Ανάλυση Ι   Διάλεξη 8   24-10-25

Θα ξεκινήσω στις 12:10

ασκήσεις σε επαγωγή, sup, inf, ακολουθίες

Σημειώσεις Νοτάρη

σελ 23 ασκ 7

$$\sum_{k=0}^{N} x^k = x^0 + x^1 + \dots + x^N$$

$$= 1 + x + \dots + x^N$$

$$\sum_{k=0}^{3} 1 = 4$$

$$(1+x+x^2+\dots+x^N)(1-x) = (1+\cancel{x}+\cancel{x^2}+\dots+\cancel{x^N}) - (\cancel{x}+\cancel{x^2}+x^3+\dots+x^{N+1})$$

γεωμετρ. σειρά

$$= 1 - x^{N+1}$$

$$1+x+x^2+\dots+x^N = \frac{1-x^{N+1}}{1-x}$$

αν $|x|<1$ $x^{N+1} \to 0$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad \text{αν } |x|<1$$

$$\sum_{k=0}^{N} x^k \underset{x=1}{=} N+1$$

σελ. 23 ασκ 6.

(β)

$$(a+b)^n = \underbrace{(a+b)(a+b)\cdots\cdots(a+b)}_{n \text{ φορές}}$$

$$= a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 \quad \binom{n}{n-1}ab^{n-1} + b^n$$

$\binom{n}{k}$: διωνυμικοί όροι του Νεύτωνος

διαβάζεται n ανά k

$$\binom{n}{1} = n \qquad \binom{n}{2} = \frac{n(n-1)}{2}$$

$n \geq k \quad \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

$\lambda! = 1 \cdot 2 \cdot \cdots \cdot \lambda$

1 2 3

12
13
23

για $n=3$ $\binom{3}{2} = 3$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \qquad \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

ερωτ(α) :

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k} \qquad = \frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 1}$$

$$\frac{n!}{k!\,(n-k)!} + \frac{n!}{(k-1)!\,(n-k+1)!}$$

$$\|$$

$$\frac{n!}{(k-1)!\,(n-k)!}\left[\underbrace{\frac{1}{k} + \frac{1}{n-k+1}}_{\frac{n+1}{k(n-k+1)}}\right] = \frac{(n+1)!}{k!\,(n-k+1)!} = \binom{n+1}{k}$$

$$\frac{4!}{3!\,1!} \qquad 24 \qquad 6$$

1 2 3 4

123, 134, 234

123, 134, 124, 234

124, 134, 234

123, 124, 234

$$\frac{12}{3} = 4$$

$$\binom{10}{2}+\binom{10}{3}=\binom{11}{3}$$

$n=10$, $k=3$

$$\binom{n}{k}+\binom{n}{k-1}=\binom{n+1}{k}$$

$$(a+b)^1 = 1a + 1b$$
$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$
$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + b^3$$

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Τριγ. Pazcal

$$(a+b)^{10} = \cdots\cdots\cdots \binom{10}{2}a^8b^2 + \binom{10}{3}a^7b^3 + \cdots$$
$$(a+b)^{11} = \cdots + \binom{11}{3}a^8b^3 + \cdots$$

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$
$$(a-b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}(-b)^k$$

$$\boxed{2^n = (1+1)^n = \sum_{k=0}^{n}\binom{n}{k}}$$

$$(1-1)^n = 0 = \sum_{k=0}^{n}\binom{n}{k}(-1)^k$$

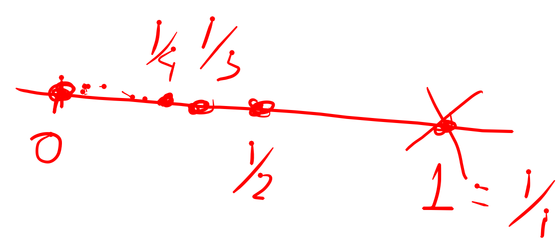
$\left[\begin{array}{l} k=0 \\ 0! = 1 \end{array}\right]$

ασκ 15 (σελ. 24)

(iii) $C = \{0, 1/2, 1/3, 1/4, \ldots, 1/n, n \in \mathbb{N} \setminus \{1\}\}$

$\forall x \in C \quad x \geq 0 \qquad \forall x \in C \quad x \leq 1/2$

$1/n \leq 1/2 \quad (n \geq 2)$

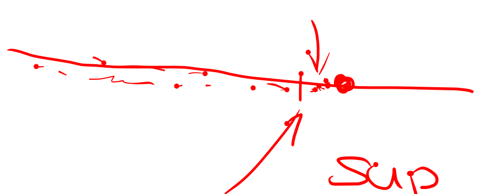


$0 \in C$

$1/2 \in C$

$\max C = 1/2 \Rightarrow \sup C = 1/2$

$\min C = 0 \Rightarrow \inf C = 0$



όλοι οι $x \in C \quad x \leq C$

$\forall \varepsilon > 0, \quad \sup - \varepsilon$

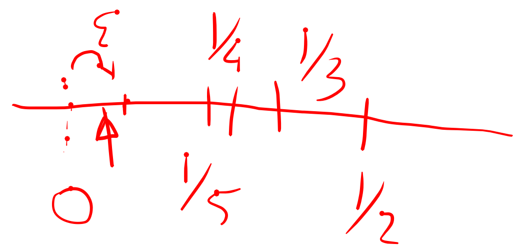
$\forall x \in C \Rightarrow$

$x \leq 1/2$

$1/2$ άνω φράγμ.

$1/2 - \varepsilon < x \in C$

$$C' = \left\{ \tfrac{1}{2}, \tfrac{1}{3}, \dots, \tfrac{1}{n}, \dots, n \geq 2, n \in \mathbb{N} \right\}$$



$0 \neq \min C'$

$\nexists \min C'$

$\exists \inf C' = 0$

(*) $0 \leq \frac{1}{n}$ ✓ 0 είναι κάτω φράγμα

$\exists x \in C'$ (**) $\underbrace{0 + \varepsilon > x = \tfrac{1}{n}}$ $\longrightarrow$ $\varepsilon > \frac{1}{n} \Rightarrow n > \frac{1}{\varepsilon}$

$n > \left[\frac{1}{\varepsilon}\right] + 1$

$\Downarrow$ $\quad n \geq \left[\frac{1}{\varepsilon}\right] + 1$

$0 = \inf C$

15(v) $E = \left\{ \frac{1}{n} + (-1)^n : n \in \mathbb{N} \right\}$

$= E_{\pi\varepsilon\rho} \cup E_{\alpha\rho\tau}$

$-1 + \frac{1}{n} \geq -2$

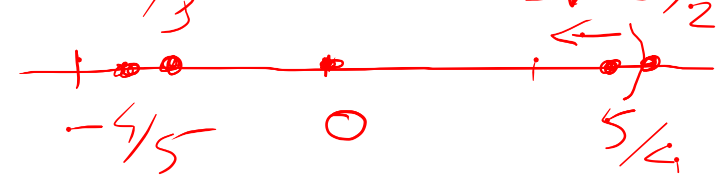
$1 + \frac{1}{n}$

$1 + \frac{1}{n} = \frac{n+1}{n}$ για n αρτιο

$-1 + \frac{1}{n} = \frac{-n+1}{n} = -\frac{(n-1)}{n}$

$-1 \quad -2/3$

$1 \quad 3/2$



για n περιτ.

$n = 1 \qquad 1 + (-1)^1 = 0$

$n = 2 \qquad 1/2 + (-1)^2 = 3/2$

$n = 3 \qquad 1/3 + (-1)^3 = -2/3$

$n = 4 \qquad 1/4 + (-1)^4 = 5/4$

$\vdots$

n: περιτ. $\qquad 1/n - 1$

n: αρτ. $\qquad 1/n + 1$

min	max	inf	sup
×	3/2	-1	3/2

$-1 \leq x \quad \forall x \in E$

για n: περιτ. $x = -1 + 1/n \leq 0$

n: αρτ. $x = 1 + 1/n > 0$

$\forall \varepsilon > 0 \quad -1 + \varepsilon > x \quad \exists x \in E$

$-1 + \varepsilon > -1 + 1/n$

$n \geq [1/\varepsilon] + 1$

ασκ 19 σελ 24

$C = A + B = \{ a+b, a \in A, b \in B \}$

A, B φραγμένα,

$\exists \sup A \quad \sup B$

$\inf A \quad \inf B$

$\underbrace{\sup C}_{s} = \underbrace{\sup A}_{s_A} + \underbrace{\sup B}_{s_B}$

(*) $s > s_A + s_B \Rightarrow$ άτοπο.

$\cancel{x} = s - (s_A + s_B) > 0$

$x > 0$

$x/2 > 0$

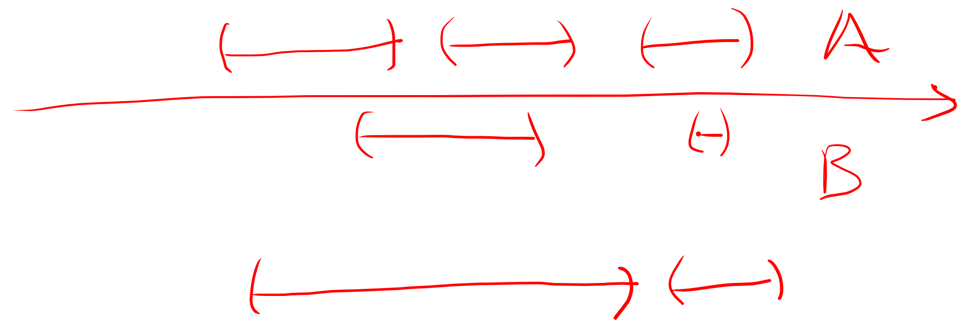
$$\underline{s - \frac{x}{2}} < y \in C$$

$= a_1 + b_1 \qquad a_1 \in A \quad b_1 \in B$

$\frac{s}{2} + \frac{(s_A + s_B)}{2} \geq \underline{s_A + s_B} \qquad a_1 \leq s_A \quad b_1 \leq s_B$

$$\frac{s}{\cancel{2}} \leq \frac{s_A + s_B}{\cancel{2}}$$

$A \cup B$ $\quad$ $\sup(A \cup B) = \max(\sup A, \sup B)$



$A \cap B$

$\sup(A \cap B) \stackrel{?}{=} \min(\sup A, \sup B)$