iy} = C_{1y} + i \cdot 0.001 \cdot Y_{step} \cdot DivY \tag{8}$$

$$QQ_{iz} = C_{1z} + i \cdot 0.001 \cdot Z_{step} \cdot DivZ \tag{9}$$

The REC3D, as previously remarked, scans the whole volume inside the field of view following the intersection procedure for N^3 computational steps. On the other hand, the total number of steps needed in the accelerated method is reduced to $\sqrt{(N_x)^2 + (N_y)^2 + (N_z)^2}$. The comparison of the two methods gives a factor of

$$\frac{N^3}{N\sqrt{3}} = \frac{N^2}{\sqrt{3}} \tag{10}$$

for the speed improvement of the accelerated method.

3. Simulation in GEANT4/GATE Environment

GATE, the Geant4 Application for Tomographic Emission [5], combines the advantages of the general-purpose Geant4 simulation code and of specific software tool implementations dedicated to emission tomography. GATE takes advantage of the well-validated physics models, of the geometry description, and of the visualization and 3D rendering tools offered by Geant4 but has a distinctive characteristic the modeling of time-dependent processes. The PET detector simulated in this study is a small-Animal

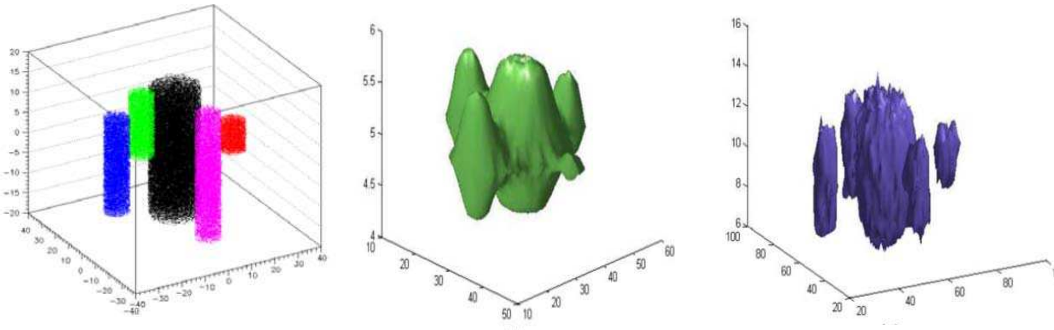

Figure 2: Cylindrical phantom geometry used in the simulation study.

PET scanner called Sherbrooke 16 ring detector [6]. The Sherbrooke Animal PET scanner is a 16-ring camera composed of 256 detectors per ring. It is based on the EG&G C30994 detector module consisting of two BGO scintillators, each coupled individually to a silicon "reachthrough" APD. The detectors are enclosed in an hermetic package of dimensions 3.8 mm x 13.2 mm x 30 mm which determines the channel packing density (4channels/cm^2). The detector modules are physically and logically grouped into cassettes which also incorporate the front-end electronics. Cassettes include 8 modules flayer forming a 2x8 detector array and they can be assembled into "blocks" of several layers. The dimensions and shape of the cassettes are such that they can be used in the construction of various diameter rings without modification. The port diameter is 135 mm, which is suitable for small laboratory animals such as rats, rabbits or the brain of small monkeys. Since each layer of modules consists of two adjacent rings of detectors, three image planes (two direct, one cross) are defined within a 10.5 mm thick transaxial slice [6].

The phantom used in this study is a asymmetrical phantom that consists of five cylindrical sources of various heights and diameters. In order to be freed from the artifacts created by positron emitting sources, due to the positron mean free path, here two gamma back-to-back sources are used. The cylinders are placed in a cross like formation inside the detector as shown in Figure 2. In more detail the phantom characteristics are shown in Table 1.

Source	Radius	Height	(x,y) Placement	Element	Activity
0	10mm	8cm	(+0.0,+0.0) [cm]	^{18}F	8.0 kBq
1	5mm	2cm	(+2.5,+0.0) [cm]	^{18}F	0.5 kBq
2	5mm	4cm	(+0.0,+2.5) [cm]	^{18}F	1.0 kBq
3	5mm	6cm	(-2.5,+0.0) [cm]	^{18}F	1.5 kBq
4	5mm	8cm	(+0.0,-2.5) [cm]	^{18}F	2.0 kBq

Table 1: Geometry and activity of the 5-cylinder phantom used in the GEANT4/GATE simulation.

4. Results and Discussion

The REC3D results where compered with widely used reconstruction software for PET image reconstruction (Figure 3). For the reconstruction image produced by REC3D

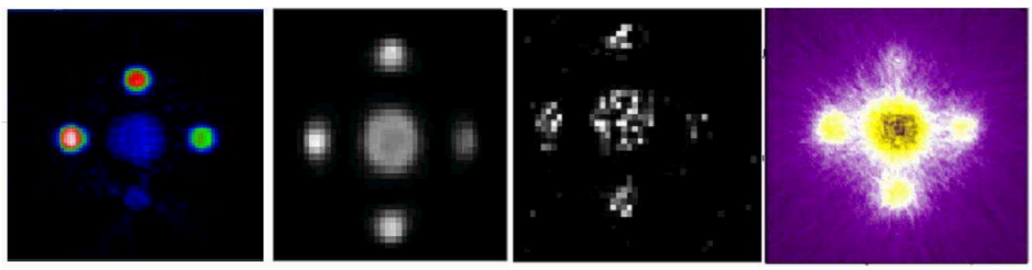

Figure 3: From Left to Right: FBP2D, MLEM, OSEM, REC3D

only 4 981 events where used and its quality is satisfying and superior from the other images. The OSEM algorithm uses 1 404 140 single events and MLEM 1 404 140 as well. The image reconstruction outcome using the FBP2D algorithm needs 4 940 290 events.

The large number of the required events for the other methods is translated to more computation time, which demands more acquisition time and more patient discomfort.

In conclusion it can be noted that REC3D can reconstruct accurately images with fewer data and less computational time. The algorithm can be potentially used in any scanner architecture, regardless the detector's geometrical characteristics. Contrary to the other iterative methods, the REC3D technique is not limited by any data quality restrictions or any kind of data filtering.

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