ΤΥΠΟΛΟΓΙΟ «ΑΣΤΡΟΦΥΣΙΚΑ ΡΕΥΣΤΑ»

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{u})=0,\quad \rho\frac{d\boldsymbol{u}}{dt}=-\nabla P+\rho\nu\nabla^2\boldsymbol{u}+\rho\frac{\nu+3\nu_\tau}{3}\nabla(\nabla\cdot\boldsymbol{u})+\rho\boldsymbol{g}+\delta\boldsymbol{E}+\frac{\boldsymbol{J}\times\boldsymbol{B}}{c},\quad \boldsymbol{g}=-\nabla\Phi_g,\quad \nabla\cdot\boldsymbol{g}=-4\pi G\rho,$$

$$\frac{de}{dt}+P\frac{d}{dt}\left(\frac{1}{\rho}\right)=\frac{\rho q+\nabla\cdot(\kappa\nabla T)+\boldsymbol{\sigma}':\boldsymbol{S}+J^2/\sigma_E}{\rho},\quad e=\frac{1}{\gamma-1}\frac{P}{\rho},\quad \boldsymbol{\sigma}'=2\rho\nu\boldsymbol{S}+\rho\frac{3\nu_\tau-2\nu}{3}(\nabla\cdot\boldsymbol{u})\boldsymbol{I},\quad S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),$$

$$\nabla\cdot\boldsymbol{E}=4\pi\delta,\quad \nabla\cdot\boldsymbol{B}=0,\quad \nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t},\quad \nabla\times\boldsymbol{B}=\frac{4\pi}{c}\boldsymbol{J}+\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t},\quad \frac{\boldsymbol{J}}{\sigma_E}=\boldsymbol{E}+\frac{\boldsymbol{u}\times\boldsymbol{B}}{c},$$

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\boldsymbol{u}\cdot\nabla,\quad \nabla\left(\frac{u^2}{2}\right)+(\nabla\times\boldsymbol{u})\times\boldsymbol{u}=(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=u\frac{\partial u}{\partial\ell}\hat{\varepsilon}+\frac{u^2}{\mathcal{R}}\hat{n},\quad (\nabla\times\boldsymbol{u})\times\boldsymbol{u}=-\nabla_\perp\left(\frac{u^2}{2}\right)+\frac{u^2}{\mathcal{R}}\hat{n},$$

$$\frac{\partial}{\partial t}(\rho\boldsymbol{u})+\nabla\cdot\boldsymbol{\Pi}^{\rm matter}=\rho\boldsymbol{g}+\delta\boldsymbol{E}+\frac{\boldsymbol{J}\times\boldsymbol{B}}{c},\quad \Pi_{ij}^{\rm matter}=\rho u_iu_j+P\delta_{ij}-\sigma'_{ij},$$

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{E}\times\boldsymbol{B}}{4\pi c}\right)+\nabla\cdot\boldsymbol{\Pi}^{\rm field}=-\left(\delta\boldsymbol{E}+\frac{\boldsymbol{J}\times\boldsymbol{B}}{c}\right),\quad \Pi_{ij}^{\rm field}=-\frac{E_iE_j+B_iB_j}{4\pi}+\frac{E^2+B^2}{8\pi}\delta_{ij},$$

$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2}+\rho e\right)+\nabla\cdot\left(\frac{\rho u^2}{2}\boldsymbol{u}+\rho e\boldsymbol{u}+P\boldsymbol{u}-\boldsymbol{\sigma}'\cdot\boldsymbol{u}-\kappa\nabla T\right)=\rho q+\rho\boldsymbol{g}\cdot\boldsymbol{u}+\boldsymbol{J}\cdot\boldsymbol{E},\quad \frac{\partial}{\partial t}\left(\frac{E^2+B^2}{8\pi}\right)+\nabla\cdot\left(\frac{c}{4\pi}\boldsymbol{E}\times\boldsymbol{B}\right)=-\boldsymbol{J}\cdot\boldsymbol{E}.$$

Καρτεσιανές: $d\boldsymbol{r}=dx\hat{x}+dy\hat{y}+dz\hat{z},\quad \nabla f=\frac{\partial f}{\partial x}\hat{x}+\frac{\partial f}{\partial y}\hat{y}+\frac{\partial f}{\partial z}\hat{z},\quad \nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2},$

$$\nabla\times\boldsymbol{u}=\left(\frac{\partial u_z}{\partial y}-\frac{\partial u_y}{\partial z}\right)\hat{x}+\left(\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}\right)\hat{y}+\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right)\hat{z},\quad \nabla\cdot\boldsymbol{u}=\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z},\quad \nabla^2\boldsymbol{u}=\nabla^2u_x\hat{x}+\nabla^2u_y\hat{y}+\nabla^2u_z\hat{z},$$

$$(\boldsymbol{u}\cdot\nabla)\boldsymbol{B}=\boldsymbol{u}\cdot\nabla B_x\hat{x}+\boldsymbol{u}\cdot\nabla B_y\hat{y}+\boldsymbol{u}\cdot\nabla B_z\hat{z}.$$

Κυλινδρικές ($x=\varpi\cos\phi,\quad y=\varpi\sin\phi,\quad \hat{x}=\hat{\varpi}\cos\phi-\hat{\phi}\sin\phi,\quad \hat{y}=\hat{\varpi}\sin\phi+\hat{\phi}\cos\phi$): $d\boldsymbol{r}=d\varpi\hat{\varpi}+\varpi d\phi\hat{\phi}+dz\hat{z},$

$$\nabla f=\frac{\partial f}{\partial\varpi}\hat{\varpi}+\frac{1}{\varpi}\frac{\partial f}{\partial\phi}\hat{\phi}+\frac{\partial f}{\partial z}\hat{z},\quad \nabla^2 f=\frac{1}{\varpi}\frac{\partial}{\partial\varpi}\left(\varpi\frac{\partial f}{\partial\varpi}\right)+\frac{1}{\varpi^2}\frac{\partial^2 f}{\partial\phi^2}+\frac{\partial^2 f}{\partial z^2},$$

$$\nabla\times\boldsymbol{u}=\left(\frac{1}{\varpi}\frac{\partial u_z}{\partial\phi}-\frac{\partial u_\phi}{\partial z}\right)\hat{\varpi}+\left(\frac{\partial u_\varpi}{\partial z}-\frac{\partial u_z}{\partial\varpi}\right)\hat{\phi}+\frac{1}{\varpi}\left[\frac{\partial(\varpi u_\phi)}{\partial\varpi}-\frac{\partial u_\varpi}{\partial\phi}\right]\hat{z},\quad \nabla\cdot\boldsymbol{u}=\frac{1}{\varpi}\frac{\partial(\varpi u_\varpi)}{\partial\varpi}+\frac{1}{\varpi}\frac{\partial u_\phi}{\partial\phi}+\frac{\partial u_z}{\partial z},$$

$$\nabla^2\boldsymbol{u}=\left(\nabla^2u_\varpi-\frac{u_\varpi}{\varpi^2}-\frac{2}{\varpi^2}\frac{\partial u_\phi}{\partial\phi}\right)\hat{\varpi}+\left(\nabla^2u_\phi+\frac{2}{\varpi^2}\frac{\partial u_\varpi}{\partial\phi}-\frac{u_\phi}{\varpi^2}\right)\hat{\phi}+\nabla^2u_z\hat{z},$$

$$(\boldsymbol{u}\cdot\nabla)\boldsymbol{B}=\left(u_\varpi\frac{\partial B_\varpi}{\partial\varpi}+\frac{u_\phi}{\varpi}\frac{\partial B_\varpi}{\partial\phi}+u_z\frac{\partial B_\varpi}{\partial z}-\frac{u_\phi B_\phi}{\varpi}\right)\hat{\varpi}+\left(u_\varpi\frac{\partial B_\phi}{\partial\varpi}+\frac{u_\phi}{\varpi}\frac{\partial B_\phi}{\partial\phi}+u_z\frac{\partial B_\phi}{\partial z}+\frac{u_\phi B_\varpi}{\varpi}\right)\hat{\phi}+\left(u_\varpi\frac{\partial B_z}{\partial\varpi}+\frac{u_\phi}{\varpi}\frac{\partial B_z}{\partial\phi}+u_z\frac{\partial B_z}{\partial z}\right)\hat{z},$$

$$S_{\varpi\varpi}=\frac{\partial u_\varpi}{\partial\varpi},\quad S_{\varpi\phi}=S_{\phi\varpi}=\frac{1}{2\varpi}\left[\frac{\partial u_\varpi}{\partial\phi}+\varpi^2\frac{\partial}{\partial\varpi}\left(\frac{u_\phi}{\varpi}\right)\right],\quad S_{\varpi z}=S_{z\varpi}=\frac{1}{2}\left(\frac{\partial u_\varpi}{\partial z}+\frac{\partial u_z}{\partial\varpi}\right),$$

$$S_{\phi\phi}=\frac{1}{\varpi}\left(u_\varpi+\frac{\partial u_\phi}{\partial\phi}\right),\quad S_{\phi z}=S_{z\phi}=\frac{1}{2}\left(\frac{\partial u_\phi}{\partial z}+\frac{1}{\varpi}\frac{\partial u_z}{\partial\phi}\right),\quad S_{zz}=\frac{\partial u_z}{\partial z}.$$

Σφαιρικές ($x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$, $\hat{z}=\hat{r}\cos\theta-\hat{\theta}\sin\theta$): $d\boldsymbol{r}=dr\hat{r}+rd\theta\hat{\theta}+r\sin\theta d\phi\hat{\phi},$

$$\nabla f=\frac{\partial f}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\hat{\phi},\quad \nabla^2 f=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2},$$

$$\nabla\times\boldsymbol{u}=\frac{1}{r\sin\theta}\left[\frac{\partial(u_\phi\sin\theta)}{\partial\theta}-\frac{\partial u_\theta}{\partial\phi}\right]\hat{r}+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial u_r}{\partial\phi}-\frac{\partial(ru_\phi)}{\partial r}\right]\hat{\theta}+\frac{1}{r}\left[\frac{\partial(ru_\theta)}{\partial r}-\frac{\partial u_r}{\partial\theta}\right]\hat{\phi},$$

$$\nabla\cdot\boldsymbol{u}=\frac{1}{r^2}\frac{\partial(r^2u_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(u_\theta\sin\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial\phi},\qquad \nabla^2\boldsymbol{u}=\left[\nabla^2u_r-\frac{2u_r}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial(u_\theta\sin\theta)}{\partial\theta}-\frac{2}{r^2\sin\theta}\frac{\partial u_\phi}{\partial\phi}\right]\hat{r}+$$

$$+\left(\nabla^2u_\theta-\frac{u_\theta}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\phi}{\partial\phi}\right)\hat{\theta}+\left(\nabla^2u_\phi-\frac{u_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial\phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\theta}{\partial\phi}\right)\hat{\phi},$$

$$(\boldsymbol{u}\cdot\nabla)\boldsymbol{B}=\left(u_r\frac{\partial B_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial B_r}{\partial\theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial B_r}{\partial\phi}-\frac{u_\theta B_\theta+u_\phi B_\phi}{r}\right)\hat{r}+\left(u_r\frac{\partial B_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial B_\theta}{\partial\theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial B_\theta}{\partial\phi}+\frac{u_\theta B_r}{r}-\frac{u_\phi B_\phi\cos\theta}{r\sin\theta}\right)\hat{\theta}+$$

$$+\left(u_r\frac{\partial B_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial B_\phi}{\partial\theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial B_\phi}{\partial\phi}+\frac{u_\phi B_r}{r}+\frac{u_\phi B_\theta\cos\theta}{r\sin\theta}\right)\hat{\phi},$$

$$S_{rr}=\frac{\partial u_r}{\partial r},\quad S_{r\theta}=S_{\theta r}=\frac{1}{2r}\left[\frac{\partial u_r}{\partial\theta}+r^2\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],\quad S_{r\phi}=S_{\phi r}=\frac{1}{2r\sin\theta}\left[\frac{\partial u_r}{\partial\phi}+r^2\sin\theta\frac{\partial}{\partial r}\left(\frac{u_\phi}{r}\right)\right],$$

$$S_{\theta\theta}=\frac{1}{r}\left(u_r+\frac{\partial u_\theta}{\partial\theta}\right),\quad S_{\theta\phi}=S_{\phi\theta}=\frac{1}{2r\sin\theta}\left[\frac{\partial u_\theta}{\partial\phi}+\sin^2\theta\frac{\partial}{\partial\theta}\left(\frac{u_\phi}{\sin\theta}\right)\right],\quad S_{\phi\phi}=\frac{1}{r\sin\theta}\left(u_r\sin\theta+u_\theta\cos\theta+\frac{\partial u_\phi}{\partial\phi}\right).$$

$$\iiint(\nabla\cdot\boldsymbol{A})\,d\tau=\oint\!\!\oint\boldsymbol{A}\cdot d\boldsymbol{a},\quad \iint(\nabla\times\boldsymbol{A})\cdot d\boldsymbol{a}=\oint\boldsymbol{A}\cdot d\boldsymbol{r},\quad \boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=(\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b}-(\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}.$$

$$\nabla\cdot(f\boldsymbol{A})=(\nabla f)\cdot\boldsymbol{A}+f(\nabla\cdot\boldsymbol{A}),\quad \nabla\times(f\boldsymbol{A})=(\nabla f)\times\boldsymbol{A}+f(\nabla\times\boldsymbol{A}),\quad \nabla\times(\nabla\times\boldsymbol{A})=\nabla(\nabla\cdot\boldsymbol{A})-\nabla^2\boldsymbol{A}.$$

$$0=2T+V+\sum_i\boldsymbol{r}_i\cdot\boldsymbol{F}_i^{\rm εξ},\quad 0=3\iiint P\,d\tau-\oint\!\!\oint P\boldsymbol{r}\cdot d\boldsymbol{a}+V,\quad V=\frac{1}{2}\iiint\Phi_g\rho\,d\tau=-\frac{1}{8\pi G}\iiint g^2\,d\tau.$$

$$\lambda_D=\sqrt{\frac{k_{\rm B}T}{8\pi ne^2}},\quad \omega_p=\sqrt{\frac{4\pi ne^2}{m_e}},\quad \sigma=8\sigma_{\rm large}\ln\Lambda,\quad \sigma_{\rm SB}T_{\rm eff}^4=\frac{3GM\dot{M}_a}{8\pi\varpi^3}\left(1-\sqrt{\frac{\varpi_{\rm in}}{\varpi}}\right),\quad c_s=\sqrt{\gamma\frac{P}{\rho}},\quad v_A=\frac{B}{\sqrt{4\pi\rho}}.$$

$$[\rho u_n]=0,\quad [\rho u_n^2+P]=0,\quad [\rho u_n\boldsymbol{u}_t]=0,\quad \left[\frac{\rho u^2}{2}u_n+\frac{\gamma}{\gamma-1}Pu_n\right]=0,\quad \frac{\rho_2}{\rho_1}=\frac{\gamma+1}{\gamma-1+2/M_{1n}^2},\quad \frac{P_2}{P_1}=\frac{2\gamma M_{1n}^2-\gamma+1}{\gamma+1}.$$

Τιμές φυσικών σταθερών στο σύστημα μονάδων cgs: $c=3\times10^{10}$, $G=6.67\times10^{-8}$, $h=6.625\times10^{-27}$, $e=4.8\times10^{-10}$, $m_e=9.1\times10^{-28}$, $m_p=1.67\times10^{-24}$, $k_{\rm B}=1.38\times10^{-16}$, $\sigma_{\rm SB}=5.67\times10^{-5}$, 1 eV $=1.6\times10^{-12}$, 1 yr $=3.1\times10^7$, 1 AU $=1.5\times10^{13}$, 1 pc$=3\times10^{18}$, $M_\odot=2\times10^{33}$, $R_\odot=6.96\times10^{10}$.