Stellar Nucleosynthesis

Reading material:

- D.C. Clayton Principles of stellar evolution and nucleosythesis, University of Chicago Press 1983
- N. Langer: Nucleosynthesis, Bonn University, 2012
- B. Pagel: Nucleosynthesis and Chemical Evolution of Galaxies , Cambridge University Press 1997
- C. Aerts, Stellar Structure and evolution notes

Nucleosynthesis

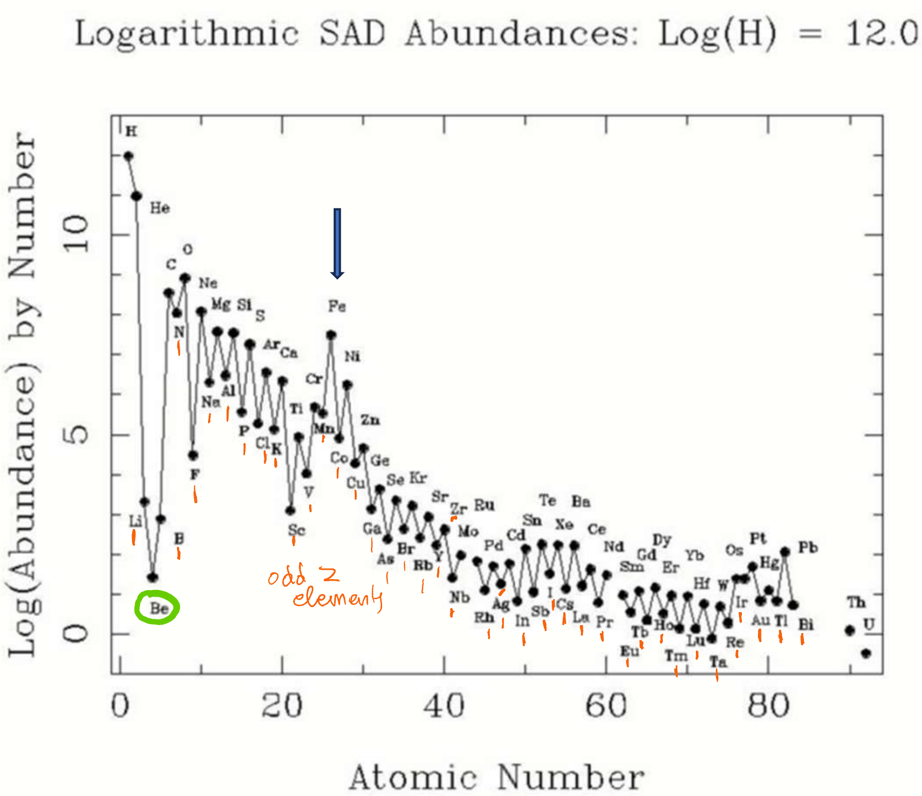
- Stellar nucleosynthesis is the process involving nuclear reactions through which fresh atomic nuclei are synthesized from pre-existing nuclei or nucleons.
- The first stage of nucleosynthesis occurred in the hot, early Universe, with the production of H, He, and traces of Li-7 (**primordial nucleosynthesis**).
- In the present-day Universe nucleosynthesis occurs through:
  - thermonuclear reactions in stellar interiors and explosions (building nuclei up to the Fe-peak)
  - neutron captures in stellar interiors and explosions (building nuclei above the Fe-peak), … –including neutron-star megrers
  - spallation reactions in the interstellar medium, whereby light nuclei (Li, Be, and B) are produced by fragmentation of heavier ones (C, N, and O).

See review by N. Prantzos & S. Ekstrom 2011

Observational constraints

A theory of nucleosynthesis should explain the measured relative abundances versus atomic number Z as well as observed isotopic ratios

Solar System



Solar abundances by number: vertical axis is $\log(X_Z/A_Z)$ + constant, where X_Z is the abundance of element Z by mass and A_Z is the mean atomic weight of that element. (C. R. Crowley, U Mich).

Data sources for the solar system abundances

1. Spectral analysis of the Sun

1.1 photospheric absorption lines → abundance ratios wtspt hydrogen

$$\log_{10}\left(\frac{N_{\text{element}}}{N_H}\right) \quad \text{where} \quad \log_{10}(N_H) = 12$$

– exceptions: He, Ne, Ar, rare heavy elements
– no information on isotopic abundances, except from some molecules (e.g. CO, CN, MgH)

1.2 emission lines from chromosphere, corona (far-UV) –→ He, Ne, Ar, but less accurate

2. Direct measurements (chemical analysis, mass spectrometry)
   - Earth (crust, oceans, atmosphere), Moon rocks : very inhomogeneous
     → little information on elemental abundances
     → but provides isotope ratios
   - meteorites (esp. chondrites): uniform atomic composition - corresponds to solar photosphere
     → represents Solar System abundances
     → also isotope ratios (with interesting anomalies)
   - solar wind (ion counters on space probes) –→ e.g. ^{3}He

Outside the Solar System

chemical analysis of spectra of other stars, gaseous nebulae and external galaxies

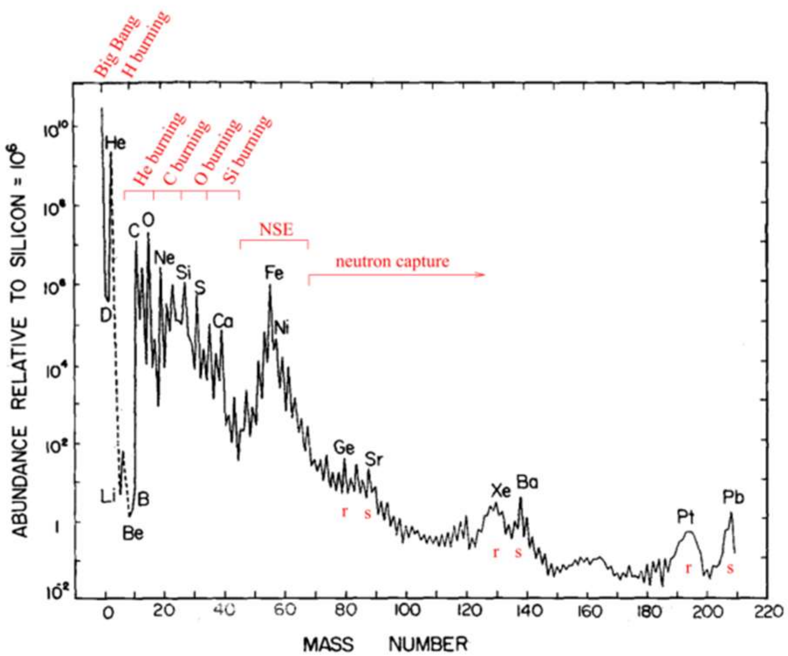


Figure 1.1: The 'local galactic' abundance distribution of nuclear species, as a function of mass number A. The abundances are given relative to the Si abundance which is set to 10^6. Peaks due to the r- and s-process are indicated. It is the main aim of this course to provide

Table 1.1: The 25 most abundant nuclei. Symbols in the last column indicate the nuclear burning stage, also 'BB': Big Bang, 'NSE': nuclear statistical equilibrium.

rank	Z	element	A	nucleon fraction	source (process)
1	1	H	1	7.057(−1)	BB
2	2	He	4	2.752(−1)	BB, H(CNO, pp)
3	8	O	16	9.592(−3)	He
4	6	C	12	3.032(−3)	He
5	10	Ne	20	1.548(−3)	C
6	26	Fe	56	1.169(−3)	NSE
7	7	N	14	1.105(−3)	H(CNO)
8	14	Si	28	6.530(−4)	O
9	12	Mg	24	5.130(−4)	C, Ne
10	16	S	32	3.958(−4)	O
11	10	Ne	22	2.076(−4)	He
12	12	Mg	26	7.892(−5)	C, Ne
13	18	Ar	36	7.740(−5)	Si, O
14	26	Fe	54	7.158(−5)	NSE, Si
15	12	Mg	25	6.893(−5)	C, Ne
16	20	Ca	40	5.990(−5)	Si, O
17	13	Al	27	5.798(−5)	C, Ne
18	28	Ni	58	4.915(−5)	Si, NSE
19	6	C	13	3.683(−5)	H(CNO)
20	2	He	3	3.453(−5)	BB, H(pp)
21	14	Si	29	3.445(−5)	C, Ne
22	11	Na	23	3.339(−5)	C, H(NeNa)[1]
23	26	Fe	57	2.840(−5)	NSE
24	14	Si	30	2.345(−5)	C, Ne
25	1	H	2	2.317(−5)	BB

[1]H-burning via the NeNa-chain.

From Langer's notes

Some basics from nuclear physics

- Atomic nucleus consists of Z protons and N neutrons

  A = Z + N = mass number

  Z = constant: **isotopes**

  A = constant: **isobars**

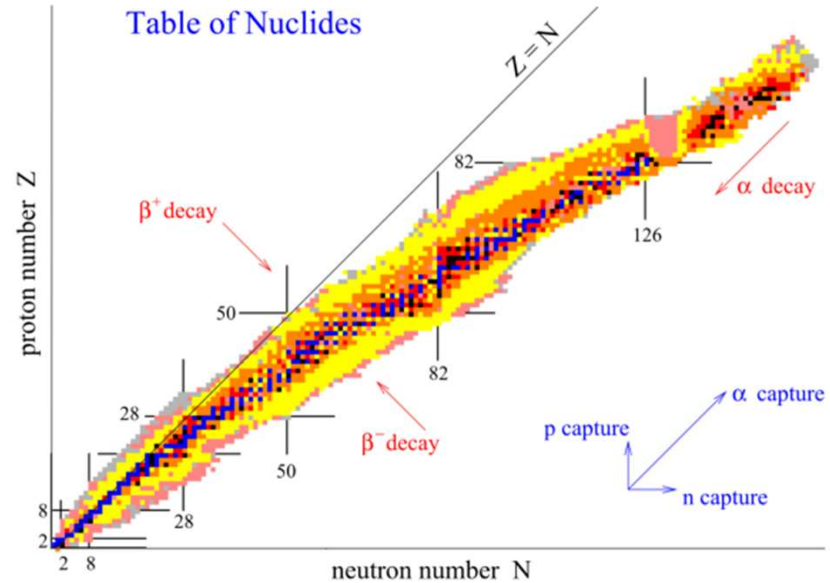
- Stable nuclei → valley of stability in the (N, Z) plane

  Outside the valley: spontaneous, radioactive decay

$\beta^-: (Z, A) \rightarrow (Z+1, A) + \mathrm{e}^- + \nu$

$\beta^+: (Z, A) \rightarrow (Z-1, A) + \mathrm{e}^+ + \bar{\nu}$

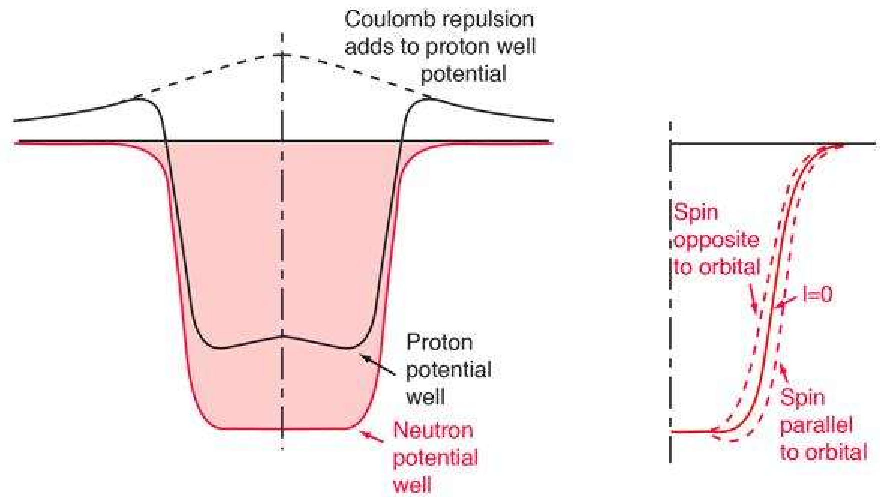
$a\colon\ (Z, A) \rightarrow (Z-2, A-4) +\ {}^4He$



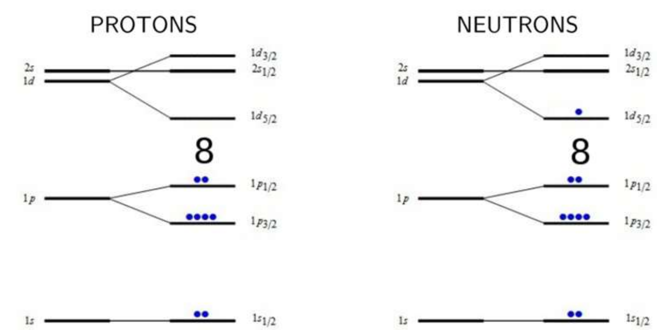
Nuclear shell model – in brief

- Nucleus = **many-body system** → exact solution impossible
- Approximation: each nucleon moves in an **average (mean-field) potential**
- Converts complex interactions → **independent single-particle motion**
- Potential: **flat interior + diffuse surface**
- Protons & neutrons fill **separate energy levels**
- **Pauli principle** → limited occupancy per state
- Total energy ≈ sum of single-particle energies
  - small **residual interactions**
- **Spin–orbit coupling ($\vec{L} \cdot \vec{S}$)** splits levels

(opposite ordering to atoms: parallel L–S → lower energy in nuclei)



Ground state in ^{17}O



Semiempirical nuclear mass formula

$$M(A,Z) = (A-Z)m_n + Z(m_p + m_e)$$

$$-a_1 A + a_2 A^{2/3} + a_3 \frac{(A/2-Z)^2}{A} + a_4 \frac{Z^2}{A^{1/3}} + a_5 \frac{\delta}{A^{3/4}}$$

where

$a_1 - 15.53, a_2 = 17.804, a_3 = 94.77, a_4 = 0.7103, a_5 = 33.6$ in $\mathrm{MeV/c^2}$

volume of nucleus $\propto A$ → $R_{nucl} \propto A^{1/3}$

① "free" rest mass of neutrons

② "free" rest mass of protons

③ "free" rest mass of electrons

④ decrease in nuclear mass (increase in BE) from nearest-neighbour interactions between nucleons

⑤ Surface energy term (at the surface a nucleon has fewer neighbours → less BE)

⑥ $$a_3 \frac{\left(\frac{A}{2} - Z\right)^2}{A} = a_3 \frac{\left(\frac{N-Z}{2}\right)^2}{A}$$ $(A = N + Z)$ **$N = Z$ most stable** (for light nuclei)

↑ caused by the fact that p & n are fermions → Pauli exclusion principle
[if e.g. you have too many n they will have to occupy higher energy levels ⇒ increasing total energy → reducing stability.]

⑦ Coulomb repulsion $\propto \frac{Z^2}{R_{nuc}} = \frac{Z^2}{A^{1/3}}$ (electrostatic energy of protons)

⑧ Pairing term → pp, nn pairs of opposite spin → lower energy configuration

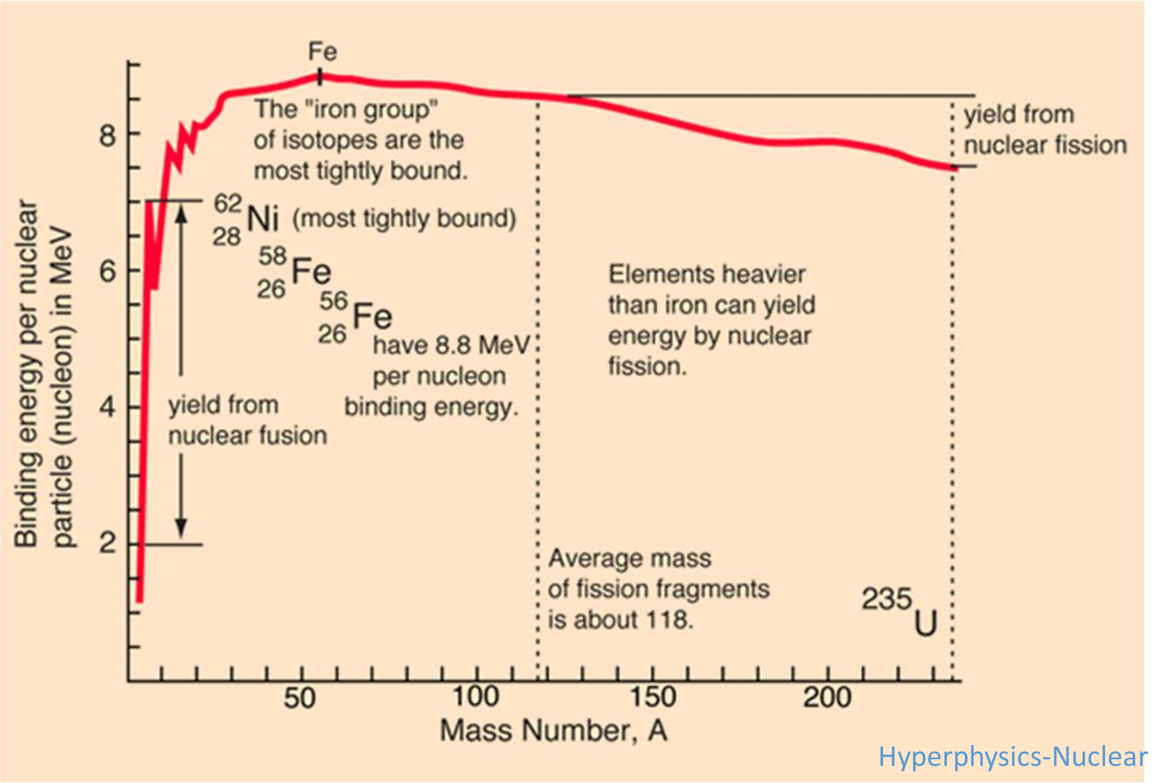
$\delta = -1$ even Z, even N most stable

$\delta = +1$ odd Z, odd N least stable

$\delta = 0$ one even, one odd (so A odd) one type has an unpaired nucleon → zero result in BE

Binding energy per nucleon

$$\frac{BE}{A} \equiv \frac{M(A,Z) - (A-Z)m_n - Z(m_p + m_e)}{A}$$



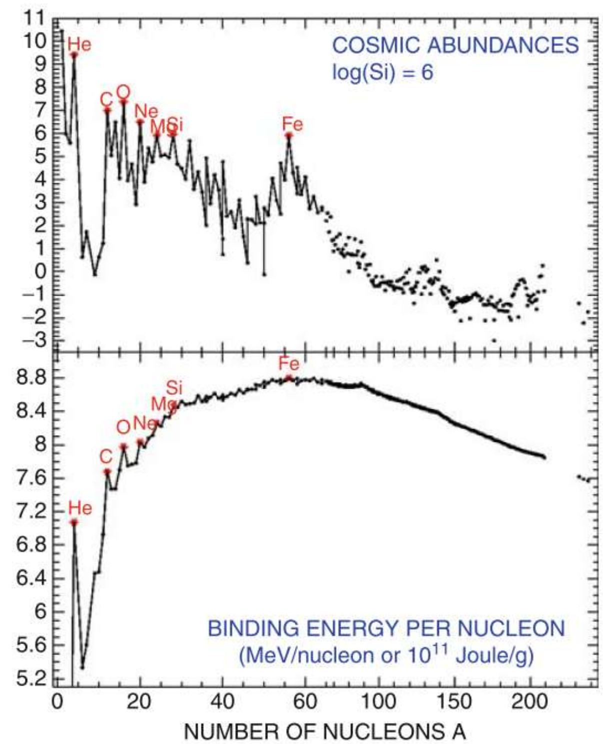
Hyperphysics-Nuclear

Remarks

Local maxima in abundances correlate with maxima in binding energy

- ➢ more stable nuclei are more abundant than their neighbors (Elements with even Z are lmoreabundant than those with odd Z; Magic number and doubly magic nuclei (2, 8, 20, 28, 50, 82, 126) more tightly bound, thus more abundant
- ➢ the fragile Li, Be, and B isotopes are extremely underabundant.
- ➢ at a global level, the light H and He are overwhelmingly more abundant than the more strongly bound C, N, and O, which in turn are more abundant than the even more stable Fe-peak nuclei.

Solar ("cosmic") abundances



Stellar Nucleosynthesis. Figure 1 Solar abundances on a logarithmic scale of the stable nuclei (*top*), and corresponding binding energies of these nuclei (*bottom*) vs. mass number

➡ **The (local) correlation between cosmic abundances and nuclear stability suggests that nuclear reactions have shaped the abundances of elements in the Universe. The fact that the correlation is only local and does not hold at a global level implies that nuclear processes have affected only a small fraction of the baryonic matter in the Universe (less than a few per cent).**

Εισαγωγικές έννοιες για τις πυρηνικές αντιδράσεις

Γενική περιγραφή μιας πυρηνικής αντίδρασης μεταξύ δύο πυρήνων α και X, που δίνουν δύο «προϊόντα», έστω Y και b

$a + X \rightarrow Y + b$ (3)

Η διατήρηση ενέργειας απαιτεί να ισχύει:

$E_{aX} + (M_a + M_X)c^2 = E_{bY} + (M_b + M_Y)c^2$ (4)

όπου E_{aX} η κινητική ενέργεια των α και X στο σύστημα κέντρου μάζας, και E_{bY} η κινητική ενέργεια στο σύστημα κέντρου μάζας του συστήματος bY. Οι υπόλοιποι όροι είναι οι μάζες ηρεμίας των α, X, b, Y.

Στις πυρηνικές αντιδράσεις διατηρούνται:

✓Το φορτίο → για τη (3) $Z_X + Z_a = Z_Y + Z_b$ (5)

✓Ο βαρυονικός αριθμός $(p, n, \bar{p}, \bar{n},..)$ → για τη (3) $A_X + A_a = A_Y + A_b$ (6)

✓Ο λεπτονικός αριθμός $(e^-, e^+, \nu_e, \overline{\nu}_e, \ldots)$ όταν στην πυρηνική αντίδραση συμμετέχουν λεπτόνια είτε ως αντιδρώντα ή/και ως προϊόντα (ασθενείς πυρηνικές αντιδράσεις)

Σημειώνουμε ότι τα σωμάτια έχουν θετικό βαρυονικό/λεπτονικό αριθμό και τα αντίστοιχα αντισωμάτια, αρνητικό.

Λόγω της διατήρησης του φορτίου μπορούμε στην εξίσωση (3) να αντικαταστήσουμε τις μάζες των πυρήνων με τις μάζες των αντίστοιχων ουδέτερων ατόμων (κάνουμε μόνο ένα μικρό λάθος λόγω διαφορών στις ενέργειες σύνδεσης των ηλεκτρονίων στα διάφορα άτομα)

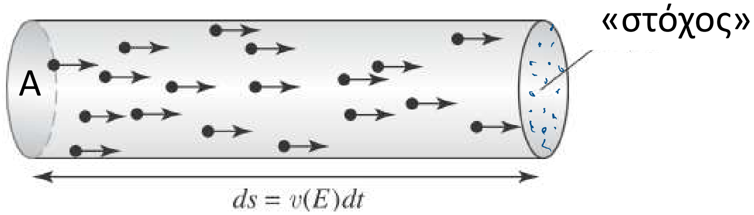
Η **ενέργεια που απελευθερώνεται (Q>0) από την (4)** είναι $\mathrm{Q} = (M_a + M_X - M_b - M_Y)c^2$ (7)
Εφόσον ο βαρυονικός αριθμός διατηρείται (εξ. 6) μπορούμε επίσης να γράψουμε την (7) ως
$Q = (\Delta M_X + \Delta M_a - \Delta M_Y - \Delta M_b)\, c^2$ (8)
(Αν Q<0 η αντίδραση είναι «ενδόθερμη»)

Ενεργός διατομή (cross section) και ρυθμός αντίδρασης (reaction rate)

Στην αντίδραση $a + X \rightarrow Y + b$, υποθέτουμε ότι τα σωματίδια X βομβαρδίζονται από σωματίδια α με συγκεκριμένη ταχύτητα υ. Ο ρυθμός με τον οποίο αντιδρούν εξαρτάται από την ενεργό διατομή της αντίδρασης σ, δηλαδή:

$$\sigma = \frac{\alpha\rho\iota\theta\mu o\varsigma\ \alpha\nu\tau\iota\delta\rho\alpha\sigma\varepsilon\omega\nu\ \alpha\nu\alpha\ \pi\upsilon\rho\eta\nu\alpha\ \mathrm{X}\ \sigma\tau\eta\ \mu o\nu\alpha\delta\alpha\ \tau o\upsilon\ \chi\rho o\nu o\upsilon}{\rho o\eta\ \pi\rho o\sigma\pi\iota\pi\tau o\nu\tau\omega\nu\ \pi\upsilon\rho\eta\nu\omega\nu\ \alpha\ \sigma\tau\eta\ \mu o\nu\alpha\delta\alpha\ \tau o\upsilon\ \chi\rho o\nu o\upsilon}$$

όπου η ροή προσπιπτόντων πυρήνων ορίζεται ως ο αριθμός των προσπιπτόντων πυρήνων ανά μονάδα επιφάνειας.



Εδώ υποθέσαμε ότι η ροή των σωματιδίων α οφείλεται σε ομοιόμορφη μετατόπιση με ταχύτητα $\boldsymbol{\upsilon}$

Έστω η αριθμητική πυκνότητα των πυρήνων α είναι n_a και των πυρήνων X, n_{X}

Ο αριθμός των σωματιδίων α που προσπίπτουν μέσα σε χρόνο dt στον «στόχο» (πυρήνες X) είναι

$\frac{n_a \mathrm{dV}}{\mathrm{dt}} = \frac{n_a \mathrm{A}ds}{\mathrm{dt}} = \frac{n_a \mathrm{A}\upsilon dt}{\mathrm{dt}} = n_a\ \mathrm{A}\upsilon$ και η αντίστοιχη ροή (αριθμός σωματιδίων ανά μονάδα επιφάνειας) θα είναι $= n_a\ \mathrm{A}\upsilon/\mathrm{A} = n_a\ \upsilon$

Ο αριθμός αντιδράσεων μεταξύ πυρήνων α και X που θα συμβούν στη μονάδα του χρόνου και στη μονάδα όγκου θα ισούται με το γινόμενο της ροής των προσπιπτόντων σωματιδίων $n_a\upsilon$ με την αριθμητική πυκνότητα των πυρήνων X, n_{X}, και την ενεργό διατομή σ

δηλαδή $r_{aX} = n_a n_X \upsilon\sigma$ (9)

Σε ένα μίγμα πυρήνων α και Χ (θεωρούμε ότι μπορούμε να τους περιγράψουμε ως μίγμα αερίων) σε Θ.Ι. υπάρχει ένα φάσμα, $\varphi(\upsilon)$, σχετικών ταχυτήτων μεταξύ των σωματιδίων (πυρήνων) τύπου α και τύπου Χ, με $\int_o^\infty \phi(\upsilon)\mathrm{d}\upsilon = 1$, δηλαδή η $\phi(\upsilon)$ δίνει την πιθανότητα η σχετική ταχύτητα να έχει μέτρο μεταξύ υ και $\upsilon + d\upsilon$.

Οπότε μπορούμε να γράψουμε την (9) ως:

$$r_{aX} = n_a n_X \int_0^\infty \phi(\upsilon)\sigma(\upsilon)\upsilon\mathrm{d}\upsilon = n_a n_X \langle\sigma\upsilon\rangle$$

όπου $\langle\sigma\upsilon\rangle = \frac{\int_0^\infty \phi(\upsilon)\sigma(\upsilon)\upsilon\mathrm{d}\upsilon}{\int_o^\infty \phi(\upsilon)\mathrm{d}\upsilon}$

Τα παραπάνω ισχύουν για την περίπτωση που $\alpha \neq \mathrm{X}$, οπότε το γινόμενο $n_a n_X$ μας δίνει τον συνολικό αριθμό μοναδικών ζευγών (α, X). Αν α $\equiv \mathrm{X}$, τότε ο αριθμός των μοναδικών ζευγών θα είναι $\frac{1}{2} n_a n_X$. Οπότε, για να συμπεριλάβουμε και τις δύο περιπτώσεις, γράφουμε

$$r_{aX} = \frac{1}{1+\delta_{\alpha\mathrm{X}}} n_a n_X \langle\sigma\upsilon\rangle \quad (10)$$

Για ιδανικό αέριο σε (τοπική) θερμοδυναμική ισορροπία η κατανομή φ(υ) δίνεται από την κατανομή Maxwell-Boltzmann,

$$\phi(\upsilon) = 4\pi\upsilon^2 \left(\frac{\mu}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{\mu\upsilon^2}{2kT}\right)$$ (11) (απόδειξη ως άσκηση – έγινε στο μάθημα)

όπου $\mu = \frac{m_\alpha m_X}{m_\alpha + m_X}$ η ανηγμένη μάζα.

οπότε $\langle\sigma\upsilon\rangle = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{\frac{3}{2}} \int_0^\infty \upsilon^3\sigma(\upsilon)\mathrm{d}\upsilon$ (12)

Για μη σχετικιστικές ταχύτητες, μπορούμε να γράψουμε την (12) συναρτήσει της κινητικής ενέργειας στο σύστημα κέντρου μάζας, $E = \frac{1}{2}\mu\upsilon^2$

Γράφοντας την κατανομή ταχυτήτων ως κατανομή κινητικών ενεργειών στο σύστημα ΚΜ, έχουμε

$$\psi(E)dE = \phi(v)dv = -\frac{2}{\sqrt{\pi}}\frac{E}{kT}\exp\left(-\frac{E}{kT}\right)\frac{dE}{(kTE)^{\frac{1}{2}}}$$

Οπότε η (12) γράφεται

$$\langle\sigma\upsilon\rangle = \int_0^\infty \sigma(E)\upsilon(E)\psi(E)dE = \left(\frac{8}{\pi\mu}\right)^{1/2}(kT)^{-3/2}\int_0^\infty \sigma(E)E\exp\left(-\frac{E}{kT}\right)\mathrm{d}E$$

Πρέπει να βρούμε το $\sigma(E)$

Πυρηνική ενεργός διατομή σ(E)

- Πως μπορούν να συμβούν οι θερμοπυρηνικές αντιδράσεις; Πρέπει να πλησιάσουν πολύ κοντά μεταξύ τους θετικά φορτισμένα σωματίδια.
- Στο διπλανό διάγραμμα βλέπουμε τη καμπύλη δυναμικής ενέργειας για την αλληλεπίδραση Coulomb μεταξύ δυο πρωτονίων.
- Για να συμβεί η πυρηνική σύντηξη των δύο πρωτονίων (γενικά δύο πυρήνων) πρέπει να υπερπηδηθεί το φράγμα Coulomb.

ΜΙΑ ΠΟΙΟΤΙΚΗ ΣΥΖΗΤΗΣΗ:

(α) Κλασική προσέγγιση

Υποθέτουμε ότι η απαιτούμενη ενέργεια προέρχεται από τη θερμική ενέργεια του αερίου δηλ.

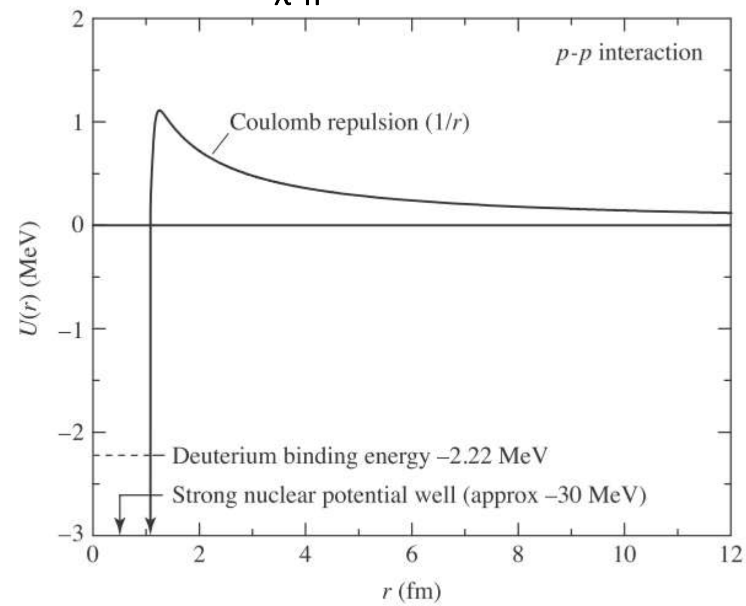
$$\frac{1}{2}\mu\overline{\upsilon^2} = \frac{3}{2}kT_{\text{classical}} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{r} \quad (14)$$

(όπου μ η ανηγμένη μάζα του συστήματος των δυο αλληλοεπιδρώντων πυρήνων, και υ η σχετική τους ταχύτητα (για σύντηξη p-p, $\mu = \frac{m_p}{2}$ και $Z_1 = Z_2 = 1$).

Άρα, $T_{\text{classical}} = \frac{Z_1 Z_2 e^2}{6\pi\epsilon_0 k r} \sim 10^{10}\,\mathrm{K} \gg T_{c,\odot} = 1.5 \times 10^7 K$ (15)

όπου θέσαμε $r \sim 1 fm$ για την ακτίνα ενός τυπικού πυρήνα.

Σχήμα από C&O



Χαρακτηριστική καμπύλη δυναμικής ενέργειας για τις πυρηνικές αντιδράσεις. Η άπωση Coulomb ανάμεσα στους θετικά φορτισμένους πυρήνες έχει ως αποτέλεσμα ένα φράγμα το οποίο είναι αντιστρόφως ανάλογο της απόστασης ανάμεσα στους πυρήνες και ανάλογο του γινομένου των φορτίων τους. Το πυρηνικό πηγάδι δυναμικού μέσα στον πυρήνα οφείλεται στην ελκτική πυρηνική δύναμη.

(β) Κβαντομηχανική προσέγγιση

Η απροσδιοριστία στη θέση του ενός πυρήνα (p για τη σύντηξη p-p) που αλληλεπιδρά με έναν άλλο πυρήνα (p για τη σύντηξη p-p) μπορεί να είναι τόσο μεγάλη ώστε, ακόμα και αν η κινητική ενέργεια στο σύστημα Κ.Μ. δεν είναι αρκετή για να ξεπεραστεί το κλασικό φράγμα Coulomb, ο πυρήνας (το p) μπορεί, παρ’ όλα αυτά να βρεθεί μέσα στο κεντρικό πηγάδι δυναμικού που ορίζεται από την ισχυρή δύναμη του άλλου πυρήνα (p) → κβαντομηχανικό φαινόμενο σήραγγας

$$\frac{1}{2}\mu \upsilon^2 = \frac{p^2}{2\mu} \Rightarrow \frac{(\frac{\hbar}{\lambda})^2}{2\mu} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{\lambda} \quad (16)$$

όπου *p* η σχετική ορμή, μ η ανηγμένη μάζα και, και $r \sim \lambda$, όπου λ το μήκος de Broglie $\lambda = \frac{\hbar}{p} = \frac{\hbar}{(2mE)^{1/2}}$ (17)

για μη σχετικιστικά σωματίδια.

Λύνουμε ως προς λ τη σχέση (16), και αντικαθιστούμε το r με λ στη σχέση (15), οπότε τελικά βρίσκουμε

$$T_{\text{quantum}} = \frac{Z_1^2 Z_2^2 e^4 \mu_m}{12\ {}^2\epsilon_0^2 h^2 k} \cong 10^7 K\ (\gamma\iota\alpha\ p - p) \quad (18)$$

Δηλαδή λαμβάνοντας υπόψη τη κβαντομηχανική συμπεριφορά των αλληλεπιδρώντων σωματιδίων βρίσκουμε θερμοκρασίες (για p-p) της τάξης μεγέθους της θερμοκρασίας στο κέντρο του ήλιου.

Προσέγγιση του $\sigma(E)$ (στα πλαίσια της κβαντομηχανικής προσέγγισης)

(i) περιμένουμε το $\sigma(E) \propto \pi\lambda^2 (\gamma\varepsilon\omega\mu\varepsilon\tau\rho\iota\kappa\eta\ \varepsilon\nu\varepsilon\rho\gamma o\varsigma\ \delta\iota\alpha\tau o\mu\eta) \cong \pi\left(\frac{\hbar}{p}\right)^2 \propto \frac{1}{E}$ $\left(E = \frac{p^2}{2\mu}, \text{μη σχετικ.}\right)$

(ii) Επίσης περιμένουμε (αποδεικνύεται αυστηρά) ότι η πιθανότητα να διαπεράσει το σωματίδιο το φράγμα Coulomb, U_c, θα σχετίζεται με το λόγο του E/U_c.

[αν το U_c=0 η πιθανότητα να ξεπεραστεί το φράγμα είναι προφανώς 1, ενώ αν το U_c τείνει στο άπειρο, η πιθανότητα τείνει εκθετικά στο 0.]

Αποδεικνύεται ότι $\sigma(E) \propto e^{-2\pi^2 U_c/E}$

όπου $\frac{U_c}{E} = \frac{Z_1 Z_2 e^2/4\pi\epsilon_0 r}{\frac{p^2}{2\mu}} = \frac{Z_1 Z_2 e^2}{2\pi\epsilon_0 h\upsilon}$ (όπου πήραμε $r \sim \lambda = \frac{h}{p}$, $p = \mu\upsilon$)

Δηλαδή:

$\sigma(E) \propto e^{-bE^{-1/2}}$ με $b \equiv \frac{\pi\mu_m^{1/2} Z_1 Z_2 e^2}{2^{1/2}\epsilon_0 h}$ (να αποδειχθεί ως άσκηση)

Οπότε μπορούμε να γράψουμε το $\sigma(\mathrm{E})$ ως:

$\sigma(E) = S(E)E^{-1}e^{-2\pi^2 U_c/E}$ οπότε $\langle\sigma\upsilon\rangle = \left(\frac{8}{\mu\pi}\right)^{1/2}(kT)^{-3/2}\int_0^\infty \sigma(E)E\exp\left(-\frac{E}{kT}\right)\mathrm{d}E$ και

$$r_{\alpha X} = \frac{1}{1+\delta_{\alpha X}} n_\alpha n_X \langle\sigma\upsilon\rangle = \left(\frac{2}{kT}\right)^{3/2}\frac{1}{1+\delta_{\alpha X}}\frac{n_\alpha n_x}{(\mu\pi)^{1/2}}\int_0^\infty S(E)e^{-bE^{-1/2}}e^{-E/kT}dE$$

όπου υποθέτουμε ότι η $S(\mathrm{E})$ («astrophysical S factor») περιλαμβάνει όλες τις εξαρτήσεις από τη δομή των πυρήνων.

Gamow peak

Ας μελετήσουμε το ολοκλήρωμα $\int_0^\infty S(E)e^{-bE^{-1/2}}e^{-E/kT}dE$

Ας υποθέσουμε ότι το S(E) είναι περίπου σταθερό για την περιοχή ενεργειών που μας ενδιαφέρουν (αυτό γενικά μπορούμε να το υποθέσουμε όταν η E είναι μακριά από συντονισμούς).

Τότε μπορούμε να γράψουμε

$$\int_0^\infty S(E)e^{-bE^{-1/2}}e^{-E/kT}dE \cong S(E_o)\int_0^\infty e^{-(bE^{-\frac{1}{2}}+\frac{\mathrm{E}}{kT})}dE$$

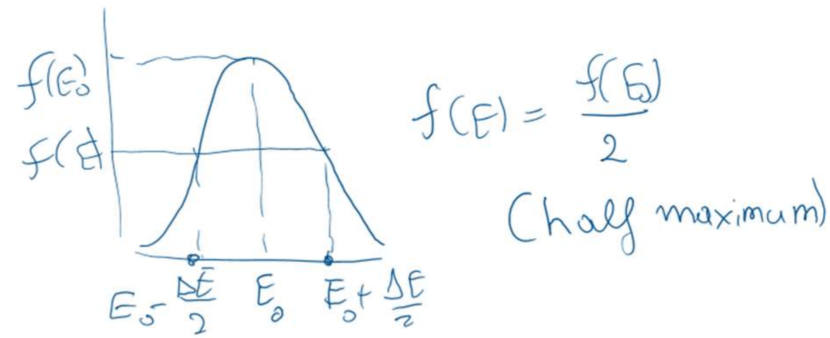
Παρατηρείστε ότι η $e^{-E/kT}$ τείνει γρήγορα στο μηδέν για μεγάλα E, ενώ η $e^{-bE^{-1/2}}$ τείνει γρήγορα το μηδέν για μικρές ενέργειες. Οπότε η μεγαλύτερη συνεισφορά στο ολοκλήρωμα θα είναι για τιμές του E που είναι κοντά στο μέγιστο της συνάρτησης $f(E) = e^{-(bE^{-\frac{1}{2}}+\frac{\mathrm{E}}{kT})}$

Το μέγιστο της συνάρτησης αυτής συμβαίνει όταν $\frac{d(bE^{-\frac{1}{2}}+\frac{\mathrm{E}}{kT})}{d\mathrm{E}} = 0 \Rightarrow \frac{1}{kT} - \frac{1}{2}bE_o^{-\frac{3}{2}} = 0 \Rightarrow$

$E_0 = \left(\frac{bkT}{2}\right)^{3/2}$ → κορυφή Gamow

(Αποδεικνύεται ότι…) Μπορούμε να προσεγγίσουμε την $f(E)$ με μία γκαουσιανή συνάρτηση της μορφής

$$f(E) \approx f(E_0)\exp\left[-\left(\frac{E-E_0}{\Delta E}\right)^2\right],$$



όπου $f(E_0) = e^{-(b\mathrm{E}_o^{-\frac{1}{2}}+\mathrm{E}_o/kT)} = e^{-(3\mathrm{E}_o/kT)}$ (ελέγξτε το)

<u>Εύρεση του ΔΕ</u>

0

$$f(E) = f(E_0) + f'(E_0)(E-E_0) + \frac{1}{2}f''(E_0)(E-E_0)^2 + \cdots$$

$$f(E) = \frac{f(E_0)}{2} = f(E_0) + \frac{1}{2}f''(E_0)(E-E_0)^2 \Rightarrow$$

$$-\frac{f(E_0)}{2} = \frac{1}{2}f''(E_0)(E-E_0)^2 \Rightarrow E - E_0 = -\left(\frac{f}{f''}\right)^{\frac{1}{2}}_{E=E_0} \Rightarrow \Delta\mathrm{E} = 2(E-E_0) = \left(\frac{4E_0kT}{3}\right)^{1/2}$$ (ελέγξτε το)

Οπότε

$$\int_0^{\infty} f(E)\mathrm{d}E \approx e^{-\left(\frac{3\mathrm{E}_O}{kT}\right)} \int_0^{\infty} \exp\left[-\left(\frac{E-E_0}{\Delta E}\right)^2\right] \mathrm{d}E \approx e^{-\left(\frac{3\mathrm{E}_O}{kT}\right)} \sqrt{\pi}\Delta E \Rightarrow$$

$$\int_0^{\infty} f(E)\mathrm{d}E \approx \sqrt{\pi} e^{-\left(\frac{3\mathrm{E}_O}{kT}\right)} \left(\frac{4E_0 kT}{3}\right)^{1/2}$$, όπου $E_0 = \left(\frac{bkT}{2}\right)^{3/2}$

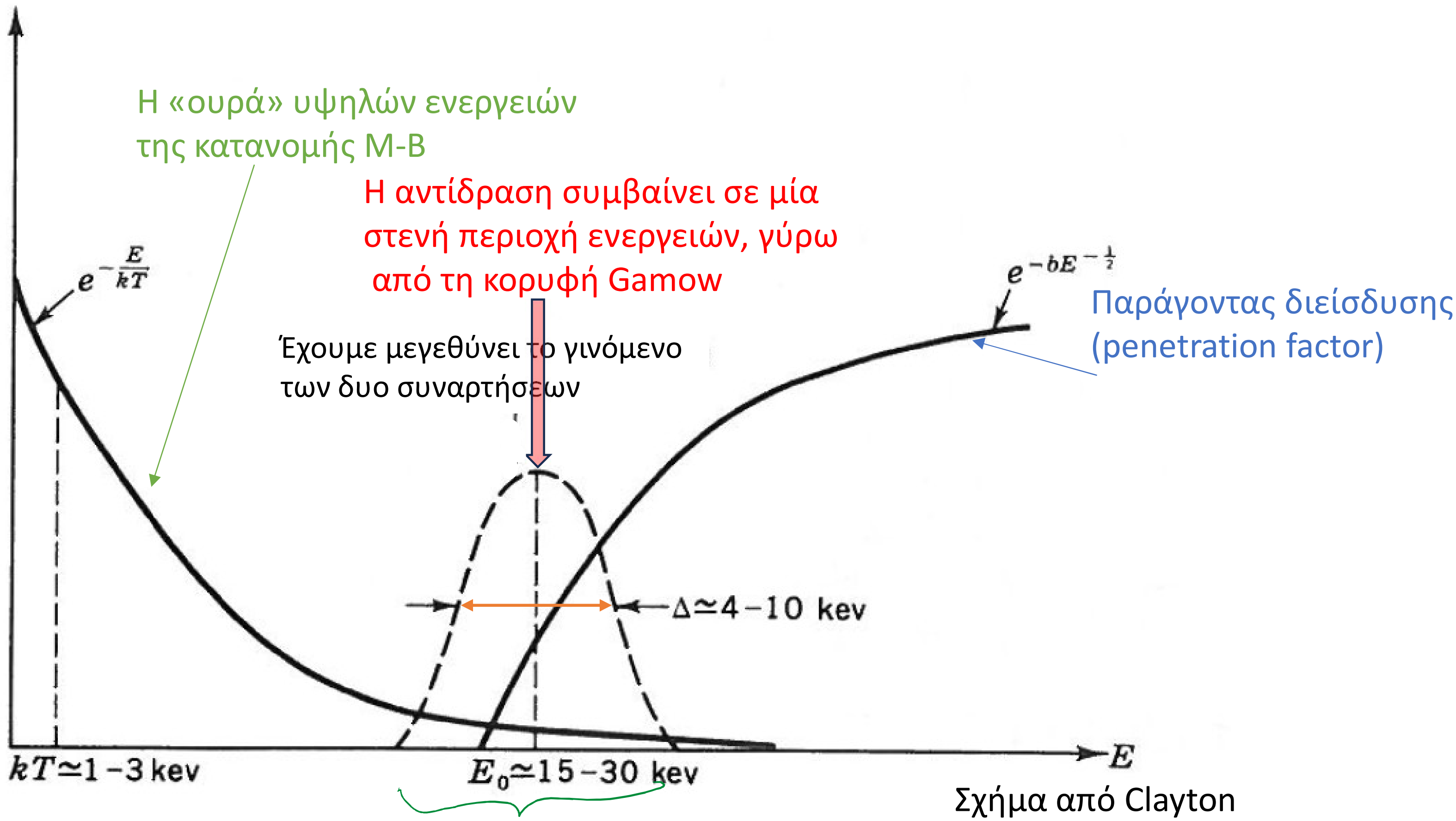
Χρησιμοποιώντας αυτές τις σχέσεις καταλήγουμε στο ότι (κάνετε τις πράξεις!)

$\langle \sigma \upsilon \rangle \propto \frac{1}{T^{2/3}} \exp\left(-\frac{C}{T^{1/3}}\right)$, όπου το C εξαρτάται από το γινόμενο των φορτίων των δύο πυρήνων που αλληλεπιδρούν (άρα από το ύψος του φράγματος Coulomb).

Συνήθως η σχέση αυτή προσεγγίζεται (για θερμοκρασίες κοντά στη T_0) από την

$$\langle \sigma \upsilon \rangle = \langle \sigma \upsilon \rangle_0 \left(\frac{T}{T_0}\right)^{\nu}$$ (άσκηση)

Gamow peak



Σχήμα από Clayton

Συνήθως παίρνουμε $S(E) \sim S(E_o) \sim$ σταθερό (όταν είμαστε μακριά από συντονισμούς)

Σχόλια

- Έχουμε υποθέσει ότι η S(E) μεταβάλλεται αργά με την E, ιδίως γύρω από την κορυφή Gamow. Υπάρχουν όμως χαρακτηριστικές ενέργειες (που αντιστοιχούν σε ενεργειακές διαφορές μεταξύ διακριτών σταθμών του πυρήνα → μοντέλο φλοιών) για τις οποίες το S(E) δεν είναι σταθερό, αλλά έχει ισχυρές κορυφές συντονισμού.

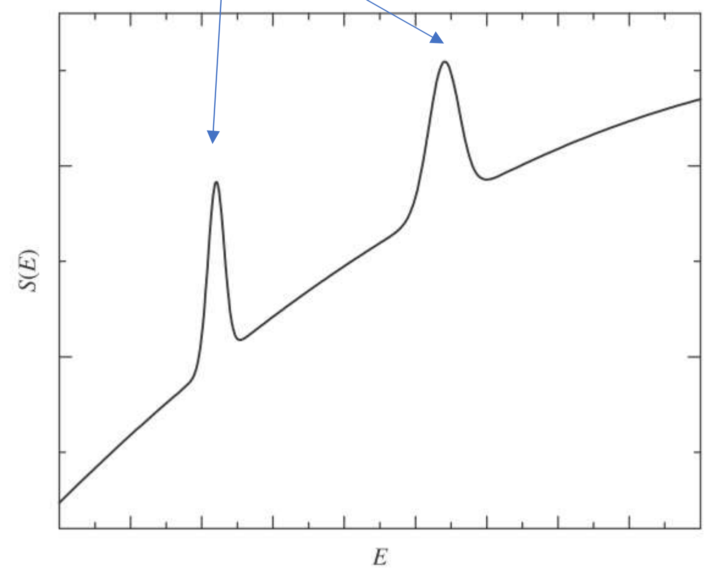
- Θωράκιση ηλεκτρονίων (electron screening): το φράγμα Coulomb υποβιβάζεται λόγω της παρουσίας διάχυτων ηλεκτρονίων γύρω από τον πυρήνα, οπότε

$$U_{\text{eff}} = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{r} + U_s(r)$$

$U_s(r) < 0$

Διόρθωση λόγω θωράκισης από ηλεκτρόνια

Κορυφές λόγω συντονισμού



- Ο παράγοντας S(E) μπορεί, κατ’ αρχήν, να υπολογιστεί θεωρητικά, αλλά στην πράξη ο προσδιορισμός του βασίζεται σε εργαστηριακές μετρήσεις της ενεργού διατομής.

Το πρόβλημα είναι ότι τέτοιες μετρήσεις είναι εφικτές μόνο σε υψηλές ενέργειες, συνήθως > 0.1 MeV, καθώς για μικρότερες ενέργειες η ενεργός διατομή είναι πολύ μικρή και καθίσταται μη μετρήσιμη στο εργαστήριο. Επομένως, απαιτείται παρεκβολή (extrapolation) του S(E) από ψηλότερες σε χαμηλότερες (για > μία τάξη μεγέθους) ενέργειες.

Υπολογισμός της παραγόμενης ενέργειας

Για να υπολογίσουμε την ενέργεια που παράγεται στη μονάδα του χρόνου από στοιχειώδη μάζα dm του αστρικού υλικού, πρέπει να πολ/σουμε το $r_{\alpha\mathrm{X}}$ (αντιδράσεις/στοιχειώδη όγκο/χρόνο) με τον στοιχειώδη όγκο dV και με την ενέργεια ανά αντίδραση $\varepsilon = Q_{aX}$.

Η ισχύς (φωτεινότητα – luminosity) που παράγεται από τη μάζα dm είναι

$$dL = \varepsilon r_{\alpha\mathrm{X}} dV \Rightarrow \frac{dL}{dm} = \frac{\varepsilon r_{\alpha\mathrm{X}}}{dm/dV} = \frac{\varepsilon r_{\alpha\mathrm{X}}}{\rho} \equiv \varepsilon_{aX} \text{ (erg/s/g)}$$

Για να βρούμε το $\frac{dL}{dm}$ πρέπει να αθροίσουμε τα ε_{aX} για όλες τις πιθανές αντιδράσεις.

$$\varepsilon_{nuc} = \sum_{\alpha,X} \varepsilon_{\alpha X}$$

Έτσι παίρνουμε τον συνολικό ρυθμό παραγωγής ενέργειας από πυρηνικές αντιδράσεις ανά μονάδα μάζας.

Χρησιμοποιώντας τη σχέση $r_{aX} = \frac{1}{1+\delta_{\alpha X}} n_a n_X \langle\sigma\upsilon\rangle$ παίρνουμε

$\varepsilon_{aX} = \frac{Q_{aX}}{\rho} \frac{1}{1+\delta_{\alpha X}} n_a n_X \langle\sigma\upsilon\rangle$

Αλλά $n_i = \frac{X_i \rho}{A_i m_H}$ (όπου X_i η μάζα του στοιχείου i προς τη συνολική μάζα, και A_i ο μαζικός αριθμός του στοιχείου i)

[Πράγματι: $n_i = \frac{N_i}{V} = \frac{N_i/m_*}{V/m_*} = \rho \frac{N_i}{m_*} = \rho \frac{N_i A_i m_H}{m_* A_i m_H} = \rho \frac{m_i}{m_* A_i m_H} = \frac{X_i \rho}{A_i m_H}$, όπου m_* η συνολική μάζα (όλων των στοιχείων που συναποτελούν τη στοιχειώδη μάζα που παράγει την ισχύ dL), m_i η συνολική μάζα του στοιχείου i (μέσα στην ίδια στοιχειώδη μάζα που παράγει την ισχύ dL), $\rho = \frac{m_*}{V}$ και $X_i = \frac{m_i}{m_*}$]

Οπότε $\varepsilon_{aX} = \frac{Q_{\alpha X}}{(1+\delta_{\alpha X}) A_\alpha A_X m_H^2} \rho X_\alpha X_X \langle\sigma\upsilon\rangle_{\alpha X}$

Θυμόμαστε ότι $\langle\sigma\upsilon\rangle = \langle\sigma\upsilon\rangle_0 \left(\frac{T}{T_0}\right)^\nu$

Άρα, μπορούμε να γράψουμε ότι

$r_{\alpha X} \simeq r_0 X_\alpha X_X \rho^2 T^\nu$ και $\varepsilon_{\alpha X} = \varepsilon_0' X_\alpha X_X \rho T^\nu$

Nucleosynthesis via r- and s- processes

- Heavy elements (A>56) **cannot be formed efficiently by charged-particle reactions**
- Neutron capture is key:

$$(Z, A) + n \rightarrow (Z, A+1) + \gamma$$

- No Coulomb barrier → occurs even at **low energies**

- Two possibilities after capture:
- If nucleus is **stable,** it remains as is until it captures another neutron
- If nucleus **unstable** then it can:
  - decay (β-decay) : $(Z, A+1) \rightarrow (Z+1, A+1) + e^- + \bar{\nu}_e$
  - capture another neutron

- The overwhelming majority of β- decay lifetimes are of order of hours

s-process (slow neutron capture)

- When neutron capture rate < β-decay rate
- Then the nucleus, after the neutron capture, decays to its stable counterpart. Then it may capture another nucleus etc.
- So the neutron capture chain will march through the stable isotopes of an element until it reaches a radioactive species , at which point β-decay will occur and the capture chain will resume in the element Z+1

r-process (rapid neutron capture)

- When neutron capture rate > β-decay rate
- Nuclei capture many neutrons before decaying
- Path moves far into neutron-rich region
- After neutron flux stops → β-decay chains back to stability

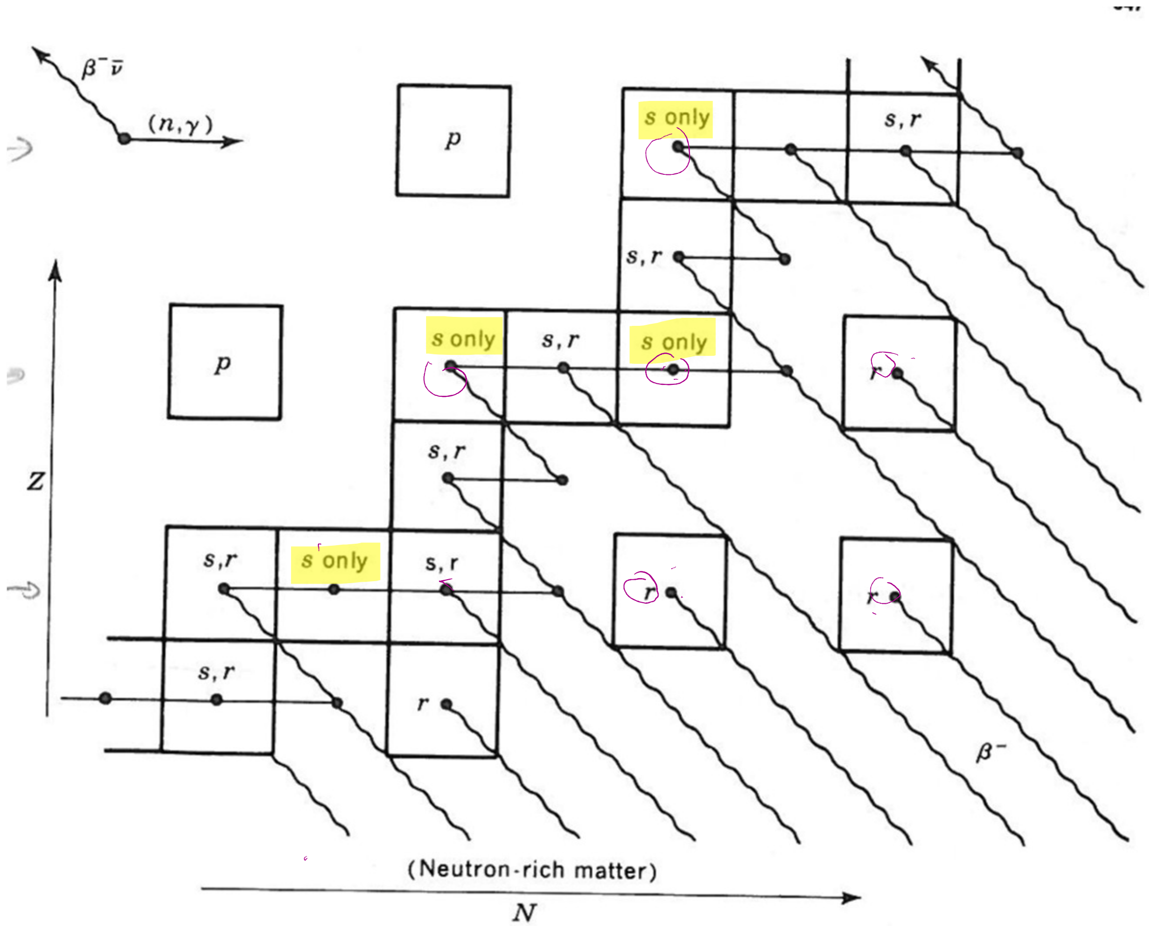


Fig. 7-14 A characterization of a portion of the chart of nuclides showing the assignment of nuclei to the classes *s*, *r*, and *p*. The *s*-process path of (n,γ) reactions followed by quick beta decays enters at the lower left and passes through each nucleus designated by the letter *s*. Neutron-rich matter undergoes a chain of beta decays terminating at the most neutron-rich of the stable isobars, which are designated by the letter *r*. Those nuclei on the *s*-process path which are shielded from *r*-process production are labeled "*s only*." The rare proton-rich nuclei which are bypassed by both neutron processes are designated by the letter *p*.

(s-only
⊙ ~ even Z
⊙ (Z−2, A)
stable)

Clayton 1983

Synthesis of p- nuclei

- Some proton rich nuclei cannot be synthesized by either s- or r- neutron capture processes. (rare isotopes like rare isotopes like: ${}^{92}Mo$, ${}^{144}Sm$,

- Begin with **pre-existing heavy nuclei** (mostly from s- and r-process), exposed to high-energy photons ($T\sim$ 2–3 GK)
- Photodisintegration → neutron removal → move horizontally to lower N

$$(Z, A) \overset{(\gamma,n)}{\rightarrow} (Z, A-1)$$

- As nuclei become proton-rich: Neutron separation energy increases (neutrons now more bound)

  →Proton / alpha emission (more loosely bound – Coulomb repulsion) becomes competitive → new channels open: $(\gamma, p), (\gamma, \alpha)$

Nucleosynthesis in stellar interiors (*) Εξέταση 1^{η} Ιουλιου 10:30

1. H- burning

- Most of the stars in the Universe shine by converting H to ^{4}He. However, only ~15% of the cosmic abundance of **^{4}He** is produced by stars (otherwise, the cosmic background of optical light would exceed by far the observed one): more than **80% of it has been produced in the Big Bang** by the primordial nucleosynthesis.
- In stars of mass < 1.2 Msolar and central temperatures below $20x10^{6}$ K, most of the H-burning energy is released by the **p-p chains**.
- In stars of higher masses and temperatures, H-burning occurs through the **CNO cycle**, where the C, N, and O isotopes (produced from previous stellar generations) act as catalysts.
- The sum of the abundances of **CNO** nuclei remains constant throughout H-burning, but there is **an internal rearrangement**: ^{12}C and ^{16} O turn into ^{14}N and, to a smaller extent, into ^{13}C and ^{17}O; these are the main nuclei produced by the CNO cycle.

Σύντηξη υδρογόνου: Αλυσίδες pp

Περιλαμβάνουν αντιδράσεις που συνολικά περιγράφονται από την:

$$4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2e^{+} + 2\nu_e + 2\gamma$$

Αλυσίδα ppI (69%)

$${}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu_e$$

(${}^{2}_{1}\mathrm{H}$ = D)

Ασθενής αλληλεπίδραση – πολύ αργή → καθορίζει το συνολικό r_{ix}

$${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma$$

$${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H}.$$

Αλυσίδα ppII (31%)

$${}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

$${}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$$

$${}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \rightarrow 2\,{}^{4}_{2}\mathrm{He}.$$

Αλυσίδα ppIII (0.3%)

$${}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

$${}^{8}_{5}\mathrm{B} \rightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu_e$$

$${}^{8}_{4}\mathrm{Be} \rightarrow 2\,{}^{4}_{2}\mathrm{He}.$$

ppI chain in more detail

reaction:	rate:	Q (MeV)	$\langle E_\nu \rangle$ (MeV)
$\mathrm{H} + \mathrm{H} \rightarrow \mathrm{D} + \mathrm{e}^+ + \nu$	$r_{\mathrm{pp}} = \frac{1}{2}\mathrm{H}^2 \langle \sigma v \rangle_{\mathrm{pp}}$	1.442	0.265
$\mathrm{D} + \mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$	$r_{\mathrm{pD}} = \mathrm{H\,D} \langle \sigma v \rangle_{\mathrm{pD}}$	5.493	
${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,\mathrm{H}$	$r_{33} = \frac{1}{2}({}^3\mathrm{He})^2 \langle \sigma v \rangle_{33}$	12.860	

$$\frac{d\mathrm{H}}{dt} = -\mathrm{H}^2 \langle \sigma v \rangle_{\mathrm{pp}} - \mathrm{H\,D} \langle \sigma v \rangle_{\mathrm{pD}} + 1({}^3\mathrm{He})^2 \langle \sigma v \rangle_{33}$$

$\left(2\mathrm{H} \times \frac{1}{2} (\text{and } r_{33}) \right)$

$$\frac{d\mathrm{D}}{dt} = \frac{\mathrm{H}^2}{2} \langle \sigma v \rangle_{\mathrm{pp}} - \mathrm{H\,D} \langle \sigma v \rangle_{\mathrm{pD}}$$

$$\frac{d\,{}^3\mathrm{He}}{dt} = \mathrm{H\,D} \langle \sigma v \rangle_{\mathrm{pD}} - ({}^3\mathrm{He})^2 \langle \sigma v \rangle_{33}$$

$$\frac{d\,{}^4\mathrm{He}}{dt} = \frac{({}^3\mathrm{He})^2}{2} \langle \sigma v \rangle_{33}$$

Deuterium abundance

$$\frac{dD}{dt} = \frac{H^2}{2}\langle\sigma v\rangle_{pp} - H\,D\,\langle\sigma v\rangle_{pD}$$

➡ Self-regulating

- if D-abundance is small ⇒ first term > second term ⇒ $dD/dt > 0$

  ⇒ D-abundance grows

- if D-abundance is large ⇒ first term < second term ⇒ $dD/dt < 0$

  ⇒ D-abundance decreases

➡ the D-abundance tends to an equilibrium abundance such that $\frac{dD}{dt} = 0$

$$\Rightarrow \frac{H^2}{2}\langle\sigma v\rangle_{pp} = HD\langle\sigma v\rangle_{pD} \Rightarrow \left(\frac{D}{H}\right)_{eq} = \frac{\langle\sigma v\rangle_{pp}}{2\langle\sigma v\rangle_{pD}} \approx 3\times 10^{-18}\ (T \sim 10^7 K)$$

Lifetime of species X against reacting with particle a

$$\tau_a(X) \equiv \left|\frac{n(X)}{[\mathrm{d}n(X)/dt]_a}\right|$$

Example: the lifetimes of H and D against proton capture

$$\mathrm{H} + \mathrm{H} \rightarrow \mathrm{D} + \mathrm{e}^+ + \nu \qquad r_{\mathrm{pp}} = \tfrac{1}{2}\mathrm{H}^2 \langle\sigma v\rangle_{\mathrm{pp}}$$

H → n_p D → n_D

$$\tau_{\mathrm{p}}(\mathrm{H}) = \frac{n_{\mathrm{p}}}{(dn_{\mathrm{p}}/dt)_{\mathrm{p}}} = \frac{n_{\mathrm{p}}}{n_{\mathrm{p}}^2\langle\sigma v\rangle_{\mathrm{pp}}} = \frac{1}{n_{\mathrm{p}}\langle\sigma v\rangle_{\mathrm{pp}}} = \frac{m_p}{X\rho_{core}\langle\sigma v\rangle_{pp}} \approx 10^{10}\mathrm{yr}$$

X → solar abund.

$$\mathrm{D} + \mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma \qquad r_{\mathrm{pD}} = \mathrm{H}\,\mathrm{D}\,\langle\sigma v\rangle_{\mathrm{pD}}$$

$$\tau_{\mathrm{p}}(\mathrm{D}) = \frac{n_{\mathrm{D}}}{(dn_{\mathrm{D}}/dt)_{\mathrm{p}}} = \frac{n_{\mathrm{D}}}{n_{\mathrm{p}}n_{\mathrm{D}}\langle\sigma v\rangle_{\mathrm{pD}}} = \frac{1}{n_{\mathrm{p}}\langle\sigma v\rangle_{\mathrm{pD}}} \approx 1.6\mathrm{sec}$$

$$\Rightarrow \quad \frac{\tau_{\mathrm{p}}(\mathrm{D})}{\tau_{\mathrm{p}}(\mathrm{H})} = \frac{\langle\sigma v\rangle_{\mathrm{pp}}}{\langle\sigma v\rangle_{\mathrm{pD}}} \Rightarrow \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{e}} = \frac{\tau_{\mathrm{p}}(\mathrm{D})}{2\tau_{\mathrm{p}}(\mathrm{H})}$$

Discussion of $\frac{\tau_\mathrm{p}(\mathrm{D})}{\tau_\mathrm{p}(\mathrm{H})} = \frac{\langle\sigma v\rangle_\mathrm{pp}}{\langle\sigma v\rangle_\mathrm{pD}} \Rightarrow \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_\mathrm{eq} = \frac{\tau_\mathrm{p}(\mathrm{D})}{2\tau_\mathrm{p}(\mathrm{H})}$

- Because hydrogen is consumed **extremely slowly** ($\tau_p(H) \sim 10^{10}$ yr), its abundance*, H*, barely changes over the short times during which deuterium evolves. So H can be treated as effectively constant.

- With H fixed:
  - ✓ deuterium is produced at an approximately constant rate (r_{pp}) $r_\mathrm{pp} = \frac{1}{2}\mathrm{H}^2 \langle\sigma v\rangle_\mathrm{pp}$
  - ✓ but destroyed at a rate proportional to how much deuterium already exists. $r_\mathrm{pD} = \mathrm{H}\,\mathrm{D}\,\langle\sigma v\rangle_\mathrm{pD}$
- This makes the deuterium equation a standard relaxation process:

$$\frac{dD}{dt} = constant - (constant) \cdot D$$

whose solution approaches equilibrium exponentially:

$$D(t) = D_\mathrm{eq} + \left(D_0 - D_\mathrm{eq}\right)e^{-t/\tau_p(D)}.$$

So regardless of the initial amount of deuterium, $D(t) \to D_\mathrm{eq}$ in a few $\tau_p(D)$ (a few seconds)

The ^{3}He abundance

Assuming $\mathrm{D/H} = (\mathrm{D/H})_{\mathrm{eq}}$, we are left with the following eqs involving H and ^{3}He

$$\frac{d\mathrm{H}}{dt} = -\mathrm{H}^2\langle\sigma v\rangle_{\mathrm{pp}} - \mathrm{H\,D}\langle\sigma v\rangle_{\mathrm{pD}} + (^3\mathrm{He})^2\langle\sigma v\rangle_{33}$$

$$\frac{d\mathrm{D}}{dt} = \frac{\mathrm{H}^2}{2}\langle\sigma v\rangle_{\mathrm{pp}} - \mathrm{H\,D}\langle\sigma v\rangle_{\mathrm{pD}} = 0$$

$$\frac{d\,^3\mathrm{He}}{dt} = \mathrm{H\,D}\langle\sigma v\rangle_{\mathrm{pD}} - (^3\mathrm{He})^2\langle\sigma v\rangle_{33}$$

$$\frac{d\mathrm{H}}{dt} = -\frac{3}{2}\mathrm{H}^2\langle\sigma v\rangle_{\mathrm{pp}} + (^3\mathrm{He})^2\langle\sigma v\rangle_{33}$$

$$\frac{d\,^3\mathrm{He}}{dt} = \frac{\mathrm{H}^2}{2}\langle\sigma v\rangle_{\mathrm{pp}} - (^3\mathrm{He})^2\langle\sigma v\rangle_{33}$$

Self-regulating

$$\frac{d\,^3\mathrm{He}}{dt} = 0 \Rightarrow \frac{\mathrm{H}^2}{2}\langle\sigma v\rangle_{pp} = (^3He)^2\langle\sigma v\rangle_{33} \Rightarrow \left(\frac{^3He}{\mathrm{H}}\right)_{\mathrm{eq}} = \left(\frac{\langle\sigma v\rangle_{\mathrm{pp}}}{2\langle\sigma v\rangle_{33}}\right)^{\frac{1}{2}} \approx 10^{-5}\ (T \approx 1.5\times10^{7}\mathrm{K})$$

(strong T dependence)

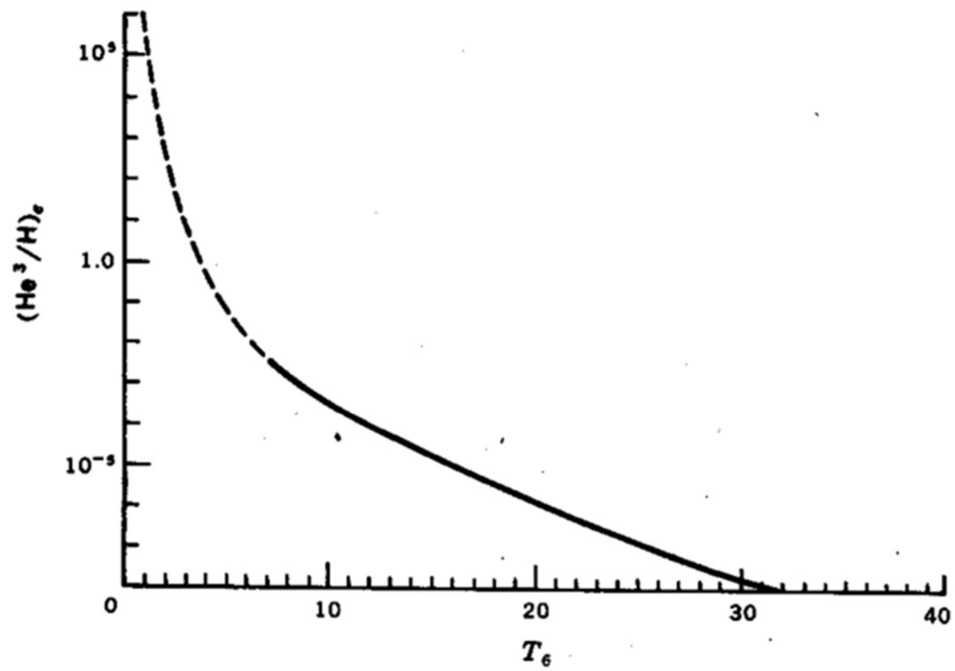


Figure 4.4: The equilibrium ratio of ^{3}He to H during hydrogen burning. The curce is dashed for $T_6 < 8$ becuase the time required for ^{3}He to reach equilibrium is longer than the H-burning lifetime at such low temperature. Figure from Clayton.

$$\frac{dH}{dt} = -\frac{3}{2}H^2\langle\sigma v\rangle_{pp} + (^3\mathrm{He})^2\langle\sigma v\rangle_{33}$$

- proton burning decreases H → increases $^3\mathrm{He}$

→ $^3\mathrm{He}$ fused via $\frac{d^3\mathrm{He}}{dt} = \frac{\mathrm{H}^2}{2}\langle\sigma v\rangle_{\mathrm{pp}} - (^3\mathrm{He})^2\langle\sigma v\rangle_{33}$

→ some protons are effectively returned to the cycle

→slowing hydrogen depletion → stabilizing feedback.

So the whole pp-chain network tends toward a quasi-steady state

Lifetime of ^{3}He against itself

$$ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} \rightarrow {}^4_2\mathrm{He} + 2\,{}^1_1\mathrm{H}. $$

- lifetime of ^{3}He against itself

  $\tau_3(^3\mathrm{He}) \approx 10^5$ yr in the center of the Sun ($T_6 = 15$)

- increasing rapidly for lower T :

  for $T_6 \lesssim 8$, $\tau_3(^3\mathrm{He}) \gtrsim 10^9\mathrm{yr}$

⇒ equilibrium is reached in the center of the Sun

⇒ equilibrium is not reached outside central parts of the Sun

If ^{3}He reaches equilibrium (e.g. in the center of the sun), then:

$$ \frac{d\mathrm{H}}{dt} = -\frac{3}{2}\mathrm{H}^2\langle\sigma v\rangle_{\mathrm{pp}} + (^3\mathrm{He})^2\langle\sigma v\rangle_{33} $$

$$ \frac{d\,^3\mathrm{He}}{dt} = 0 = \frac{\mathrm{H}^2}{2}\langle\sigma v\rangle_{\mathrm{pp}} - (^3\mathrm{He})^2\langle\sigma v\rangle_{33} $$

$$ \frac{d\mathrm{H}}{dt} = -\mathrm{H}^2\langle\sigma v\rangle_{\mathrm{pp}} = -2r_{\mathrm{pp}} $$

$$ \frac{d\,^4\mathrm{He}}{dt} = \frac{(^3\mathrm{He})^2}{2}\langle\sigma v\rangle_{33} \longrightarrow \frac{d^4\mathrm{He}}{dt} = \frac{\mathrm{H}^2}{4}\langle\sigma v\rangle_{\mathrm{pp}} = \tfrac{1}{2}r_{\mathrm{pp}} $$

The rate of H-burning is then set completely by the pp reaction, the slowest in the chain.

.

ppI summary

- Effective energy release of the ppI chain: $Q_{\rm eff} = Q - 2\langle E_\nu \rangle = 26.21\,{\rm MeV}$ per $^4{\rm He}$

  → effective energy generation rate in erg g^{-1} s^{-1}:

$$\epsilon_{\rm ppI} = \frac{Q_{\rm eff}}{\rho}\frac{d^4{\rm He}}{dt} = Q_{\rm eff}\frac{r_{\rm pp}}{2\rho}$$ (valid for ^{3}He equilibrium)

- D is destroyed (in fact already on the pre-main sequence)
- ^{3}He is destroyed in the center, enriched above the core of the Sun
- the pp-reaction is the slowest, and (when ^{3}He is in equilibrium) sets the pace of the overall rate of the ppI chain

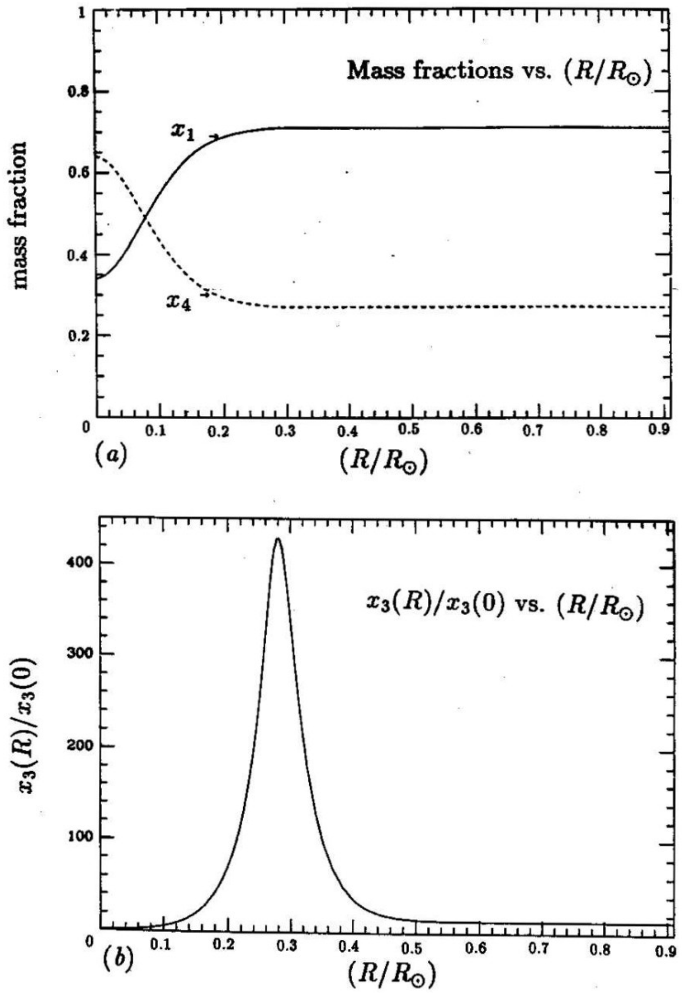


Figure 4.6: Composition profiles in a typical solar model. (a) Mass fractions of ^{1}H (X_1) and of ^{4}He (X_4) as a function of radius. (b) Mass fraction of ^{3}He (X_3) relative to its central value $X_3(0)$, as a function of radius. Figure from Bahcall (1989).

ppII chain in more detail

- If ^4He is abundant, and the temperature is not too low, the reaction
  $^3He + ^4He \rightarrow ^7Be + \gamma$
- Larger reduced mass of reacting nuclei → stronger T sensitivity
  → reaction starts to dominate at high temperatures
- followed by e^- capture by ^7Be: $^7Be + e^- \rightarrow ^7Li + \nu$ (weak interaction)
  $\tau(^7\mathrm{Be}) \simeq 0.3\mathrm{yr}$
  If Be is neutral (in the lab!) $\tau(^7\mathrm{Be}) \simeq 76.9\mathrm{d}$....$e^-$ is captured usually from K shell
  In the stellar core Be is fully ionized, so e- capture depends on n_e and T $\tau_e(^7\mathrm{Be}) \simeq 7.06 \times 10^8 \frac{T_6^{1/2}}{\rho(1+X)}$
- Q-value of 0.862 MeV, emitted as one 0.862 MeV neutrino (89.6% of the time), or one 0.384 MeV neutrino and one 0.478 MeV photon (10.4%)
- and then
  $^7Li + ^1H \rightarrow ^4He + ^4He$

ppIII chain in more detail

$${}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

$${}^{8}_{5}\mathrm{B} \rightarrow {}^{8}_{4}\mathrm{Be}^{*} + e^{+} + \nu_e$$

$${}^{8}_{4}\mathrm{Be} \rightarrow 2\,{}^{4}_{2}\mathrm{He}.$$

Lifetimes of radioactive species

$$\tau({}^{8}\mathrm{B}) \simeq 1 \text{ s}$$
$$\tau({}^{8}\mathrm{Be}) \simeq 10^{-22} \text{ s}$$

The neutrinos produced are very energetic. $E_\nu^{\max} \approx 14\text{–}15\ \mathrm{MeV}$

In comparison to:

- pp neutrinos: up to **0.42 MeV**
- neutrinos from ^{7}Be e^{-} capture : **0.86 MeV**

Solar Neutrino detectors (Homestake, Super-K, SNO) were mostly sensitive to ppIII neutrinos

Comparing ppI, ppII and ppIII

- ppII, III start with $^3He(\alpha, \gamma)^7Be$, but the α-particle is used only as a catalyst: it is liberated in the end, by $^7Li(p, \alpha)^4He$ or by $^8B(\alpha)^4He$
- ppII, III need the (slow) pp-reaction only once to produce one ^4He nucleus, whereas ppI needs two pp-reactions per ^4He nucleus

⇒ at high temperature: ppII, III may dominate

reaction	Q (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (keV barn)	dS/dE (barn)	τ (yr)
$^1H(p, e^+\nu)^2H$	1.442	0.265	3.94×10^{-22}	4.61×10^{-24}	10^{10}
$^2H(p, \gamma)^3He$	5.493		2.5×10^{-4}	7.9×10^{-6}	10^{-8}
$^3He(^3He, 2p)^4He$	12.860		5.18×10^3	-1.1×10^1	10^5
$^3He(\alpha, \gamma)^7Be$	1.587		5.4×10^{-1}	-3.1×10^{-4}	10^6
$^7Be(e^-, \nu)^7Li$	0.862	0.814			10^{-1}
$^7Li(p, \alpha)^4He$	17.347		5.2×10^1	0	10^{-5}
$^7Be(p, \gamma)^8B$	0.137		2.4×10^{-2}	-3×10^{-5}	10^2
$^8B(e^+\nu)^8Be^*(\alpha)^4He$	18.071	6.710			10^{-8}

D, ^7Be, ^8B, ^7Li very short lifetimes

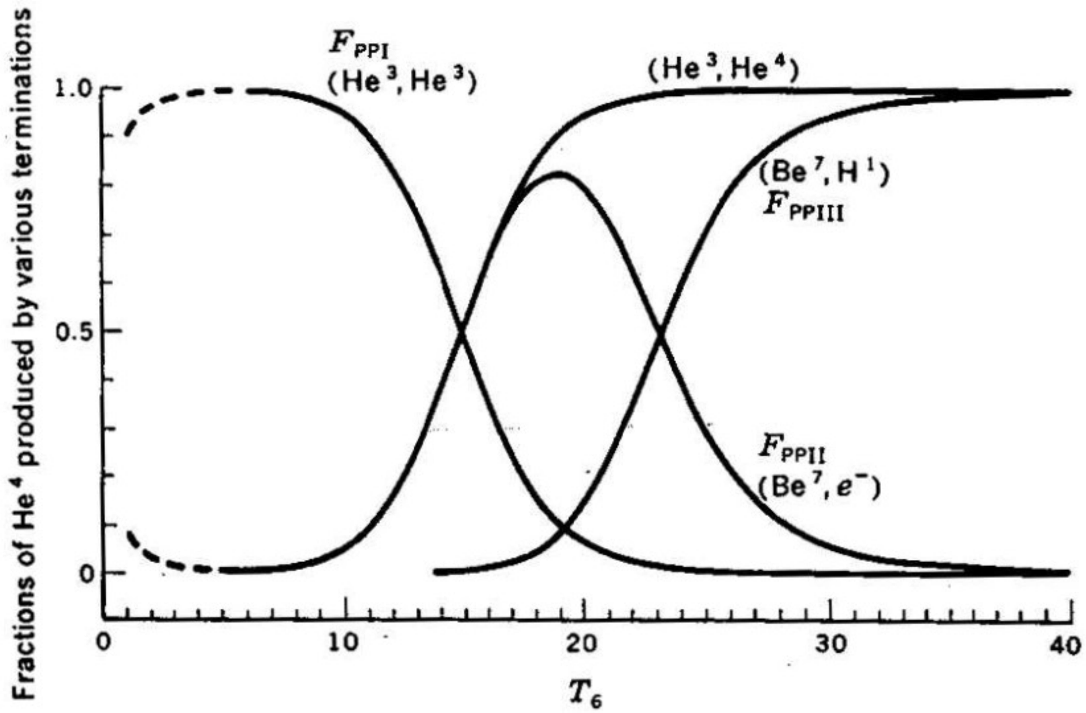
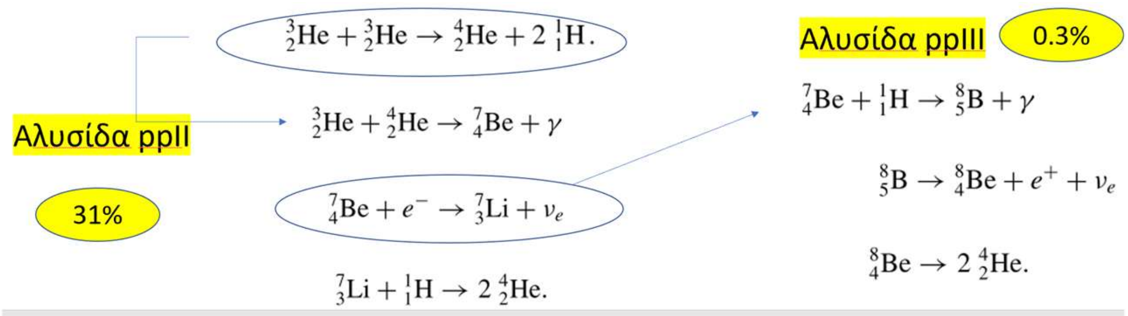


Figure 4.7: Fraction of ^{4}He produced by the ppI, ppII and ppIII chains, as a function of temperatute. The chains are assumed to be in equilibrium, and it was assumed that $Y = X$. Figure from Clayton.

Branching ratios



➢ $$\frac{f_{\text{ppI}}}{f_{\text{ppII}}+f_{\text{ppIII}}} = \frac{r_{33}}{r_{34}} = \frac{\langle\sigma v\rangle_{33}}{2\langle\sigma v\rangle_{34}}\frac{^3\text{He}}{^4\text{He}}$$

⇒ ppI dominates over ppII, III for small ^{4}He and/or large ^{3}He abundance

⇒ ppI also dominates at low T , because $\langle\sigma v\rangle_{34}$ increases more steeply with T $\langle\sigma v\rangle_{33}$

➢ $$\frac{f_{\text{ppII}}}{f_{\text{ppIII}}} = \frac{r_{\text{e7}}}{r_{\text{p7}}} = \frac{\lambda_{\text{e7}}}{\langle\sigma v\rangle_{\text{p7}}}\frac{n_{\text{e}}}{n_{\text{p}}} = \frac{\lambda_{\text{e7}}}{\langle\sigma v\rangle_{\text{p7}}}\frac{1+X}{2X}$$

λ_{e7} = probability per unit time that one ^{7}Be nucleus captures an electron

$$n_p = \frac{X\rho}{m_H},\ n_e = \frac{\rho}{\mu_e m_H},\ \frac{1}{\mu_e} = \frac{1}{2}(1+X)$$ (assuming only H and He)

(caution: μ_e mean molecular weight per electron not per particle)

$$\mu = \frac{\sum_j N_j A_j}{\sum_j N_j (Z_j + 1)}$$, while $$\mu_e = \frac{\sum_j N_j A_j}{\sum_j N_j Z_j}$$

Differential equations for ppII-ppIII

- D, ^{8}B, ^{7}Li and ^{7}Be have very short lifetimes ⇒ they can be assumed to be always in equilibrium ⇒ only H, ^{3}He and ^{4}He are important:

$$\frac{d\mathrm{H}}{dt} = -\frac{3}{2}\mathrm{H}^2 \langle\sigma v\rangle_{\mathrm{pp}} + (^3\mathrm{He})^2 \langle\sigma v\rangle_{33} - {}^3\mathrm{He}\,{}^4\mathrm{He}\,\langle\sigma v\rangle_{34}$$

$$\frac{d\,^3\mathrm{He}}{dt} = \frac{\mathrm{H}^2}{2} \langle\sigma v\rangle_{\mathrm{pp}} - (^3\mathrm{He})^2 \langle\sigma v\rangle_{33} - {}^3\mathrm{He}\,{}^4\mathrm{He}\,\langle\sigma v\rangle_{34}$$

$$\frac{d\,^4\mathrm{He}}{dt} = \frac{(^3\mathrm{He})^2}{2} \langle\sigma v\rangle_{33} + {}^3\mathrm{He}\,{}^4\mathrm{He}\,\langle\sigma v\rangle_{34}$$

PROVE AS EXERCISE

^{3}He ⇒ self-regulating as for ppI, but equilibrium ^{3}He abundance now depends on ^{4}He/H as well as T

PROVE AS EXERCISE

- If ^{3}He in equilibrium → $\frac{d\,^3\mathrm{He}}{dt} = 0$ → $\frac{d\,^4\mathrm{He}}{dt} = \frac{\mathrm{H}^2}{4}\langle\sigma v\rangle_{\mathrm{pp}} + \frac{^3\mathrm{He}\,^4\mathrm{He}}{2}\langle\sigma v\rangle_{34} \equiv \frac{1}{2} r_{\mathrm{pp}}\Phi(\alpha)$, where $\Phi(\alpha)$ depends on temperature and on the ^{4}He/H ratio

Net energy production rate– Net nucleosynthesis in pp chain

$$\frac{d\,^4\mathrm{He}}{dt} = \frac{1}{2} r_{\mathrm{pp}} \Phi(\alpha)$$

The function Φ(α) depends on temperature and on the ^{4}He/H ratio

- for small α (low T and/or small ^{4}He/H) ⇒ ppI dominates, Φ ≈ 1 $\frac{d\,^4\mathrm{He}}{dt} = \frac{1}{2} r_{\mathrm{pp}}$
- for large α (high T and/or large ^{4}He/H) ⇒ ppII or ppIII dominate, Φ ≈ 2 $\frac{d\,^4\mathrm{He}}{dt} = r_{\mathrm{pp}}$

  (⇒ one pp reaction required per ^{4}He)

➢ The energy generated by ppI, ppII and ppIII is always the same

(Q = 26.73 MeV per net 4 p → ^{4}He reaction). However, the ν-losses are different .

➢ when ^{3}He in equilibrium

$$\epsilon_{\mathrm{pp}} = \frac{r_{\mathrm{pp}}}{2\rho} \cdot (4m_{\mathrm{p}} - m_{\alpha})c^2 \cdot \Phi(\alpha)\left(0.981 f_{\mathrm{ppI}} + 0.961 f_{\mathrm{ppII}} + 0.739 f_{\mathrm{ppIII}}\right)$$

➢ Total pp-cycle nucleosynthesis ^{4} He, ^{3}He

Σύντηξη Η: Κύκλος CNO

Οι πυρήνες CNO δρουν σαν «καταλύτες».

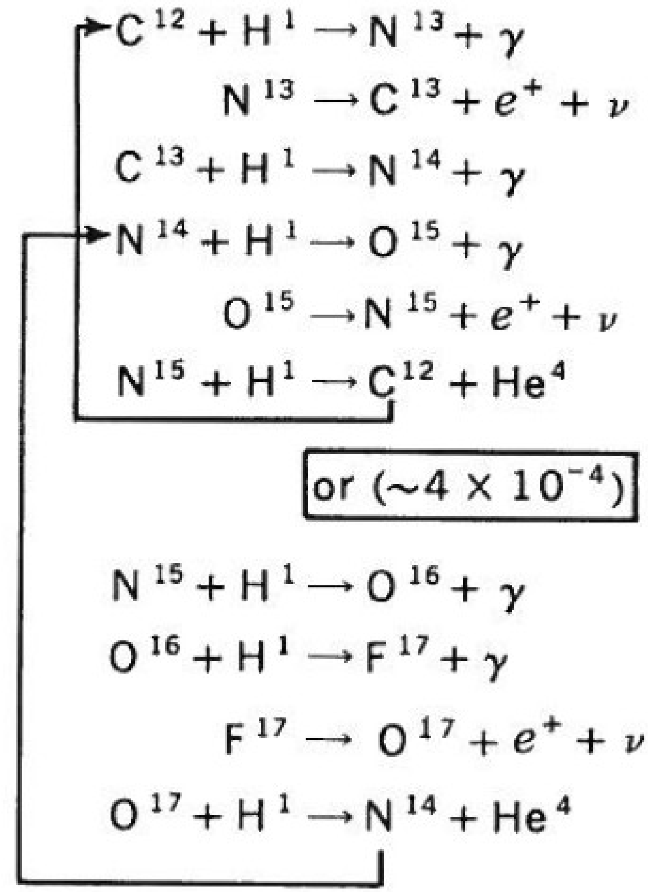
Ο κύκλος μπορεί να ξεκινήσει από οπουδήποτε ανάλογα με τον διαθέσιμο πυρήνα .

$$\varepsilon_{\mathrm{CNO}} = 8.67 \times 10^{20} \rho X X_{\mathrm{CNO}} C_{\mathrm{CNO}} T_6^{-2/3} e^{-152.28 T_6^{-1/3}} \mathrm{Wkg}^{-1}$$

Γύρω από τη θερμοκρασία $T = 1.5 \times 10^7 \mathrm{K}$

$$\varepsilon_{\mathrm{CNO}} \simeq \varepsilon'_{0,\mathrm{CNO}} \rho X X_{\mathrm{CNO}} T_6^{19.9}$$

με $\varepsilon'_{0,\mathrm{CNO}} = 8.24 \times 10^{-31} \mathrm{Wm}^3 \mathrm{kg}^{-2}$



Το συνολικό ε από την αλυσίδα pp υπολογίζεται ότι είναι:

$$\epsilon_{pp} = 0.241\rho X^2 f_{pp}\psi_{pp}C_{pp}T_6^{-2/3}e^{-33.80T_6^{-1/3}}\mathrm{Wkg^{-1}}$$

(όπου f_{pp} ο παράγοντας θωράκισης, ψ_{pp}~1 είναι ένας παράγοντας διόρθωσης, ο οποίος έχει να κάνει με την ταυτόχρονη ύπαρξη των PP I, PP II και PP III και C_{pp} ~1 περιλαμβάνει διορθωτικούς όρους ανώτερης τάξης. Επίσης, $\mathrm{T_6} \equiv \frac{\mathrm{T}}{10^6\mathrm{K}}$.

Κοντά στη θερμοκρασία $T = 1.5 \times 10^7\mathrm{K}$

$\epsilon_{pp} \simeq \varepsilon'_{0,pp}\rho X^2 f_{pp}\psi_{pp}C_{pp}T_6^4$ με $\varepsilon'_{0,pp} = 1.08 \times 10^{-12}\mathrm{Wm^3kg^{-2}}$

Προσέξτε τη σχετικά ασθενή (σε σύγκριση με άλλες αντιδράσεις που θα δούμε αργότερα) εξάρτηση από τη T.

the CNO cycle in more detail

The two β decays are very fast

$$^{13}\mathrm{N}(e^+\nu): \ \tau \simeq 10 \text{ min}, \ E_\nu \simeq 0.71 \text{ MeV}$$
$$^{15}\mathrm{O}(e^+\nu): \ \tau \simeq 2 \text{ min}, \ E_\nu \simeq 1.00 \text{ MeV}$$

→ Assume
$\tau_\beta(^{13}\mathrm{N}) = \tau_\beta(^{15}\mathrm{O}) \cong 0$

→
$$^{12}\mathrm{C}(p, \gamma e^+\nu)^{13}\mathrm{C}$$
$$^{14}\mathrm{N}(p, \gamma e^+\nu)^{15}\mathrm{N}$$

$$\mathrm{C}^{12} + \mathrm{H}^1 \longrightarrow \mathrm{N}^{13} + \gamma$$
$$\mathrm{N}^{13} \longrightarrow \mathrm{C}^{13} + e^+ + \nu$$
$$\mathrm{C}^{13} + \mathrm{H}^1 \longrightarrow \mathrm{N}^{14} + \gamma$$
$$\mathrm{N}^{14} + \mathrm{H}^1 \longrightarrow \mathrm{O}^{15} + \gamma$$
$$\mathrm{O}^{15} \longrightarrow \mathrm{N}^{15} + e^+ + \nu$$
$$\mathrm{N}^{15} + \mathrm{H}^1 \longrightarrow \mathrm{C}^{12} + \mathrm{He}^4$$

proton capture

Differential equations for the CN cycle:

$$\frac{d\,^{12}\mathrm{C}}{dt} = \mathrm{H}\left(^{15}\mathrm{N}\,\langle\sigma v\rangle_{15} - {}^{12}\mathrm{C}\,\langle\sigma v\rangle_{12}\right)$$
$$\frac{d\,^{13}\mathrm{C}}{dt} = \mathrm{H}\left(^{12}\mathrm{C}\,\langle\sigma v\rangle_{12} - {}^{13}\mathrm{C}\,\langle\sigma v\rangle_{13}\right)$$
$$\frac{d\,^{14}\mathrm{N}}{dt} = \mathrm{H}\left(^{13}\mathrm{C}\,\langle\sigma v\rangle_{13} - {}^{14}\mathrm{N}\,\langle\sigma v\rangle_{14}\right)$$
$$\frac{d\,^{15}\mathrm{N}}{dt} = \mathrm{H}\left(^{14}\mathrm{N}\,\langle\sigma v\rangle_{14} - {}^{15}\mathrm{N}\,\langle\sigma v\rangle_{15}\right)$$
$$\frac{d\mathrm{H}}{dt} = -\mathrm{H}\left(^{12}\mathrm{C}\,\langle\sigma v\rangle_{12} + {}^{13}\mathrm{C}\,\langle\sigma v\rangle_{13} + {}^{14}\mathrm{N}\,\langle\sigma v\rangle_{14} + {}^{15}\mathrm{N}\,\langle\sigma v\rangle_{15}\right)$$
$$\frac{d\,^{4}\mathrm{He}}{dt} = \mathrm{H}\,^{15}\mathrm{N}\,\langle\sigma v\rangle_{15}$$

- ✓ total mass (nucleon number) is conserved
- ✓ total number of CNO-nuclei is conserved
- ✓ $dH/dt < 0$ and $d\,^{4}He/dt > 0$ always
- ✓ equations for CN nuclei are self-regulating ⇒ ^{12}C, ^{13}C, ^{14}N and ^{15}N will seek equilibrium abundance

Lifetimes of CN-nuclei against p-capture

$C^{12} + H^{1} \longrightarrow N^{13} + \gamma$

$$\tau_p(^{12}C) = \frac{n(^{12}C)}{(dn(^{12}C)/dt)_p} = \frac{n(^{12}C)}{n_p n(^{12}C)\langle\sigma v\rangle_{12}} = \frac{1}{n_p\langle\sigma v\rangle_{12}}$$

$\langle\sigma v\rangle_{12}$ = Maxwellian-averaged cross section of $^{12}C(p,\gamma)^{13}N$

Similarly:

$C^{13} + H^{1} \longrightarrow N^{14} + \gamma$

$$\tau_p(^{13}C) = \frac{1}{n_p\langle\sigma v\rangle_{13}}$$

$N^{14} + H^{1} \longrightarrow O^{15} + \gamma$

$$\tau_p(^{14}N) = \frac{1}{n_p\langle\sigma v\rangle_{14}}$$

$N^{15} + H^{1} \longrightarrow C^{12} + He^{4}$

$$\tau_p(^{15}N) = \frac{1}{n_p\langle\sigma v\rangle_{15}}$$

At $T \cong 2 \times 10^7 K$

$$\tau_p(^{15}N) \ll \tau_p(^{13}C) < \tau_p(^{12}C) \ll \tau_p(^{14}N) \ll \tau_{star}!$$

$$35 \text{ yr} \ll 1600 \text{ yr} < 6600 \text{ yr} \ll 9 \times 10^5 \text{ yr}$$

Equilibrium will be achieved as all lifetimes are << stellar lifetime, i.e.

$$\frac{d}{dt}{}^{12}C = \frac{d}{dt}{}^{13}C = \frac{d}{dt}{}^{14}N = \frac{d}{dt}{}^{15}N = 0$$

Equilibrium abundance ratios

e.g.

$$\frac{d^{13}\mathrm{C}}{dt} = \mathrm{H}(^{12}\mathrm{C}\langle\sigma v\rangle_{12} - {}^{13}\mathrm{C}\langle\sigma v\rangle_{13}) = 0 \Rightarrow \qquad \left(\frac{^{12}\mathrm{C}}{^{13}\mathrm{C}}\right)_{eq} = \frac{\langle\sigma v\rangle_{13}}{\langle\sigma v\rangle_{12}} = \frac{\tau_\mathrm{p}(^{12}\mathrm{C})}{\tau_\mathrm{p}(^{13}\mathrm{C})}$$

CN nucleosynthesis:

Lifetimes $^{15}\mathrm{N} : {}^{13}\mathrm{C} : {}^{12}\mathrm{C} : {}^{14}\mathrm{N} \simeq 1 : 45 : 190 : 26\,000$

→ CN equilibrium abundance distribution $\quad {}^{15}\mathrm{N} : {}^{13}\mathrm{C} : {}^{12}\mathrm{C} : {}^{14}\mathrm{N} \simeq \frac{1}{26\,000} : \frac{1}{600} : \frac{1}{140} : 1$

independent of the initial composition

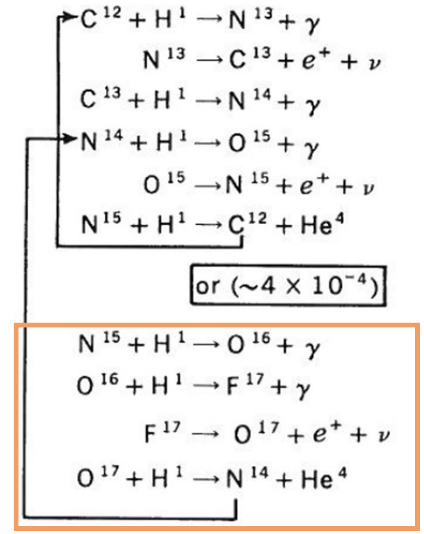
⇒ ^{14}N is the major nucleosynthesis product of the CN-cycle (except for ^{4}He).

The rest of the CNO cycles

CNOII

$$^{15}\mathrm{N}\,(\mathrm{p},\gamma)\,^{16}\mathrm{O}\,(\mathrm{p},\gamma)\,^{17}\mathrm{F}\,(\mathrm{e}^{+}\nu)\,^{17}\mathrm{O}\,(\mathrm{p},\alpha)\,^{14}\mathrm{N}\,(\mathrm{p},\gamma)\,^{15}\mathrm{O}\,(\mathrm{e}^{+}\nu)\,^{15}\mathrm{N} \quad (\mathrm{CNO\text{-}II})$$

- second cycle not relevant for energy generation
- but: very relevant for nucleosynthesis

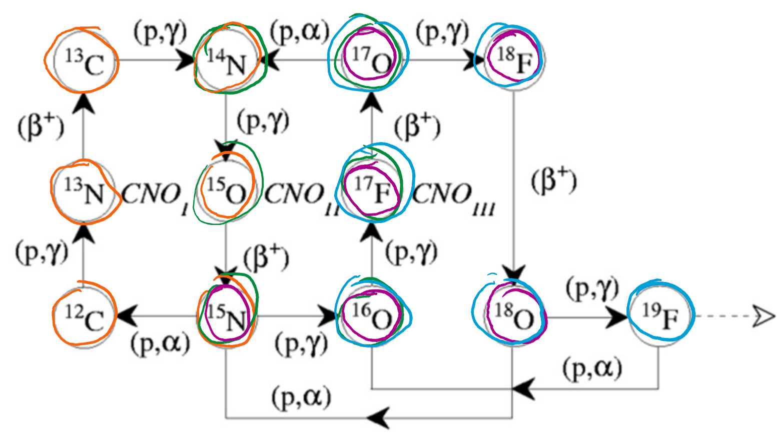


CN-equilibrium is achieved much faster (typically in $\sim 10^4$ yr) than CNO-equilibrium.
Reason: $\tau_{\mathrm{CN-eq.}} \simeq \tau_{\mathrm{p}}(^{12}\mathrm{C}) \ll \tau_{\mathrm{p}}(^{16}\mathrm{O}) \simeq \tau_{\mathrm{CNO-eq.}}$

Two more cycles

$$^{17}\mathrm{O}\,(\mathrm{p},\gamma)\,^{18}\mathrm{F}\,(\mathrm{e}^{+}\nu)\,^{18}\mathrm{O}\,(\mathrm{p},\alpha)\,^{15}\mathrm{N}\,(\mathrm{p},\gamma)\,^{16}\mathrm{O}\,(\mathrm{p},\gamma)\,^{17}\mathrm{F}\,(\mathrm{e}^{+}\nu)\,^{17}\mathrm{O} \quad (\mathrm{CNO\text{-}III})$$

$$^{18}\mathrm{O}\,(\mathrm{p},\gamma)\,^{19}\mathrm{F}\,(\mathrm{p},\alpha)\,^{16}\mathrm{O}\,(\mathrm{p},\gamma)\,^{17}\mathrm{F}\,(\mathrm{e}^{+}\nu)\,^{17}\mathrm{O}(\mathrm{p},\gamma)\,^{18}\mathrm{F}\,(\mathrm{e}^{+}\nu)\,^{18}\mathrm{O} \quad (\mathrm{CNO\text{-}IV})$$



- General rule: the (p, α)-reactions – if leading to a stable nucleus – are faster than the (p, γ)-reactions (strong vs. electromagnetic force!)

- Only exception: $^{13}C(p, \gamma)^{14}N$ is faster than $^{13}C(p, \alpha)^{10}B$.

  ⇒ speed of CNO-cycles: CN >> CNO-II >> CNO-III >> CNO IV

- When the isotopic abundances reach a steady-state (nuclear equilibrium) we get the CNO-equilibrium abundances

  1) ^{14}N is strongly produced

  2) ^{12}C, ^{15}N, ^{16}O, ^{18}O, ^{19}F are depleted

  3) ^{13}C and ^{17}O can be produced by the CNO-cycle

εδω

CNO-equilibrium abundances

	^{12}C	^{13}C	^{14}N	^{15}N	^{16}O	^{17}O	^{18}O	^{19}F
equil. mass fraction ($Z = 2\,\%$, $T = 3 \cdot 10^7\,K$)	10^{-4}	$6{\cdot}10^{-5}$	10^{-2}	$3 \cdot 10^{-7}$	$3 \cdot 10^{-4}$	$4 \cdot 10^{-6}$	10^{-9}	10^{-9}
solar mass fraction	$3.5 \cdot 10^{-3}$	$4 \cdot 10^{-5}$	10^{-3}	4.10^{-6}	10^{-2}	4.10^{-6}	2.10^{-5}	4.10^{-7}
ratio	$\frac{1}{30}$	1.5	10	$\frac{1}{10}$	$\frac{1}{30}$	1	$\frac{1}{20\,000}$	$\frac{1}{400}$

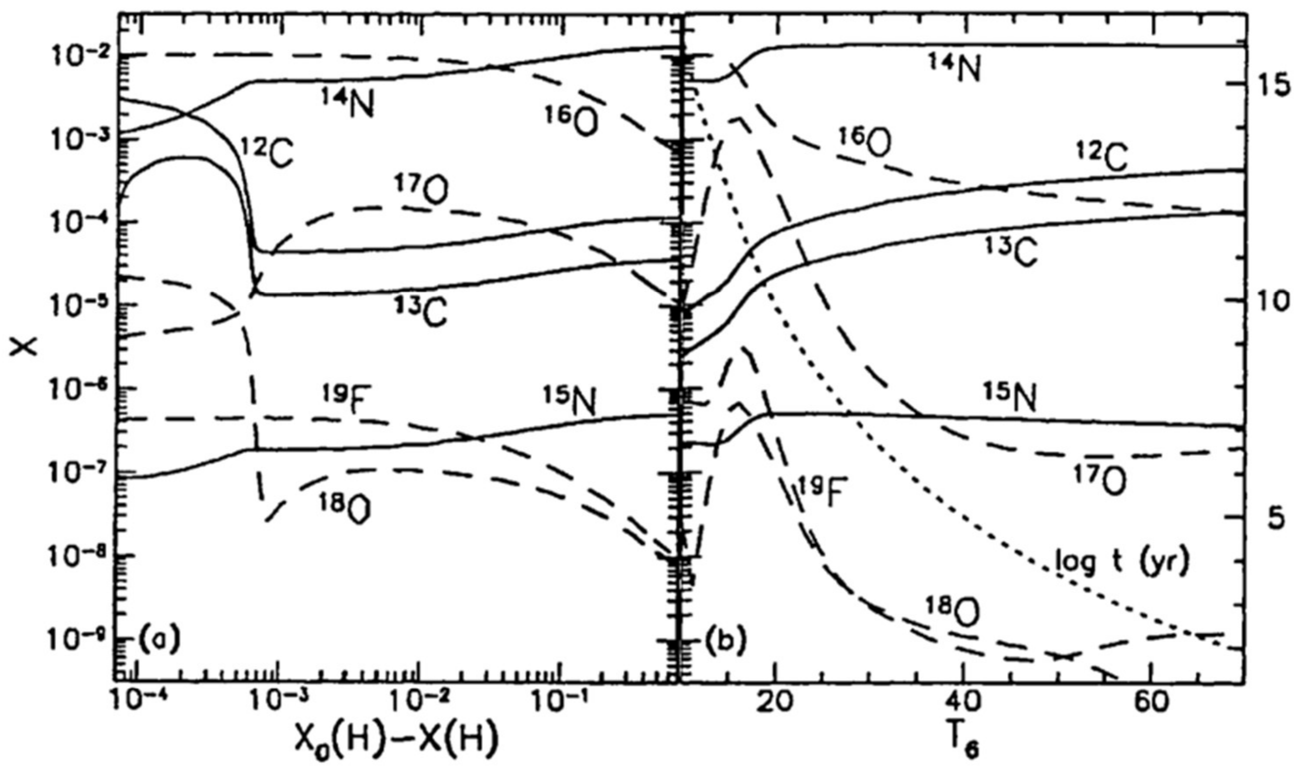


Figure 4.11: *(left)* Time evolution of the mass fractions of stable nuclides involved in the CNO cycles, versus the amount of H burnt at a constant temperature $T = 25 \times 10^6$ K and density $\rho = 100\,\mathrm{g\,cm^{-3}}$. $X_0(\mathrm{H}) = 0.7$ is the initial H mass fraction. *(right)* Final values of these mass fractions (at $X(\mathrm{H}) = 10^{-9}$) versus T_6, the temperature in 10^6 K. The dotted line indicates the logarithm of the H-burning time (in years) to be read on the right-hand side ordinate. Figure from Arnould & Mowlavi (1993).

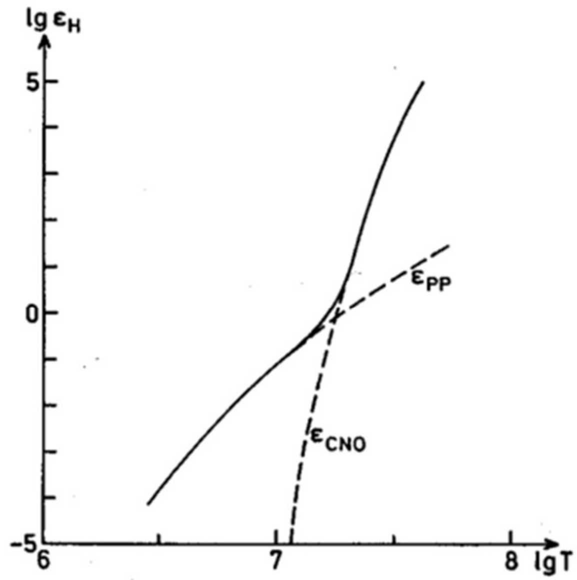
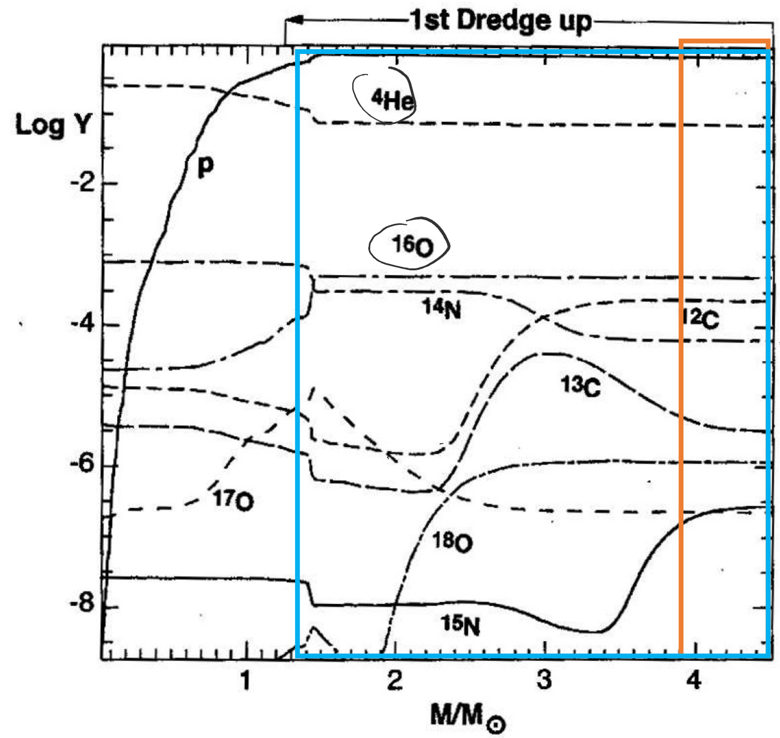


Figure 4.13: Temperature dependence of nuclear energy generation by the pp reactions and the CN cycle. Figure from Kippenhahn & Weigert (1990).

First dredge-up: occurs after core H exhaustion, as the star ascends the red giant branch (RGB), when the convective envelope deepens and mixes CN-processed material from the interior to the surface.



After 1st dredge up:
no change on surface
no change
depleted
increased
~ same on average
depleted
depleted

Figure 4.14: Abundance profiles of several light nuclei (plotted as the logarithm of $Y = X/A$, where X is the usual mass fraction) in the inner regions of a $5\,M_\odot$ star after core H-burning. The region that is subsequently mixed by the first dredge-up is indicated by the arrow along the top. Before the first dredge-up, the original composition is unchanged down to $\sim 4\,M_\odot$. Below $\sim 4\,M_\odot$, ^{13}C increases and ^{15}N decreases due to CN cycling, and somewhat deeper down, where the CN cycle has been more effective, ^{14}N increases while ^{12}C and ^{13}C decrease. After the first dredge-up, the envelope composition is homogeneously mixed to the average value down to the indicated depth. This means an increase in the surface $^{14}N/^{12}C$ and $^{13}C/^{12}C$ abundance ratios. Figure from Busso et al. (1999).

2. He- burning

- Starting composition: ashes of hydrogen burning

  For example $(Z = Z_{\odot})$: $X = 0.70, Y = 0.28, Z = 0.02(\text{ with } X(\text{O}) \approx 0.01)$

  After H burning, in the core: $X = 0, Y = 0.98, Z = 0.02(\, with\, X(\text{N}) \approx 0.014)$

- Main reaction of helium burning: so called 3α-reaction

  <u>1st step</u>: $^{4}He + ^{4}He \rightarrow ^{8}Be$ (Q = − 92 keV)

  Endothermic ⇒ ^{8}Be is unstable against breakup in 2α, but $\tau_{1/2}(^{8}Be) \cong 3 \times 10^{-16}$ s is much larger than the timescale of an α + α scattering (10^{-21}- 10^{-20}s)

  ⇒ a small concentration of ^{8}Be can build up until equilibrium is reached:

  $\alpha + \alpha \leftrightarrow ^{8}\text{Be}$

  At $T \simeq 10^{8}\text{K}, ^{8}\text{Be}/^{4}\text{He} \simeq 10^{-9}$

  <u>2nd step</u> $^{8}\text{Be} + \alpha \rightarrow ^{12}\text{C} + \gamma$ resonant reaction $^{8}Be(\alpha)^{12}C^{*}(\gamma)^{12}C$.

Further reactions

$^{12}C(\alpha, \gamma)^{16}O$ very important – reaction rate not well known – unclear whether the main product of He burning is C or O

- the $^{12}C(a, \gamma)^{16}O$ reaction <u>is not resonant</u>, so that at central He-exhaustion a considerable amount of ^{12}C is leftover in the stellar core.
- ^{16}O is usually dominant at that stage, but the exact $^{16}O/^{12}C$ ratio depends on the rate of the $^{12}C(\alpha, \gamma)^{16}O$ reaction, which is still uncertain.

$^{16}O(\alpha, \gamma)^{20}Ne$ less important (only at very high T)

<u>Secondary nucleosynthesis during helium burning:</u>

$$^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O(\alpha,\gamma)^{22}Ne \quad (\text{already at } T \simeq 10^8 \text{ K})$$

production of ^{18}O*, ^{22}Ne

$$^{22}Ne(\alpha,n)^{25}Mg \quad (\text{for } T \geq 3 \cdot 10^8 \text{ K})$$

production of n → s process elements

* **He-burning constitutes the production mode of ^{18}O in the Universe**, since some amount of it survives in the He-shell the subsequent reaction $^{18}O(\alpha, \gamma)^{22}Ne$

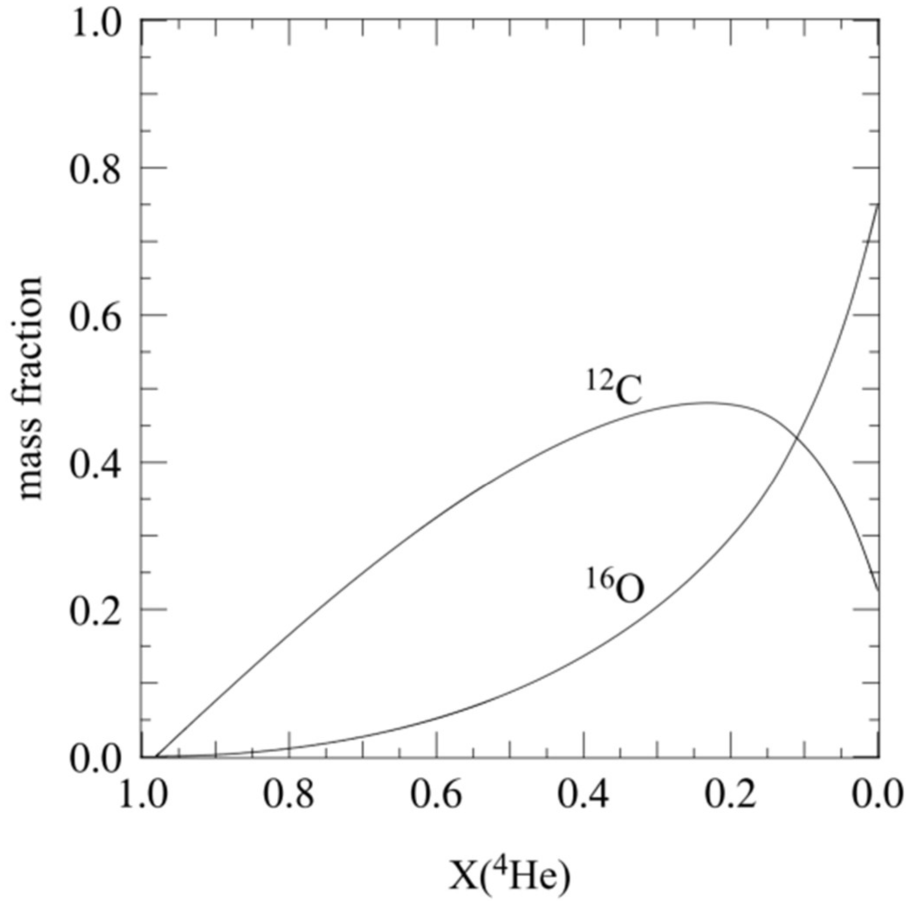


Figure 4.19: Mass fractions of ^{12}C and ^{16}O as a function of the decreasing He mass fraction, calculated for a $5\,M_\odot$ star with $Z = 0.02$.

s-process elements

- In massive stars, towards the end of core He-burning, at T~250x10^6K, **neutrons** are released through $^{22}Ne(\alpha,n)^{25}Mg$.
- These neutrons are easily **captured** by all nuclei in the star (in proportion to the corresponding neutron capture cross sections).
- A part of those nuclei is destroyed in subsequent stages of the evolution of massive stars, but those that survive are ejected by the final supernova explosion.
- This "**weak s-process**" leads to the production of the light s-nuclei, **with mass number A between 60 and 90.**

Advanced Nuclear Burning in Massive Stars:
C, Ne, and O Burning

- C-burning in massive stars occurs at temperature T ~ 8x1

- The core composition at C-ignition is dominated by the
  90% of the total) in a proportion that decreases with the

Many important secondary reactions:

$^{20}Ne(\alpha,\gamma)^{24}Mg$	$^{23}Na(\alpha,p)^{26}Mg$	$^{26}Mg(p,\gamma)^{27}Al$
$^{23}Na(p,\gamma)^{24}Mg$	$^{23}Na(p,\alpha)^{20}Ne$	$^{25}Mg(p,\gamma)^{26}Al$
$^{22}Ne(\alpha,n)^{25}Mg$	$^{25}Mg(\alpha,n)^{28}Si$	$^{23}Mg(n,p)^{23}Na$
$^{25}Mg(n,\gamma)^{26}Mg$	$^{23}Mg(e^{+}\nu)^{23}Na$	$^{21}Na(e^{+}\nu)^{21}Ne$
$^{20}Ne(p,g)^{21}Na$	$^{21}Ne(p,g)^{22}Na$	$^{22}Na(e^{+}\nu)^{22}Ne$

and dozens (hundreds?) more

- The exact proportion of $^{12}C/^{16}O$ and the exact initial fraction of ^{12}C (which is crucial for the energetics of C-burning) depend sensitively on the – still uncertain – value of the $^{12}C(\alpha,\gamma)^{16}O$ rate and on the adopted criterion of convection during He-burning.

- Channels for fusion of two ^{12}C nuclei:

$$^{12}C + ^{12}C \rightarrow ^{24}Mg^{*} \rightarrow ^{20}Ne + ^{4}He(Q = +4.616\,\mathrm{MeV}),$$

$$^{12}C + ^{12}C \rightarrow ^{24}Mg^{*} \rightarrow ^{23}Na + p(Q = +2.238\,\mathrm{MeV}),$$

$$^{12}C + ^{12}C \rightarrow ^{24}Mg^{*} \rightarrow ^{23}Mg + n(Q = -2.605\,\mathrm{MeV}).$$

endothermic reaction with a small probability (0.1% at T = 1GK)

Final products of C-burning
^{20}Ne (mostly), ^{23}Na, ^{24}Mg

After C exhaustion, stellar core consists of ^{16}O and ^{20}Ne (more than 90% of the total by mass) and few % of ^{23}Na and ^{24}Mg

- Despite its smaller Coulomb barrier, ^{16}O is not the next fuel to burn, since it is exceptionally stable (being a doubly magic nucleus with Z = N = 8).
- The photodisintegration of ^{20}Ne nuclei $^{20}\mathrm{Ne} + \gamma \longleftrightarrow {}^{16}\mathrm{O} + \alpha$ becomes energetically feasible at T ~ 1.5 GK, i.e., before the fusion temperature of ^{16}O nuclei (T ~2 GK) is reached.
- The alpha particles released are captured by both ^{16}O (to restore ^{20}Ne) and ^{20}Ne (to form ^{24}Mg).

$$^{20}\mathrm{Ne} + \alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$$

- The net result of the operation is that this **^{20}Ne "melting"** can be described by:

  2 ^{20}Ne → ^{16}O +^{24}Mg.

- The photodisintegration of ^{20}Ne is endoergic, but the exoergic α-captures on ^{16}O and ^{20}Ne more than compensate for the energy lost.
- ^{20}Ne burning produces 0.1 MeV/nucleon or about one-fourth the specific energy released by C-burning.
- Several other energetically unimportant reactions, induc burning:

^{23}Na (a product of C-burning) disappears through ^{23}Na(p,

Other secondary reactions:

$^{24}Mg(\alpha,\gamma)^{28}Si$	$^{27}Al(\alpha,p)^{30}Si$
$^{25}Mg(\alpha.n)^{29}Si$	$^{30}Si(p,\gamma)^{31}P$
$^{26}Mg(\alpha,n)^{30}Si$	etc.

- After Ne "melting", the stellar core consists mainly of ^{16}O, ^{24}Mg, and ^{28}Si. ^{29}Si, ^{30}Si, and ^{32}S also present at the 10^{-2} level (in mass fraction).
- The subsequent **fusion of two ^{16}O** nuclei produces a compound nucleus of **^{32}S** which **decays** through the **p, α, (d) and n channels**;

$$
\begin{aligned}
{}^{16}O + {}^{16}O \rightarrow ({}^{32}S)^* &\rightarrow {}^{31}S + n + 1.45\,\text{MeV} && 5\% \\
&\rightarrow {}^{30}P + d - 2.41\,\text{MeV} && \leq 5\% \\
&\rightarrow {}^{31}P + p + 7.68\,\text{MeV} && 56\% \\
&\rightarrow {}^{28}Si + \alpha + 9.59\,\text{MeV} && 34\%
\end{aligned}
$$

proceeds through the ^{32}S compound nucleus with a high density of resonances. Very like carbon burning.

- The energy released by oxygen burning is 0.5 MeV/nucleon. As in the previous stages, dozens of n, p, and α induced reactions occur.

- In addition, the increased temperature and density introduce two novel features:
  - Electron captures occur mainly on ^{31}S, ^{30}P, ^{33}S, ^{33}Cl and ^{37}Ar → neutron-rich composition
  - Photodesintegration reactions become also important and destroy most of the heavier than Fe nuclei that have been built through n captures in the previous burning phases (mostly by s-process during He-burning).

Si-Melting and Nuclear Statistical Equilibrium (NSE)

- At O-exhaustion, the composition of the stellar core is dominated by ^{28}Si (30–40% by mass fraction) and, either ^{32}S and ^{38}Ar (in the more massive cores, where low densities do not favor electron captures) or ^{30}Si and ^{34}S (in the lower mass stars, below 15 M_{solar})
- The reaction $^{28}Si + {}^{28}Si \rightarrow ({}^{56}Ni)^*$ is strongly Coulomb suppressed.
- Rather, a portion of the silicon (and sulfur, argon, etc.) "melts" by **photodisintegration** reactions into a sea of neutrons, protons, and alpha-particles. These lighter constituents add onto the remaining silicon and heavier elements, gradually increasing their mean nuclear mass number until species in the iron group are most abundant.

$$^{28}Si(\gamma,\alpha)^{24}Mg(\gamma,\alpha)^{20}Ne(\gamma,\alpha)^{16}O(\gamma,\alpha)^{12}C(\gamma,2\alpha)\alpha$$

Other photodisintegration reactions releasing p and n also occur, especially in material with neutron excess

Carbon burning ("melting")	Heavy ion fusion
Neon Burning	Photodisintegration rearrangement
Oxygen burning	Heavy ion fusion
Silicon burning ("melting")	Photodisintegration rearrangement

In the conditions of NSE, for not too high temperatures (T < 10 GK), the most abundant nuclei are those that minimize the free energy for the given neutron excess η

$$\eta = \frac{N_{tot} - Z_{tot}}{N_{tot} + Z_{tot}},$$

N_{tot} = all neutrons (free + bound)

Z_{tot} =all protons (free + bound)

Nuclear Statistical Equilibrium (NSE)

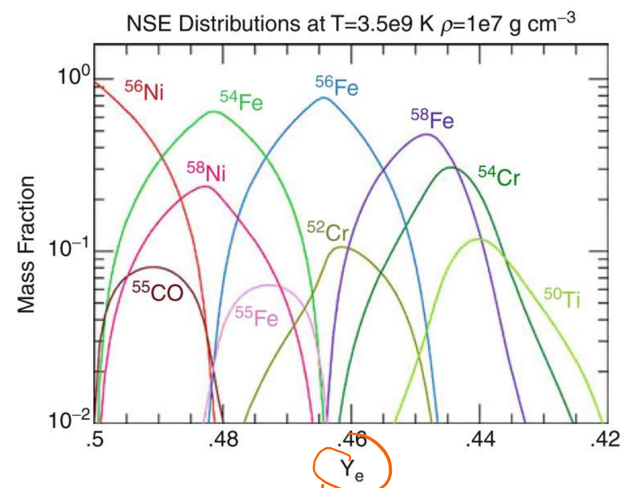
- Example of a quasi-equilibrium cluster of reactions

$$^{28}Si \rightleftarrows {}^{29}Si \rightleftarrows {}^{30}Si \rightleftarrows {}^{31}P \rightleftarrows {}^{32}S \rightleftarrows {}^{28}Si$$

n n p p α

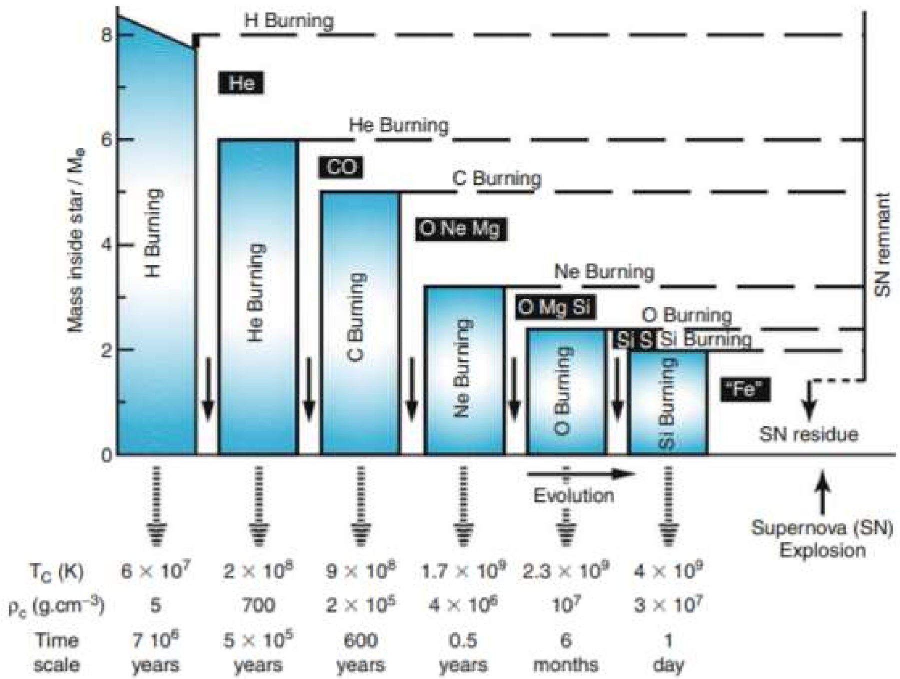
Basically, silicon burning in the star's core turns the products of oxygen burning (Si, S, Ar, Ca, etc.) into the most tightly bound nuclei (in the iron group) for a given neutron excess, η.

- ➢ Towards end of Si-melting all electromagnetic and strong nuclear interactions are in equilibrium with their inverses.
- ➢ neutrinos still escape freely from the stellar core, so, neutrino-producing weak interactions never come into equilibrium with their inverses and weak equilibrium is not established.
- ➢ last reactions to reach equilibrium are those linking ^{24}Mg to ^{20}Ne, ^{16}O to ^{12}C, and, finally, the reaction 3α to ^{12}C.
- ➢ Nuclear Statistical Equilibrium (NSE) is reached (assuming that all nuclides obey the statistics of an ideal Maxwell–Boltzmann gas)



Stellar Nucleosynthesis. Figure 3 Abundances of major nuclides in nuclear statistical equilibrium (NSE), as a function of the electron mole number Y_e

$$Y_e = \frac{\text{total number of protons}}{\text{total number of baryons}}$$



Stellar Nucleosynthesis. Figure 2 Schematic evolution of the interior composition of a massive star of 25 $M_\odot$. The central temperatures (T_c) and densities (ρ_c), as well as the approximate durations of the various burning stages appear in the bottom. After the final supernova (SN) explosion, a neutron star or a black hole is left behind (SN residue), while the rest is ejected in the interstellar medium, forming the SN remnant

Explosive Nucleosynthesis in Supernovae

Explosive Nucleosynthesis in Core-Collapse Supernovae (CCSN)

- Iron core collapse proceeds on a timescale of milliseconds. Due to increasingly high temperatures, photodisintegrations tear down Fe nuclei to nucleons and alpha particles, while higher densities favor electron captures and conversion of protons to neutrons. When the density of nuclear matter is reached (~10^{14}gcm^{-3}), the repulsive component of the strong nuclear force brings the collapse of the inner core abruptly to a halt.

- At the boundary of the inner core a shock wave is formed, with an energy approximately equal to the kinetic energy of the core at the moment of the bounce (~5×10^{51} erg, according to results of numerical simulations).

- As the shock wave propagates outwards, into the still infalling outer core, it photodisintegrates its Fe nuclei (tearing them down to nucleons) and consequently loses ~ 8.8 MeV/nucleon or 1.5×10^{51} erg per 0.1 M_{solar}

- As the shock wave propagates through the stellar envelope, it heats the various layers to temperatures higher than in the corresponding central burning stages.

- If the nuclear burning timescale at radius r is smaller than the hydrodynamical timescale t_{HD} (r), then explosive nucleosynthesis will take place ($\tau_{HD} \sim 0.446\, \rho_6^{-1/2}$ s).

    - vigorous explosive Si- and O-burning → same major reactions as for quiescent burning

  The difference is that the beta decay lifetimes of unstable nuclei are often larger than the timescales of explosive nucleosynthesis, meaning that the cross sections of nuclear reactions on those unstable nuclei are also required,

    - Ne and C layers are less affected

    - He and H layers not affected at all by the explosion

Explosive Nucleosynthesis in Thermonuclear Supernovae (SNIa)

- The high density of the WD (~10^9g/cm^3) leads to a very efficient screening of the nuclear reactions, whereby repulsive Coulomb barriers between positively charged nuclei are lowered by the presence of negatively charged electrons. **^{12}C +^{12}C fusion occurs then at considerably lower temperatures** and **at much higher rates** than in massive stars. **Ignition occurs when the ^{12}C +^{12}C reaction releases energy faster than neutrino losses**

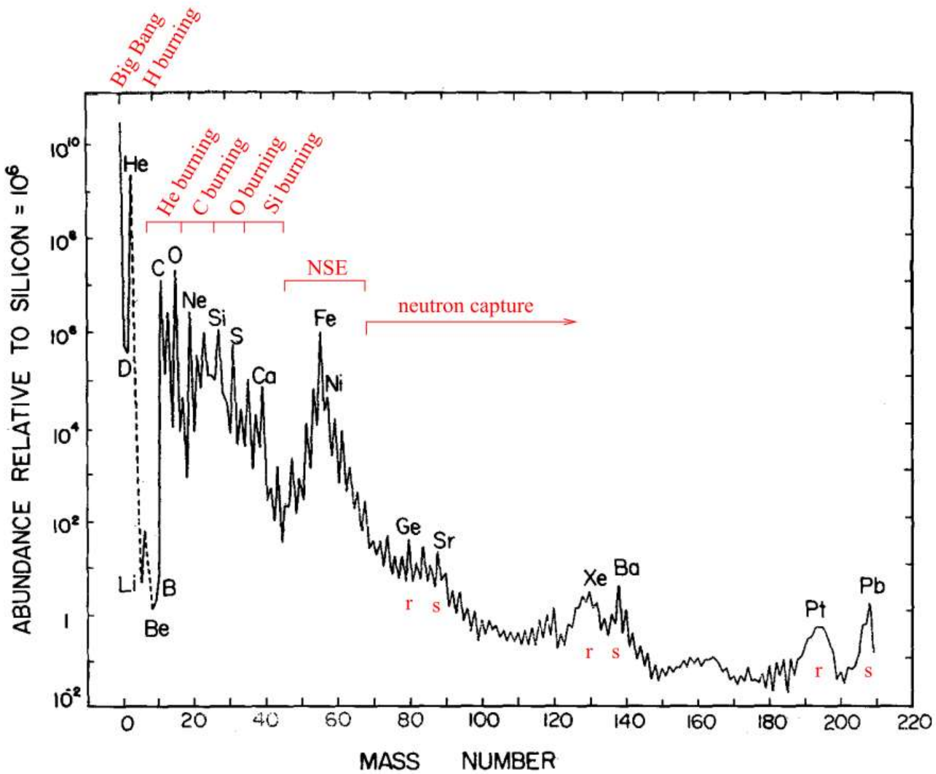
- WD are composed of **degenerate gas**: internal pressure depends only on density, not on temperature. Thus, heating of the medium (through the energy released by ^{12}C +^{12}C is not accompanied by pressure increase, which would lead to gas expansion and cooling; instead, **temperature increases steadily**, and so does the ^{12}C +^{12}C reaction rate.

- In those conditions, a "**carbon deflagration**" (subsonic burning front) burns explosively the material of the star, all the way towards **Nuclear Statistical Equilibrium (NSE)**.

- The timescale of the explosion is too short to allow for important electron captures, and thus the final outcome is essentially $^{56}_{28}Ni$ (~0.7 M_{solar})
  Only in the central regions the high densities favor the production of $^{54}_{26}Fe$ and $^{58}_{28}Ni$ through electron captures.

- In the outer layers, NSE is not reached because of lower temperatures that lead to the production of intermediate mass nuclei, like Si, Ca, etc.

SNIa play a very important role, since they produce between 50% and 65% of the Fe-peak nuclei.


Big Bang
H burning
He burning
C burning
O burning
Si burning
NSE
neutron capture
ABUNDANCE RELATIVE TO SILICON = 10^6
MASS NUMBER
He
D
Li
Be
B
C
O
Ne
Si
S
Ca
Fe
Ni
Ge
Sr
Xe
Ba
Pt
Pb
r
s
r
s
r
s