Τρίτη 1 Ιουνίου 2021

(23) $\frac{dx}{dt} = Ax$

$$A = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$(\lambda+1)^2 = 0, \quad (A+I)^2 = 0$$

$$A^2 = -2A - I \qquad \text{C-H.}$$

$$\begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix}$$

$$-y = -y$$

$$-\cancel{x} + 2y = -\cancel{x} \Rightarrow y = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A_\varepsilon = \begin{pmatrix} -1 & 2 \\ \varepsilon & -1 \end{pmatrix}$$

$$e^{At} = I + At + \frac{A^2 t^2}{2!} + \cdots$$

$$A^2 = -2A - I$$

$$A^3 = -2A^2 - A = \cdots\cdot$$

$$e^{At} = \alpha(t) I + \beta(t) A \qquad \alpha$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$e^{-t} = \alpha(t) \quad - \beta(t)$$

$$e^{At}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{-t}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

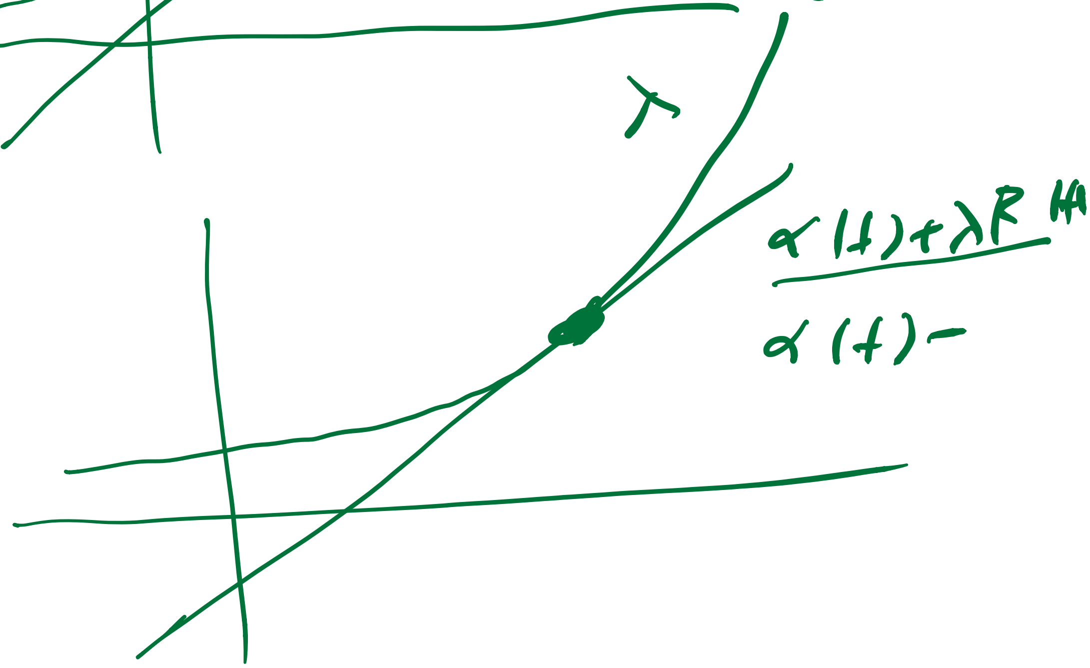
λ $\quad e^{At}\vec{e}_\lambda = e^{\lambda t}\vec{e}_\lambda$

$e^{\lambda t} = \alpha(t) + \lambda\beta(t)$

$\alpha(t) + \lambda\beta(t)$

$e^{\lambda t}$

λ



$te^{-t} = \beta(t)$

$\alpha(t) = e^{-t} + te^{-t}$ $\qquad A = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$

$$e^{At} = \left(e^{-t} + te^{-t}\right) I + te^{-t} A$$

$$\begin{pmatrix} e^{t} + t\cancel{e^{-t}} - \cancel{te^{-t}} & 2te^{-t} \\ 0 & e^{-t} \end{pmatrix}$$

$$e^{At} = \begin{pmatrix} e^{-t} & 2te^{-t} \\ 0 & e^{-t} \end{pmatrix}$$

$$\rightarrow \frac{dx}{dt} = -x + 2y$$

$$\frac{dy}{dt} = -y \rightarrow y(t) = e^{-t} y_0$$

$$\frac{dx}{dt} = -x + 2e^{-t}y_0$$

$$x(t) = e^{-t}x_0 + \int_0^t e^{-(t-s)}\,2e^{-s}x_0\,ds$$

$$y(t) = e^{-t}y_0$$

$$e^{At}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$x(t) = e^{-t}x_0 + 2e^{-t}\int_0^t y_0\,ds$$

$$= e^{-t}x_0 + 2te^{-t}y_0$$

$$y(t) = e^{-t}y_0$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \underbrace{\begin{pmatrix} e^{-t} & 2te^{-t} \\ 0 & e^{-t} \end{pmatrix}}_{e^{At}} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$A = \underline{X}\, \Delta\, X^{-1}$$

$$X$$

$$e^{At} = I + At + \frac{A^2 t^2}{2!} + \cdots$$

$$A^2 = X \Delta \cancel{X^{-1}} \cancel{X} \Delta X^{-1} = X \Delta^2 X^{-1}$$

$$A^n = X \Delta^n X^{-1}$$

$$I + \Delta t + \frac{\Delta^2 t^2}{2!} + \cdots$$

$$\boxed{e^{At} = X e^{\Delta t} X^{-1}} \leftarrow \Delta = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$e^{\Delta t} = \begin{pmatrix} e^{\lambda_1 t} & & 0 \\ & \ddots & \\ 0 & & e^{\lambda_2 t} \end{pmatrix}$$

να τη βρίσκω

$$\begin{pmatrix} -1 & 2 \\ \varepsilon & -1 \end{pmatrix}$$

2×2

η μέθοδος της διαγώνια κεφ 2ο §

A

$$X \to \boxed{t} \to X'$$

Μηχαν

t.

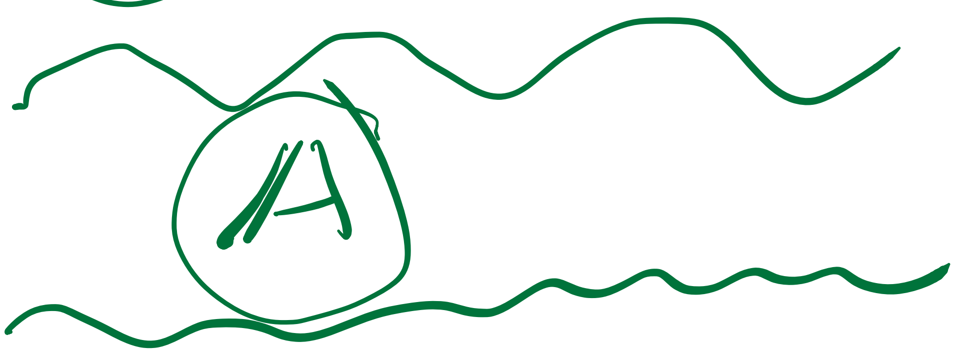
$$e^{At} \approx I + At$$

$$A = \lim_{t \to 0} \frac{B - I}{t}$$

$$\begin{pmatrix} \alpha_{11} & \alpha_{21} \\ \alpha_{12} & \alpha_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha_{11} \\ \alpha_{12} \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{11} & \alpha_{21} \\ \alpha_{12} & \alpha_{22} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_{21} \\ \alpha_{22} \end{pmatrix}$$

$$e^{At} \to \log\left(e^{At}\right) \sim A$$



$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \cdots$$

$$\log(1+x) = x$$

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$\delta V = y$

$$\frac{d}{dt}(\delta V) = (\nabla \cdot \vec{v})\, \delta V$$

δV

$$\boxed{\frac{dy}{dt} = f(\bar{x})\, y}$$

$y = 0$

$\delta V = 0$

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$A =$

$$\frac{dx}{dt} = \begin{pmatrix} -1 & 10 \\ 0 & -2 \end{pmatrix} x \qquad (\lambda+1)(\lambda+2) = 0$$

$\lambda = -1 \quad \lambda = -2$

$-1 \quad \begin{pmatrix}1\\0\end{pmatrix} \quad \begin{pmatrix}-1 & 10\\0 & -2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = -\begin{pmatrix}1\\0\end{pmatrix}$

$$\begin{pmatrix}-1 & 10\\0 & -2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = -2\begin{pmatrix}x\\y\end{pmatrix}$$

$$-x + 10y = -2x$$

$$y = -\frac{x}{10}$$

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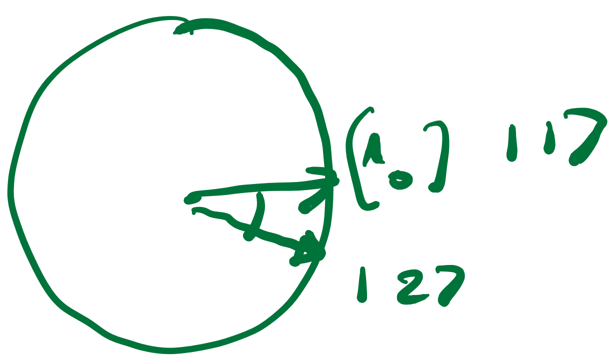
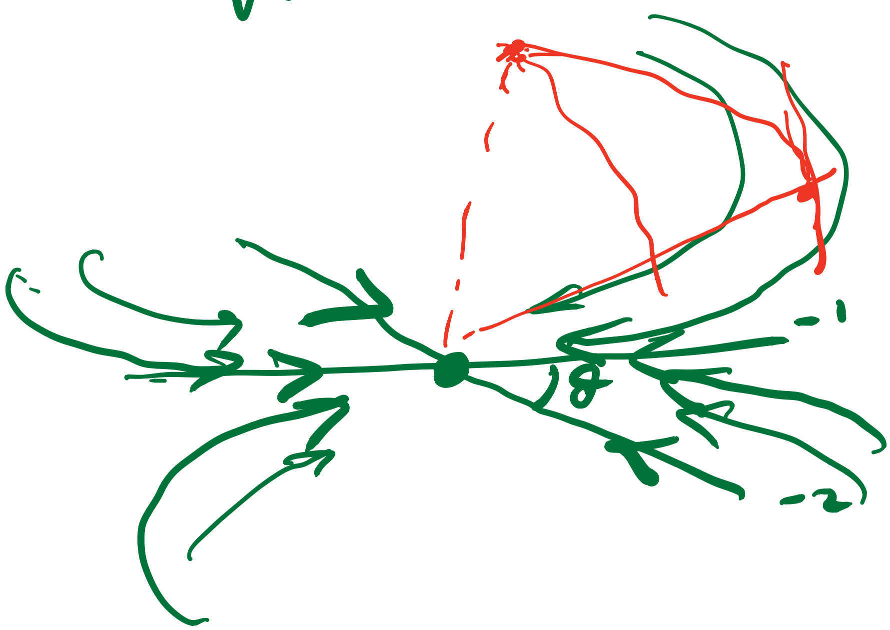
$-1 \quad \begin{bmatrix}1\\0\end{bmatrix} \qquad -2 \quad \begin{bmatrix}1\\-1/10\end{bmatrix}\frac{1}{\sqrt{1+\frac{1}{100}}}$



$$\cos\theta = \frac{10}{\sqrt{101}}, \quad \theta = \cos^{-1}\frac{10}{\sqrt{101}}$$

$$\sin\theta \sim \frac{10}{\sqrt{101}} \sim 0.1$$

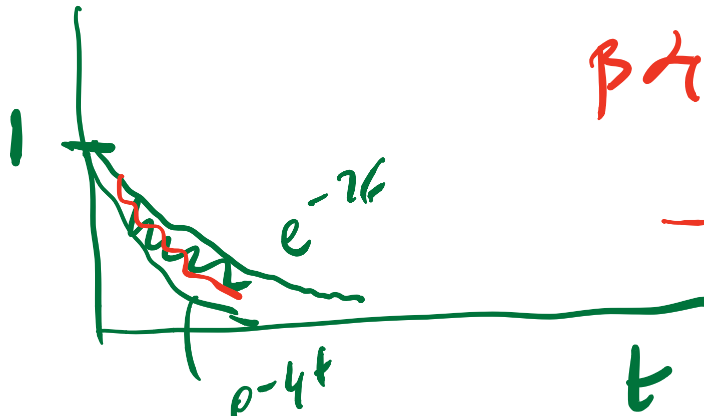


$$E = x_1^2 + x_2^2$$

$$\underline{X(0) = |1\rangle} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$X(t) = e^{-t}|1\rangle \qquad E_1(t) = e^{-2t}$$

$$X(t) = e^{-2t}|2\rangle \qquad E_2(t) = e^{-4t}$$



Βελτιώσαμε!?

$|x\rangle = \alpha|1\rangle + \beta|2\rangle$

α πέφτει

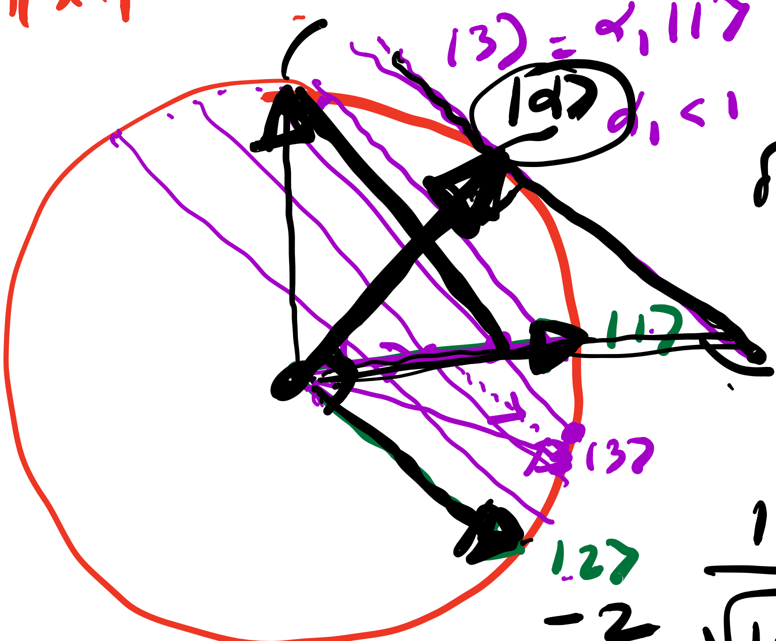
$|x\rangle$ για ποιο $\|x\| = 1$

$\begin{pmatrix}0\\1\end{pmatrix} = |\beta\rangle$

$|3\rangle = \alpha_1|1\rangle + \alpha_2|2\rangle$

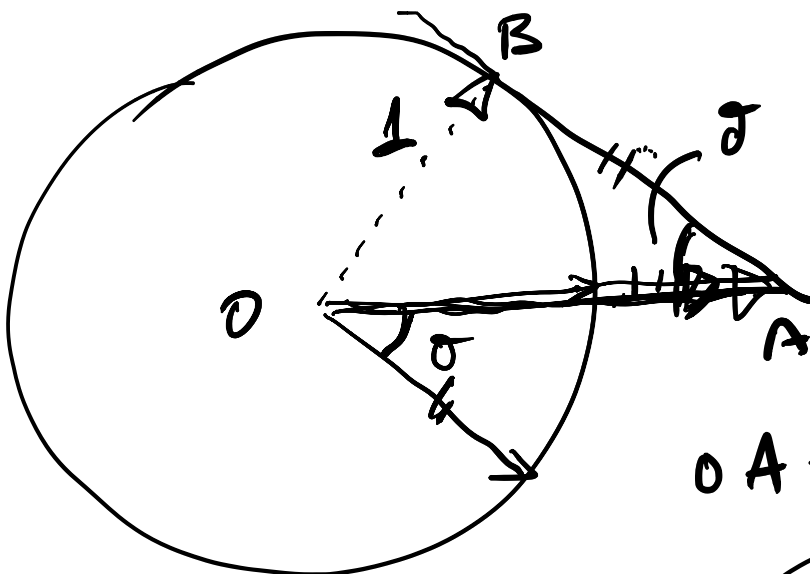
$\alpha_1 < 1 \quad \alpha_2 < 1$

διορθογώνια διά



$\frac{1}{\sqrt{1+\frac{1}{100}}}(1, -\frac{1}{10})$

$$|\alpha\rangle = \frac{1}{\sqrt{1+\frac{1}{100}}}\left(\frac{1}{10}, 1\right)$$



$$OA = \frac{1}{\sin\theta}$$

$$|\alpha\rangle = \frac{1}{\sin\theta}|1\rangle + \cot\theta|2\rangle$$

$$\left(\frac{OB}{AB} = \tan\theta\right)$$

$$\sim$$

$$\ddot{x} + x + \boxed{\varepsilon \dot{x}\left(x^2-1\right)} = 0$$

$\underline{\varepsilon > 0}$

$\underline{\varepsilon=0}$

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x - \varepsilon y(x^2-1) \end{cases}$$

Van der Pol

$\varepsilon=0$

$$\left. \begin{aligned} \dot{x} &= y \\ \dot{y} &= -x \end{aligned} \right]$$

Σημ. ισορροπίας $\quad y=0 \quad x=0$

Γραμμ.

$$\dot{x} = y$$
$$\dot{y} = -x + \varepsilon y$$

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & \varepsilon \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\lambda^2 - \varepsilon\lambda + 1 = 0$$

$$T = \varepsilon > 0$$ απωθητική σπείρα

$$\lambda = \frac{\varepsilon}{2} \pm \sqrt{\frac{\varepsilon^2}{4} - 1}$$

$$= \frac{\varepsilon}{2} \pm i\underbrace{\sqrt{1 - \varepsilon^2/4}}_{\omega}$$

Περιοδική κίνηση τι οριακός **κύκλος**.

$$E = \frac{\dot{x}^2}{2} + \frac{x^2}{2}$$

$$\underline{\ddot{x} + x} = -\varepsilon \dot{x}(x^2-1)$$

$$\dot{x}\ddot{x} + x\dot{x} = -\varepsilon \dot{x}^2(x^2-1)$$

$$\frac{dE}{dt} = -\varepsilon \dot{x}^2(x^2-1)$$

$$E(t) = E(0) - \varepsilon \int_0^t \dot{x}^2(x^2-1)\,dt$$

επί της τροχιάς

t

$$E(t) = E(0) + \varepsilon \int_0^t \underbrace{\dot{x}^2(1-x^2)}_{>0} \, dt$$

$\frac{1}{\varepsilon}$

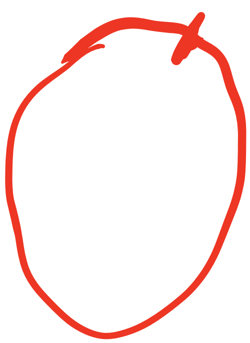
∃ τροχιά κλειστή

$\varepsilon \ll 1$

$$\boxed{\ddot{x} + x} + O(\varepsilon) = 0$$

για μικρά εt

$x = 1$

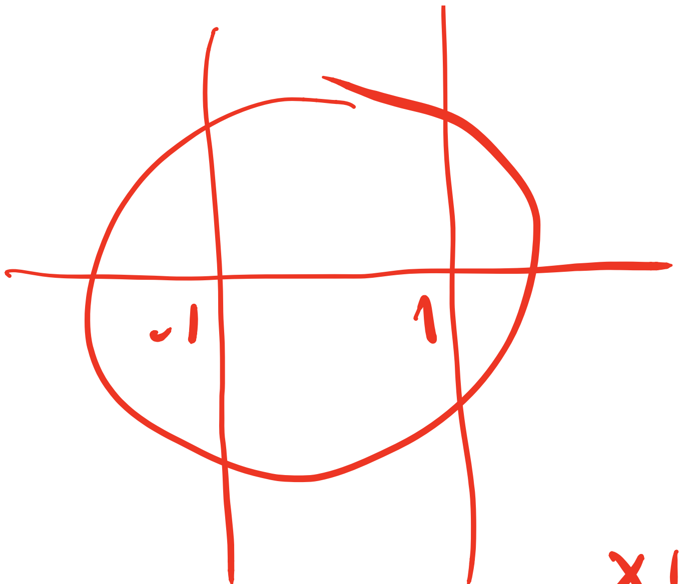


γιατί

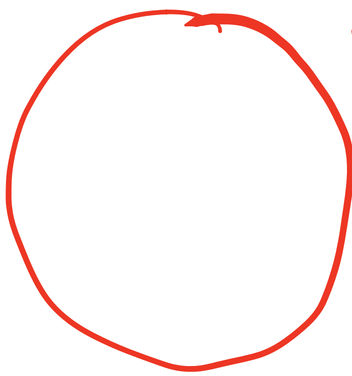
$$E(T) = E(0)$$

$$\boxed{0 = \int_0^T \dot{x}^2 \underline{(1-x^2)} \, dt}$$

$x_\gamma(t)$



$x(t) = A\sin t$



$$\int_0^{2\pi} A^2\cos^2 t\,(1 - A^2\sin^2 t)\,dt = 0$$

$$\int_0^{2\pi} \cos^2 t\,(1 - A^2\sin^2 t)\,dt = 0$$

$$2\pi\left(\frac{1}{2} - \frac{A^2}{4}\,\frac{1}{2}\right) = 0$$

$A^2 = 4$, $A = 2$!

$x \simeq 2 \sin t$

$O(\mu)$ $\quad \varepsilon << 1$