Τρίτη 25 Μαΐου

$$\boxed{\dot{x} = Ax}$$

$\dot{\vec{x}} = \vec{v}(x,t)$

$\vec{x} \in \mathbb{R}^2$

δA

$$\frac{1}{\delta A}\frac{d}{dt}(\delta A) = \nabla\cdot\vec{v}$$

για 2D $\quad \nabla\cdot(Ax) =$

$= \mathrm{Trace}(A)$

$$\det\left(e^{At}\right) = e^{\mathrm{Trace}(A)\,t}$$

$$Ax = \lambda x, \quad (A-\lambda I)x = 0$$

$$\det(A - \lambda I) = 0$$

$$\lambda^2 - \mathrm{Trace}(A)\,\lambda + \det(A) = 0$$

$\lambda_1, \lambda_2 \qquad x_1, x_2$

ιδιοτιμές ιδιοδ.

$$x = \alpha x_1 + \beta x_2$$

$$\boxed{A = X \Lambda X^{-1}}$$

$\lambda_1 \quad \lambda_2$

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$X^{-1} x = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \longrightarrow \begin{pmatrix} \lambda_1 \alpha \\ \lambda_2 \beta \end{pmatrix}$$

$$A x$$

$$\frac{dx}{dt} = A x$$

$$\sim \quad \frac{d}{}$$

$$y = X^{-1} x, \quad x = X y$$

$$\frac{dy}{dt} = \underbrace{X^{-1} A X}_{\Lambda} \, y$$

$$A = X \Lambda X^{-1}$$

$$X^{-1} A X = \Lambda$$

$$\boxed{\frac{dy}{dt} = \Lambda y}$$

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

αν είναι ορθογώνια
ή φιλα οπότε

X , $X^{-1} = X^{+}$ είναι
ισομετρία

$\lambda_1, \lambda_2 \in \mathbb{R}$

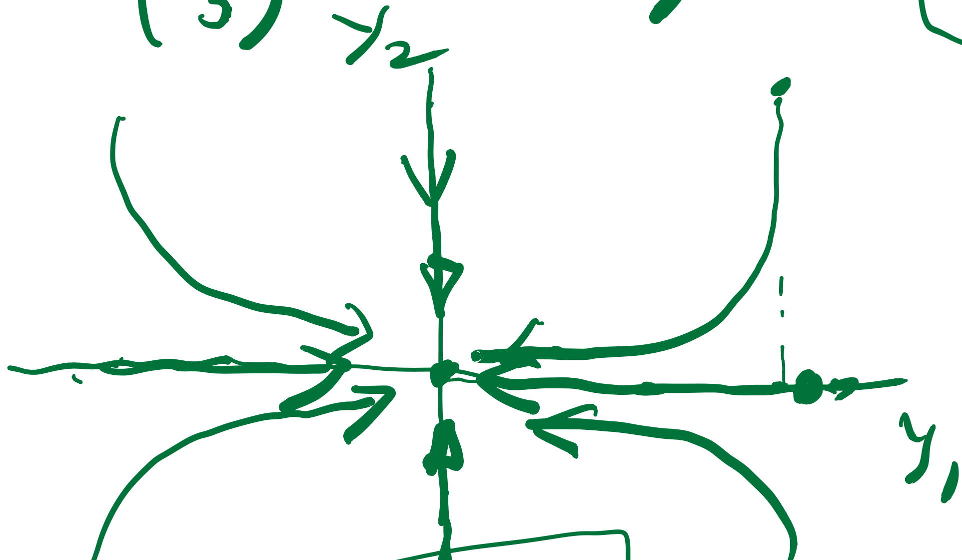
At

$$\frac{dy}{dt} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} y$$

$\lambda_1 = -1$

$\lambda_2 = -2$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$\alpha \in \mathbb{R}$

$$y(0) = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frac{dy}{dt} = \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}}_{A} y$$

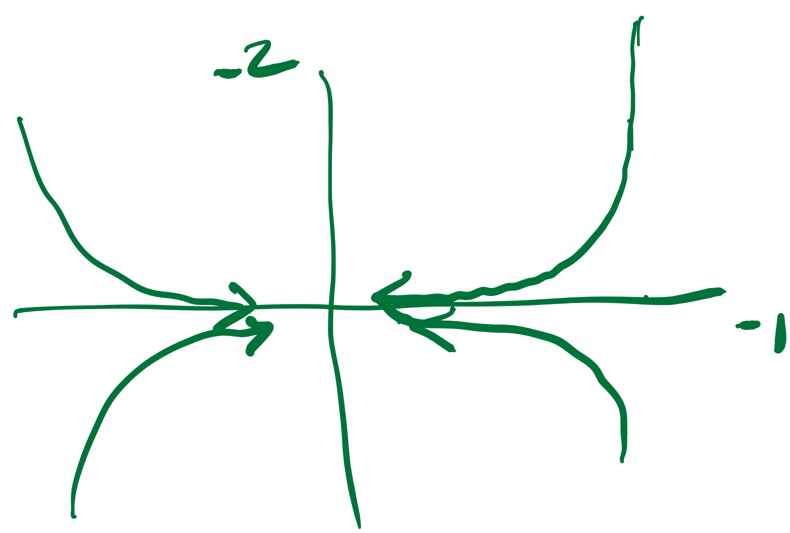
$$y(t) = e^{At} y(0) = \alpha e^{-t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$e^{\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} t} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$$

$$\alpha e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$t = 0$ $\qquad\qquad$ t

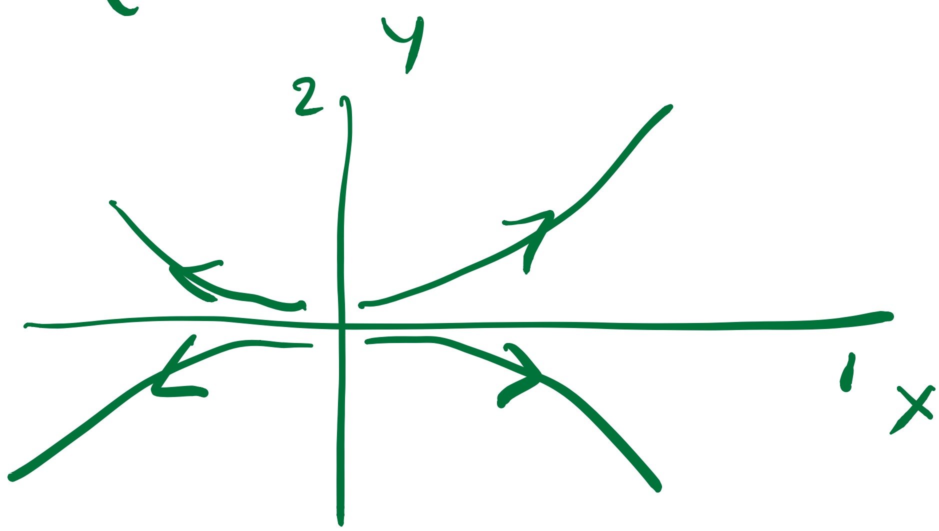


$$A = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\frac{dy}{dt} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} y$$

$$t \to -t = \tau$$



$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

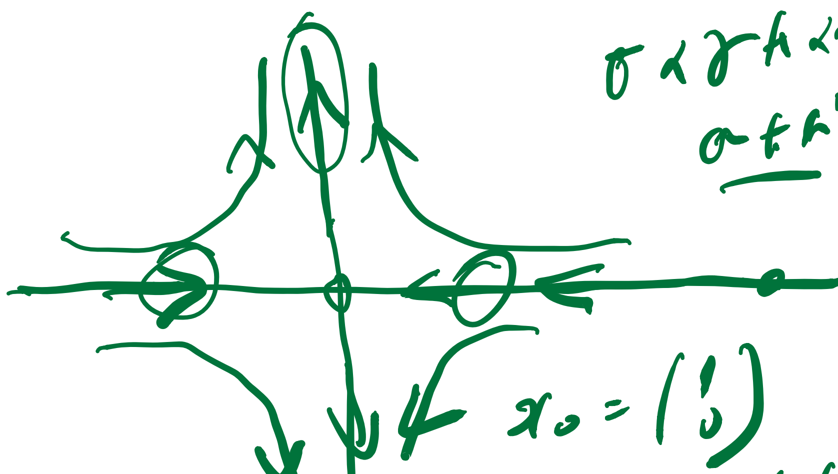
$$x = x_0 e^{t}$$

$$y = y_0 e^{2t}$$

$$\frac{x^2}{y} = \frac{x_0^2}{y_0}$$

$$y = x^2 \overbrace{\left(\frac{x_0^2}{y_0}\right)}^{\alpha}$$

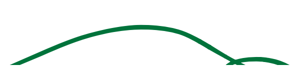
$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} x \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



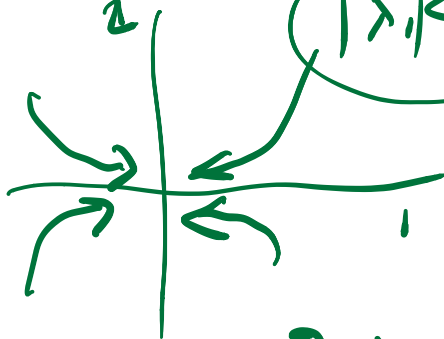
σαγματικό
σημείο

$$x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\to e^{-t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

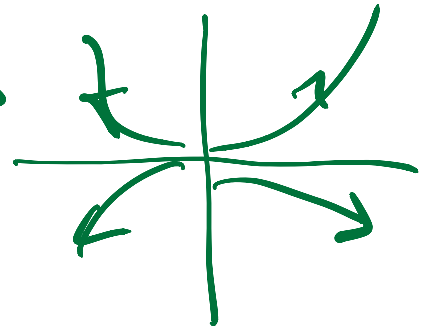


$|\lambda_1|<|\lambda_2|$



$\lambda_1, \lambda_2 < 0$

② καταβόθρα

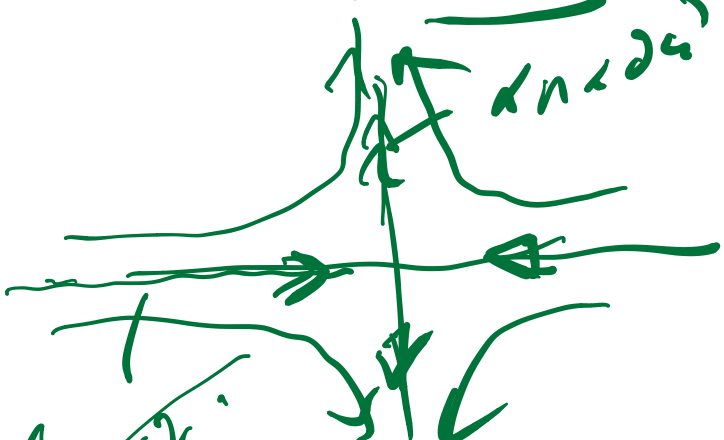


πηγή

$\lambda_1, \lambda_2 > 0$

Σάγμα

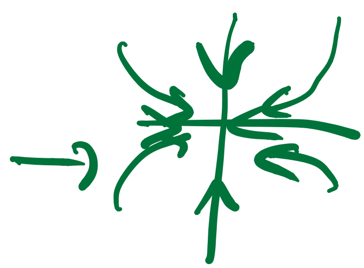
ασταθής πολλαπλότητα



ευσταθής πολλαπλότητα

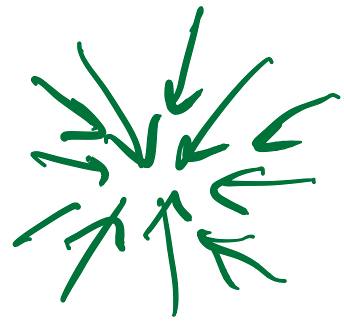
$y = X^{-1}x$

②



$\lambda_1 = \lambda_2$

$\left(\lambda I \right.$

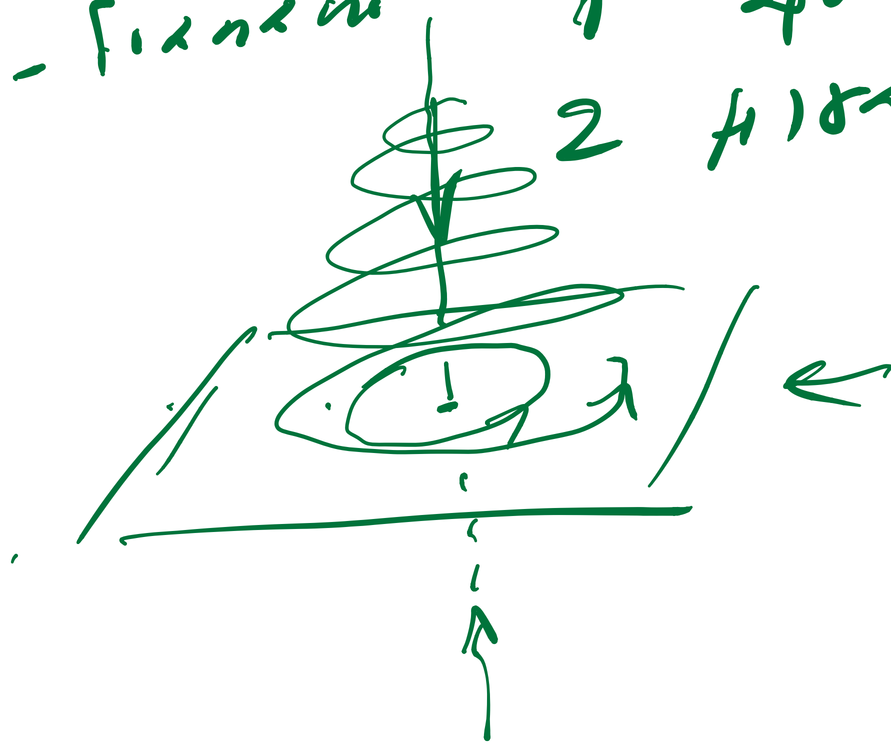


$\lambda < 0$

καί έχω
δύο ιδιοδιανύσματα

μιγαδική έχω αναλλοίωτο **επίπεδο**

3-διάστατα 1 πραγ
2 μιγαδικές

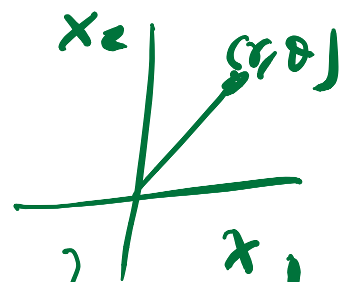


$$A = \begin{pmatrix} \sigma & -\omega \\ \omega & \sigma \end{pmatrix}$$

$$\frac{dx}{dt} = \begin{pmatrix} \sigma & -\omega \\ \omega & \sigma \end{pmatrix} x$$

$$(\sigma - \lambda)^2 = -\omega^2$$

$$\underline{\lambda = \sigma \pm i\omega}$$



$$\left.\begin{aligned} \frac{dx_1}{dt} &= \sigma x_1 - \omega x_2 \\ \frac{dx_2}{dt} &= \omega x_1 + \sigma x_2 \end{aligned}\right\}$$

$$r^2 = x_1^2 + x_2^2 \qquad (1)$$

$$\tan\theta = \frac{x_2}{x_1} \qquad (2)$$

$$r\dot{r} = x_1\dot{x}_1 + x_2\dot{x}_2$$

$$\boxed{\dot{r} = \frac{x_1\dot{x}_1 + x_2\dot{x}_2}{r}}$$

$$\sec^2\theta\,\dot{\theta} = \frac{x_1\dot{x}_2 - x_2\dot{x}_1}{x_1^2}$$

$$\dot{\theta} = \frac{x_1\dot{x}_2 - x_2\dot{x}_1}{r^2}$$

$$x_1 = r\cos\theta, \quad r = x_1\sec\theta$$

$$\dot{x}_1 = \sigma x_1 - \omega x_2, \quad \dot{x}_2 = \omega x_1 + \sigma x_2$$

$$\dot{r} = \frac{1}{r}\left[\sigma x_1^2 - \omega \cancel{x_1 x_2} + \omega \cancel{x_2 x_1} + \sigma x_2^2\right]$$

$$= \sigma r \qquad\qquad \dot{r} = \sigma r$$

$$\dot{\theta} = \frac{\omega x_1^2 + \sigma \cancel{x_1 x_2} - \sigma \cancel{x_2 x_1} + \omega x_2^2}{r^2}$$

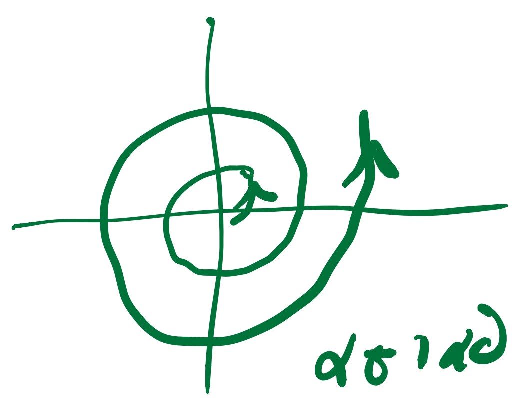
$$\dot{\theta} = \omega$$

$$\boxed{\begin{aligned} \dot{r} &= \sigma r \\ \dot{\theta} &= \omega \end{aligned}}$$

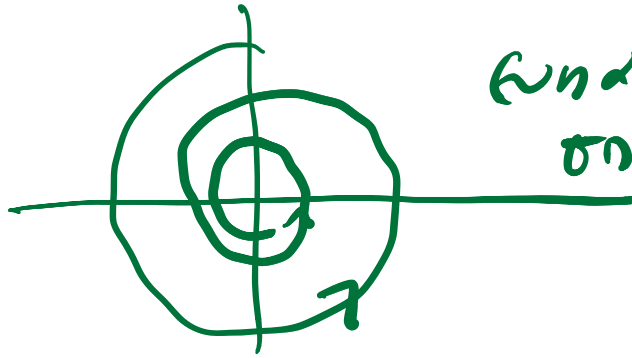
$$r = e^{\sigma t} r_0$$

$$\theta = \omega t + \theta_0$$

$$\sigma > 0, \quad \omega > 0$$



$$\lambda = \sigma \pm i\omega$$



ευσταθής σπείρα

$$A = \begin{bmatrix} \sigma & -\omega \\ \omega & \sigma \end{bmatrix} \qquad \lambda = \sigma \pm i\omega$$

$\sigma = 0$

κέντρο

όλοι έχουν μιγαδικές ιδιοτιμές $\sigma \pm i\omega$ τότε

$$A = T \begin{bmatrix} \sigma & -\omega \\ \omega & \sigma \end{bmatrix} T^{-1}$$

υπάρχει T πραγματικός μετασχηματισμός

A είναι πραγματικός πίνακας

$x_1 \qquad x_2$

$\sigma \pm i\omega$

$x_2 = x_1^*$

$A x_1 = (\sigma + i\omega) x_1$

$A x_1^* = (\sigma - i\omega) x_1^*$

$x_1 = x_{1r} + i x_{1i}$

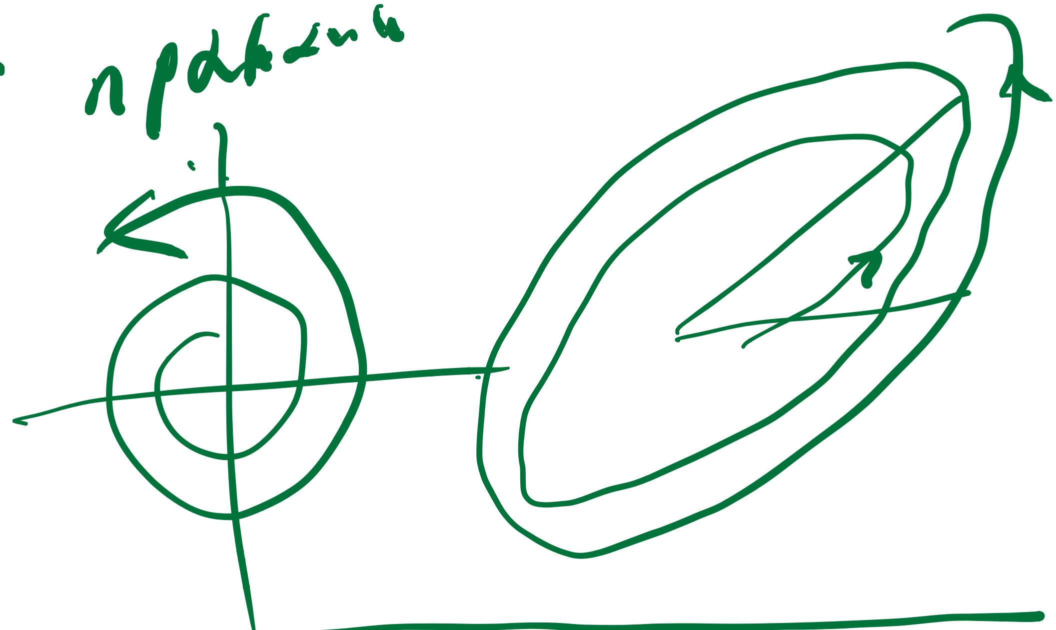
$x_2 = x_{2r} + i x_{2i}$

$$X = [x_1, x_1^*]$$

$$A = X \begin{bmatrix} \sigma + i\omega & 0 \\ 0 & \sigma - i\omega \end{bmatrix} X^{-1}$$

$$A = T \underbrace{\begin{bmatrix} \sigma & -\omega \\ \omega & \sigma \end{bmatrix}}_{} T^{-1}$$

T πραγματικό



α) $x_1 = x_{1r} + i\, x_{1i}$, $x_{1r}, x_{1i} \in \mathbb{R}$

να x_{1r} & x_{1i} είναι γραμμικά ανεξάρτητα

$$T = [\, x_{1r},\ x_{1i} \,]$$

$$A = T \begin{bmatrix} \sigma & -\omega \\ \omega & \sigma \end{bmatrix} T^{-1}$$

$$x_{1r} = \lambda\, x_{1i} \qquad \lambda \in R$$

$$A\, x_1 = A(x_{1r} + i\, x_{1i})$$

$$= A(\lambda + i)\, x_{1i} =$$

$$= (\lambda + i)\, A\, x_{1i}$$

$$A(x_{1r} + i\, x_{1i}) = (\sigma + i\omega)(x_{1r} + i\, x_{1i})$$

$$= (\sigma + i\omega)(\lambda + i)\, x_{1i} = (\lambda + i)\, A x_{1i}$$

$$\Rightarrow \; A x_{1i} = (\sigma + i\omega)\, x_{1i}$$

Είναι αδύνατον

$A x_{1i} \in \mathbb{R}$ οπότε άτοπο

$$A(x_{1r} + i x_{1i}) = (\sigma + i\omega)(x_{1r} + i x_{1i})$$

$$A x_{1r} = \sigma x_{1r} - \omega x_{1i}$$

$$A x_{1i} = \omega x_{1r} + \sigma x_{1i}$$

$$A = T \begin{pmatrix} \sigma & -\omega \\ \omega & \sigma \end{pmatrix} T^{-1}$$

$$A = \begin{pmatrix} -1 & R \\ 0 & -1 \end{pmatrix} \quad R \neq 0$$

δεν είναι διαγωνοποιήσιμο

$$(\lambda+1)^2 = 0 \quad , \lambda = -1$$

$$\begin{pmatrix} -1 & R \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix}$$

$$-x + Ry = -x \Rightarrow y = 0$$

$$-y = -y \quad \checkmark \ \forall y$$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ μόνο μια ιδιοκατάσταση.

$$A_\varepsilon = \begin{bmatrix} -1 & R \\ \varepsilon & -1 \end{bmatrix} \leftarrow$$

$$\dot{x}_\varepsilon = A_\varepsilon x_\varepsilon$$

$$\lim_{\varepsilon \to 0} x(t,\varepsilon) = \tilde{x}(t)$$

$$\frac{d\tilde{x}}{dt} = \begin{bmatrix} -1 & R \\ 0 & -1 \end{bmatrix} \tilde{x}$$

$R > 0$

$\varepsilon > 0$

$$(\lambda + 1)^2 = \varepsilon R$$

$$\lambda = -1 \pm \sqrt{\varepsilon R}$$

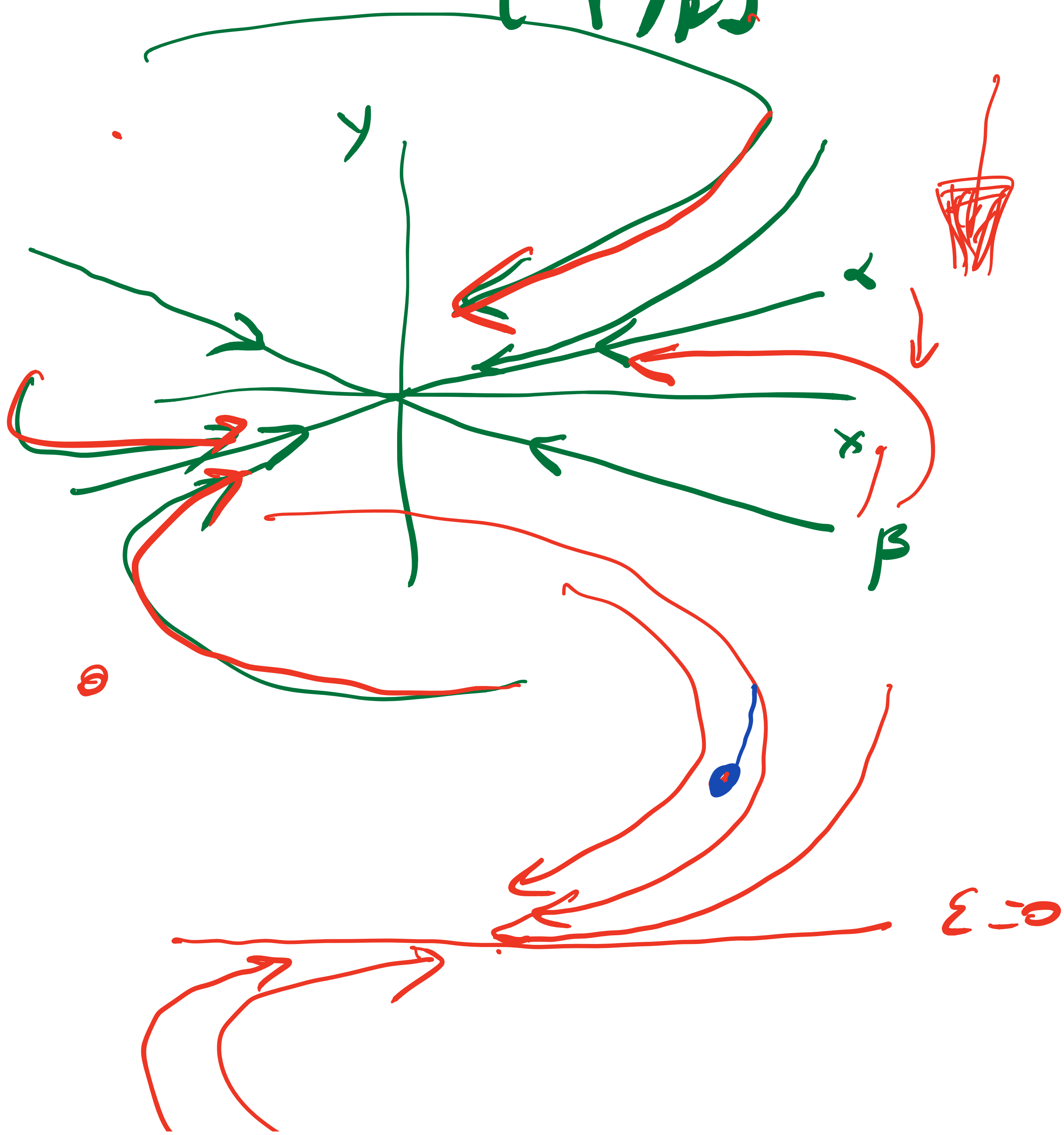
$$\begin{bmatrix} \alpha & r \\ & R \end{bmatrix} \qquad \begin{bmatrix} -1 & R \\ \varepsilon & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = (-1 \pm \sqrt{\varepsilon R}) \begin{pmatrix} x \\ y \end{pmatrix}$$

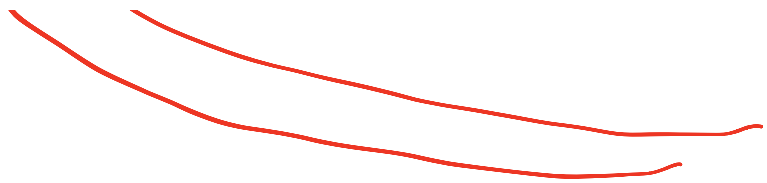
$$-x + Ry = (-1 \pm \sqrt{\varepsilon R})\, x$$

$$y = \pm \sqrt{\frac{\varepsilon}{R}}\, x$$

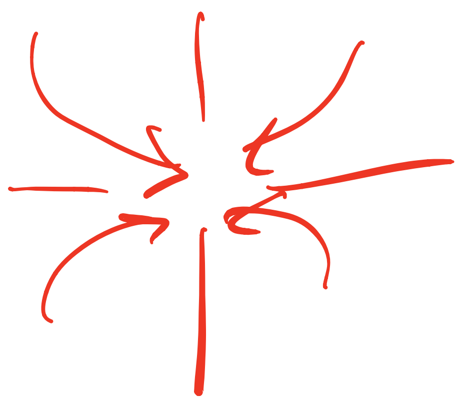
$-1+\sqrt{\varepsilon R}$ $\quad \begin{bmatrix} 1 \\ \sqrt{\varepsilon/R} \end{bmatrix}$ α

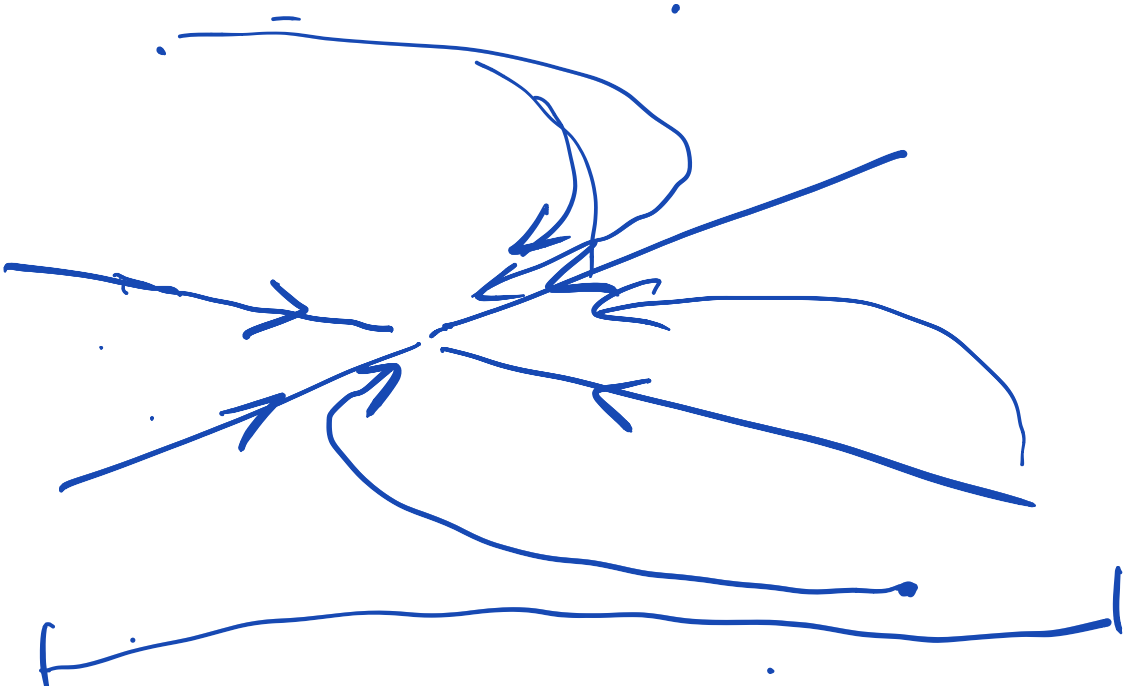
$-1-\sqrt{\varepsilon R}$ $\quad \begin{bmatrix} 1 \\ -\sqrt{\varepsilon/R} \end{bmatrix}$ β

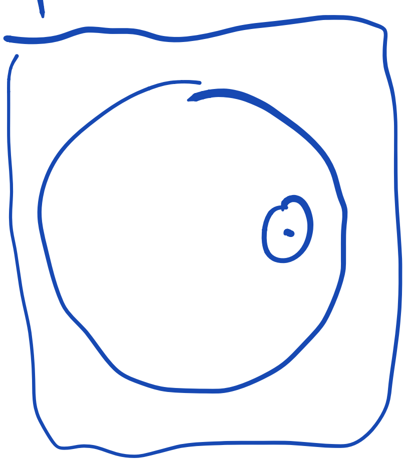




$$T \to \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$







Topo fish
monopole

(r, θ, φ)

(r, θ)

monopole

φ

$$\vec{B} = \underbrace{B_r \vec{e}_r + B_\theta e_\theta}_{} + \underbrace{B_\phi \vec{e}_\phi}_{}$$

$$\boxed{\begin{aligned} \frac{dT}{dt} &= -T + RP \\ \frac{dP}{dt} &= -P + \varepsilon T \end{aligned}}$$

$$\lambda^2 - T\lambda + D = 0$$

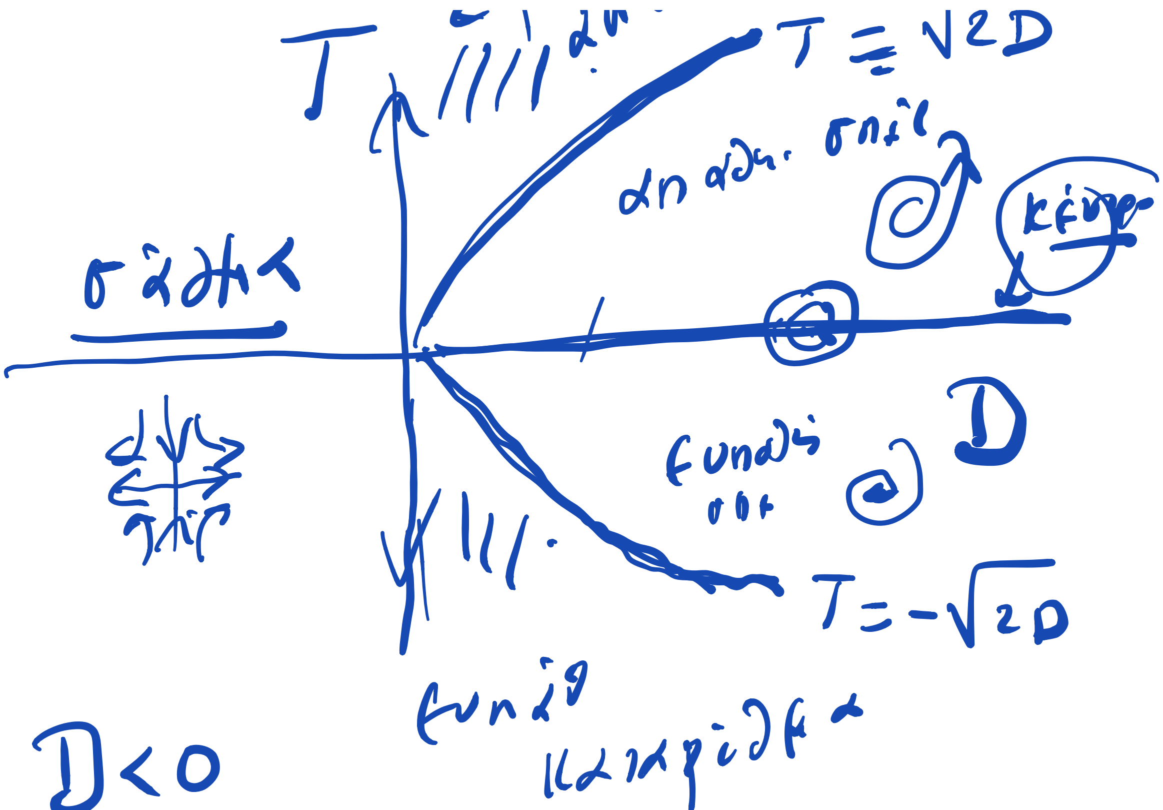
$$T = \text{trace}(A)$$

$$D = \det(A)$$

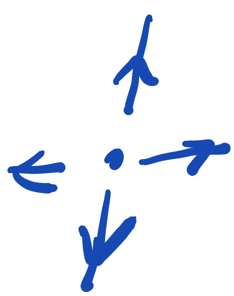
A^-

nyas

$$\lambda = \frac{T}{2} \pm \frac{\sqrt{T^2 - 4D}}{2}$$



α) $\underline{D<0}$



εδώ
δεν είναι
<u>κέντρο</u>
η γραφική
ποιο-
τική
Hartman
-Grobman

$$\dot{x} = -y$$
$$\dot{y} = x$$

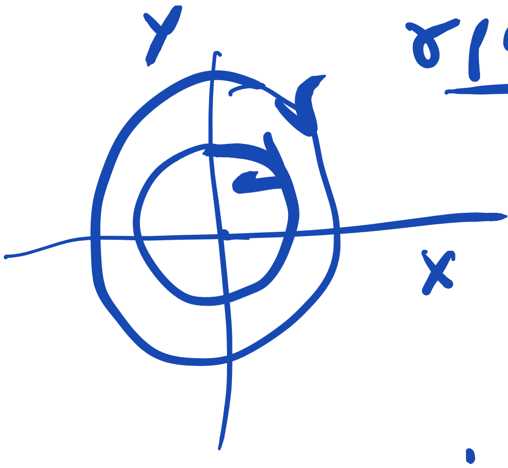
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\lambda = \pm i$$

$$\dot{x} = -y + \varepsilon x (x^2 + y^2)$$
$$\dot{y} = x + \varepsilon y (x^2 + y^2)$$

$$r^2 = x^2 + y^2$$

$$y = \varepsilon x r^2$$
$$x = -\varepsilon y r^2$$
$$\varepsilon y r^2 = \varepsilon^2 x r^4$$
$$-x = \varepsilon^2 x r^4 \Rightarrow x = 0$$
$$(0,0)$$



[illegible]

$$r^2 = x^2 + y^2$$

$$r\dot{r} = x\dot{x} + y\dot{y}$$

$$= x(-y + \varepsilon x r^2) + y(x + \varepsilon y r^2)$$

$$= \varepsilon r^4$$

$$\boxed{\dot{r} = \varepsilon r^3}$$

$$\boxed{\dot{\theta} = 1}$$

$$\dot{\theta} = \frac{x\dot{y} - y\dot{x}}{r^2}$$

$$= \frac{x(x + \varepsilon y r^2) - y(-y + \varepsilon x r^2)}{r^2}$$

$$= 1$$

$\varepsilon > 0$