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Endless fun in high dimensions—A quantum card game

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Quantum technologies enable new ways to distribute and process information. The enormous progress over the recent decades has led to an urgent need for new educational programs to train professionals to work in this field. Here, we present a card game that teaches students the building blocks of quantum computing through strategic gameplay. Participants start from the lowest quantum state and play cards that change their state and/or their opponents' state, aiming to build an algorithm that achieves the highest possible quantum state. Players can utilize several different strategies that rely on quantum features such as randomness, superposition, interference, and entanglement. Our game expands on the existing $Q|\text{Cards}\rangle$ game, originally developed using traditional qubits (with 2-level states), by including an option to play with qutrits (with 3-level states), and by developing cooperative and single player modes in addition to the existing competitive mode. The presented game contributes to the ongoing efforts on gamifying quantum physics education with a particular focus on the counter-intuitive features that make quantum computing powerful.

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I. INTRODUCTION

Quantum physics is one of the most successful branches of physics. At the very heart of quantum physics are principles such as randomness, superposition, interference, and entanglement. These concepts are difficult to grasp as they often contradict our intuition, which we developed through experiencing our large-scale, classical world. It is these counter-intuitive features that have enabled novel technologies, which would not be possible in a classical setting. This emerging technological thrust is often termed the second quantum revolution,[1] since it differs significantly on the fundamental and application level from the first set of technologies developed through applying quantum physics, e.g., lasers or semiconductor devices.

One of the most prominent examples of quantum technology is a quantum computer. Compared to their classical counterparts, quantum computers promise a faster completion of certain tasks and would enable computational algorithms that are impossible in a classical setting. Recent progress has led to the first large-scale quantum computer that outperforms all classical computers for one specific computational task.[2,3]

In this article, we present a card game that provides a fun and engaging introduction to the concepts of quantum computing. The game, *Endless Fun in high dimensions*, is designed to be a low threshold introduction to quantum computing that encourages people to look into the fundamentals of quantum physics. The game implements basic quantum mechanical concepts such as randomness, superposition, interference, and entanglement. By playing cards that correspond to quantum logic operations, the participants are manipulating the quantum state of a virtual quantum computer with the aim of achieving the highest value of their own quantum state. The final result is evaluated using an included computer program. We note that although current quantum computers work with two-level quantum states, this card game goes a step further by also including the programming of three-dimensional quantum systems, i.e., high-dimensional quantum system. Such higher dimensional states, called qudits rather than qubits, are considered promising candidates for next generation quantum computers and other technologies.[4–6] They are also beneficial in the gaming setting as they increase the complexity of the game play enabling longer lasting (maybe endless) fun.

II. BACKGROUND

Building intuition often leads to a deeper understanding of difficult concepts and has been the focus of various gamification efforts.[7,8] Games offer an entertaining way to loosen up the atmosphere in a course while promoting learning.[9,10] Education in quantum physics and, in particular, modern quantum information science can benefit from ideas developed through gamification methods.

Many quantum physicists hope that these games could be more than just tools for learning, as expressed by John Preskill who said "[p]erhaps kids who grow up playing quantum games will acquire a visceral understanding of quantum phenomena that our generation lacks."[11] Popular examples of quantum games are the online computer game *Particle in a Box*,[12] and "quantized" adaptations of well-known games: *Quantum TiqTaqToe*,[13,14] *Quantum Chess*,[15,16] *Quantum Minigolf*,[17] as well as the quantum version of Minecraft—*qCraft*.[18] In addition to online games, there are also educational board games, e.g., *Entanglion* by IBM.[19] The idea of gamification in quantum sciences has been the center of focus in various quantum game jams, i.e., events in which instructive and entertaining quantum games are developed.[20]

At the Quantum Wheel game jam in Helsinki in 2019,[20] the quantum card game $Q|\text{Cards}\rangle$ was developed and introduced.[21] The game presented here builds upon and extends $Q|\text{Cards}\rangle$. The gameplay in the basic version is similar and only varies in small details. However, in contrast to $Q|\text{Cards}\rangle$, *Endless Fun* aims at building a high-dimensional quantum computer, i.e., a quantum computer operating on states with three possible values instead of two. The game was also extended to three difficulty levels and two additional game modes.

III. ENDLESS FUN IN HIGH DIMENSIONS

Endless Fun is a strategic multi-player card game that introduces the players to quantum computational logic gates. The players aim to increase their quantum state value and decrease the quantum state values of the other players. After all cards are played, the final quantum state values are calculated by evaluation software. Comparing the expected outcome to the actual outcome allows one to reflect on misunderstandings and retrace the effects of each operation. The goal of this game is to provide a platform to practice and engage with quantum logic operations while providing varying difficulty levels without the necessity of fully understanding the underlying mathematical framework. The instruction manual, the Python-based evaluation software, the cards, and the riddles of the single-player game mode are provided in the supplementary material.[22]

Although so far most quantum computers have used binary-valued quantum systems to encode bit-valued quantum information, the increase in possible outcomes when using high-dimensional quantum systems enhances the complexity of the game and offers options to adjust the difficulty. Thus, we anticipate a longer-lasting interest in playing the game. Furthermore, the game includes a cooperative and a single-player game mode to address different learning objectives.

A. Player states

In classical computation, information is saved as bit values: either 0 or 1. In the simplest forms of quantum computing, information is encoded into two-level systems called quantum bits, or *qubits*. Similar to classical bits, qubits can take values of $|0\rangle$ or $|1\rangle$. However, the quantum nature also allows for superpositions, i.e., being in both states at the same time, loosely speaking. A superposition is written as a sum over both possible states, e.g., $1/\sqrt{2}(|0\rangle + |1\rangle)$, which means that any measurement can result in either outcome, 0 or 1, with equal probability.[23] Note that we only allow for an equal superposition with a probability amplitude of $1/\sqrt{2}$ in the game; however, in general, all normalized amplitudes are possible. High-dimensional quantum states, often called *qudits*, go a step further in complexity, as they do not only allow two but d possible outcomes. The game can be played with either two- or three-dimensional quantum states. In 2D, only the qubit values $|0\rangle$ and $|1\rangle$ are available; in 3D an additionally qudit value $|2\rangle$ is available.

In the game, each player is assigned a quantum state. During gameplay, the quantum states will evolve when cards, i.e., quantum logic operations, are played on these states.

B. Cards

Playing cards is the only way to change a player's quantum state. During gameplay, the players can play cards either on their own or other players states to manipulate the state. An overview over the playable cards is given in Table I. The detailed truth tables for all quantum gates can be found in the instruction manual.[22] The game also includes cards that do not correspond to quantum operations, e.g., a steal card that allows a player to steal a card from another player. Note further, that the choice of dimension affects the set of quantum operations. In 2D, for example, only one X-gate is defined, whereas in 3D, two X-gates with different cyclic shifts exist. The cards do not form a universal set of gates.

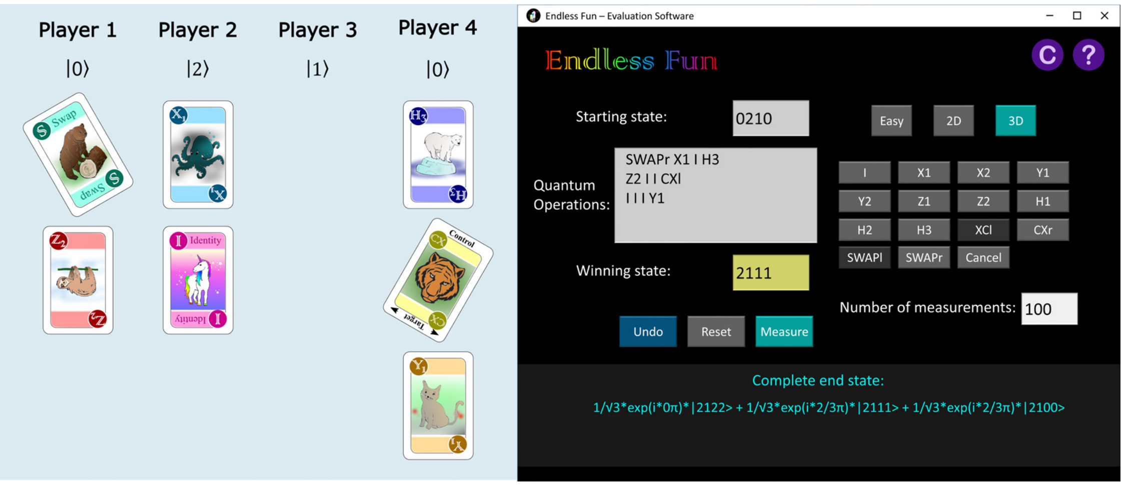
C. Gameplay

The goal of the game is to play cards, i.e., quantum logic operations, in such a way that the player's own qudit value is as high as possible and the opponents' values as low as possible. An example gameplay where the players have already played their cards is shown in Fig. 1.

The players can choose between three different difficulty levels. To not overwhelm the players who do not have a solid background in quantum information, it is recommended to start the game in the *Easy* version. In this simplified 2D-version, beginners can familiarize themselves with the rules and basic quantum logic operations without the phase properties of the operations, i.e., without the Y and Z gates and only with a single Hadamard gate. The game is more complex in the standard *2D* version, in which the cards are added that modulate the phase of single states, thus allowing the players to control quantum interference. In this version, all cards are used. Finally, the *3D* version is played with three-dimensional qudits; thus, it includes the most complex quantum states and the gameplay reaches its maximal

Table I. Overview of the playable cards and their consequences.

Actions	Cards	Description
Single-qudit gates	I	Identity operation. It does nothing.
	X	X-gate. It increases the qudit value by 1 (1 or 2 in 3D) cyclically.
	Y	Y-gate. It acts like the X-gate with an additional phase shift.
	Z	Z-gate. It adds phase shifts to the state.
	H	Hadamard-gate. It generates a superposition of all possible qudit values with equal probabilities. The game has 2 (3 in 3D) different Hadamard gates with different phase values.
Multi-qudit gates	CX	CX-gate. The CX operation is controlled by one qudit and acts as an X-gate on another.
	SWAP	SWAP-gate. Swaps the value of two qudits.
Other	Cancel	The card replaces a played quantum operation by an identity operation.
	Steal	The player can steal a card from another player, but must play it immediately.



Starting state: 0210

Quantum Operations:
SWAPr X1 I H3
Z2 I I CXl
I I I Y1

Winning state: 2111

Number of measurements: 100

Complete end state:

1/√3*exp(i*0π)*|2122> + 1/√3*exp(i*2/3π)*|2111> + 1/√3*exp(i*2/3π)*|2100>

Fig. 1. (Color online) A possible gameplay with three-dimensional qudits and four players. Player 1 starts with the qudit value of $|0\rangle$ obtained in the first round. Players 2, 3, and 4, start with their qudit values of $|2\rangle$, $|1\rangle$, and $|0\rangle$, respectively. The software on the right evaluates the winning state as $|2,1,1,1\rangle$, where the position of qudit values corresponds to the players. For a better understanding of the underlying mechanisms, the generated end state is also displayed on the bottom of the window, before it was measured. The other possible outcomes can be found by inspecting the overall state. Here, the states $|2,1,2,2\rangle$ and $|2,1,0,0\rangle$ could also have been obtained with the same probability.

difficulty. Here, the X, Y, and Z gates have two and the Hadamard gates have three variations. More details on the cards used in each version can be found in the manual.[22]

There are three game modes: competitive, cooperative, and single-player. In the cooperative mode, the goal is not to win against the other players but to reach the highest possible values summed up over all qudits as a group. In the single-player mode, the player can solve six ready-made riddles that guide the player to discover specific quantum effects. The riddles have different levels of difficulty, starting with easy ones that help the player learn about quantum interference effects. The difficulty is then gradually increased, with more quantum effects being gradually introduced. In addition, the significance of each quantum effect in quantum computing is briefly discussed along with the solution of the riddle, such that students can put quantum operations in a better context. Instructors can extend this set with their own riddles. A detailed description and the riddle sheets are available in the supplementary material.[22]

D. Strategies and quantum effects

To know what strategy to employ, a player must keep track of the evolution of the state. Playing in the single-player mode will help players learn to keep track of the state, and how it is changed by different cards. Furthermore, three quantum effects can be used in different game strategies:

- Quantum superpositions, which demonstrate the probabilistic nature of quantum measurements, i.e., quantum randomness.
- Quantum interference, which demonstrates the effect of phases on measurement outcomes.
- Quantum entanglement, which leads to strong correlations between the measurement outcomes of different systems.

In the following, we give simple examples of how the three effects can be observed in different game strategies. For simplicity, we explain the effect in detail with qubits; however, the high-dimensional counterparts follow in an analogous manner.

1. *Quantum superposition*

A Hadamard gate acting on the quantum states $|0\rangle$ or $|1\rangle$ generates a superposition of both states, as shown in Fig. 2(a). In the game, the software "collapses" the state by simulating a measurement and gives the random outcome weighted by the quantum probability. This quantum randomness adds an element of luck to the gameplay. Additionally, the superpositions can be used as a strategic element. For example, if one of the players is leading the round, the others can reduce the leader's chances of winning by setting them into a superposition. When the game is played in the 3D version, similar Hadamard operations can be performed, however, with the superposition having three possible outcomes. Note that the variety of Hadamard gates only differ from each other in phase (see manual for more details[22]), which becomes important when considering interference effects.

2. *Interference effects*

The phase of a quantum state is a physical property which does not have a direct effect on its qubit values. However, as it affects the outcome of quantum interference it can indirectly be used to change the qubit value of a state. When playing the game, it is possible to learn how to control interference through phase manipulations. Controlling phase is an important underlying working principle of quantum computations and almost always the reason behind its quantum advantage. Let's assume that we have a qubit $|0\rangle$ on which we play two H_1-Hadamard operations. In this process, the second Hadamard allows interference to occur, resulting in the state $|0\rangle$ again. If we add a Z-gate before or after the two H_1-gates, the outcome doesn't change as the players state is still $|0\rangle$. If, however, we first play the H_1 gate, then the Z-gate, and then the second H_1-gate, the resulting state is $|1\rangle$. A quick look at the state evolution shows that after the first card, we obtain the superposition state $1/\sqrt{2}(|0\rangle+|1\rangle)$. Then, the Z-gate changes the phase between the two terms, i.e., changes the state to $1/\sqrt{2}(|0\rangle-|1\rangle)$, which leads to the final state $|1\rangle$, when another H_1-gate is applied. This state evolution is outlined in detail in Fig. 2(b). Thus, with the

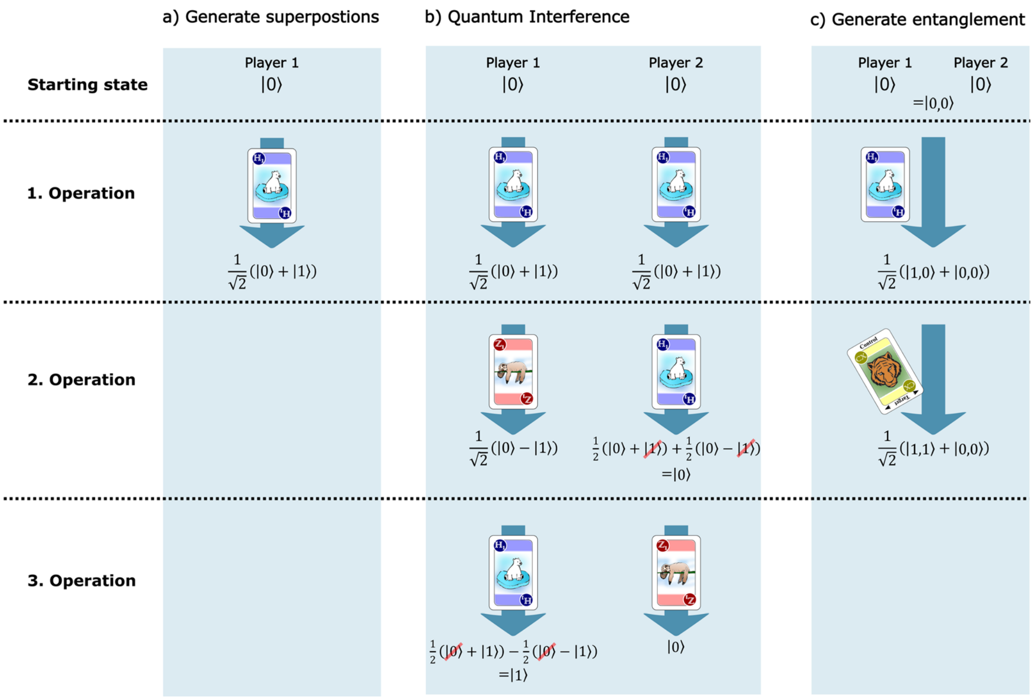


Fig. 2. (Color online) By playing the depicted cards, the players can explore superpositions, interference, and entanglement with qubits as described in detail in Sec. III D. The state after each cards operation is applied is written beneath it. (a) The Hadamard card is generating a qubit superposition of the state of player 1. Player 1's starting state $|0\rangle$ turns into $1/\sqrt{2}(|1\rangle+|0\rangle)$. (b) Quantum interference can be controlled by phase. Player 1's value after the logic operations is $|1\rangle$, Player 2's is $|0\rangle$. (c) Entangling the qubits of players 1 and 2. The starting state $|0,0\rangle$ is transformed to the entangled state $1/\sqrt{2}(|1,1\rangle+|0,0\rangle)$.

phase, we can manipulate the evolution of a superposition to obtain a desired state, thereby controlling the probability of measuring it. In this way, the phase gates can be used to control the state and the measurement outcome, through quantum interference. Interference effects can also be observed when the game is played in 3D, where the increased complexity of the states allows a larger variety of different phase manipulations and interference effects. In this game, the Y, Z, and Hadamard cards can be used to change the phase of a quantum state. Interference effects are explored in a guided manner by the two easy riddles provided in the supplementary material for single-player game mode.[22]

3. *Quantum entanglement*

Entanglement is another fundamental feature of quantum mechanics. Quantum entanglement correlates the value of one qubit with the value of another qubit. Counterintuitively, the correlations between entangled qubits still exist when they are not in a single state, but a superposition. Hence, when an entangled state is measured, the outcome of entangled players will be random due to being in a superposition, but still perfectly correlated. By using quantum entanglement in the game, you can, for example, ensure that a certain opponent does not get more points than you, as in the example in the next paragraph. A plethora of other possible strategies open up when considering tuning the correlation through other gates, e.g., phase gates.

In a quantum computation process and, thus, in the game, entanglement is generated by playing a Hadamard-gate and consecutively a CX-gate on one qubit. As an example, let's assume players 1 and 2 both have a qubit value of $|0\rangle$, as displayed in Fig. 2(c). A Hadamard-card is played on qubit 1, generating a superposition, i.e., the two-qubit state becomes $1/\sqrt{2}(|0,0\rangle+|1,0\rangle)$. Here, the position in the ket-vector corresponds to the qubit number, e.g., the state $|x,y\rangle$ refers to player 1 being in state x and player 2 in y. We then play a CX-gate (X gate controlled by qubit 1 and targeting qubit 2), which results in the state $1/\sqrt{2}(|0,0\rangle+|1,1\rangle)$. This means, that both players' states will randomly have either the value 0, or 1 after a measurement is performed. However, due to entanglement, both qubits will always end up with the same random value. The same entangling operation also works in 3D. In the single-player mode, the player is guided through instructive examples of entanglement in three different riddles with varying difficulty.

IV. FEEDBACK AND FUTURE IMPROVEMENTS

First trials with graduate and undergraduate students have shown good indications of the educational value of the game. Discussions and self-reports of the participants after playing the game allowed us to draw the following conclusions: Student understanding of quantum operations improves considerably after only a few trial games. The students not only understood how the states were evolving but they also conceived and tested better strategies to achieve the highest possible qudit values to win the game. The positive feedback shows promise for enhanced student involvement in future quantum information courses. An enthusiastic student, for example, stated that he "learned about quantum logic in an engaging, fun way." However, we also observed that students without any background in quantum mechanics struggled to develop better strategies. Hence, using the game in undergraduate studies will require more time to introduce the game and the basic concepts of quantum state and their evolutions. We also note that a more controlled and quantitative study would be needed to evaluate the educational effectiveness of the game.

The current version of the game explores some fundamental quantum mechanical effects by mimicking the operation of a quantum computer. Other interesting quantum mechanical effects, such as non-equally weighted states, are beyond the scope of this game.

Although the first trial games have given positive feedback overall, the game can be further improved. An additional operation that could be introduced to the game is a state measurement card which measures the state of one or more qudits, individually. This mechanism would add an extra layer of complexity and would allow the game to introduce simple quantum algorithms, such as superdense coding, quantum teleportation, or entanglement swapping.

V. CONCLUSION

Teaching quantum mechanical concepts is a challenging task, because quantum mechanics is non-intuitive and perceived to be highly demanding. To loosen up the atmosphere and promote student engagement, methods from gamification can be applied. The strategic card game *Endless Fun in high dimensions* offers multiple game modes to facilitate learning new quantum computational concepts. Additionally, the underlying fundamental quantum features, namely, randomness, superposition, interference, and entanglement can be experienced and understood in a quantum computing setting. Initial trial games with students have shown the potential the game has for supporting conventional teaching methods. Together with the evaluation software, the card game is a powerful tool which is not only suitable for players with background knowledge but also for introducing players to quantum operations in an easy-going way. Thus, it can also be used for outreach purposes where interested laymen can experience fundamental quantum physical features.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts of interest to disclose.

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