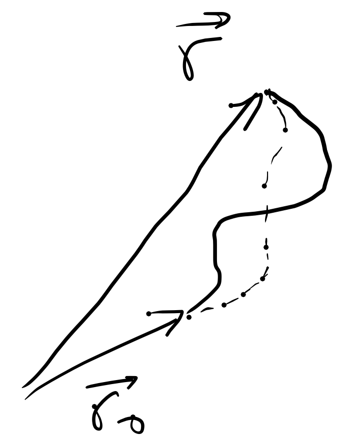
Μηχανική Ι - Διάλεξη 7 - 24/10/25

Θα ξεκινήσω στις ~9:10

- 1D κινήσεις υπό την επήρεια $F(x)$
- Διάγραμμα φάσης - επιτρεπτές κινήσεις - παράμετρος χρόνος
- ευστάθεια - αστάθεια

Συντηρητικές δυνάμεις $\vec{\nabla}\times\vec{F}=\vec{0}$

$$V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{r}$$



$$\frac{1}{2}m\vec{v}^2 + V(\vec{r}) = E$$

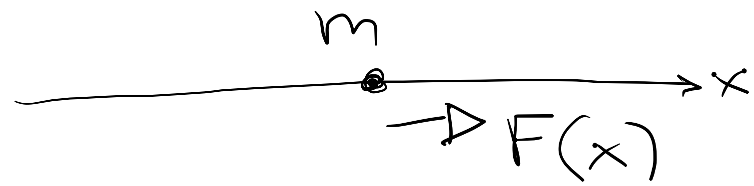
$$0=\frac{d}{dt}\Big[\quad " \quad\Big] = m\vec{v}\cdot\frac{d\vec{v}}{dt} + \frac{d}{dt}V(\vec{r}(t))$$

$E=$ σταθ.

$$\frac{d}{dt}V(\vec{r}(t)) = \vec{v}\cdot\vec{\nabla}V(\vec{r}) = \frac{dx}{dt}\frac{\partial V}{\partial x}+\frac{dy}{dt}\frac{\partial V}{\partial y}+\frac{dz}{dt}\frac{\partial V}{\partial z}$$

$$=\vec{v}\cdot\underbrace{\Big[m\frac{d\vec{v}}{dt}+\underbrace{\vec{\nabla}V}_{-\vec{F}}\Big]}_{\vec{0}}$$

σε 1-D



2ος ν.N. $F(x) = m\frac{du}{dt} = m\frac{d^2x}{dt^2} = m\frac{d}{dt}\frac{dx}{dt}$

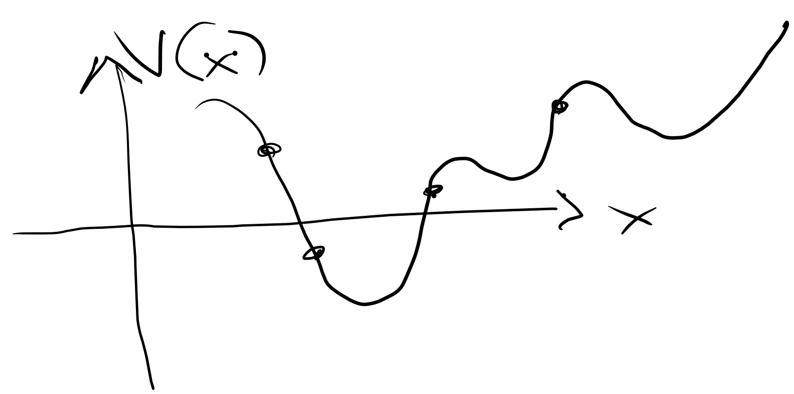
$\exists V(x): \quad F(x) = -\frac{dV}{dx}$

$$\int_{V(x_0)}^{V(x)} dV = -\int_{x_0}^{x} F(x')dx' \Rightarrow V(x) - V(x_0) = -\int_{x_0}^{x} F(x')dx'$$

$$V(x) \equiv V(x_0) - \int_{x_0}^{x} F(x')dx'$$

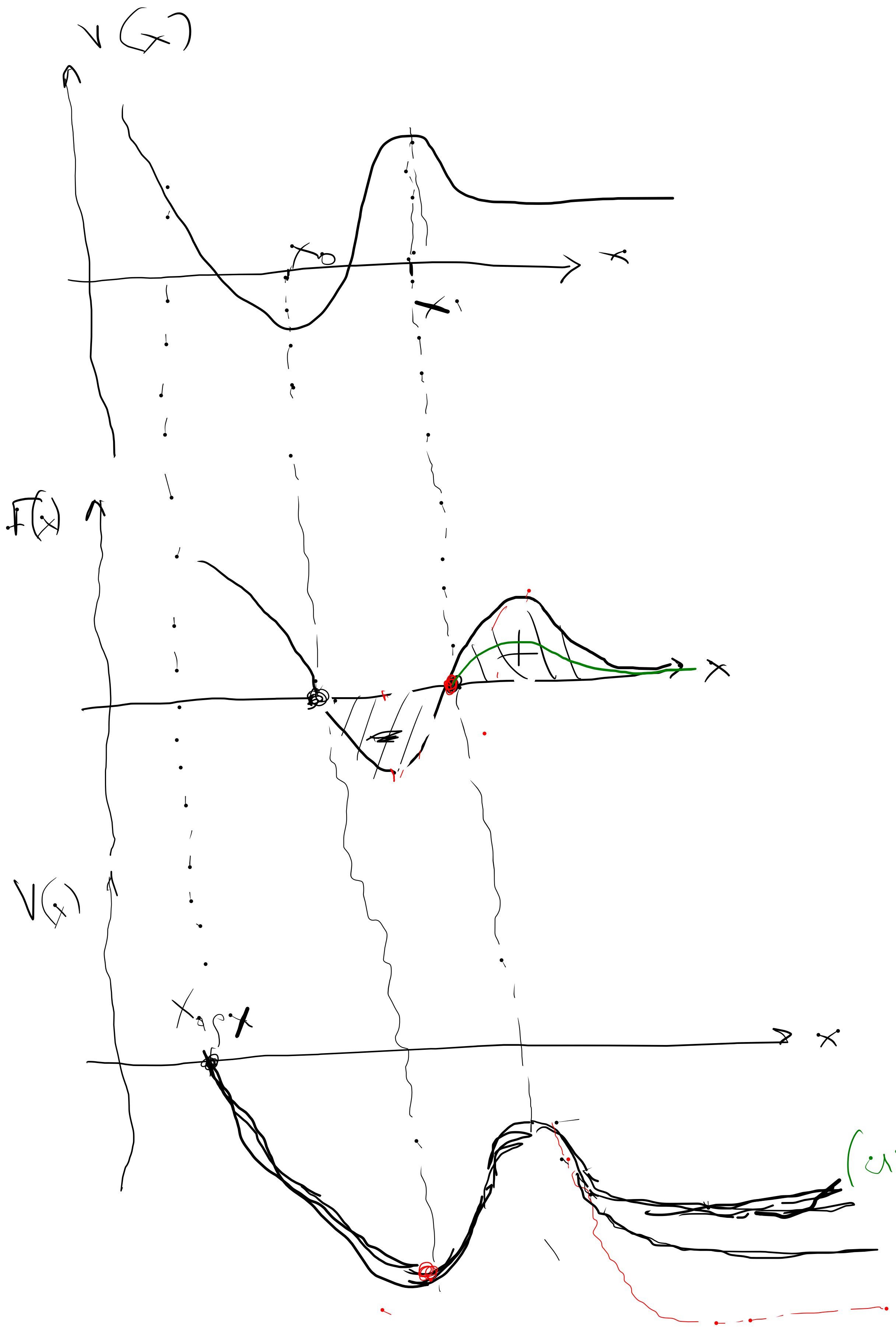
έχοντας κατασκ. την $V(x)$

$$\frac{1}{2}mu^2 + V(x) = E = \frac{1}{2}mu^2(x_0) + V(x_0)$$



- $V(x_0) = V(x_0)$ $\qquad \int_{x_0}^{x_0} = 0$
- δεν υπάρχει άλλη διαδρομή από $x_0 \to x$

$\Rightarrow$ μονοσήμαντα ορισμ. το ολοκλήρωμα της $F(x)$



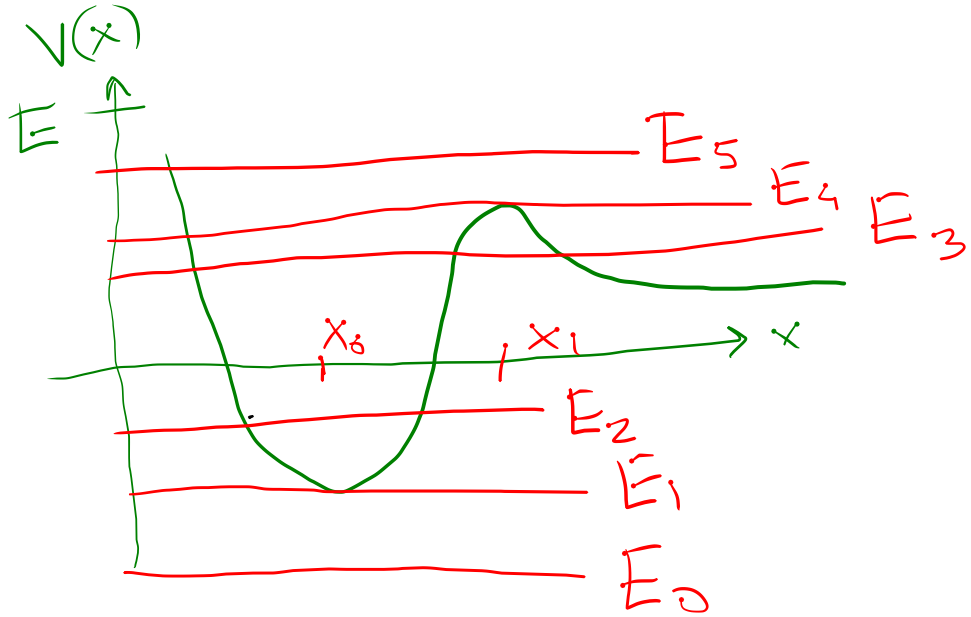
στην προηγ. διαφ.

$V(x_0)$: αυθαίρετο

Εδώ θα θεωρ. αντίστοιχως ότι

$$V(x_{αρχ}) = 0$$

$$V(x) = \underbrace{V(x_{αρχ})}_{0} - \int_{x_{αρχ}}^{x} F(x')\,dx'$$



Ποια η κίνηση αν η τιμή της E =κάτι?

$V'(x_0)=0 \quad V'(x_1)=0 \quad V''(x_0)>0 \quad V''(x_1)<0$

1) $E=E_0$ $\quad \frac{1}{2}mv^2 + V(x) = E_0 < V(x_0) = \min V(x)$

$\underbrace{\frac{1}{2}mv^2}_{\text{εξ ορισμού } \geq 0} < \underbrace{\min V(x) - V(x)}_{<0}$

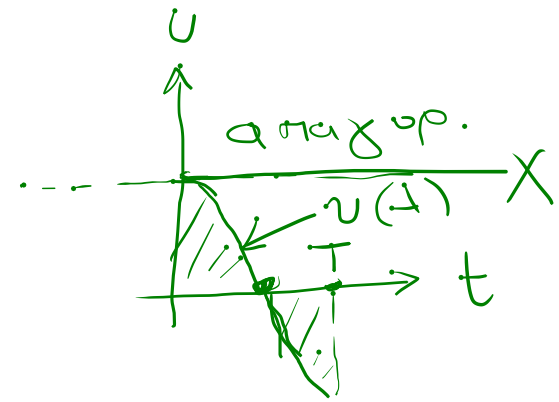
ΔΕΝ υπάρχει κίνηση με αυτή την $E=E_0$

2) $E=E_1$ $\quad \underbrace{\frac{1}{2}mv^2}_{\geq 0} = \underbrace{V(x_0)}_{E_1} - \underbrace{V(x)}_{\geq E_1} \Rightarrow$

Μόνο $x=x_0$ επιτρεπτό

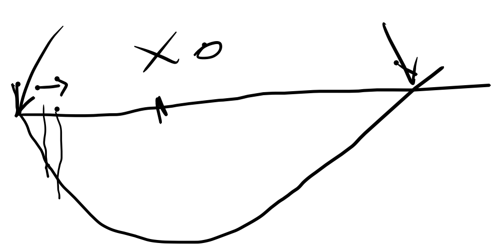
$x(t)=x_0$

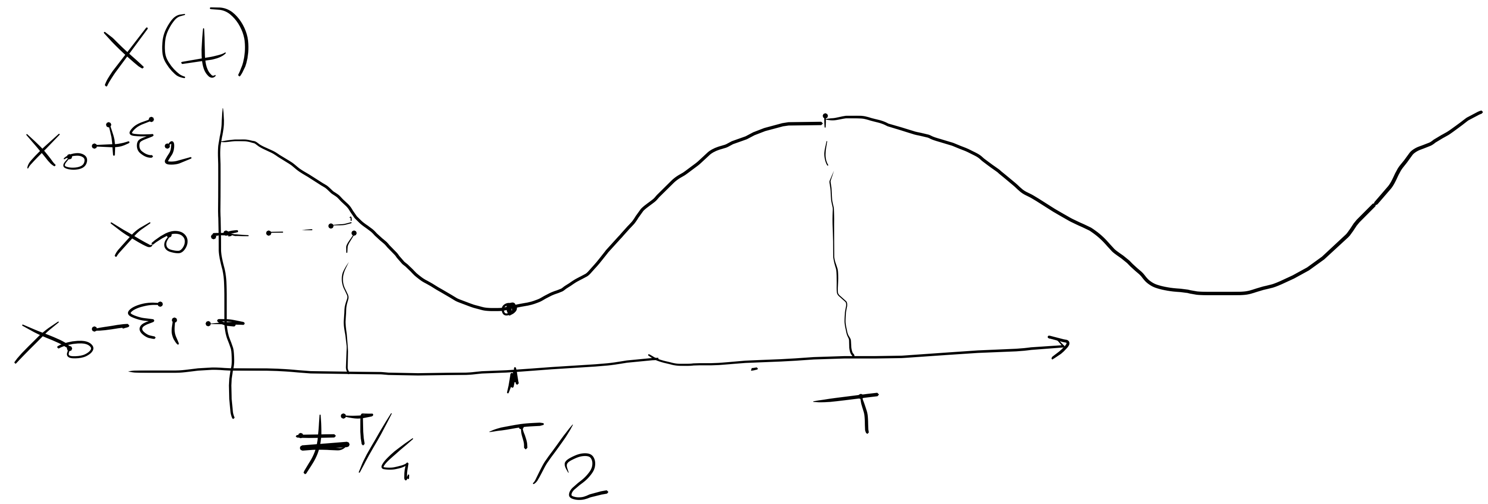
$$\frac{dv}{dt} = -\sqrt{1-v}$$



$$\int_0^T v(t)\,dt = 0$$

3) $E = E_2$ επιτρεπτή κίνηση $x \in [x_0 - \varepsilon_1, x_0 + \varepsilon_2]$





$$\frac{1}{2} m v^2 + V(x) = E_2$$

$$v(x) = \pm \sqrt{(E_2 - V(x)) \frac{2}{m}}$$

$$dt_{\rightarrow} = \frac{dx}{\text{"}+v\text{"}}$$

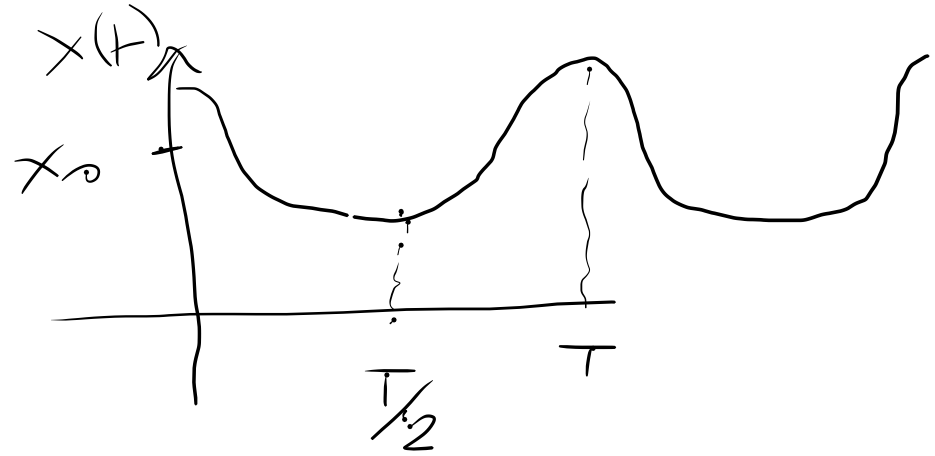
$$\|$$

$$dt_{\leftarrow} = \frac{-dx}{\text{"}-v\text{"}}$$

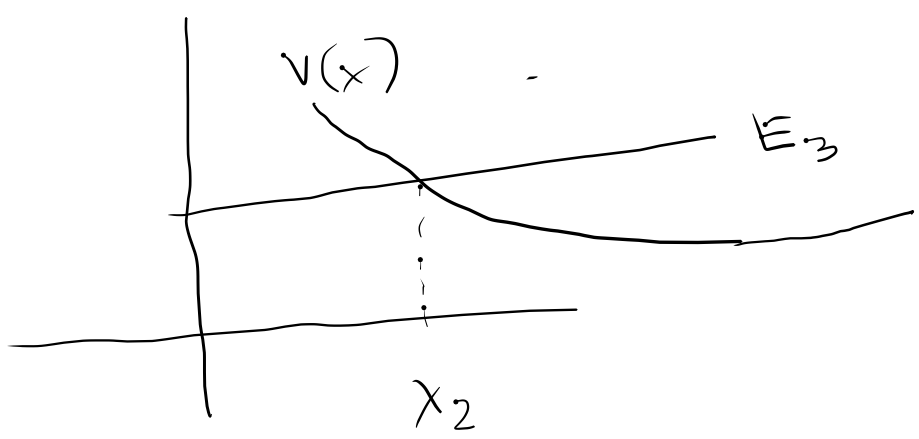
τικ = τακ
κάθε ρολόγι
που βασίζεται σε
1-D ταλάντωση

4) $E = E_3$

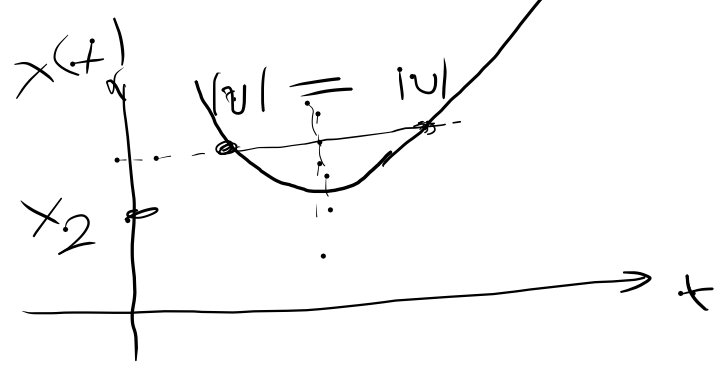
αν $x'' \simeq x_0$ τότε ταλάντωση γύρω από το x_0



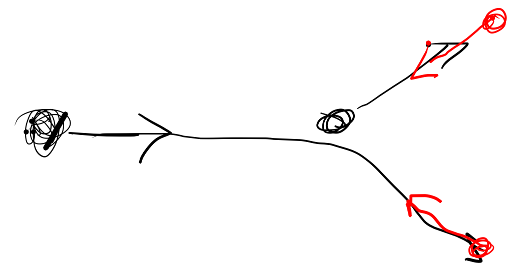
αν $x'' > x_1$



$$U = \pm\sqrt{\frac{2}{m}(E_3 - V(x))}$$



5) $E = E_4$



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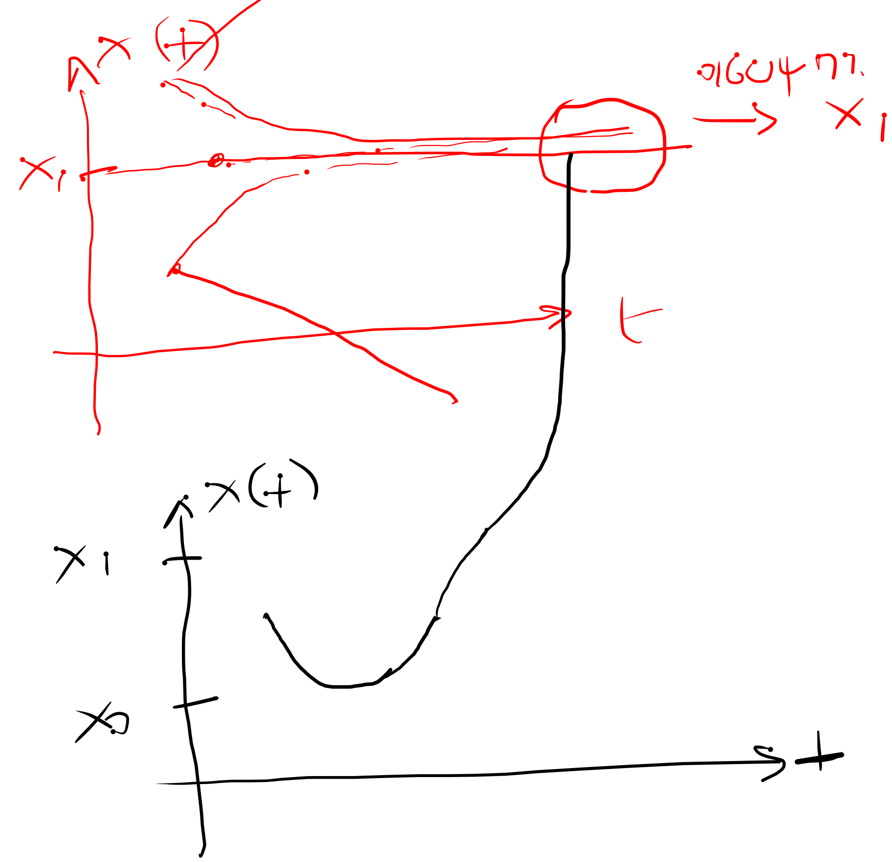
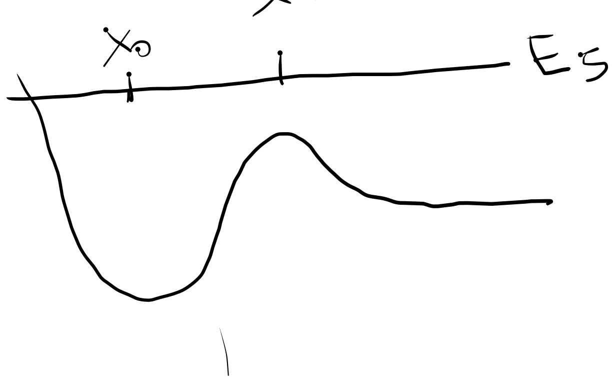
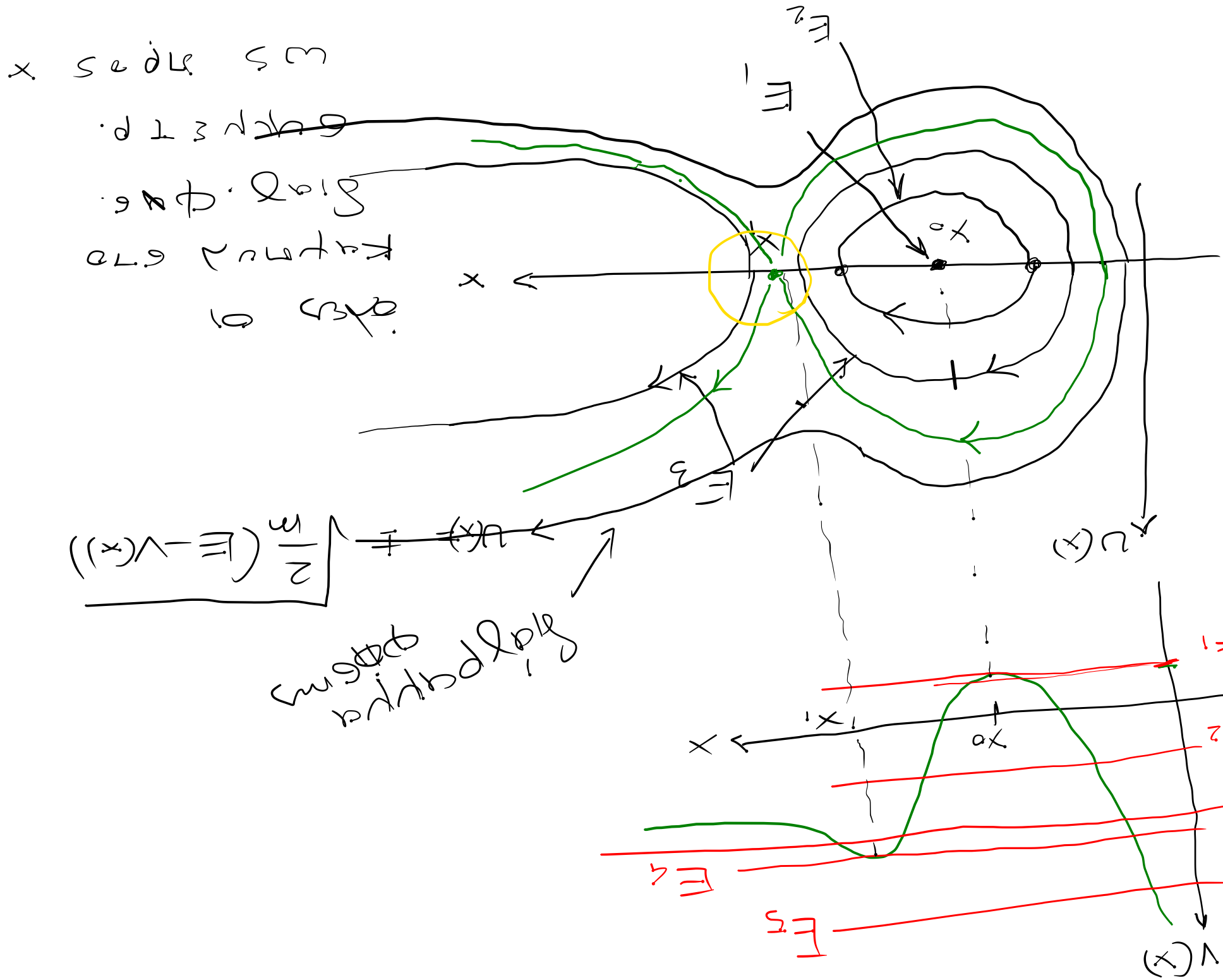


6) $E \geq E_5$




E_1
E_2
x_0
x
$\lambda U(x)$
E_3
E_4
E_5
$V(x)$
x_1
x_0
x
E_1
E_2
E_3