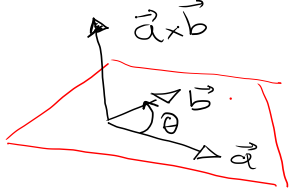
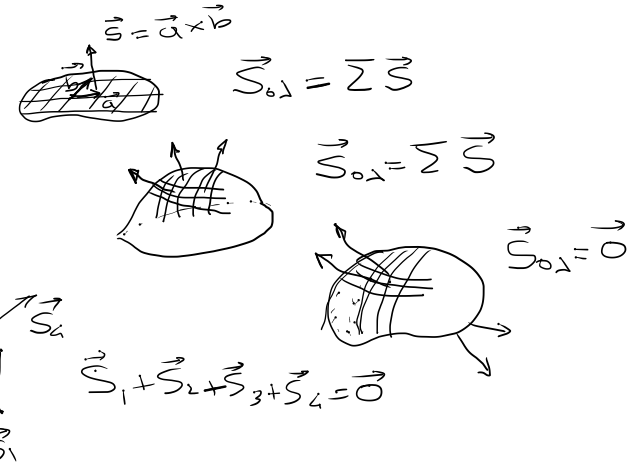


$$|\vec{a}\times\vec{b}| = \underbrace{|\vec{a}||\vec{b}|\sin\theta|}_{S_{\vec{b},\vec{a}}}$$



Πέμπτη 23-10-25 Διάλεξη 6

Θα ξεκινήσω ~12:10 !

- 3-πλά γινόμενα (σύνδεση με ανάλυση διανύσματος)
- όγκος
- σφαιρικές συντεταγμ. — καρτ. συντετ.
- σφαιρικές γωνίες.

$$(\vec{a} \times \vec{b}) \times \vec{c} =$$

$$\underbrace{(\vec{a} \times \vec{b})}_{\vec{A}}$$

$\vec{A} \perp \vec{a}, \vec{b}$

$\vec{A} \times \vec{c} \perp \vec{A}$

$\underbrace{(\vec{a} \times \vec{b}) \times \vec{c}}_{\lambda_1 \vec{a} + \lambda_2 \vec{b}} = \vec{A} \times \vec{c}$ επι του επιπέδου $\vec{a}, \vec{b}$

$$k_1 (\vec{b} \cdot \vec{c}) \vec{a} + k_2 (\vec{a} \cdot \vec{c}) \vec{b} \quad *$$

$$(\hat{i} \times \hat{j}) \times \hat{i} = \hat{k} \times \hat{i} = \hat{j} \ ✓$$

$$k_1 \underbrace{(\hat{j} \cdot \hat{i})}_{0} \hat{i} + k_2 \underbrace{(\hat{i} \cdot \hat{i})}_{1} \hat{j} = k_2 \hat{j} \Rightarrow k_2 = +1$$

$$(\hat{i} \times \hat{j}) \times \hat{j} = \hat{k} \times \hat{j} = -\hat{i}$$

$$k_1 \underbrace{(\hat{j} \cdot \hat{j})}_{1} \hat{i} + k_2 \underbrace{(\hat{i} \cdot \hat{j})}_{0} \hat{j} = k_1 \hat{i}$$

$\Rightarrow k_1 = -1$

$(\vec{a} \times \vec{b}) \times \vec{c} = -(\vec{b} \cdot \vec{c}) \vec{a} + (\vec{a} \cdot \vec{c}) \vec{b}$ γενική φόρμουλα

$$(\vec a\times\vec b)\times\vec c = \begin{vmatrix} \hat i & \hat j & \hat k \\ \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} & -\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} & \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \\ c_x & c_y & c_z \end{vmatrix}$$

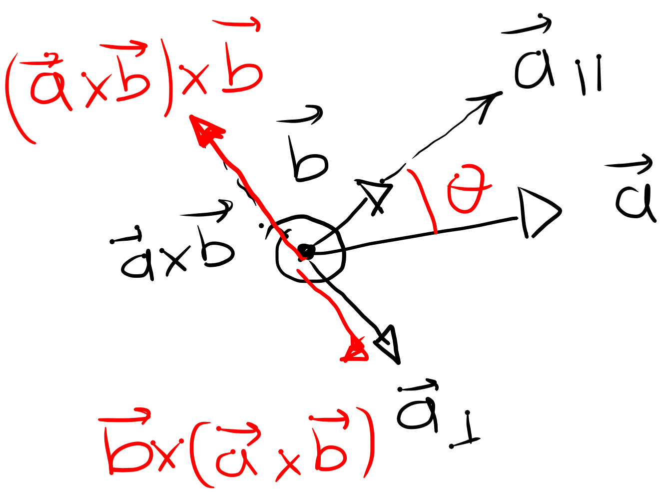
$(\vec a\times\vec b)_x$, $(\vec a\times\vec b)_y$, $(\vec a\times\vec b)_z$

$\parallel, \perp$ ως προς το $\vec b$

$$\vec a = \vec a_{\parallel} + \vec a_{\perp}$$

$$= \frac{\vec a\cdot\vec b}{b^2}\,\vec b + \underbrace{(\vec a - \vec a_{\parallel})}_{\vec b\times(\vec a\times\vec b)/b^2}$$

$$\vec a = \frac{\vec a\cdot\vec b}{b^2}\,\vec b + \frac{\vec b\times(\vec a\times\vec b)}{b^2}$$



$\vec b\times(\vec a\times\vec b)$ $/|\vec b|^2 = \vec a_{\perp}$

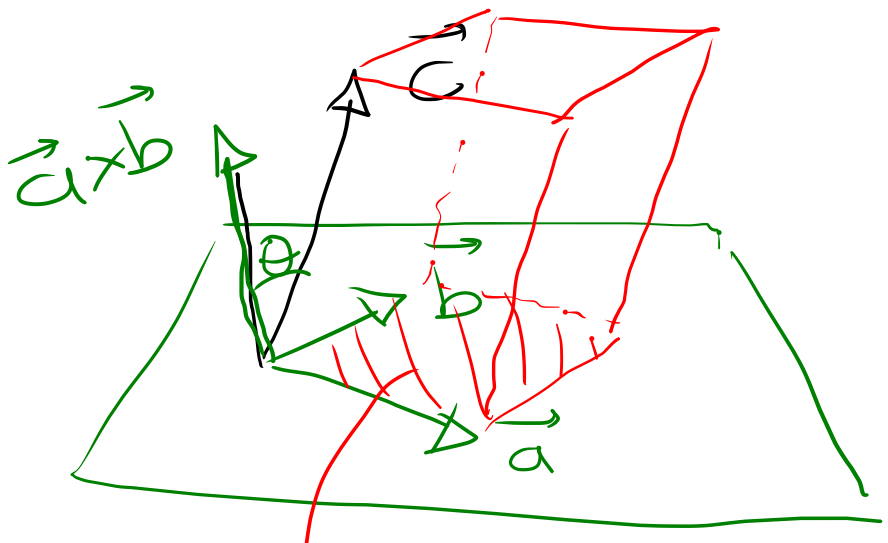
$|\vec a_{\perp}| = \underline{a\sin\theta}$

$|\vec b\times(\vec a\times\vec b)| = |\vec b||\vec a\times\vec b|$

$= \underline{|\vec b||\vec a||\vec b|\sin\theta}$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = |\vec{a}\times\vec{b}|\,|\vec{c}|\cos\theta = V_{\text{παραλληλεπ.}}\ \vec{a},\vec{b},\vec{c}$$

$$= S_{\vec{a}\times\vec{b}}\cdot H$$



$\vec{c}\parallel\vec{a}\times\vec{b}$

$\vec{c}\perp$ στο επίπεδο

Η ύψος του παραλληλεπ.

$S_{\vec{a},\vec{b}}$

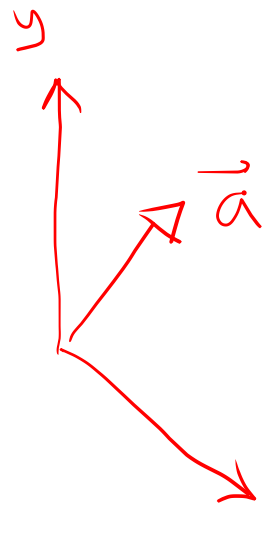
$(\vec{a}\times\vec{b})\cdot\vec{c}$ = προσημασμένος όγκος του παραλληλεπ. των $\vec{a},\vec{b},\vec{c}$

Δείξτε το!

θ: $\vec{B}$, $\vec{S}$

ροή $\vec{B}\cdot\vec{S} = \vec{B}\cdot(\vec{a}\times\vec{b}) = (\vec{a}\times\vec{b})\cdot\vec{B}$

$\cos\theta$



$\vec{a} = a_x \hat{x} + a_y \hat{y}$

$\hat{x}, \hat{y}, \vec{a}$

$a_x = f(\hat{x}\cdot\hat{y}, \hat{x}\cdot\vec{a}, \hat{y}\cdot\vec{a})$