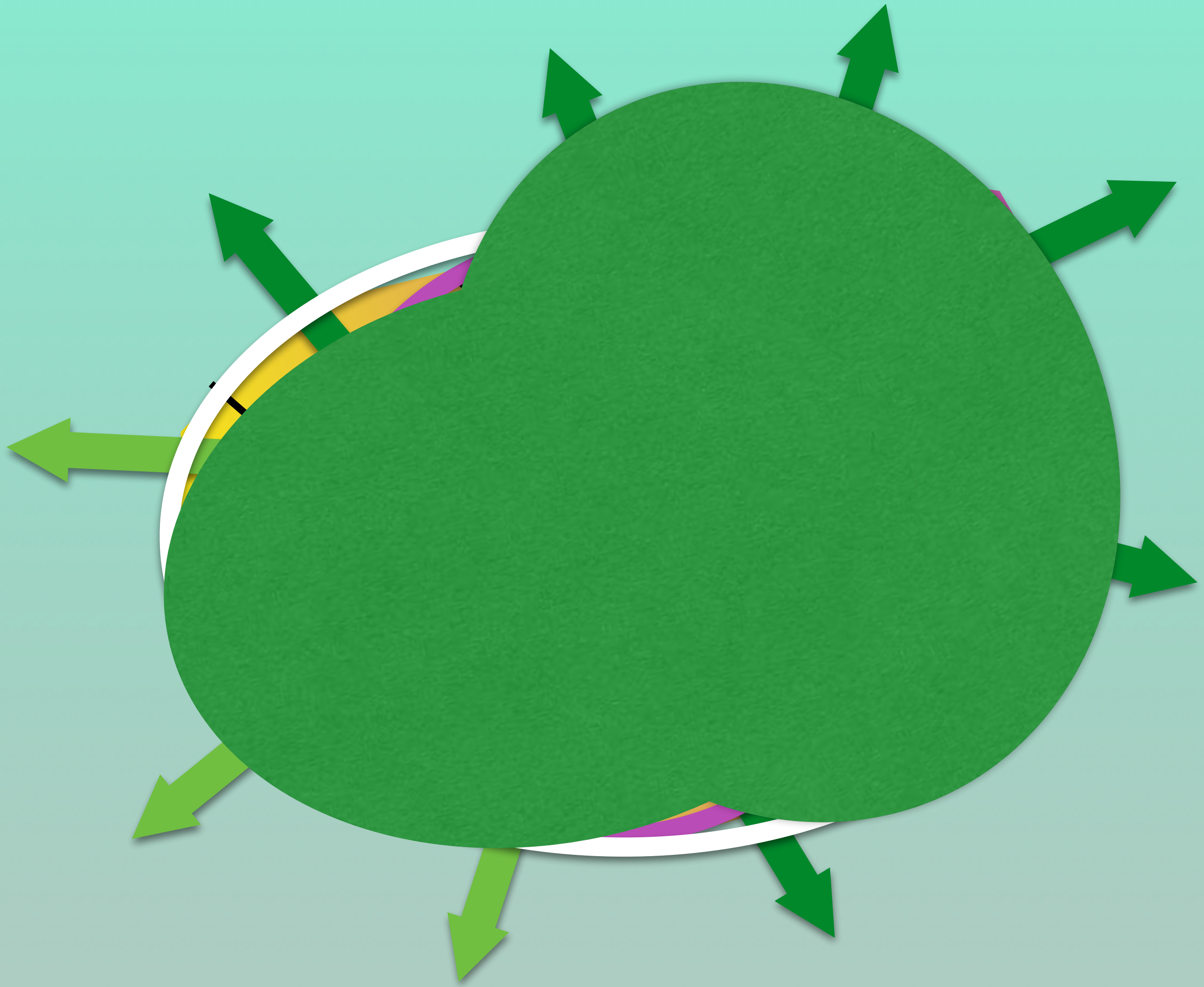
Charge/Current Distribution

$\partial_\mu F^{\mu\nu} = J^\nu$



E and M fields

- Maxwell E/M equations general solution:

$$\vec{B} = \sum_{l,m}\left[\alpha_E(l,m)f_l(k\vec{r})\vec{X}_{lm} - \frac{i}{k}\alpha_M(l,m)\vec{\nabla}\times g_l(k\vec{r})\vec{X}_{lm}\right]$$

$$\vec{E} = \sum_{l,m}\left[\frac{i}{k}\alpha_E(l,m)\vec{\nabla}\times f_l(k\vec{r})\vec{X}_{lm} + \alpha_M(l,m)g_l(k\vec{r})\vec{X}_{lm}\right]$$

- Radiation emitted: Poynting vector S

In detail

$$\vec{B} = \sum_{l,m}\left[\alpha_E(l,m) f_l(k\vec{r})\vec{X}_{lm} - \frac{i}{k}\alpha_M(l,m)\vec{\nabla}\times g_l(k\vec{r})\vec{X}_{lm}\right]$$

$$\vec{E} = \sum_{l,m}\left[\frac{i}{k}\alpha_E(l,m)\vec{\nabla}\times f_l(k\vec{r})\vec{X}_{lm} + \alpha_M(l,m) g_l(k\vec{r})\vec{X}_{lm}\right]$$

- X_{lm}: generalized spherical harmonics
- $\alpha_{E,M}$: coefficients of *E,M* fields
- *f,g*: linear combinations of Hankel functions

Multipoles

$$\alpha_E(l,m) \simeq \frac{4\pi k^{l+2}}{i(2l+1)!!}\left(\frac{l+1}{l}\right)^{1/2}(Q_{lm}+Q'_{lm})$$

$$\alpha_M(l,m) \simeq \frac{4\pi i k^{l+2}}{(2l+1)!!}\left(\frac{l+1}{l}\right)^{1/2}(M_{lm}+M'_{lm})$$

Multipoles

- The Generalized Spherical Harmonics carry the information on the order of multipolarity
- Multipoles are used to approximate the behavior of current densities in the presence of *E* & *B* fields
- They are described by the order of the term in the expansion

In more detail

- Magnetic *l*-order moment

$$M_{lm} = \frac{1}{l+1} \int d^3x \; r^l Y^*_{lm} \vec{\nabla} \cdot \left(\frac{\vec{r} \times \vec{J}}{c} \right)$$

- Electric *l*-order moment

$$Q_{lm} = \int d^3x \; r^l Y^*_{lm} \rho$$

Interaction energy

$$H_{EM} = q\Phi - \vec{p}\cdot\vec{E} + \frac{1}{6}\sum_{i=1}^{3}\sum_{j=1}^{3} Q_{ij}\frac{\partial E_j}{\partial x_i} + \ldots - \vec{\mu}\cdot\vec{H} + \ldots$$

Most common (and typically stronger) terms are:

- Electric Dipole Moment [E]
- Electric Quadrupole Moment [Q]
- Magnetic Dipole Moment [μ]

The EM moments

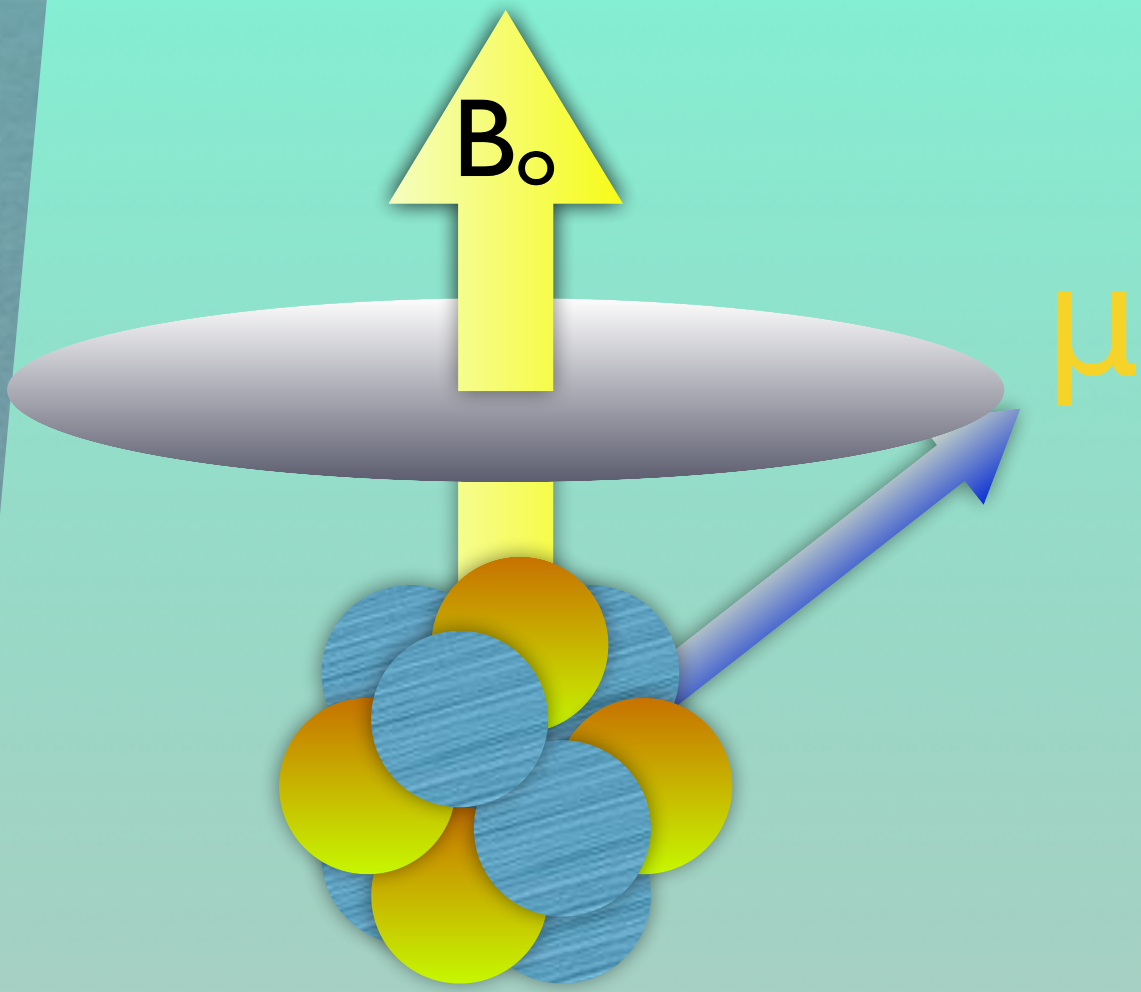
$$H_{EM} = q\Phi - \vec{p}\cdot\vec{E} + \frac{1}{6}\sum_{i=1}^{3}\sum_{j=1}^{3} Q_{ij}\frac{\partial E_j}{\partial x_i} + \ldots - \vec{\mu}\cdot\vec{H} + \ldots$$

$$\vec{p}(\vec{r}) = \int d^3r' \rho(\vec{r'})\vec{r'}$$

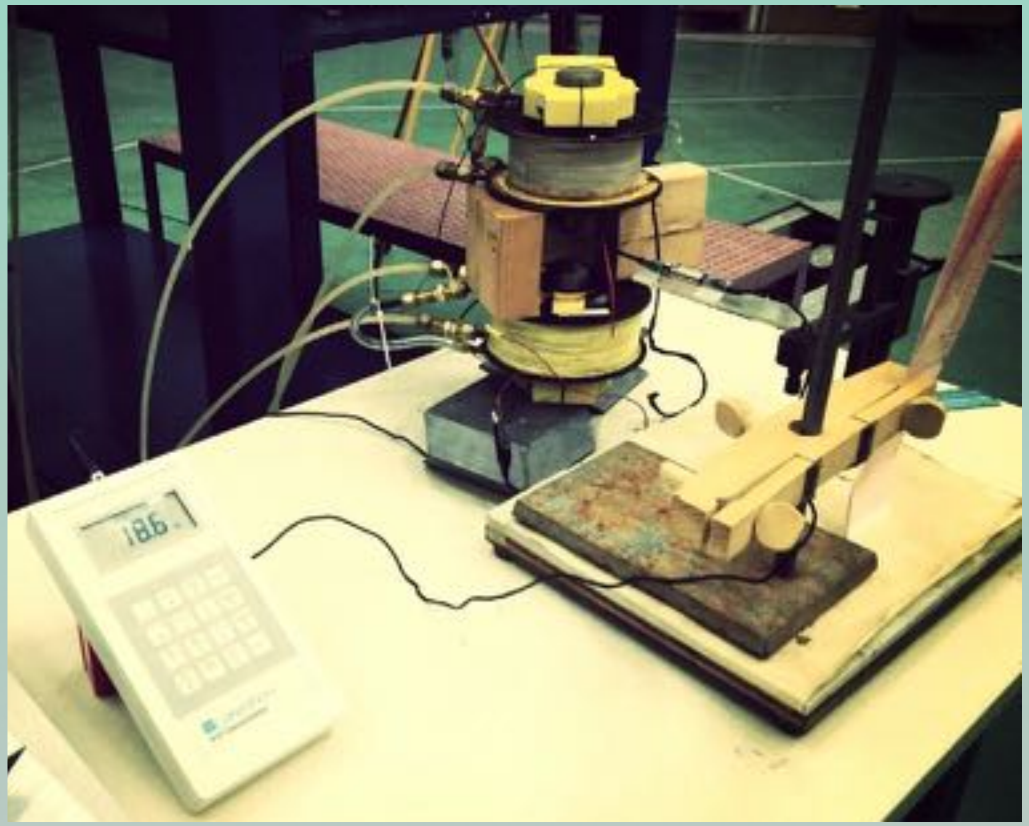
$$\vec{\mu}(\vec{r}') = \int d^3r' \vec{r}' \times \vec{j}(\vec{r})$$

$$Q_{ij}(r) = \int d^3r' \rho(r')(3x_i'x_j' - \delta_{ij}r'^2)$$

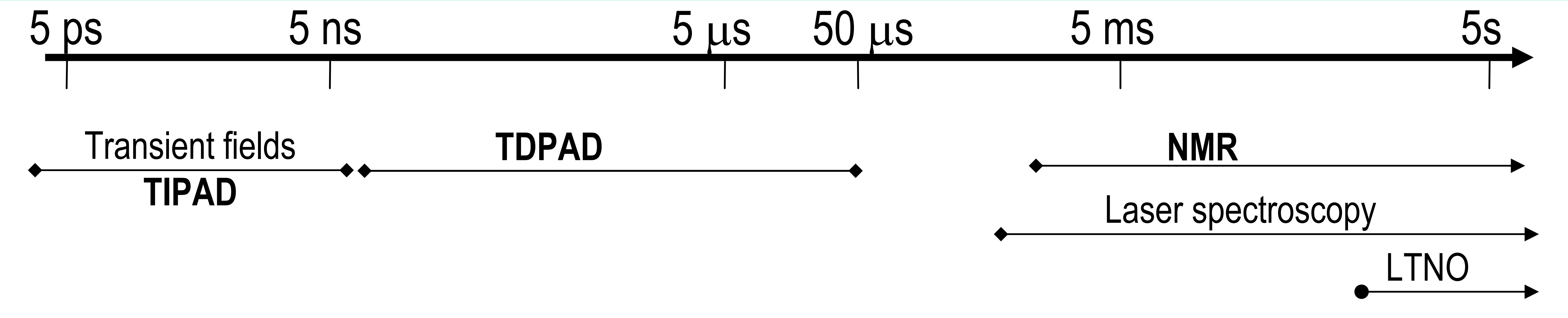
It’s all on the field



- Static (big magnet!)
- Mössbauer
- Hyperfine fields
- NMR



Lifetime is also important

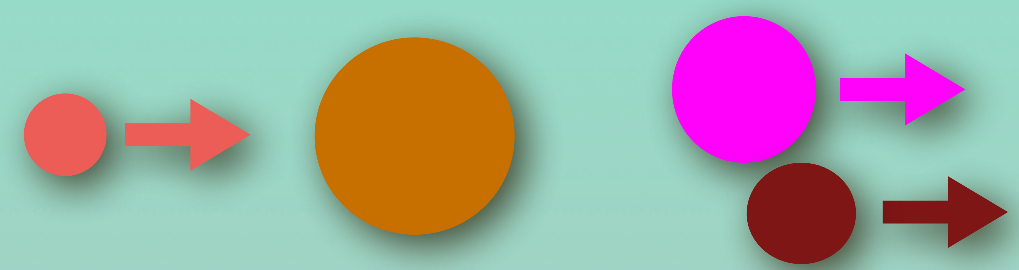


- The production mechanism is related to the method of producing spin-orientation
- Coulex, fusion-evaporation, fragmentation etc

Excitations mechanisms

- Spallation

- Induced fission

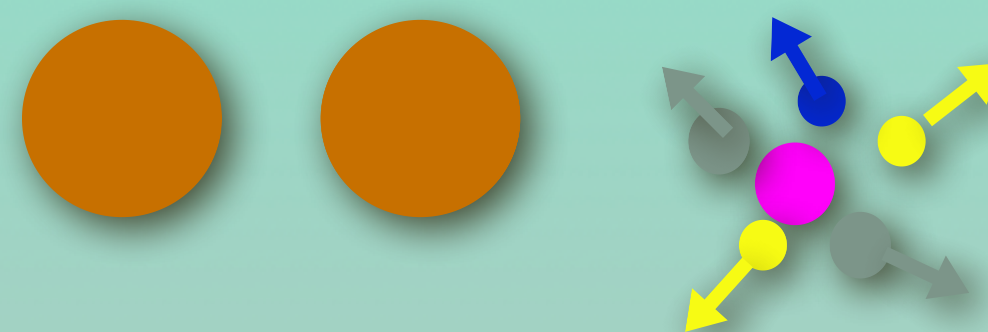


- Fragmentation

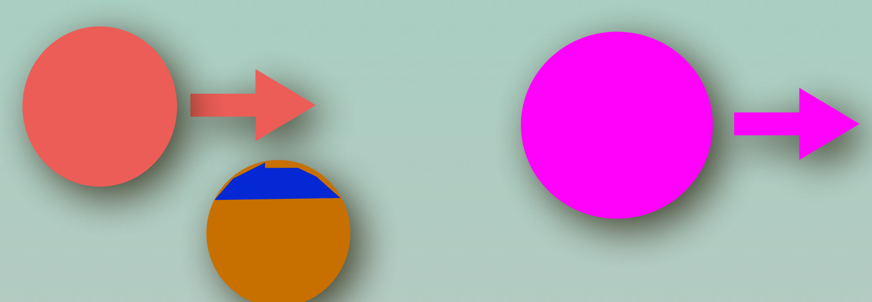
Excitation mechanisms

- Multi-fragmentation

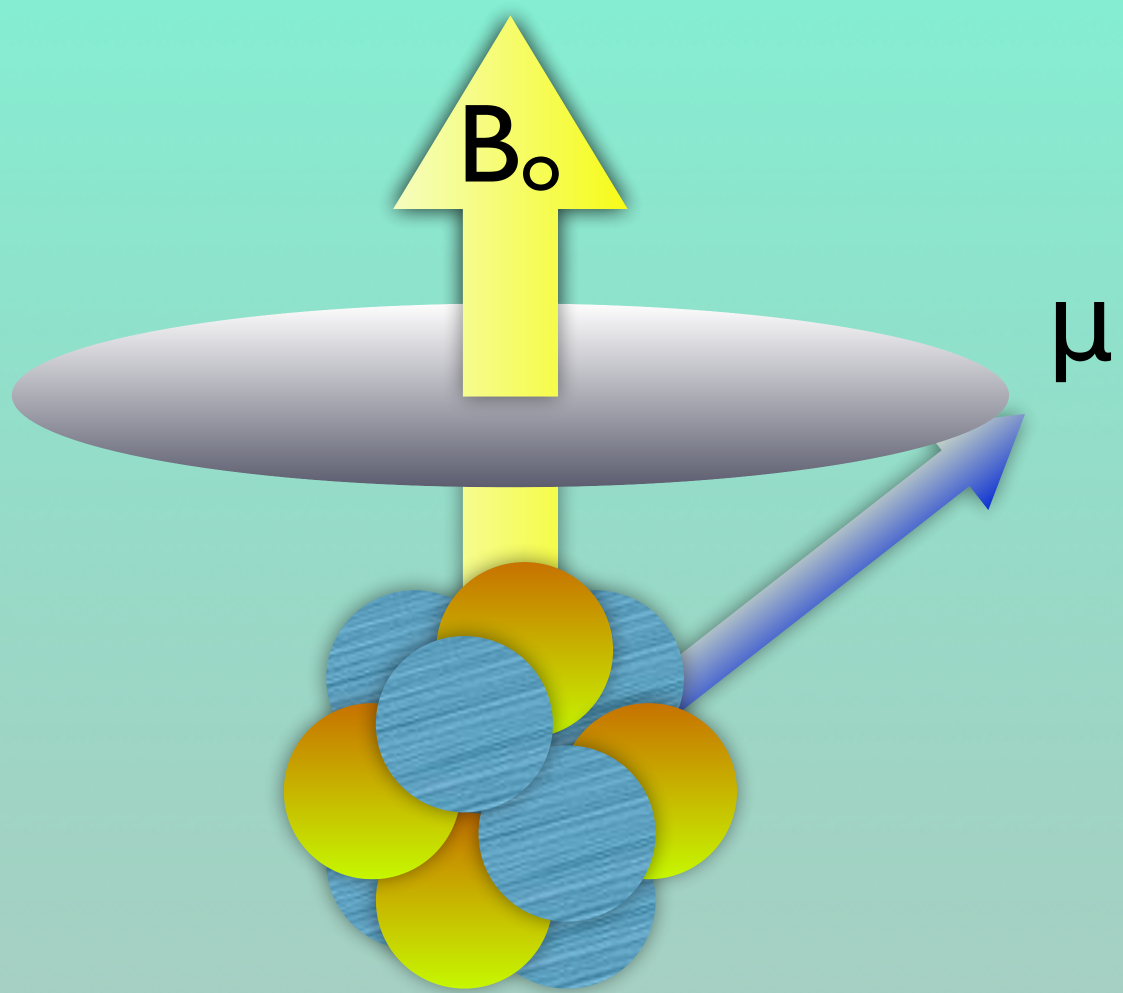
- Vaporization



- Charge pickup



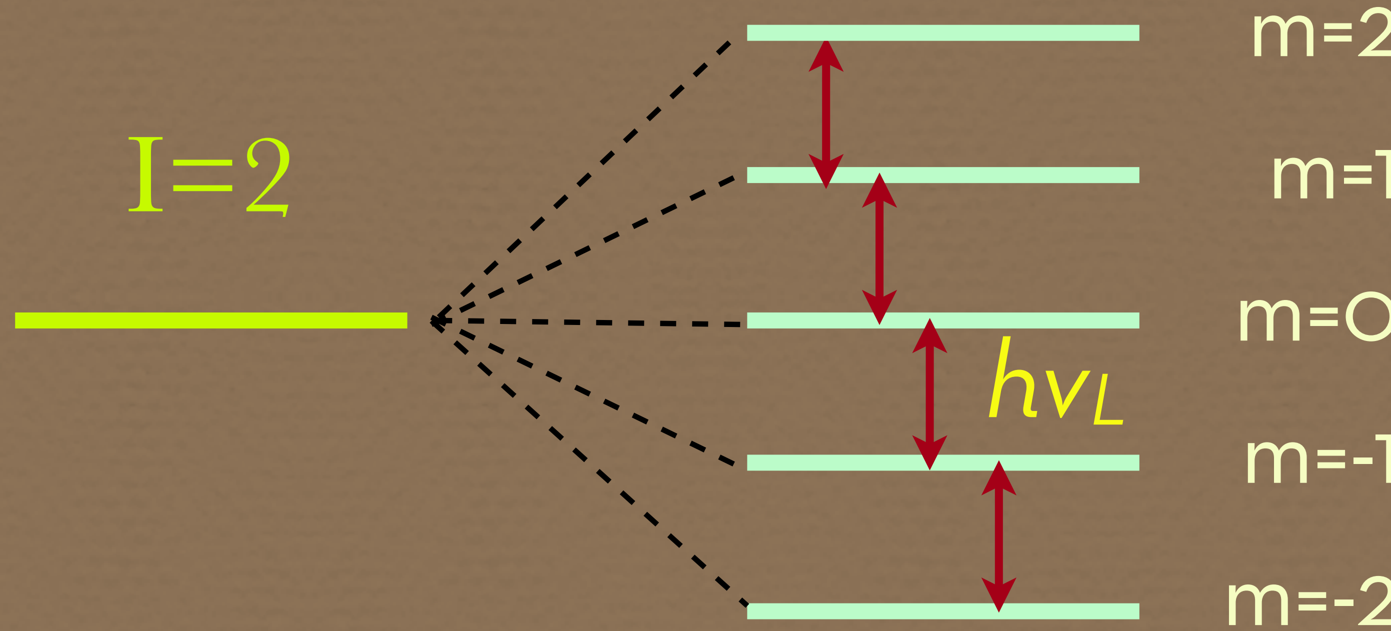
Hamiltonian term



- Zeeman splitting

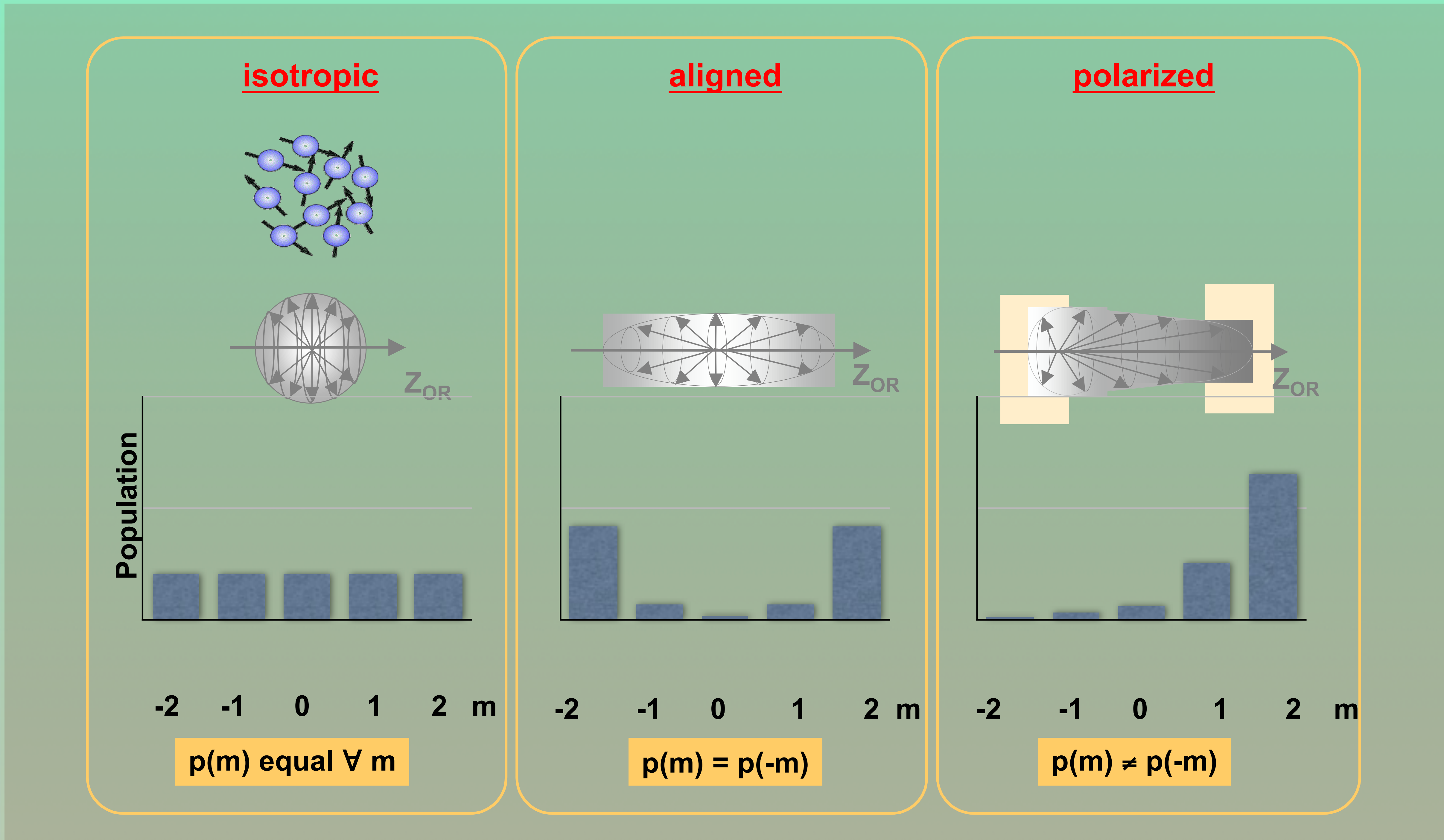
$$H_B = -\vec{\mu}\cdot\vec{B}_o = -g\mu_N\vec{J}\cdot\vec{B}_o = -\omega_L J_z$$

Zeeman levels

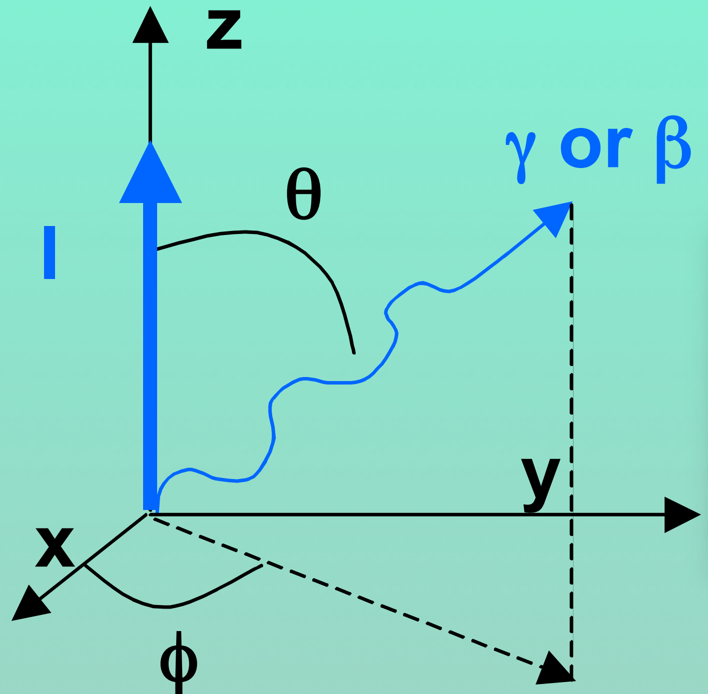


$\nu_L = g\mu_n B/h$:Larmor frequency

Level population

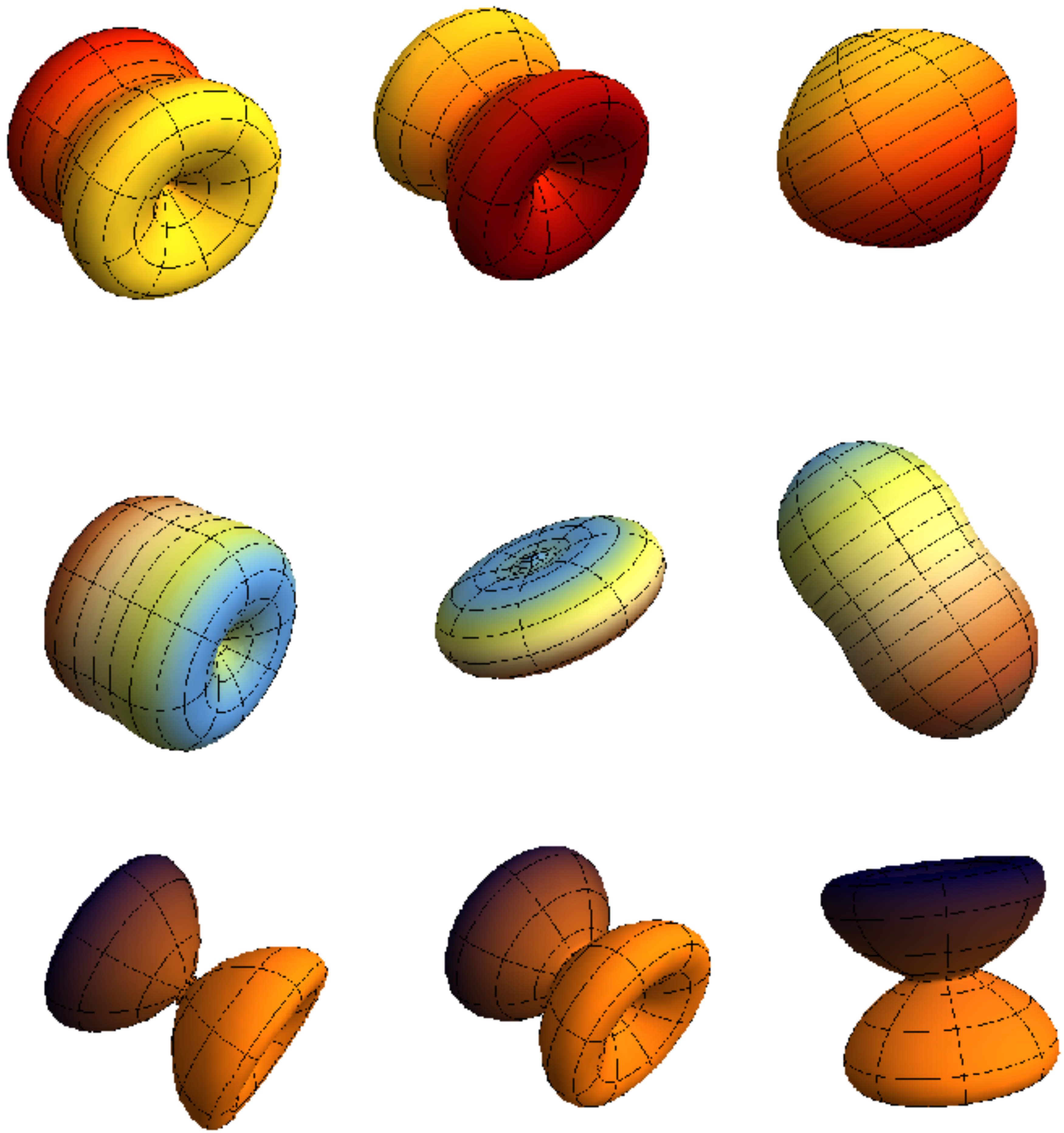


The angular correlation



$$W(\theta,\phi,t) = \sum_{k,n} \frac{\sqrt{4\pi}}{2k+1} A_k B_k^n(t) Y_k^n(\theta,\phi)$$

- Isotropic ensemble: $B_o=1$. All others equal 0
- Aligned ensemble: B_k^o for k even survive
- Polarized ensemble: B_k^o for k odd survive

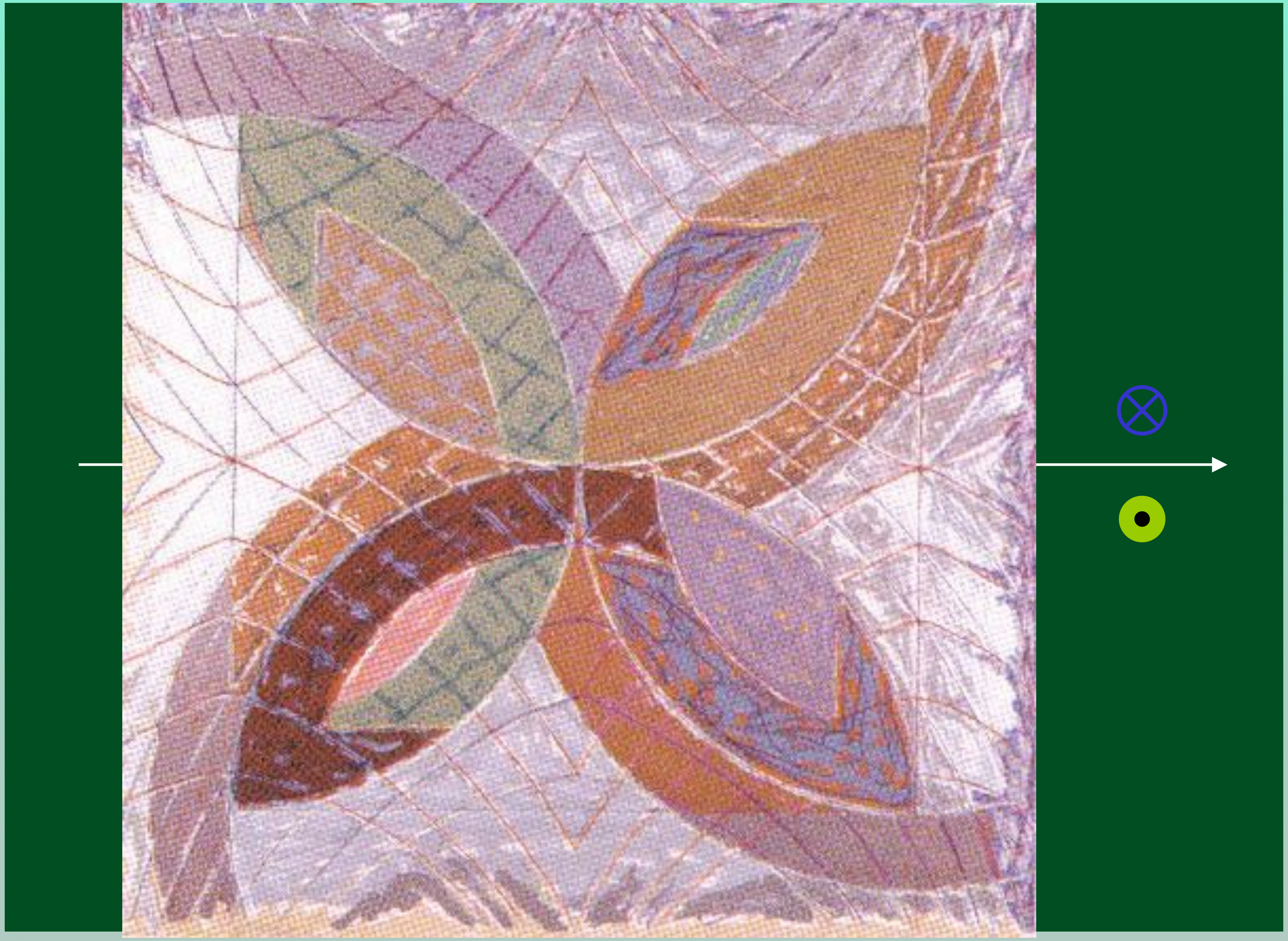


@tmertzi

What is the observable?

- The magnetic moment precesses around the field
- This changes the decay pattern of the emitted radiation (angular correlation is perturbed)
- Detection of the perturbation is detected in detectors

Going Perturbed



Frank Stella, Polar Coordinates II

Quantum Picture

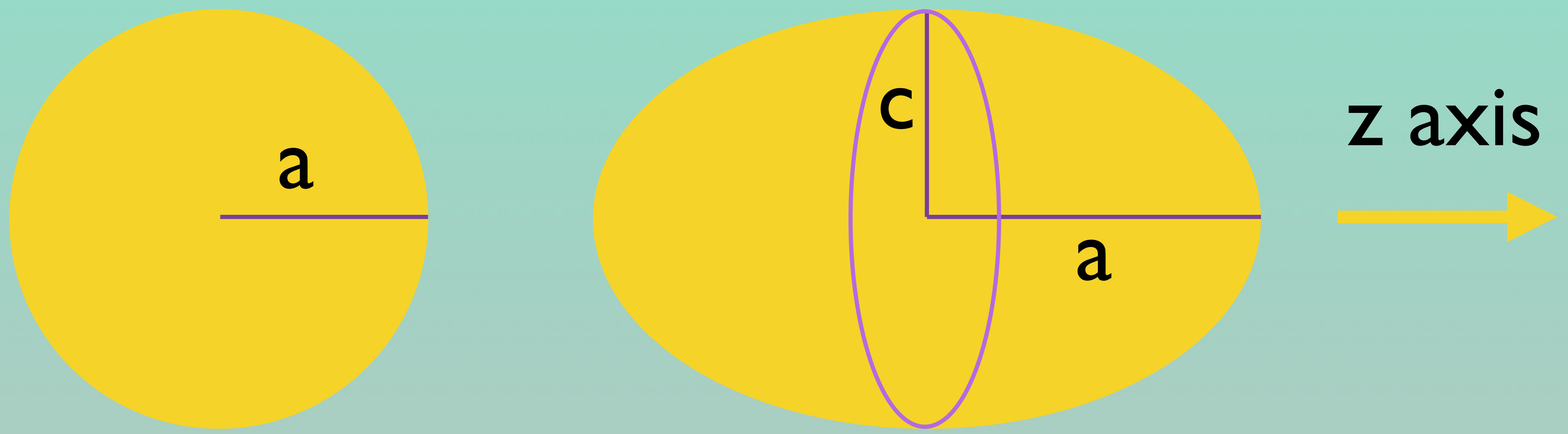
- Observables become expectation values of operators
- Magnetic Dipole Moment

$$\vec{\mu}(\vec{r}) = \frac{1}{2}\int d^3r' \; \vec{r'} \times \vec{j}(r')$$

$$\mu(I) = \langle I, m = I | \mu_z | I, m = I \rangle$$

The Electric Quadrupole

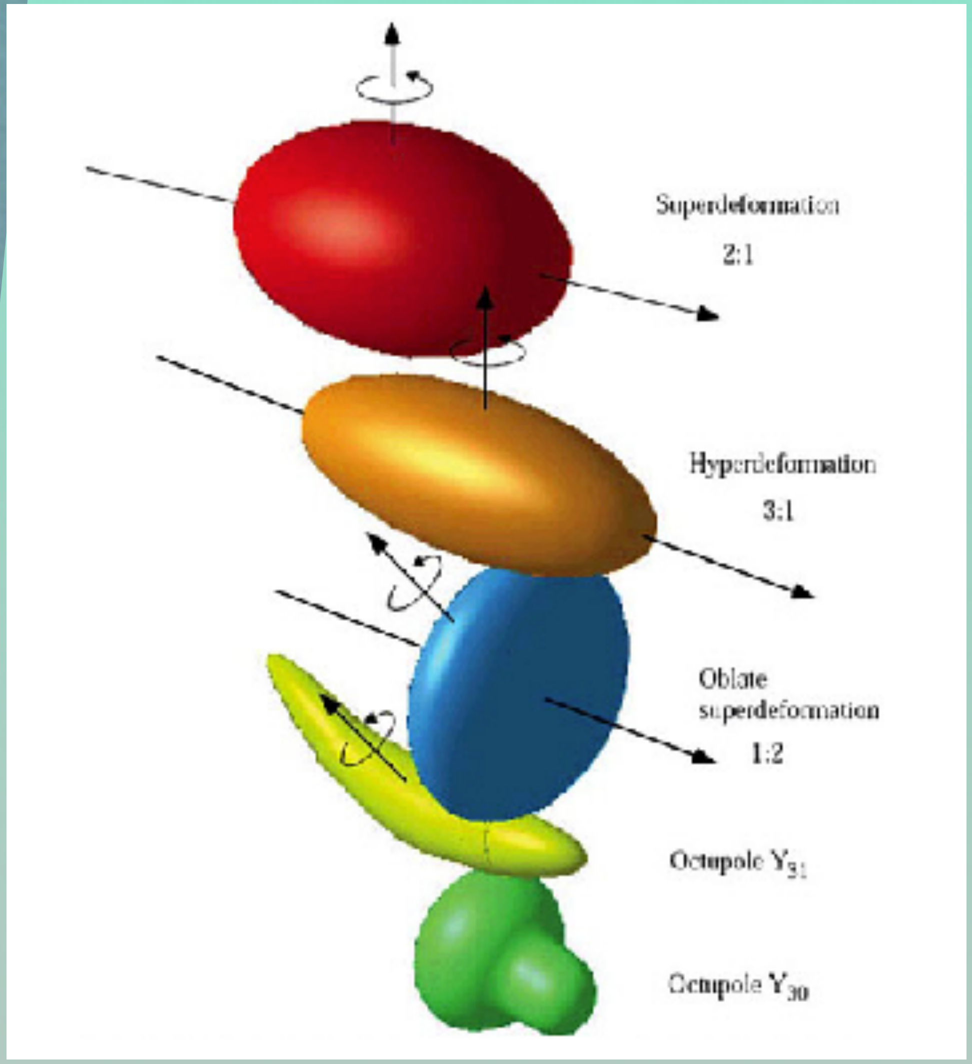
- A deviation from the spherical shape of the nucleus in one direction results in an inhomogeneous charge distribution

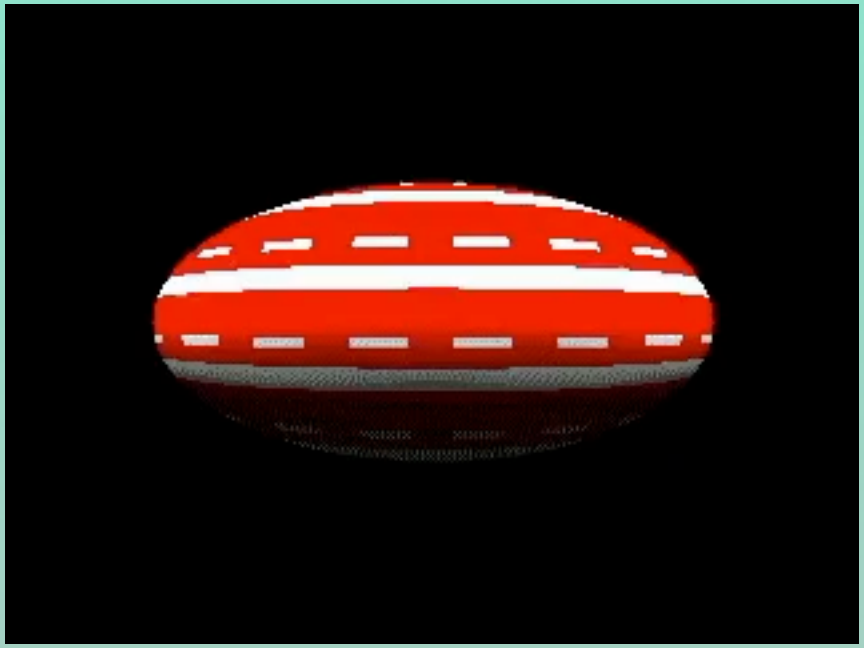


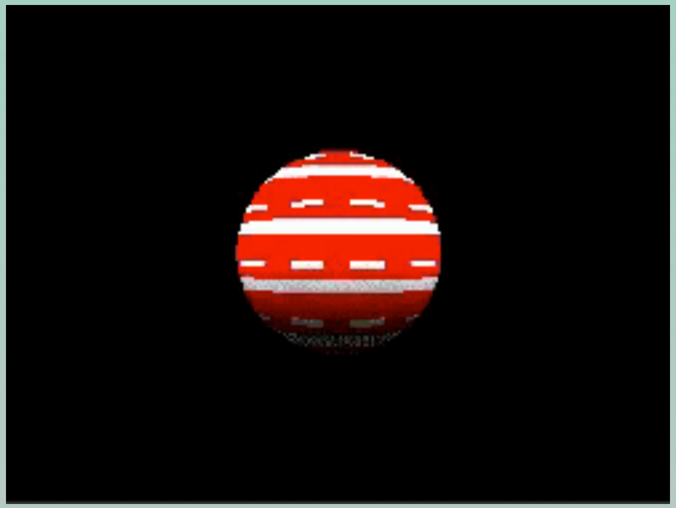
$$Q = \left(\frac{2Ze}{5}\right)\left(a^2 - c^2\right)$$

Q and shapes

- Q is a direct probe of the nuclear shape

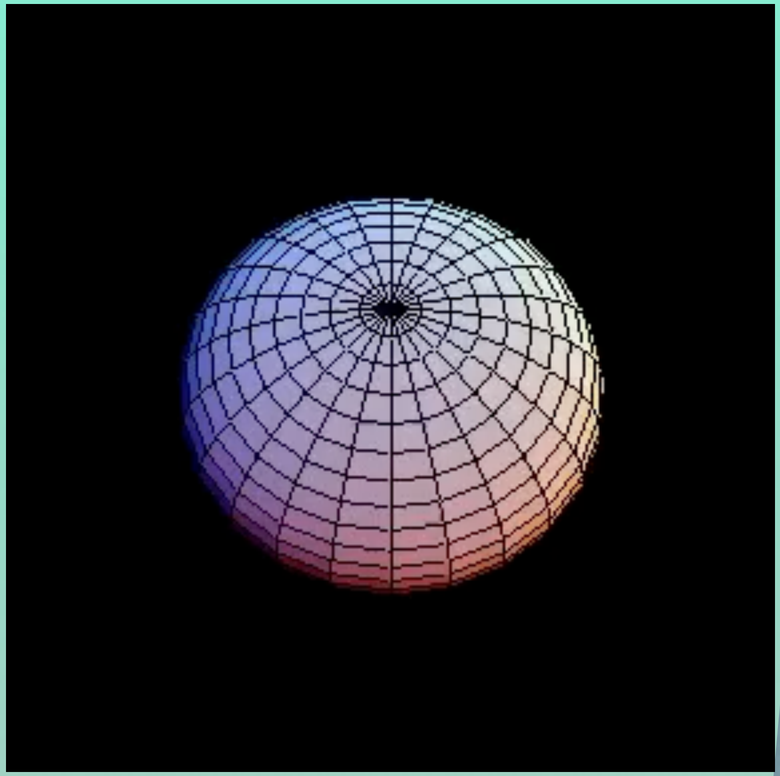


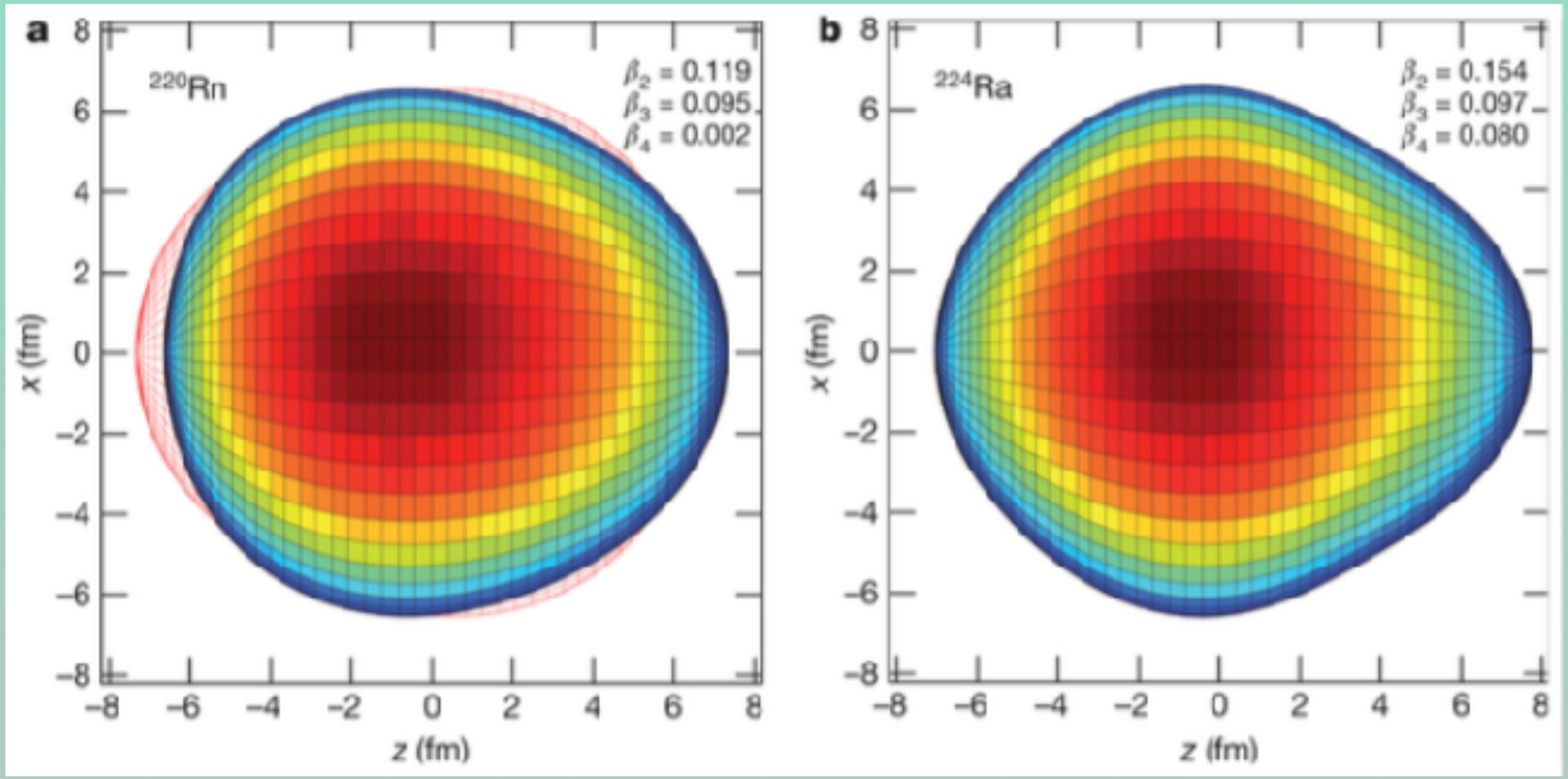




Higher-Order Moments

- Are there any other moments?
- Can we measure them?





Nature 497, 199 (2013)

The Anomalous μ

First seen with electrons

$$g_e = 2(1 + \alpha/2\pi)$$

We know why!

Important for nuclear magnetic moments

Can you guess the cause of the anomalous μ?

Adopted values

$$\mu(proton) = +2.792847356(23)\ \mu_N$$

$$\mu(neutron) = -1.9130427(5)\ \mu_N$$

Particle Data Group
Chin. Phys. C, 38, 090001 (2014) and 2015 update

The distinction in both ***sign*** and ***magnitude*** is of great importance

The *g* factor

- The magnetic moment can be directly connected to the spin of the level, *J* (units of μ_n)

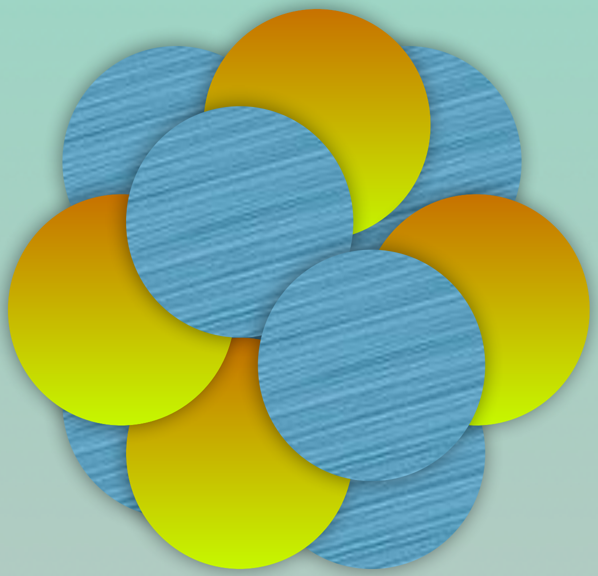
$$\vec{\mu} = g\vec{J}$$

$$\vec{\mu} = g_l\vec{l} + g_s\vec{s}$$

Generalize to A nucleons

- The magnetic moment is a one-body operator
- It can be easily expanded to a system of A nucleons

$$\vec{\mu} = \sum_{i=1}^{A} g_l^i l^i + \sum_{i=1}^{A} g_s^i s^i$$



The *g* factor of proton

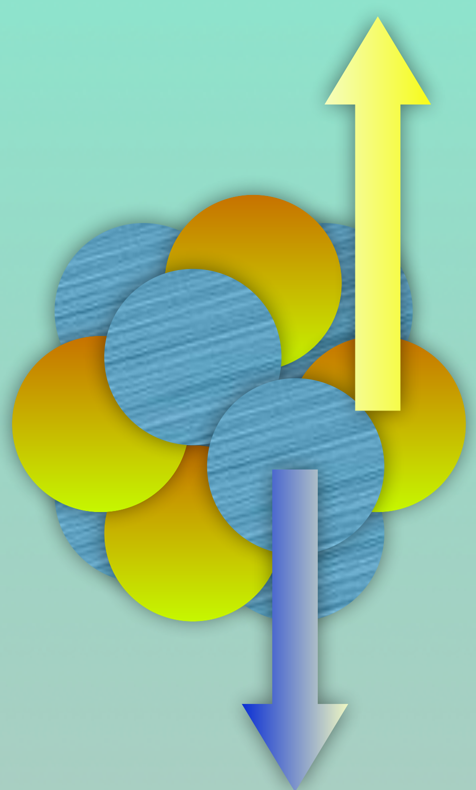
- For a perfectly charged sphere: $g_s = 1$
- If proton is a Dirac particle: $g_s = 2$

However...

$$g_s = +5.587$$

NO IDEA WHY!

Nucleonic System



Important Detail:

Protons and Neutrons have different g-factor values

	g_l	g_s
proton	1	+5.587
neutron	0	-3.826

The Deuteron μ

OCTOBER 15, 1939 PHYSICAL REVIEW VOLUME 56

The Magnetic Moments of the Proton and the Deuteron

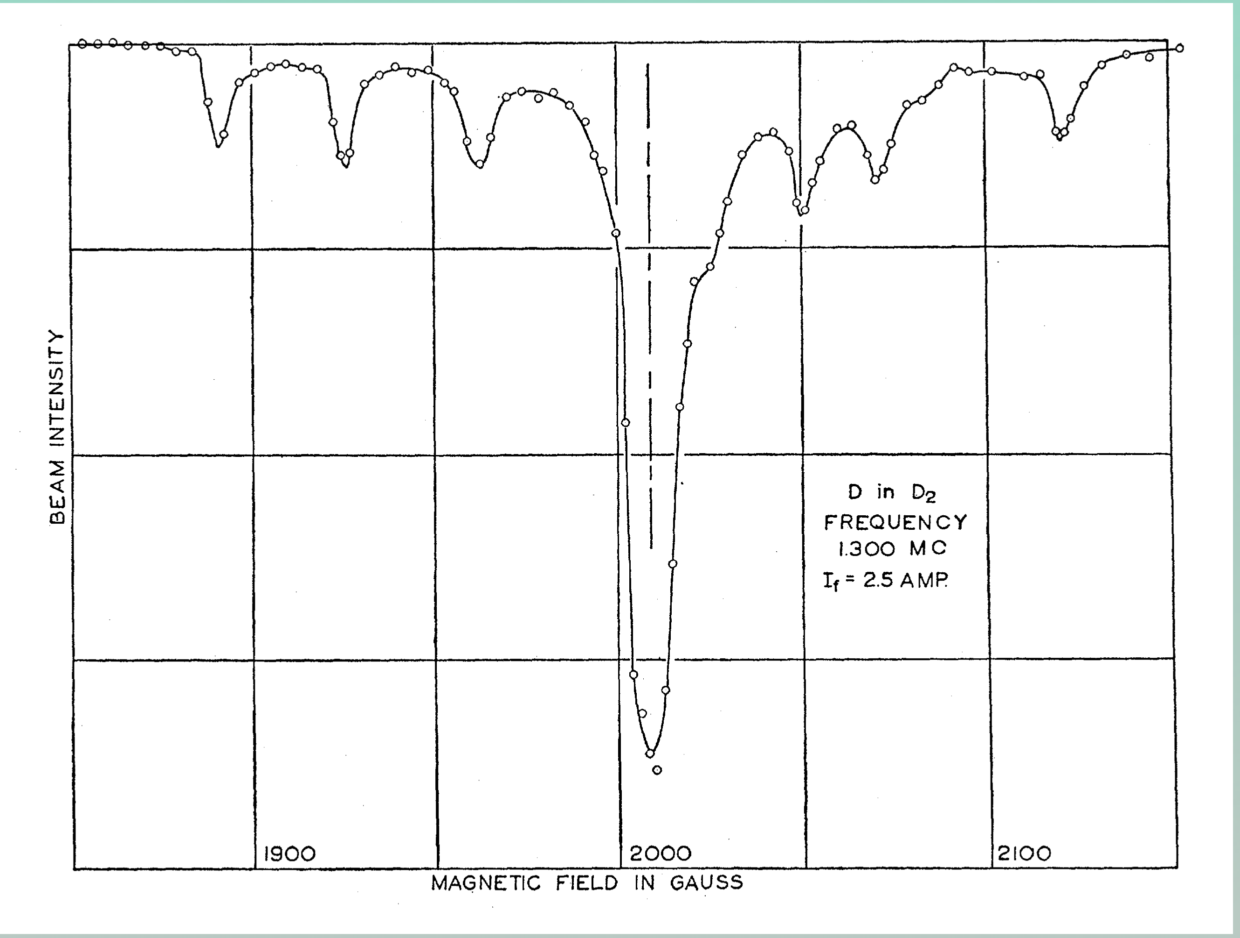
The Radiofrequency Spectrum of H_2 in Various Magnetic Fields *

J. M. B. KELLOGG, I. I. RABI AND N. F. RAMSEY, JR.
Columbia University, New York, New York

AND

J. R. ZACHARIAS
Hunter College, New York, New York

(Received July 31, 1939)



Non-Additivity

- From experimental deuteron data we know:

$\mu_p = +2.792847356(23)$
$\mu_n = -1.9130427(5)$
$\mu_{pn} = \mu_p + \mu_n = +0.8798046(5)$
$\mu_D\ (exp) = +0.857438240(12)$

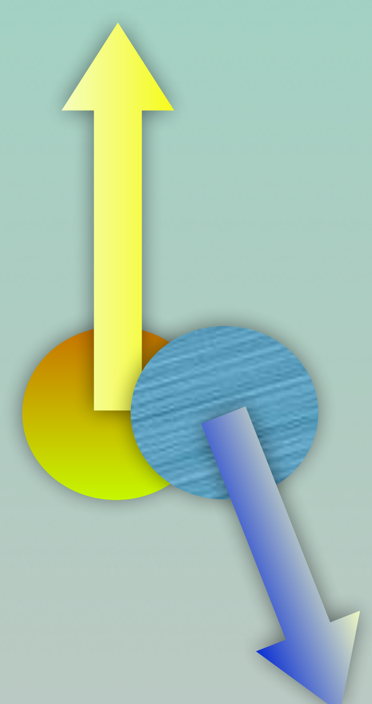
- So the deuteron is **NOT** exactly a proton and a neutron (in terms of the w.f.)

$$\mu(D) \neq \mu(p) + \mu(n)$$

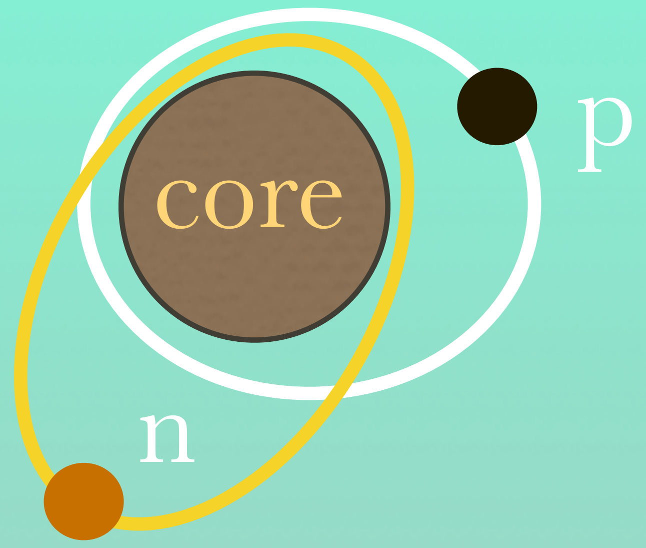
Addition Theorem

- We can add moments (or g's) using vector analysis:

$$g\,(I) = \frac{1}{2}\left[(g_1 + g_2) + (g_1 - g_2)\,\frac{I_1(I_1+1) - I_2(I_2+1)}{I(I+1)}\right]$$



Illations



- Use $l_1 = l_2 = l$

$$g(j) = g(I)$$

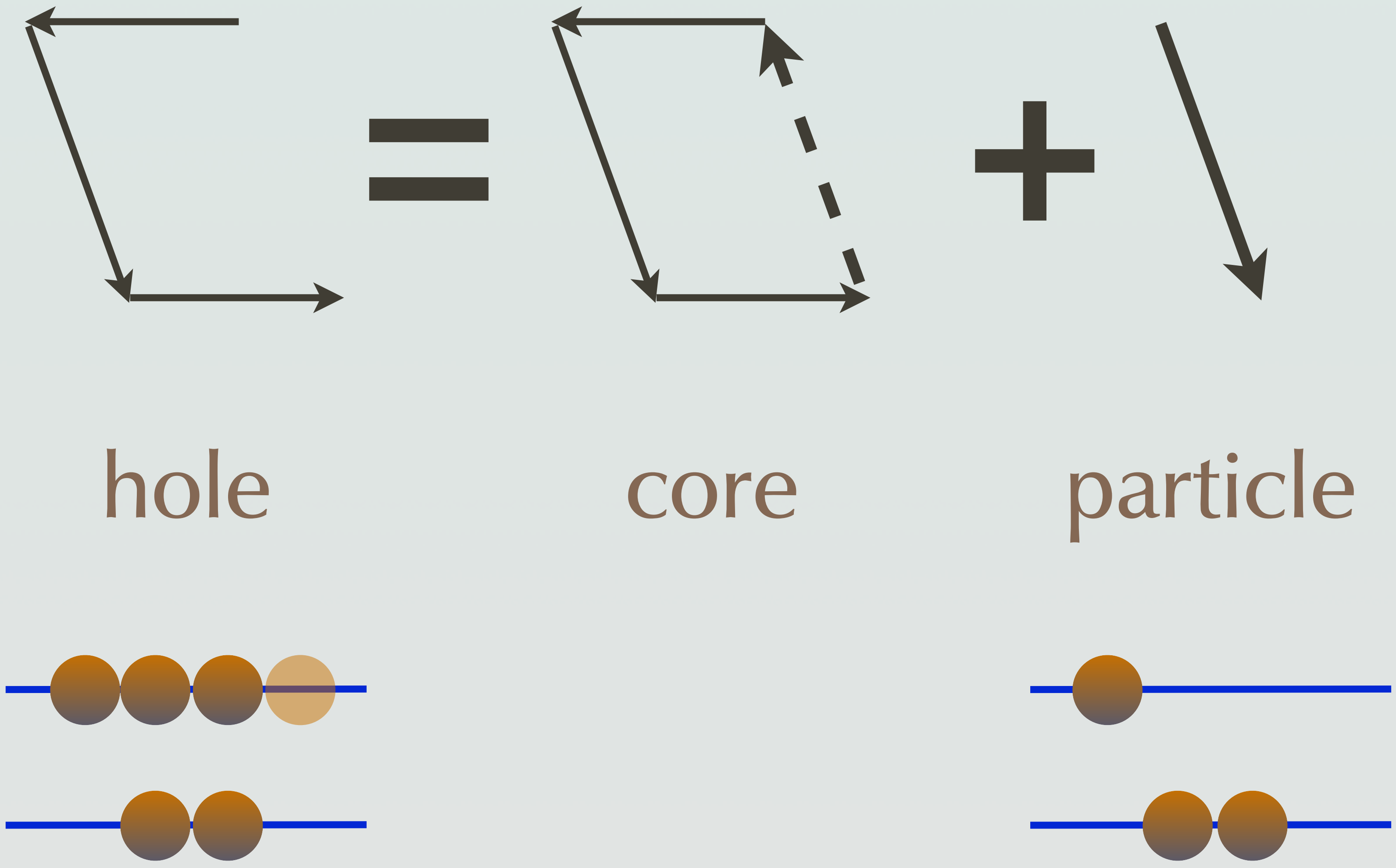
- The result may be generalized for **N** nucleons
- We may apply this for the case of **L** and **S** degrees of freedom of an individual nucleon, e.g. $g_l + g_s$ of proton

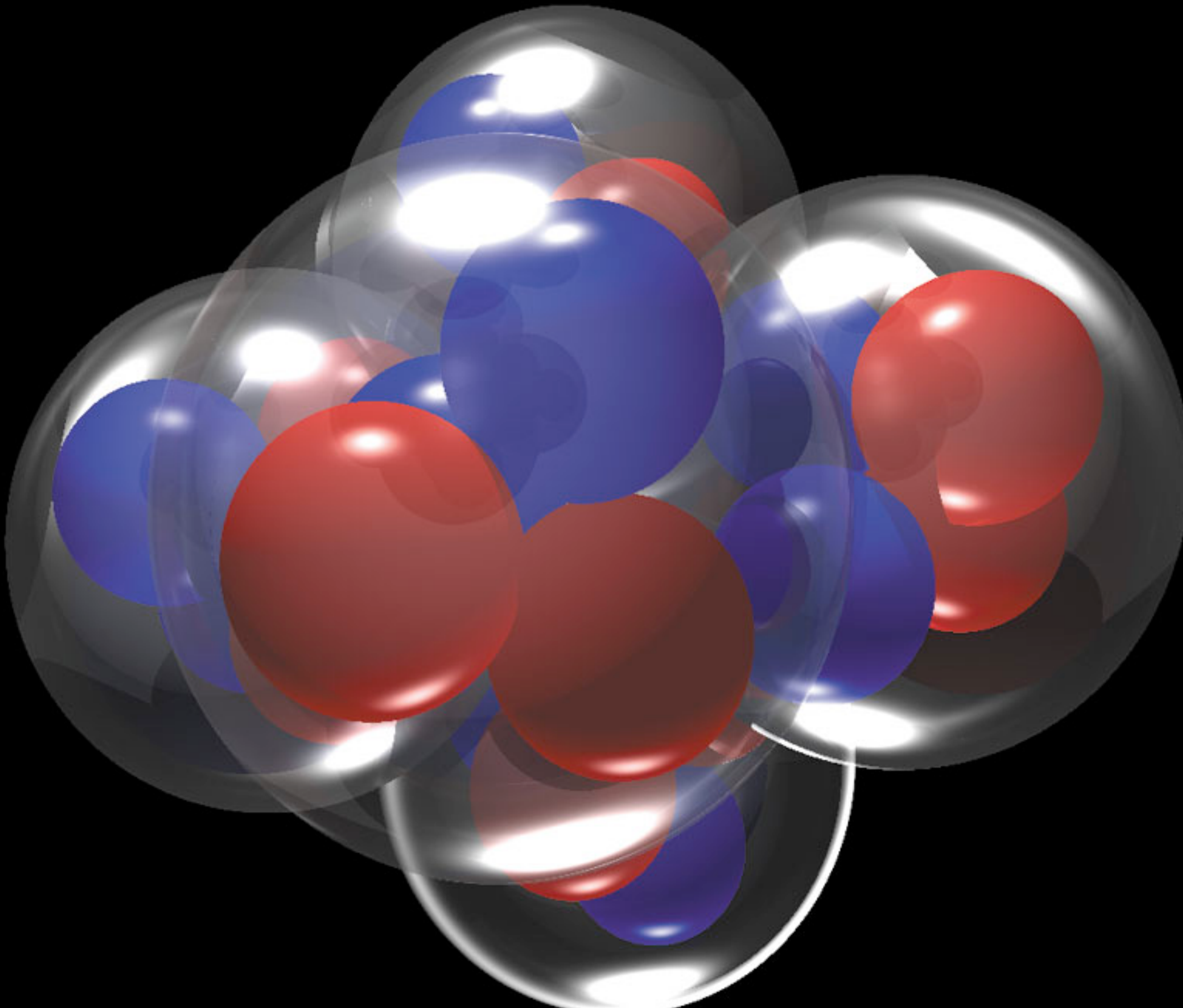
Direct application

- In a series of isotopes (or isotones), the spin of a certain state may be determined by simply measuring the g factor (exotic nuclei!)
- Within one nucleus, the g factor is a very sensitive tool to check whether the configuration within a sequence of spin-states (0,2,4,6,8,...), produced by the gradual alignment of two identical nucleons, is pure down to the lowest excitation energy.

Holes vs. Particles

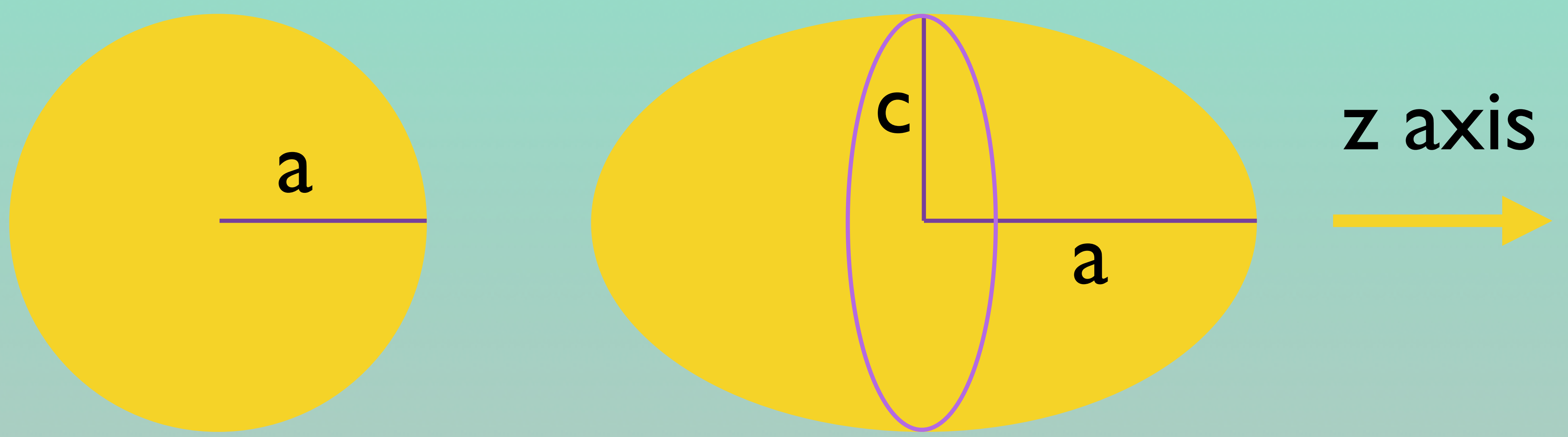
A particle is... a hole!





The Electric Quadrupole

- A deviation from the spherical shape of the nucleus in one direction results in an inhomogeneous charge distribution



$$Q = \left(\frac{2Ze}{5}\right)\left(a^2 - c^2\right)$$

Dynamical effects

Meson-current exchange

Core Polarization

Tensor Effects

Free vs. effective

- In the dynamic nuclear environment, the bare values change

$$g_s^{eff}(p,n) \approx 0.75 \cdot g_s^{free}(p,n)$$

- and for p, n respectively:

$$g_l^{eff} \approx 1.1 \quad or \quad -0.1$$

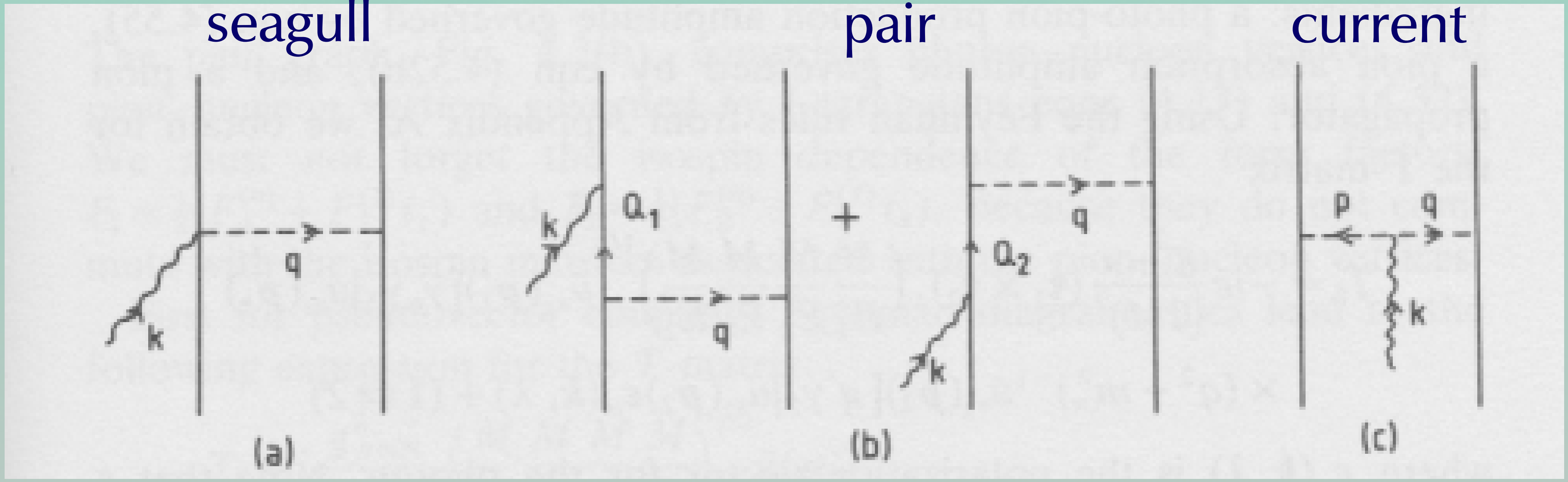
- pion clouds are mainly responsible for the alteration

Core polarization

- The closed shells seize to be inert and p-h excitations are allowed
- Coupling between the core and the valence nucleons alter the matrix elements
- Corrections may be significant

Meson Exchange Currents

- There are effective interactions due to pion exchange
- In a more fundamental picture, quark currents are responsible for the effective field
- Contributions are typically ~10%



Tensor effects

- The magnetic moment is a rank-1 tensor by construction
- In case coupling 2-body or 3-body operators, the tensor effects become significant for the expectation values

How do models treat moments?

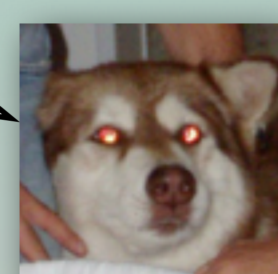
- Liquid-Drop Model
- Collective Models (rotational etc)
- Shell Models
- ...

Liquid-Drop Model

- Simplified picture: A lump of protons and neutrons (indistinguishable)
- protons carry the charge
- neutrons contribute only to the volume
- A simplified prediction:

$$g = \frac{Z}{A}$$

Can you prove it?



Quick proof

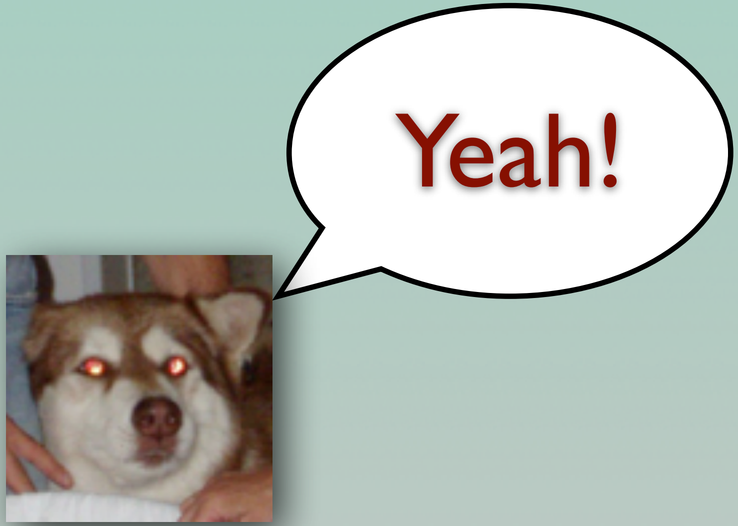
$$\vec{\mu} = \frac{1}{2c}\int d^3r\, \vec{r}\times\vec{j}(\vec{r}) = \frac{1}{2c}\int d^3r\, \vec{r}\times\rho\vec{v}$$

$\rho = (Ze)/V = (Ze)/(Am/d_m)$

$$\begin{aligned}\vec{\mu} &= \frac{1}{2c}\int d^3r\, \vec{r}\times\rho\vec{v} \\ &= \frac{Ze}{2Amc}\int d^3r\, \vec{r}\times d_m\vec{v} \\ &= \frac{\mu_N}{\hbar}\left(\frac{Z}{A}\right)\int d^3r\, d_m\vec{r}\times\vec{v}\end{aligned}$$

$$\mu = \frac{\mu_N}{\hbar}\left(\frac{Z}{A}\right)J$$

$$g = \frac{Z}{A}$$

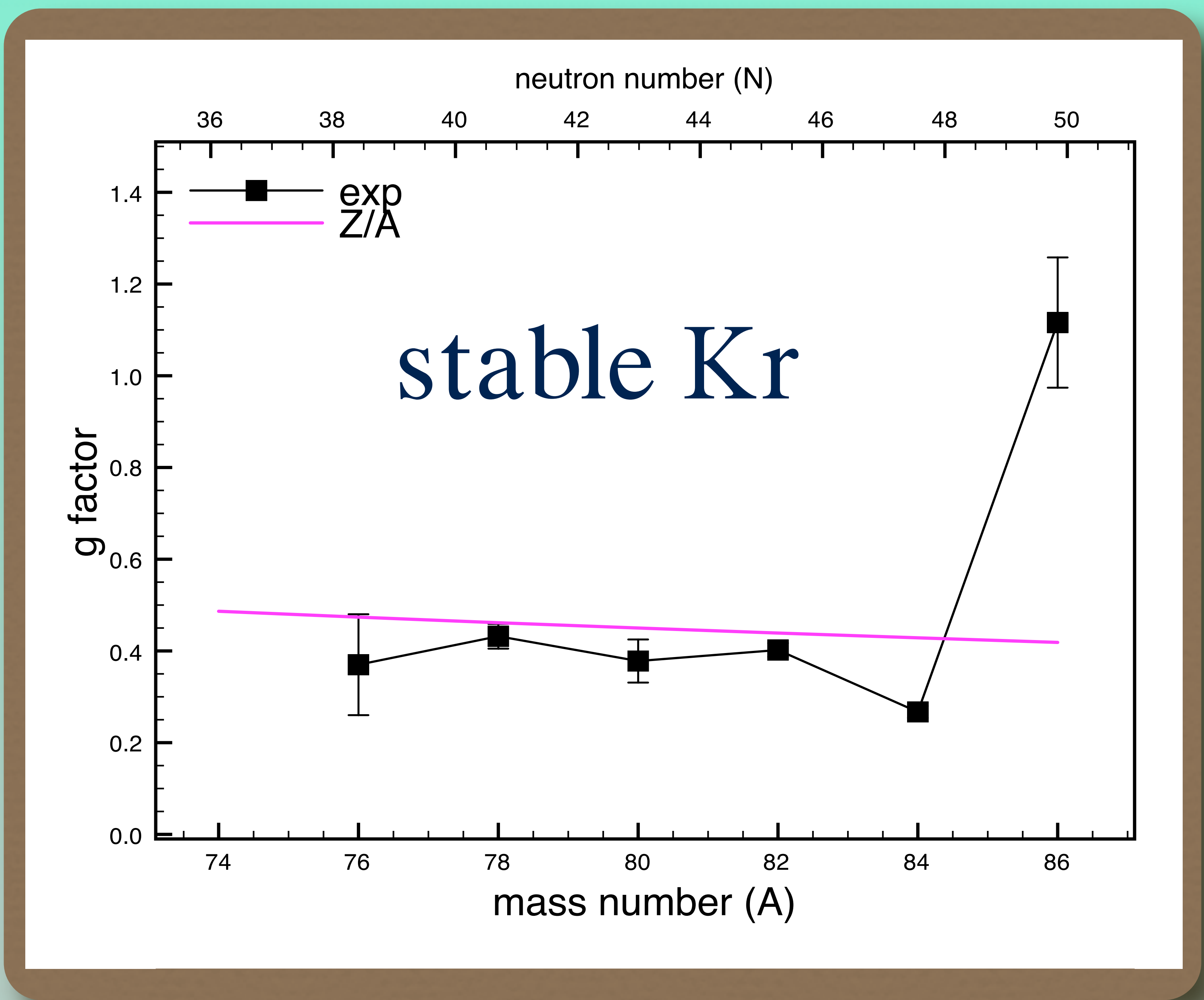


Does It Work?

- For collective states, it usually does
- However, most levels deviate significantly
- Mainly responsible for those deviations are shell effects that break collectivity

- Of Great Value: starting point to look
- Misconception: All g's are Z/A ...

Are they all =Z/A?



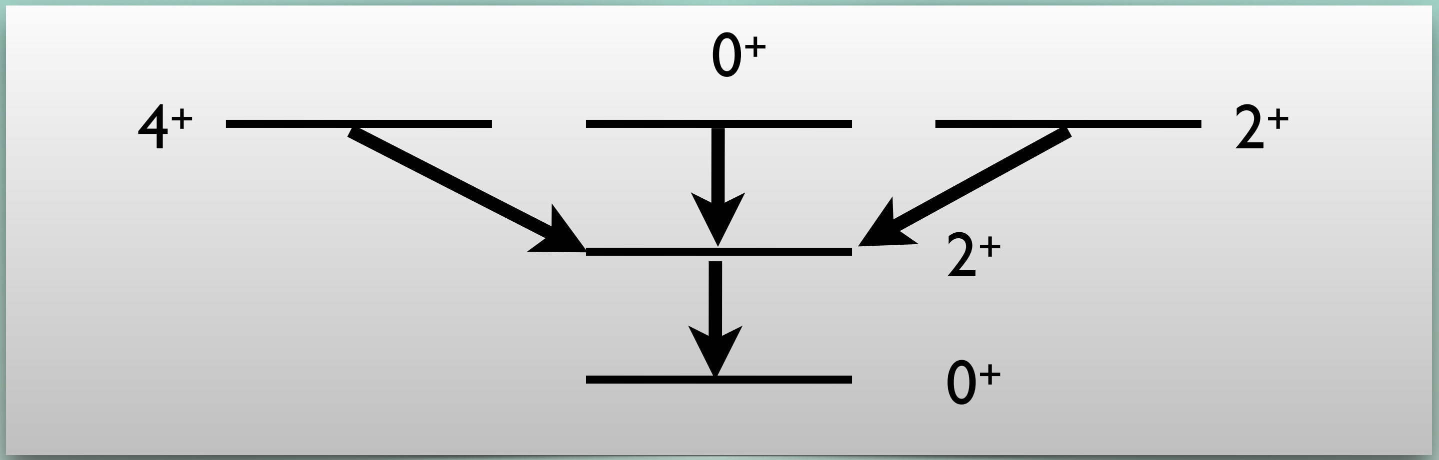
doi: 10.1103/PhysRevC.64.024314

Collective models

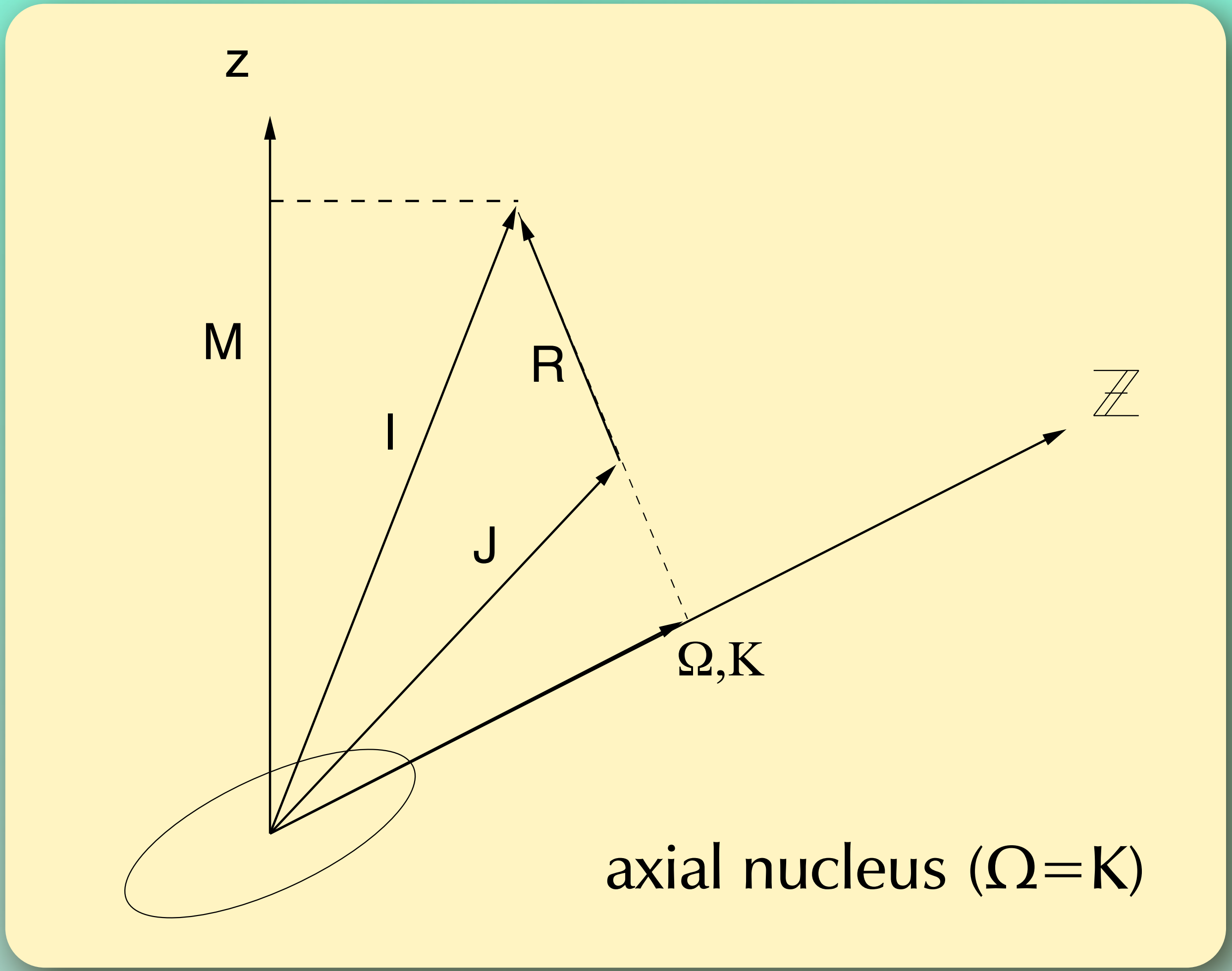
- Collectivity is not the best playground for the magnetic moment
- The observable is rather insensitive
- Best operator is probably the electric quadrupole moment, Q

Vibrational

- One-phonon states
- Two-phonon states
- Prediction falls in the Z/A value
- Application to vibrational nuclei e.g. Cd or Pd



Rotational



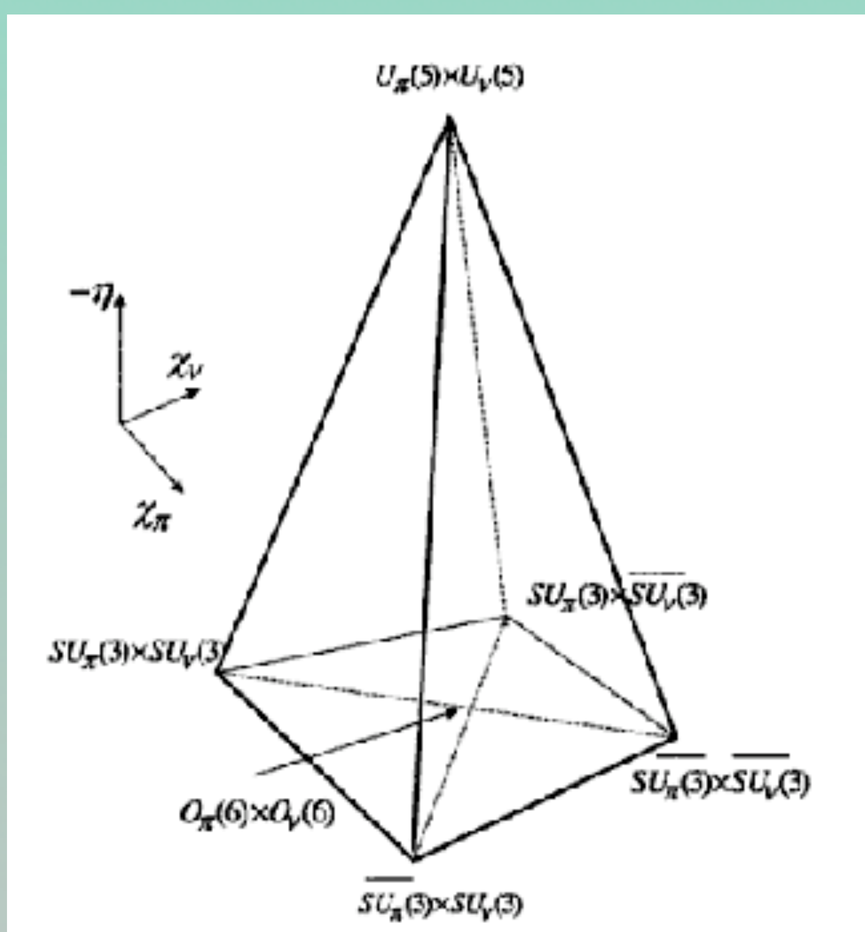
- Intrinsic frame

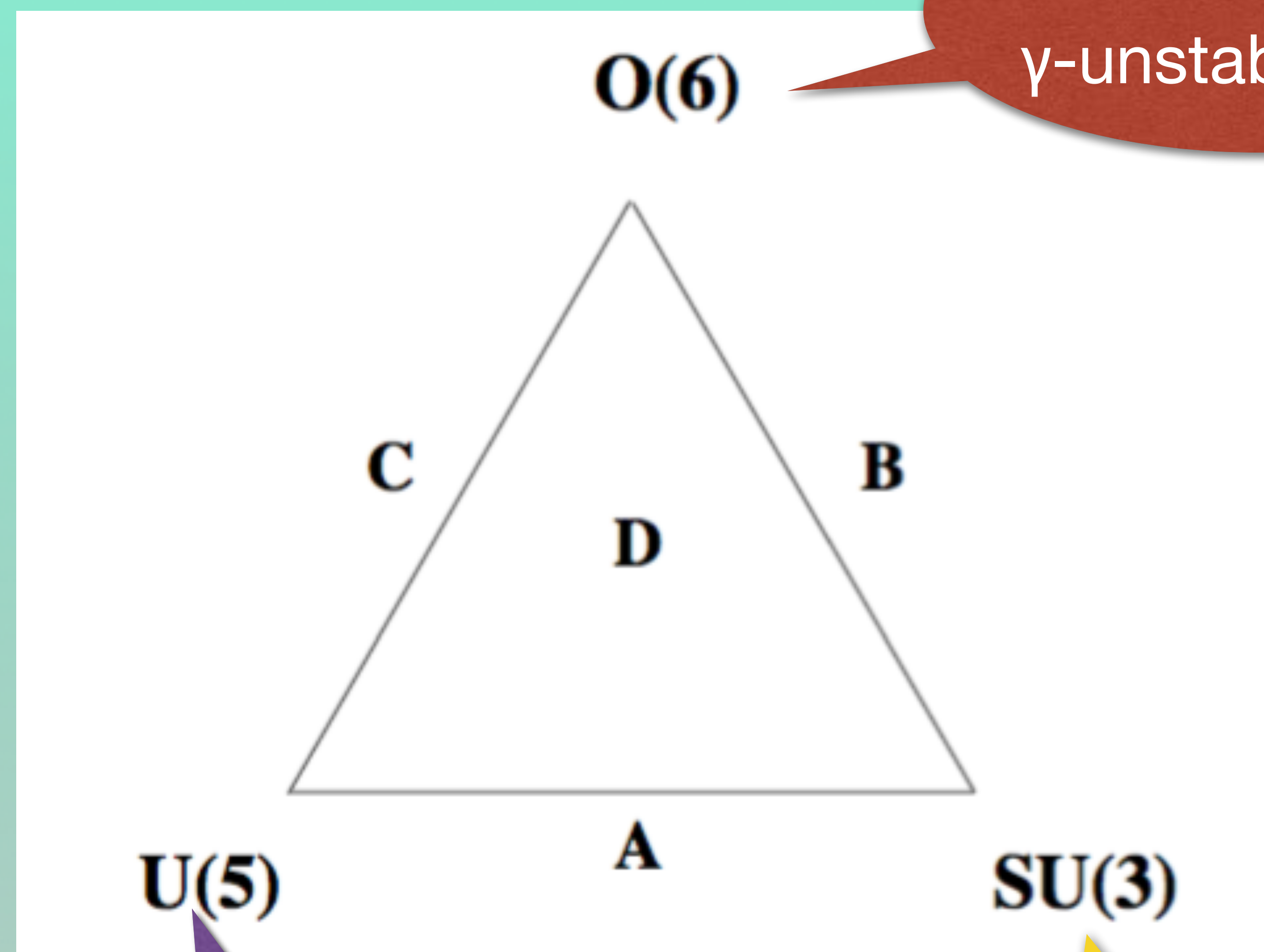
$$\mu = g_\Omega \Omega = \langle \Psi | \sum g_l l_z + g_s s_z | \Psi \rangle$$

- Lab frame (take into account the nuclear rotation)

$$\mu = g_R I + (g_\Omega - g_R) \frac{\Omega^2}{I+1}$$

Algebraic Models





deformed
γ-unstable rotor

spherical anharmonic quadrupole vibrator

deformed axially symmetric rotor

IBA-I

2+ STATES

- In IBA-I, the lowest-order transition operator may be expressed as:

$$T_{1\mu}(M) = \beta_1 \left[d^\dagger \otimes \tilde{d}\right]^{(1)}_\mu$$

$$T_{2\mu}(E) = \alpha_2 \left[d^\dagger \otimes \tilde{s} + s^\dagger \otimes \tilde{d}\right]^{(2)}_\mu + \beta_2 \left[d^\dagger \otimes \tilde{d}\right]^{(2)}_\mu$$

- And in terms of the angular momentum L:

$$T_{1\mu} = \left(\frac{3}{4\pi}\right)^{1/2} g_B L_\mu$$

- For all limits in the Casten triangle

$$\mu = \left(\frac{4\pi}{3}\right)^{1/2} \langle L, M_L = L | T_{10}(M) | L, M_L = L\rangle = g_B L$$

IBA-II

- In IBA-II, the transition operator distinguished between protons and neutrons

$$T_1(M1) = \sqrt{\frac{3}{4\pi}}(g_\pi L_\pi + g_\nu L_\nu)$$

- The g factor in IBA-II is:

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu}$$

- If $g_\pi=1$ and $g_\nu=0$ then $g=Z/A$

Single-Particle

Assume closed shells

+

odd (l,s) nucleon

Trend along the nuclear chart

Schmidt limits

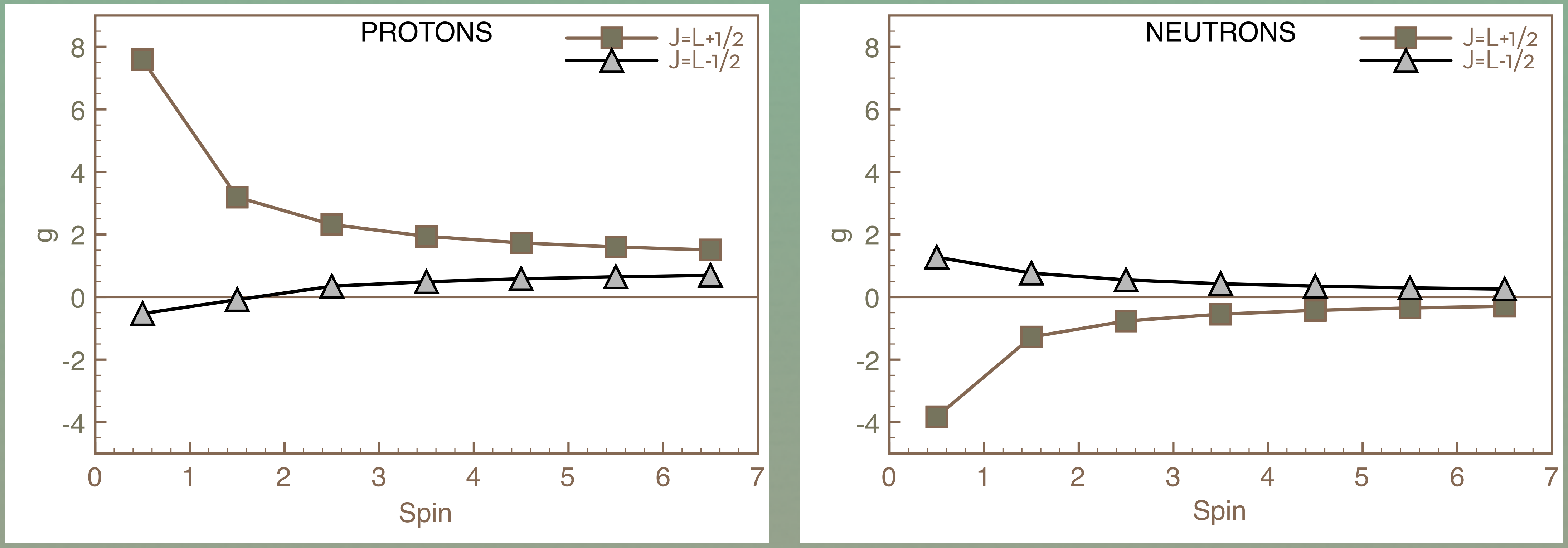
Schmidt limits

- From the addition properties, if one couples an odd nucleon with the even core

$$g_j = g_l \pm \frac{g_s - g_l}{2l+1}, \quad j = l \pm \frac{1}{2}$$

- However, there are deviations from these values throughout the nuclear chart

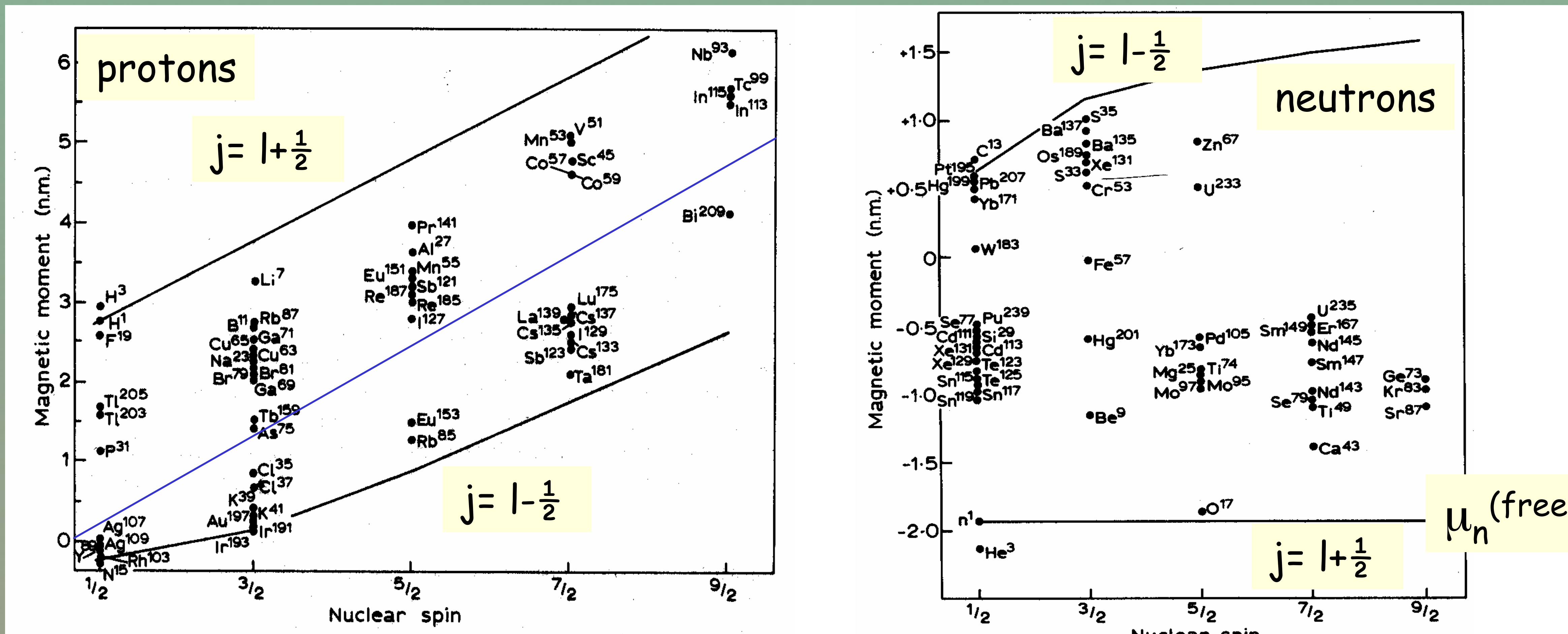
Schmidt limits



$$\mu\left(l+\frac{1}{2}\right)=\left[(j-\frac{1}{2})g_l+\frac{1}{2}g_s\right]\mu_N$$

$$\mu\left(l-\frac{1}{2}\right)=\frac{j}{j+1}\left[(j+\frac{3}{2})g_l-\frac{1}{2}g_s\right]\mu_N$$

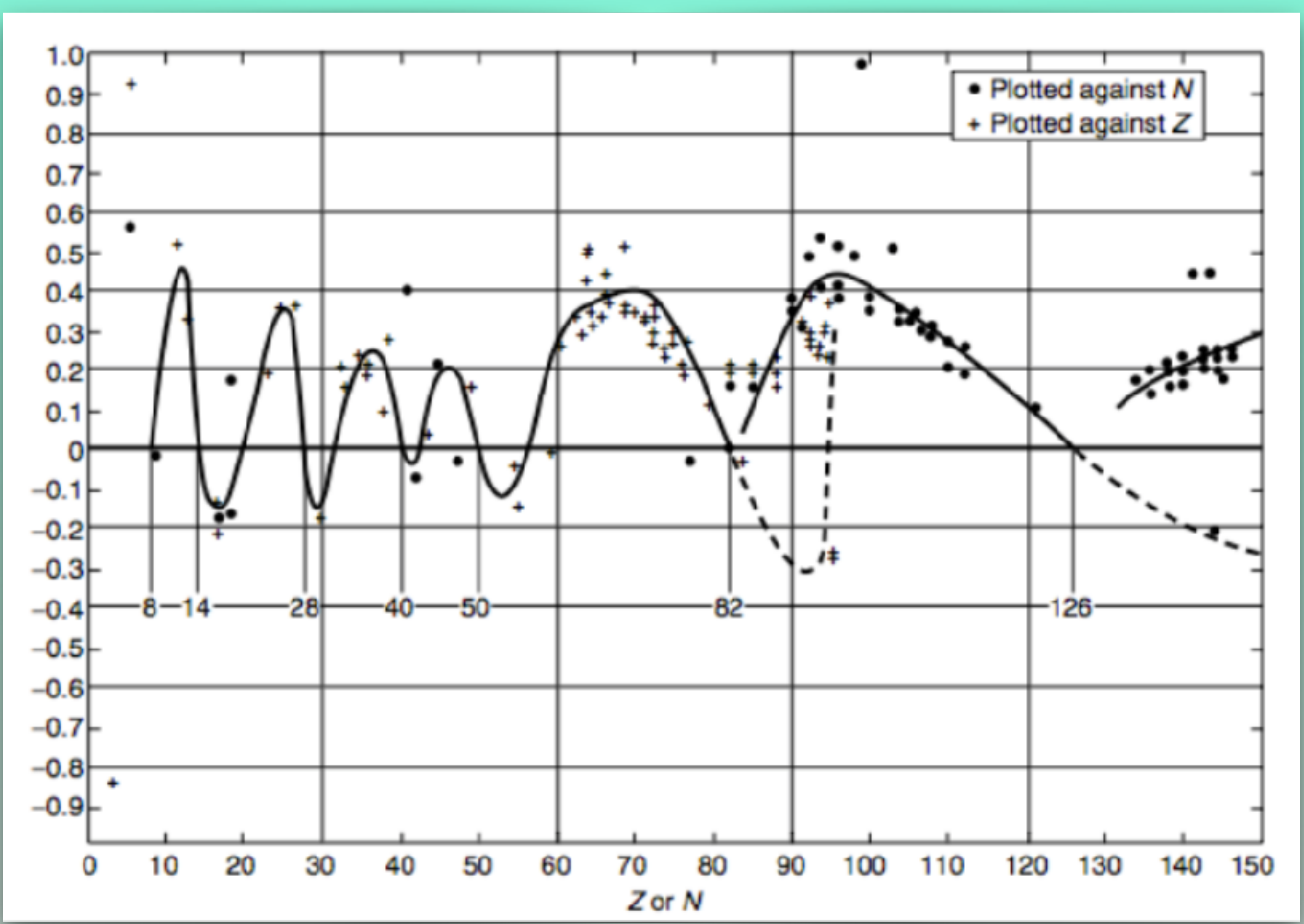
Experimental Data



Almost all data deviate from Schmidt limits.

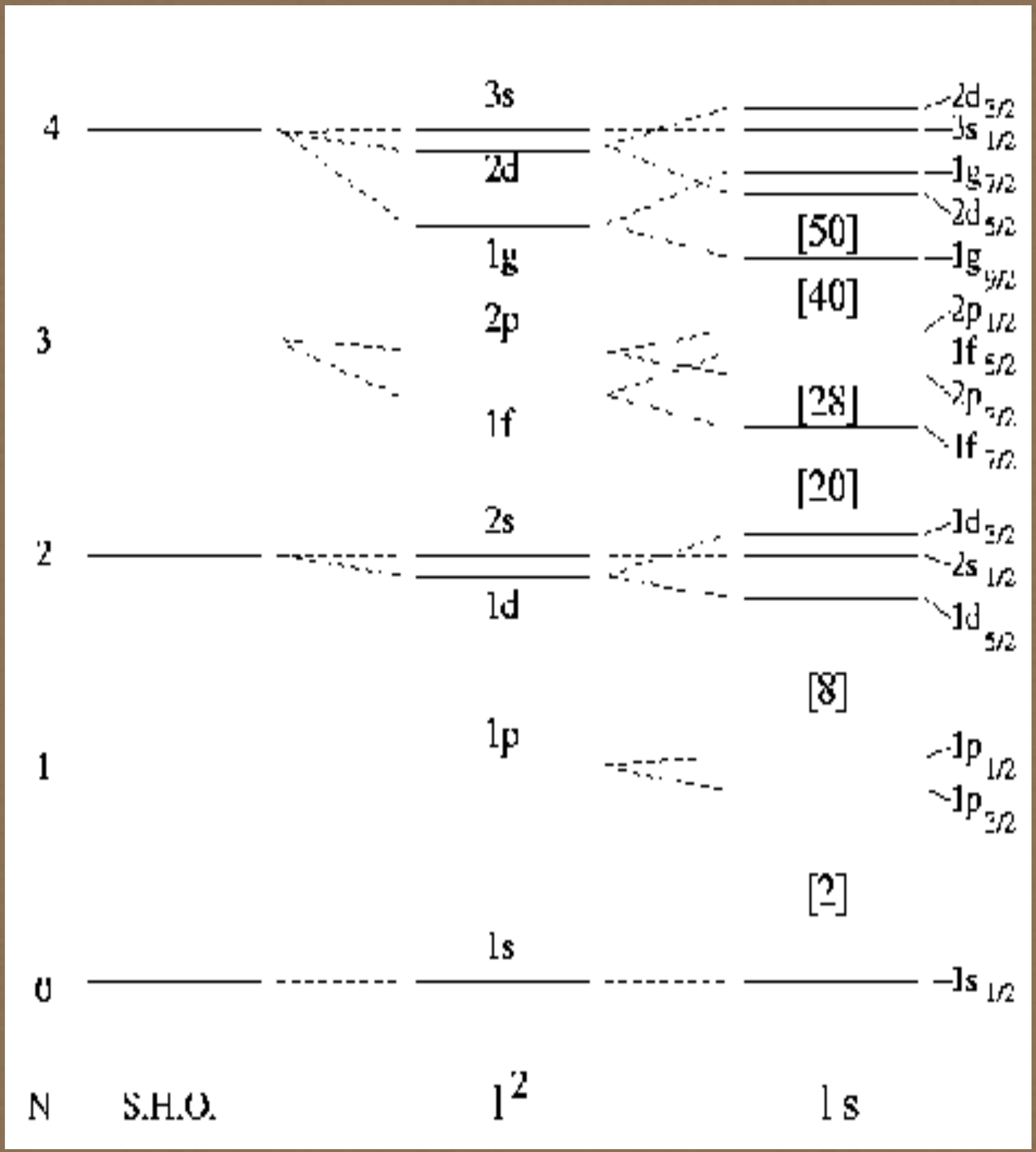
Almost all data deviate inwards

Q data



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
Z or N
8
14
28
40
50
82
126
Plotted against N
Plotted against Z

Single-particle orbits



For each energy level, a corresponding g factor may be predicted

It is interesting to study effects of coupling between different orbits

An online database for nuclear EM moments

- Official server (IAEA Nuclear Data Section):

http://www-nds.iaea.org/nuclearmoments

- Updated database:

https://magneticmoments.info

https://magneticmoments.info

Welcome to NUMOR, the Nuclear Moments and Charge Radii Database

A compilation of non-evaluated experimental data | Database cut-off date: 2019.03.31

Search	Periodic Table	Z-Helix	Particles
Blog	Cite the DB	Send us data	Help

you may search for (Z), (A) or (Z and A)

type Z

type A

Search Clear

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< **Ruthenium (Z=44)** >

Ru X-rays | Ru Atom Data | Ru Ionic Radius | Ru@NIH | Ru history | Ru@Wiki

^{93}Ru | ^{94}Ru | ^{95}Ru | ^{96}Ru | ^{97}Ru | ^{98}Ru | ^{99}Ru | ^{100}Ru | ^{101}Ru | ^{102}Ru
^{103}Ru | ^{104}Ru | ^{105}Ru | ^{106}Ru | ^{108}Ru | ^{109}Ru | ^{110}Ru | ^{112}Ru

isotope

Isotope	Mass Excess [keV]	Energy [keV]	$t_{1/2}$	Spin/Parity	μ [nm]	Q [b]	R [fm]	Ref. Std	Method	NSR keyword	doi	Comment
^{98}Ru	-88225 ± 6	0.	stable	0^+			4.4229(55)			2013AN02	10.1016/j.adt.2011.12.006	
		653.	5.9 ps	2^+	+0.82(6)				TF	2011CH23	10.1103/PhysRevC.32.1707	
					+0.94(6)				TF	2011TA06	10.1103/PhysRevC.32.1707	
					+0.8(6)				IMPAC	1974HU01	10.1103/PhysRevC.31.190	
						0.21(8) or -0.01(9) R			CER	1991HA04	10.1103/PhysRevC.43.2140	
						-0.20(9) or -0.01(9)			CER	1980LA01	10.1103/PhysRevC.32.532	
						-0.03(14)		[^{102}Ru 475	CER	1977MA41	10.1103/PhysRevC.32.532	

mass

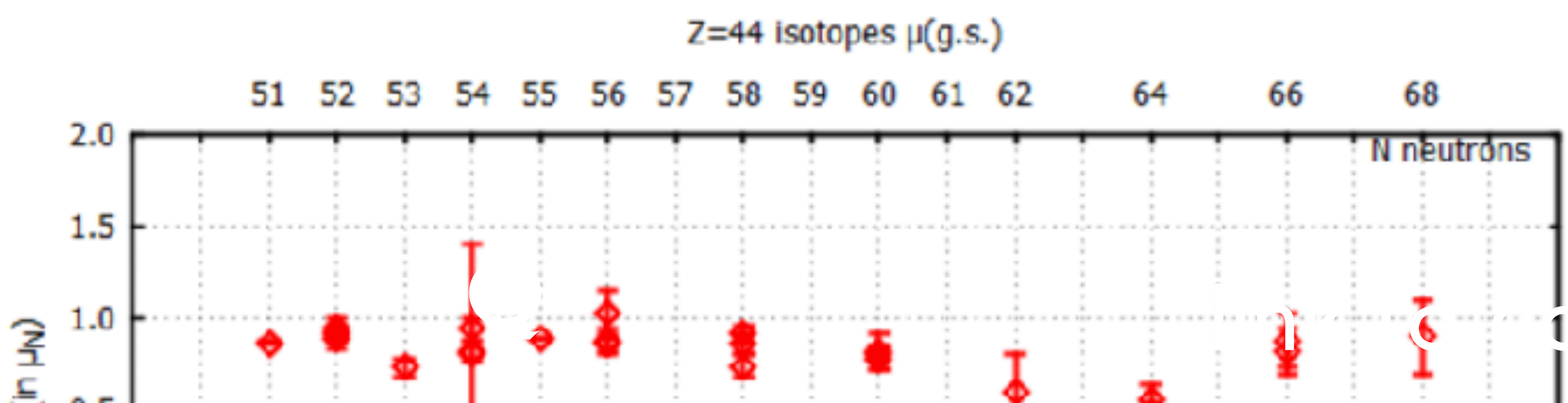
[▶InPACS] Shapes and deformations

μ

Q

radius

Plotting systematics is still in beta, use with caution.



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  - http://www-nds.iaea.org/nuclearmoments
- doi:
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  - R. Neugart & G. Neyens, 10.1007/3-540-33787-3_4
  - K.-H. Speidel et al., 10.1016/S0146-6410(02)00144-8
  - N. Benczer-Koller et al., 10.1088/0954-3899/34/9/R01
  - tjm, 10.1016/j.nima.2015.10.096
- More refs:
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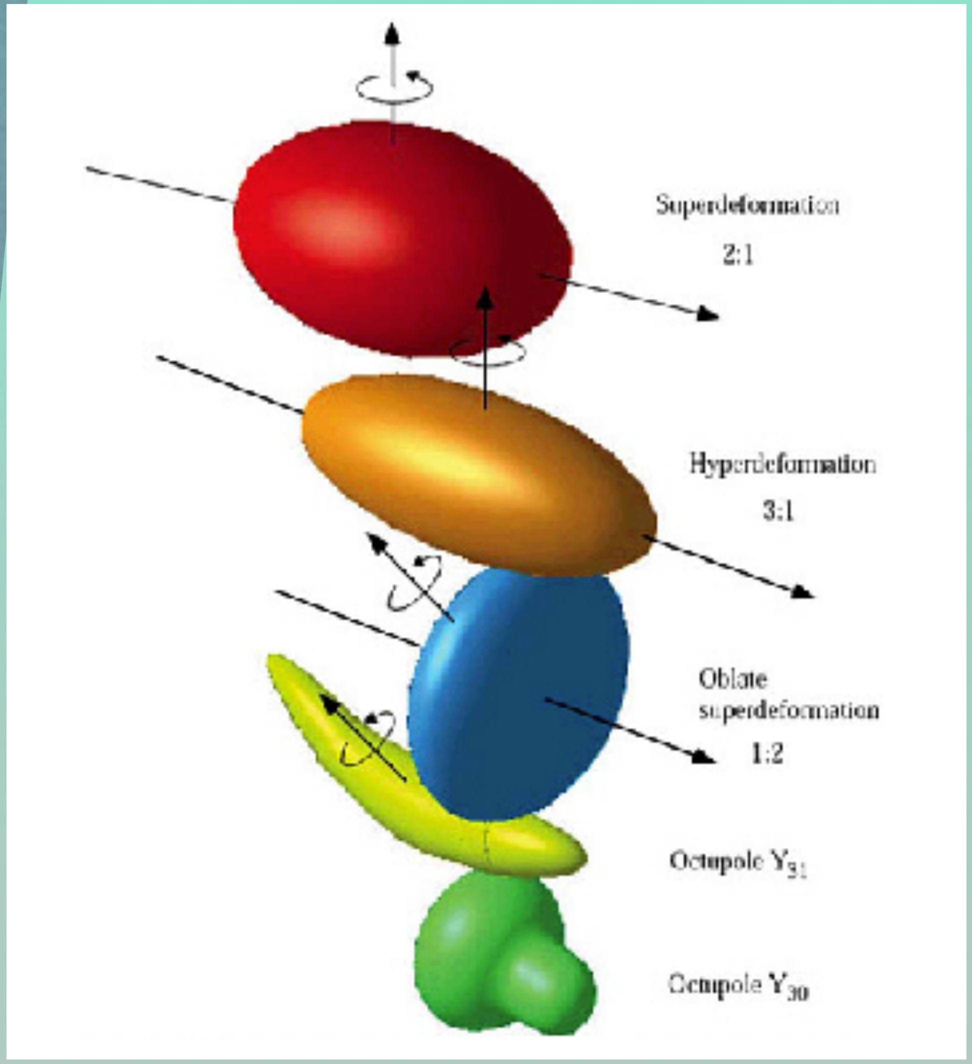
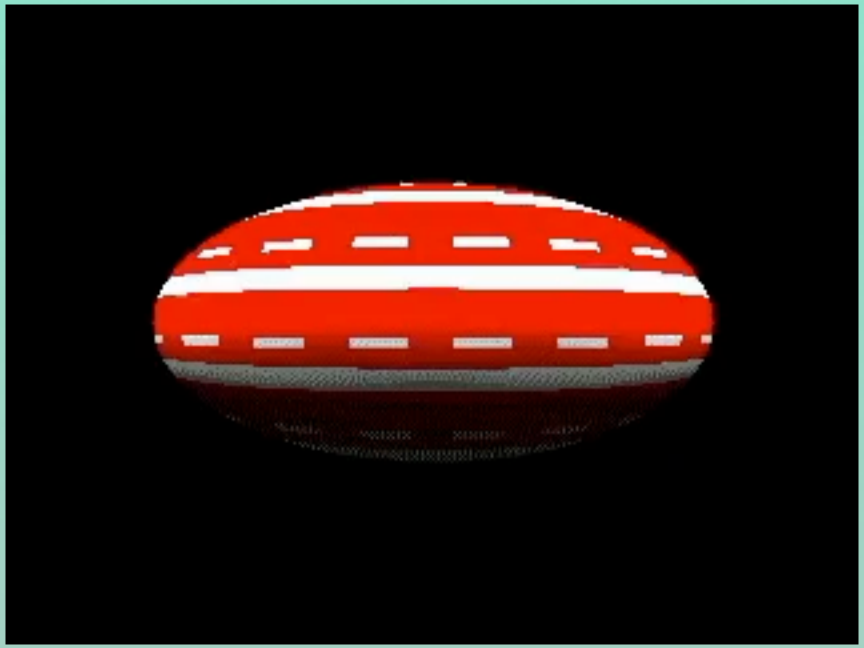
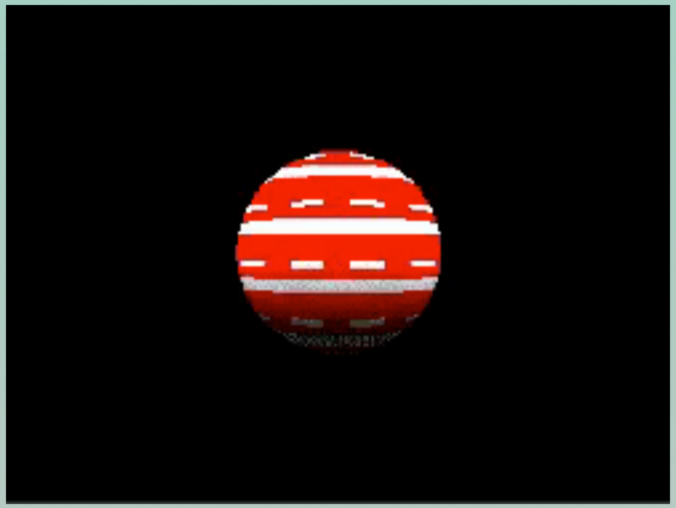
Q and shapes

- Q is a direct probe of the nuclear shape

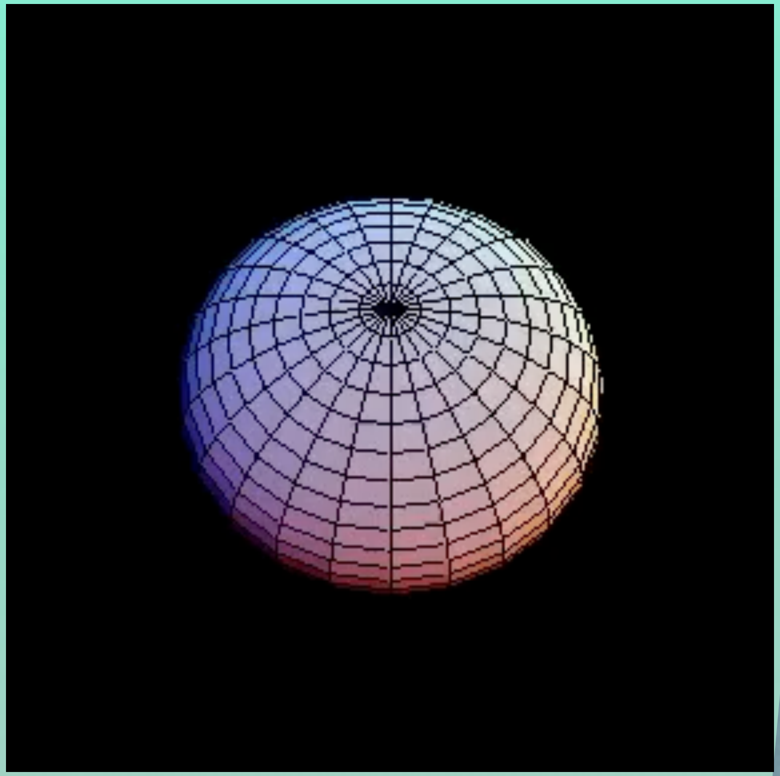


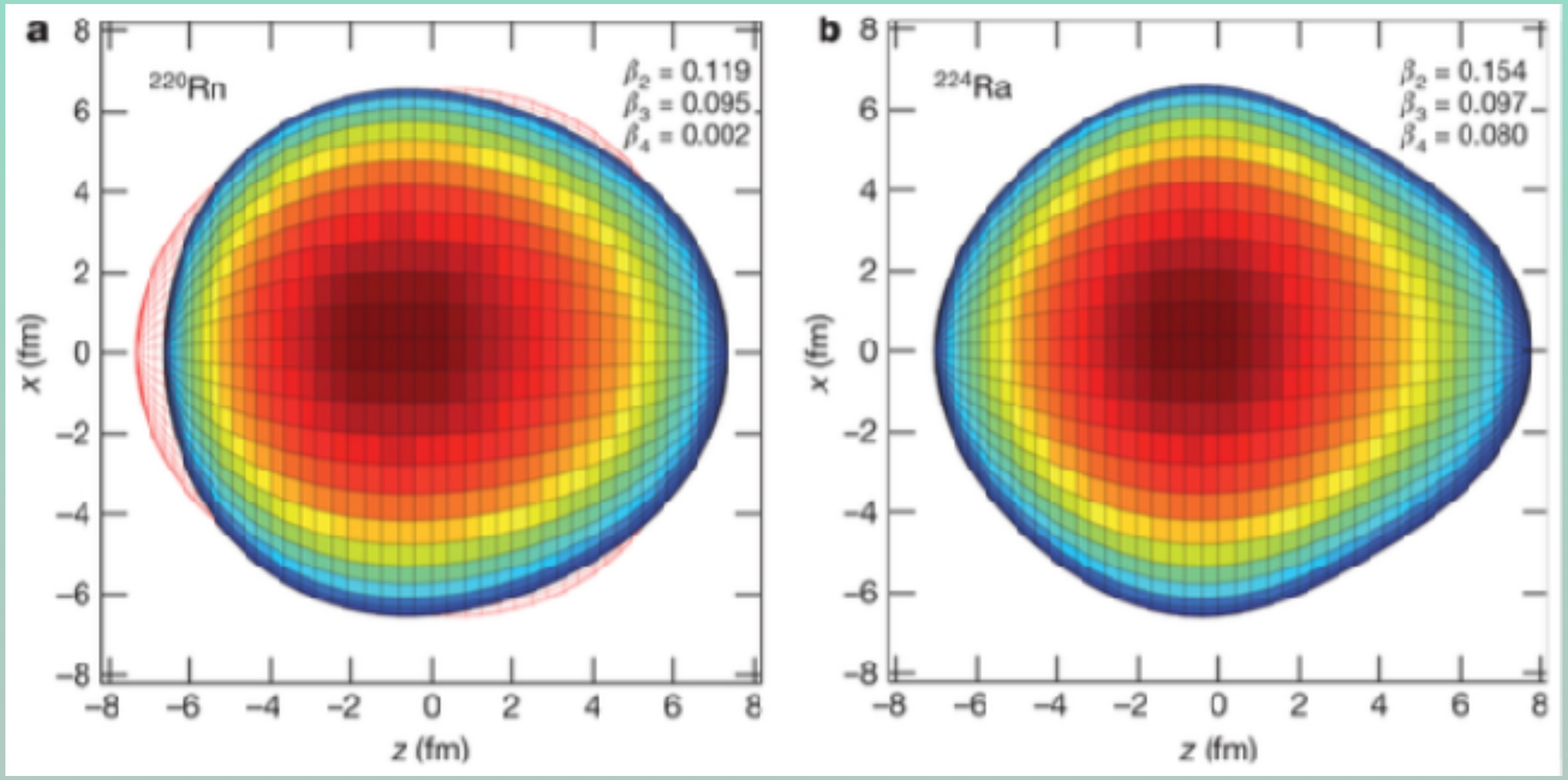




Higher-Order Moments

- Are there any other moments?
- Can we measure them?





Nature 497, 199 (2013)