16 The Nuclear Force

> Unfortunately, nuclear physics has not profited as much from analogy as has atomic physics. The reason seems to be that the nucleus is the domain of new and unfamiliar forces, for which men have not yet developed an intuitive feeling.
>
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The enormous richness of complex structures that we see all around us (molecules, crystals, amorphous materials) is due to chemical interactions. The short distance forces through which electrically neutral atoms interact can and do produce large scale structures.

The interatomic potential can generally be determined from spectroscopic data about molecular excited states and from measuring the binding energies with which atoms are tied together in chemical substances. These potentials can be quantitatively explained in non-relativistic quantum mechanics. We thus nowadays have a consistent picture of chemical binding based upon atomic structure.

The nuclear force is responsible for holding the nucleus together. This is an interaction between colourless nucleons and its range is of the same order of magnitude as the nucleon diameter. The obvious analogy to the atomic force is, however, limited. In contrast to the situation in atomic physics, it is not possible to obtain detailed information about the nuclear force by studying the structure of the nucleus. The nucleons in the nucleus are in a state that may be described as a degenerate Fermi gas. To a first approximation the nucleus may be viewed as a collection of nucleons in a potential well. The behaviour of the individual nucleons is thus more or less independent of the exact character of the nucleon-nucleon force. It is therefore not possible to extract the nucleon-nucleon potential directly from the properties of the nucleus. The potential must rather be obtained by analysing two-body systems such as nucleon-nucleon scattering and the proton-neutron bound state, i.e., the deuteron.

There are also considerably greater theoretical difficulties in elucidating the connection between the nuclear forces and the structure of the nucleon than for the atomic case. This is primarily a consequence of the strong coupling constant α_s being two orders of magnitude larger than α, its electromagnetic equivalent. We will therefore content ourselves with an essentially qualitative explanation of the nuclear force.

16.1 Nucleon–Nucleon Scattering

Nucleon-nucleon scattering at low energies, below the pion production threshold, is purely elastic. At such energies the scattering may be described by non-relativistic quantum mechanics. The nucleons are then understood as point-like structureless objects that nonetheless possess spin and isospin. The physics of the interaction can then be understood in terms of a potential. It is found that the nuclear force depends upon the total spin and isospin of the two nucleons. A thorough understanding therefore requires experiments with polarised beams and targets, so that the spins of the particles involved in the reaction can be specified, and both protons and neutrons must be employed.

If we consider nucleon-nucleon scattering and perform measurements for both parallel and antiparallel spins perpendicular to the scattering plane, then we can single out the spin triplet and singlet parts of the interaction. If the nucleon spins are parallel, then the total spin must be 1, while for opposite spins there are equally large (total) spin 0 and 1 components.

The algebra of angular momentum can also be applied to isospin. In proton-proton scattering we always have a state with isospin 1 (an isospin triplet) since the proton has $I_3 = +1/2$. In proton-neutron scattering there are both isospin singlet and triplet contributions.

Scattering phases. Consider a nucleon coming in "from infinity" with kinetic energy E and momentum $\boldsymbol{p}$ which scatters off the potential of another nucleon. The incoming nucleon may be described by a plane wave and the outgoing nucleon as a spherical wave. The cross section depends upon the phase shift between these two waves.

For states with well defined spin and isospin the cross section of nucleon-nucleon scattering into a solid angle element $\mathrm{d}\Omega$ is given by the scattering amplitude $f(\theta)$ of the reaction

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\theta)|^2 . \tag{16.1}$$

For scattering off a short ranged potential a *partial wave decomposition* is used to describe the scattering amplitude. The scattered waves are expanded in terms with fixed angular momentum ℓ. In the case of elastic scattering the following relation holds at large distances r from the centre of the scattering:

$$f(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell+1)\, \mathrm{e}^{i\delta_\ell}\, \sin\delta_\ell\, P_\ell(\cos\theta) , \tag{16.2}$$

where

$$k = \frac{1}{\lambda\!\!\!^{-}} = \frac{|\boldsymbol{p}|}{\hbar} = \frac{\sqrt{2ME}}{\hbar} \tag{16.3}$$

is the wave number of the scattered nucleon, δ_ℓ a phase shift angle and P_ℓ, the angular momentum eigenfunction, an ℓ-th order Legendre polynomial. The phase shifts δ_ℓ describe the phase difference between the scattered and unscattered waves. They contain the information about the shape and strength of the potential and the energy dependence of the cross section. The fact that δ_ℓ appears not only as a phase factor but also in the amplitude ($\sin\delta_\ell$) follows from the conservation of the particle current in elastic scattering. This is also known as *unitarity*. The partial wave decomposition is especially convenient at low energies since only a few terms enter the expansion. This is because for a potential with range a we have

$$\ell \leq \frac{|\boldsymbol{p}| \cdot a}{\hbar} . \tag{16.4}$$

The phase shift δ_0 of the partial waves with $\ell = 0$ (i.e., s waves) is decisive for nuclear binding. From (16.4) we see that the s waves dominate proton-proton scattering (potential range 2 fm) for relative momenta less than 100 MeV/c. The Legendre polynomial P_0 is just 1, i.e., independent of θ. The phase shifts δ_0 as measured in nucleon-nucleon scattering are separately plotted for spin triplet and singlet states against the momentum in the centre of mass frame in Fig. 16.1. For momenta larger than 400 MeV/c δ_0 is negative, below this it is positive. We learn from this that the nuclear force has a repulsive character at short distances and an attractive nature at larger separations. This may be simply seen as follows.

Consider a, by definition, spherically symmetric s wave $\psi(\boldsymbol{x})$. We may define a new radial function $u(r)$ by $u(r) = \psi(r) \cdot r$ which obeys the Schrödinger equation

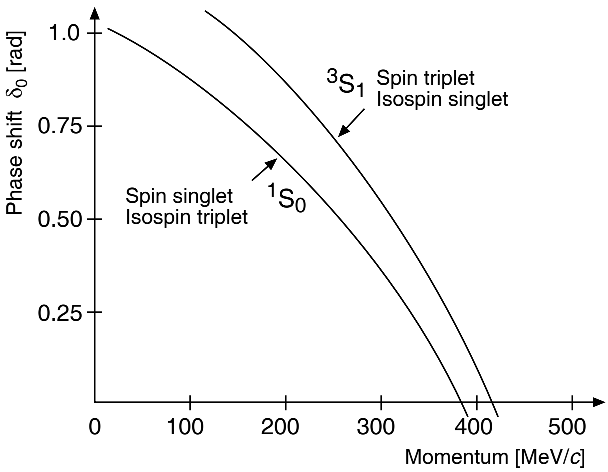

Fig. 16.1. The phase shift δ_0 as determined from experiment both for the spin triplet-isospin singlet $^3S_1$ and for the spin singlet-isospin triplet $^1S_0$ systems plotted against the relative momenta of the nucleons. The rapid variation of the phases at small momenta is not plotted since the scale of the diagram is too small.

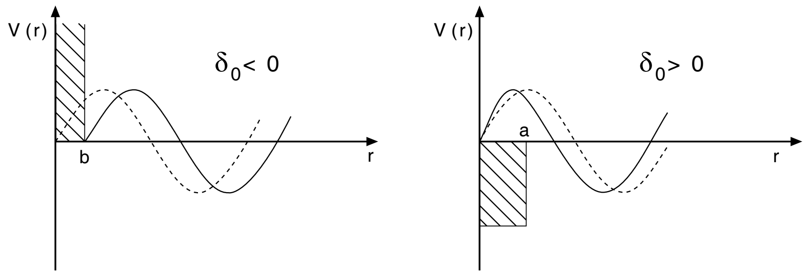

Fig. 16.2. Sketch of the scattering phase for a repulsive (*left*) and an attractive (*right*) potential. The dashed curves denote unscattered waves, the continuous ones the scattered waves.

$$\frac{\mathrm{d}^2 u(r)}{\mathrm{d}r^2} + \frac{2m(E-V)}{\hbar^2}\, u(r) = 0\,. \tag{16.5}$$

If we now solve this equation for a repulsive rectangular potential V with radius b and $V \to \infty$ (Fig. 16.2), we find

$$\delta_0 = -kb\,. \tag{16.6}$$

The scattering phase is negative and proportional to the range of the potential. A negative scattering phase means that the scattered wave lags behind the unscattered one.

For an attractive potential the scattered wave runs ahead of the unscattered one and δ_0 is positive. The size of the phase shift is the difference between the phase of the wave scattered off the edge of the potential a and that of the unscattered wave:

$$\delta_0 = \arctan\left(\sqrt{\frac{E}{E+|V|}}\tan\frac{\sqrt{2mc^2(E+|V|)}\cdot a}{\hbar c}\right) - \frac{\sqrt{2mc^2 E}\ \cdot a}{\hbar c}\,. \tag{16.7}$$

The phase shift δ_0 is then positive and decreases at higher momenta. If we superimpose the phase shifts associated with a short ranged repulsive potential and a longer ranged attractive one we obtain Fig. 16.3, where the effective phase shift changes sign just as the observed one does.

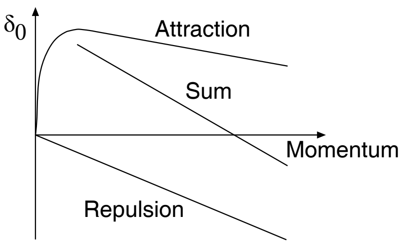

Fig. 16.3. Superposition of negative and positive scattering phases δ_0 plotted against the relative momenta of the scattered particles. The resulting effective δ_0 is generated by a short distance repulsive and a longer range attractive nucleon-nucleon potential.

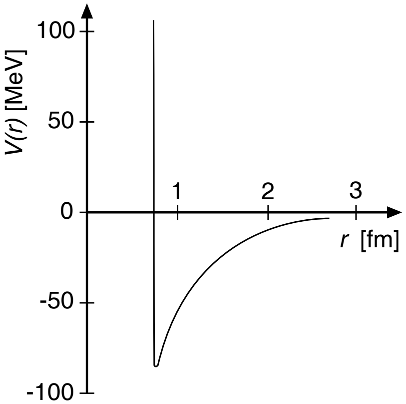

Fig. 16.4. Sketch of the radial dependence of the nucleon-nucleon potential for $\ell = 0$. Note that the spin and isospin dependence of the potential is not shown.

The relationship between the scattering phase δ_0 and the scattering potential V is contained, in principle, in (16.6) and (16.7) since the wave number k in the region of the potential depends both upon the latter's size and shape and upon the initial energy E of the projectile. A complete scattering phase analysis leads to the nuclear potential shown in Fig. 16.4 which has – as remarked above – a short ranged repulsive and a longer ranged attractive nature. Since the repulsive part of the potential increases rapidly at small r it is known as the *hard core*.

The nucleon–nucleon potential. We may obtain a general form of the nucleon-nucleon potential from a consideration of the relevant dynamical quantities. We will, however, neglect the internal structure of the nucleons, which means that this potential will only be valid for nucleon-nucleon bound states and low energy nucleon-nucleon scattering.

The quantities which determine the interaction are the separation of the nucleons $\boldsymbol{x}$, their relative momenta $\boldsymbol{p}$, the total orbital angular momentum $\boldsymbol{L}$ and the relative orientations of the spins of the two nucleons, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. The potential is a scalar and must at the very least be invariant under translations and rotations. Furthermore it should be symmetric under exchange of the two nucleons. These preconditions necessarily follow from various properties, such as parity conservation, of the underlying theory of the strong force and they limit the scalars which may appear in the potential. At the end of the day the potential, for fixed isospin, has the form [Pr63]:

$$
\begin{aligned}
V(r) = \quad & V_0(r) \\
& + V_{\rm ss}(r)\, \boldsymbol{s}_1 \cdot \boldsymbol{s}_2/\hbar^2 \\
& + V_{\rm T}(r)\, \left(3(\boldsymbol{s}_1 \cdot \boldsymbol{x})(\boldsymbol{s}_2 \cdot \boldsymbol{x})/r^2 - \boldsymbol{s}_1\boldsymbol{s}_2\right)/\hbar^2 \\
& + V_{\rm LS}(r)\, (\boldsymbol{s}_1 + \boldsymbol{s}_2) \cdot \boldsymbol{L}/\hbar^2 \\
& + V_{\rm Ls}(r)\, (\boldsymbol{s}_1 \cdot \boldsymbol{L})(\boldsymbol{s}_2 \cdot \boldsymbol{L})/\hbar^4 \\
& + V_{\rm ps}(r)\, (\boldsymbol{s}_2 \cdot \boldsymbol{p})(\boldsymbol{s}_1 \cdot \boldsymbol{p})/(\hbar^2 m^2 c^2)\,. \qquad (16.8)
\end{aligned}
$$

V_0 is a standard central potential. The second term describes a pure spin-spin interaction, while the third term is called the *tensor potential* and describes a non-central force. These two terms have the same spin dependence as the interaction between two magnetic dipoles in electromagnetism. The tensor term is particularly interesting, since it alone can mix orbital angular momentum states. The fourth term originates from a spin-orbit force, which is generated by the strong interaction (the analogous force in atomic physics is of magnetic origin). The final two terms in (16.8) are included on formal grounds, since symmetry arguments do not exclude them. They are, however, both quadratic in momentum and thus mostly negligible in comparison to the LS-term.

The significance of this ansatz for the potential is not that the various terms can be merely formally written down, but rather that, as we will see in Sect. 16.3, the spin and isospin dependence of the nuclear force can be explained in meson exchange models. Attempts to fit the potential terms to the experimental data have not fixed it exactly, but a general agreement exists for the first four terms. It should be also noted that many body forces need to be taken into account for conglomerations of nucleons.

The central potential for the $S = 0$ case is applicable to the low energy proton-proton and neutron-neutron interactions. The attractive part is, however, not strong enough to create a bound state. For $S = 1$ on the other hand this potential together with the tensor force and the spin-spin interaction is strong enough to present us with a bound state, the deuteron.

16.2 The Deuteron

The deuteron is the simplest of all the nucleon bound states i.e., the atomic nuclei. It is therefore particularly suitable for studying the nucleon-nucleon interaction. Experiments have yielded the following data about the deuteron ground state:

Binding energy	$B = 2.225$ MeV
Spin and parity	$J^P = 1^+$
Isospin	$I = 0$
Magnetic moment	$\mu = 0.857\,\mu_N$
Elec. quadrupole moment	$Q = 0.282\,e{\cdot}\text{fm}^2$.

The proton-neutron system is mostly made up of an $\ell=0$ state. If it were a pure $\ell=0$ state then the wave function would be spherically symmetric, the quadrupole moment would vanish and the magnetic dipole moment would be just the sum of the proton and neutron magnetic moments (supposing that the nucleonic magnetic moments are not altered by the binding interaction). This prediction for the deuteron magnetic moment

$$\mu_p + \mu_n = 2.792\,\mu_N - 1.913\,\mu_N = 0.879\,\mu_N \tag{16.9}$$

differs slightly from the measured value of $0.857\,\mu_{\mathrm{N}}$. Both the magnetic dipole moment and the electric quadrupole moment can be explained by the admixture of a state with the same J^P quantum numbers

$$|\psi_{\mathrm{d}}\rangle = 0.98 \cdot |\,^3\mathrm{S}_1\rangle + 0.20 \cdot |\,^3\mathrm{D}_1\rangle\,. \tag{16.10}$$

In other words there is a 4 % chance of finding the deuteron in a $^3\mathrm{D}_1$ state. This admixture can be explained from the tensor components of the nucleon-nucleon interaction.

We now want to calculate the nucleon wave function inside a deuteron. Since the system is more or less in an $\ell = 0$ state, the wave function will be spherically symmetric. We will need the depth V of the potential well (averaged over the attractive and repulsive parts) and its range, a. The binding energy of the deuteron alone gives us one parameter – the "volume" of the potential well, i.e., Va^2. The solutions of the Schrödinger equation (16.5) are

$$\begin{aligned} &\text{if } r<a: \; u_{\mathrm{I}}(r) = A\sin kr \quad \text{where } k = \sqrt{2m(E-V)}/\hbar, \; (V<0),\\ &\text{if } r>a: \; u_{\mathrm{II}}(r) = C\mathrm{e}^{-\kappa r} \quad \text{where } \kappa = \sqrt{-2mE}/\hbar, \quad (E<0), \end{aligned} \tag{16.11}$$

and $m \approx M_{\mathrm{p}}/2$ is the reduced mass of the proton-neutron system.

Continuity of $u(r)$ and $\mathrm{d}u(r)/\mathrm{d}r$ at the edge of the well, i.e., $r = a$, implies that [Sc95]

$$k\cot ka = -\kappa \qquad\qquad ak \approx \frac{\pi}{2} \tag{16.12}$$

and

$$Va^2 \approx Ba^2 + \frac{\pi^2}{8}\frac{(\hbar c)^2}{mc^2} \approx 100\ \mathrm{MeV\,fm}^2\,. \tag{16.13}$$

Current values for the range of the nuclear force, and hence the effective extension of the potential $a \approx 1.2 \cdots 1.4$ fm, imply that the depth of the

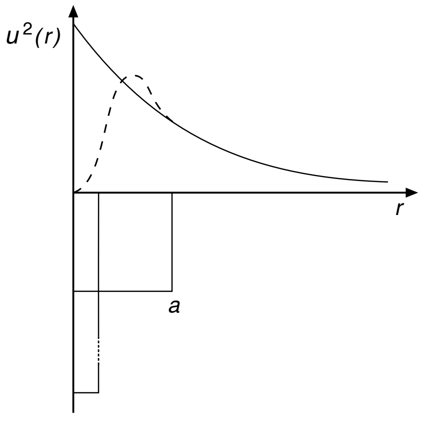

Fig. 16.5. Radial probability distribution $u^2(r) = r^2|\psi|^2$ of the nucleons in deuterium for an attractive potential with range a (*dashed curve*) and for the range $a \to 0$ with a fixed volume Va^2 for the potential well (*continuous curve*).

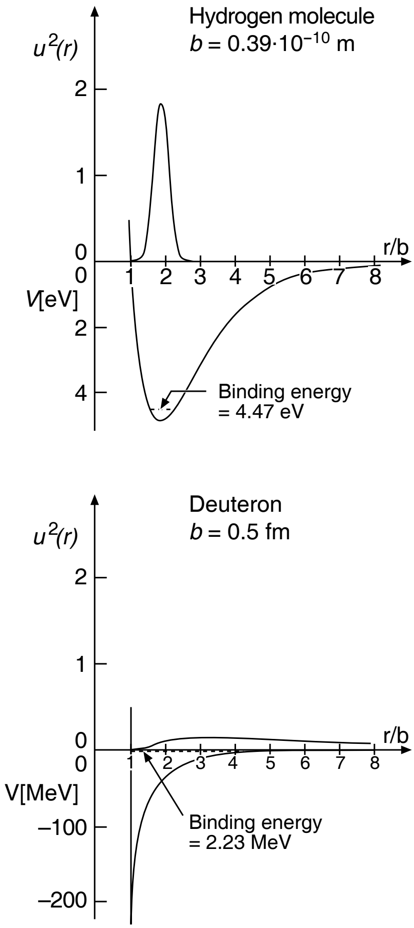

Fig. 16.6. The radial probability distribution $u^2(r)$ of the hydrogen atoms in a hydrogen molecule (*top*) [He50] and of nucleons in a deuteron (*bottom*) in units of the relevant hard cores (from [Bo69]). The covalent bond strongly localises the H atoms, since the binding energy is comparable to the depth of the potential. The weak nuclear bond, since the potential energy is comparable in size to the kinetic energy, means that the nucleons are delocalised.

potential is $V \approx 50$ MeV. This is much greater than the deuteron binding energy B (just 2.25 MeV). The tail of the wave function, which is characterised by $1/\kappa \approx 4.3$ fm, is large compared to the range of the nuclear force.

The radial probability distribution of the nucleons is sketched in Fig. 16.5 for two values of a but keeping the volume of the potential well Va^2 constant. Since deuterium is a very weakly bound system the two calculations differ only slightly, especially at larger separations.

A more detailed calculation which takes the repulsive part of the potential into account only changes the above wave function at separations smaller than 1 fm (cf. Fig. 16.5). In Fig. 16.6 the probability distribution of nucleons in deuterium and of hydrogen atoms in a hydrogen molecule are given for comparison. The separations are in both cases plotted in units of the spatial extension of the relevant hard core. The hard core sizes are about $0.4 \cdot 10^{-10}$ m for the hydrogen molecule and roughly $0.5 \cdot 10^{-15}$ m for the deuteron. The atoms in the molecule are well localised – the uncertainty in their separation ΔR is only about 10 % of the separation (cf. Fig. 16.6). The nuclear binding in deuterium is relatively "weak" and the bound state is much more spread out. This means that *the average kinetic energy is comparable to the average depth of the potential* and so the binding energy, which is just the sum of the kinetic and potential energies, must be very small.

The binding energy of the nucleons in larger nuclei are somewhat greater than that in deuterium and the density is accordingly larger. Qualitatively we still have the same situation: a relatively weak effective force is just strong enough to hold nuclei together. The properties of the nuclei bear witness to this fact: it is a precondition both for the description of the nucleus as a degenerate Fermi gas and for the great mobility of the nucleons in nuclear matter.

16.3 Nature of the Nuclear Force

We now turn to the task of understanding the strength and the form of the nuclear force from the structure of the nucleons and the strong interaction of the quarks inside the nucleons. In the following discussion we will employ qualitative arguments. The structure of the nucleon will be approached via the nonrelativistic quark model where the nucleons are built out of three constituent quarks. The nuclear force is primarily transmitted by quark-antiquark pairs, which we can only introduce ad hoc through plausibility arguments. A consistent theory of the nuclear force, based upon the interaction of quarks and gluons, does not yet exist.

Short distance repulsion. Let us begin with the short distance repulsive part of the nuclear force and try to construct some analogies to better understood phenomena. That atoms repel each other at short distances is a consequence of the Pauli principle. The electron clouds of both atoms occupy the lowest possible energy levels and if the clouds overlap then some electrons must be elevated into excited states using the kinetic energy of the colliding atoms. Hence we observe a repulsive force at short distances.

The quarks in a system of two nucleons also obey the Pauli principle, i.e., the 6 quark wave function must be totally antisymmetric. It is, however, possible to put as many as 12 quarks into the lowest $\ell = 0$ state without violating the Pauli principle, since the quarks come in three colours and have two possible spin ($\uparrow$, $\downarrow$) and isospin (u-quark, d-quark) directions. The spin-isospin part of the complete wave function must be symmetric since the colour part is antisymmetric and, for $\ell = 0$, the spatial part is symmetric. We thus see that the Pauli principle does not limit the occupation of the lowest quark energy levels in the spatial wave function, and so the fundamental reason for the repulsive core must be sought elsewhere.

The real reason is the spin-spin interaction between the quarks [Fa88]. We have already seen how this makes itself noticeable in the baryon spectrum: the Δ baryon, where the three quark spins are parallel to one another, is about 350 MeV/c^2 heavier than the nucleon. The potential energy then increases if two nucleons overlap and all 6 quarks remain in the $\ell = 0$ state since the number of quark pairs with parallel spins is greater than for separated nucleons. For each and every quark pair with parallel spins the potential energy increases by half the Δ-nucleon energy difference (15.11).

Of course the nucleon-nucleon system tries to minimise its "chromomagnetic" energy by maximising the number of antiparallel quark spin pairs. But this is incompatible with remaining in an $\ell = 0$ state since the spin-flavour part of the wave function must be completely symmetric. The colourmagnetic energy can be reduced if at least two quarks are put into the $\ell = 1$ state. The necessary excitation energy is comparable to the decrease in the chromomagnetic energy, so the total energy will in any case increase if the nucleons strongly overlap. Hence the effective repulsion at short distances is in equal parts a consequence of an increase in the chromomagnetic and the excitation energies (Fig. 16.7). If the nucleons approach each other very closely ($r = 0$) one finds in a non-adiabatic approximation that there is an 8/9 probability of two of the quarks being in a p state [Fa82, St88]. This configuration expresses itself in the relative wave function of the nucleons through a node at 0.4 fm. This together with the chromomagnetic energy causes a strong, short range repulsion. The nuclear force may be described by a nucleon-nucleon potential which rises sharply at separations less than 0.8 fm.

Attraction. Let us now turn to the attractive part of the nuclear force. Again we will pursue analogies from atomic physics. As we know the bonds between atoms are connected to a change in their internal structure and we expect something similar from the nucleons bound in the nucleus. Indeed a change in the quark structure of bound nucleons compared to that of their free brethren has been observed in deep inelastic scattering off nuclei (EMC effect, see Sect. 7.4).

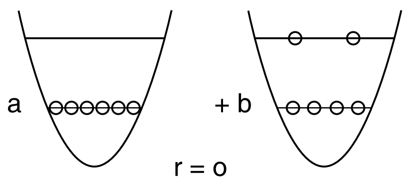

Fig. 16.7. a,b. The quark state for overlapping nucleons. This is composed of (**a**) a configuration with 6 quarks in the ℓ=0 state and (**b**) a configuration with 2 quarks in the ℓ=1 state. In a non-adiabatic approximation it is found that the state (**b**) dominates at separation $r = 0$ (probability 8/9) [Fa82, St88]. For larger distances this state becomes less important and disappears as $r \to \infty$.

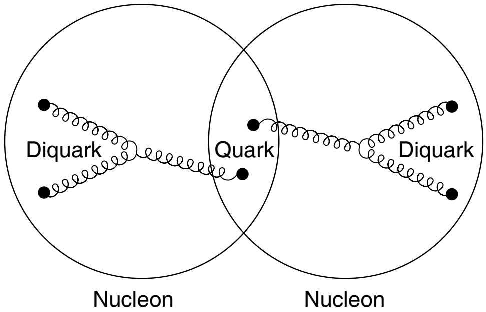

Fig. 16.8. Quark configurations in a covalent bond picture. At large separations, when the nucleons just overlap, we may understand them as each being diquark-quark systems.

It is clear upon a moments reflection that the nuclear force is not going to be well described by an *ionic bond*: the confining forces are so strong that it is not possible to lend a quark from one nucleon to another.

A *Van der Waals force*, where the atoms polarise each other and then stick to each other via the resulting dipole-dipole interaction can also not serve us as a paradigm. A Van der Waals force transmitted by the exchange of two gluons (in analogy to two photon exchange in the atomic case) would be too weak to explain the nuclear force at distances where the nucleons overlap and confinement does not forbid gluon exchange. At greater separations gluons cannot be exchanged because of confinement. Although colour neutral gluonic states (glueballs) could still be exchanged, none which are light enough have ever been experimentally observed.

The only analogy left to us to explain the nuclear force is a *covalent bond*, such as that which is, e.g., responsible for holding the H_2 molecule together. Here the electrons of the two H atoms are continually swapped around and can be ascribed to both atoms. The attractive part of the nuclear force is strongest at distances of around 1 fm and indeed reminds us of the atomic covalent bond. To simplify what follows, let us assume that the nucleon is made up of a two quark system (diquark) and a quark (see Fig. 16.8). Such a description has proven to be very successful in describing many phenomena. The most energetically favourable configuration is that where a u- and a d-quark combine to form a diquark with spin 0 and isospin 0. The alternative spin 1 and isospin 1 diquark is not favoured. The covalent bond is then expressed by the exchange of the "single" quarks, as sketched in Fig. 16.9. To push home the analogy we also show the equivalent covalent binding of the hydrogen molecule.

Since the nuclear attraction is strongest at distances of the order of 1 fm we do not need to worry about confinement effects. The covalent bond contribution to this force can be worked out analogously to the molecular case. However, the depth of the potential that is found in this way is only about one

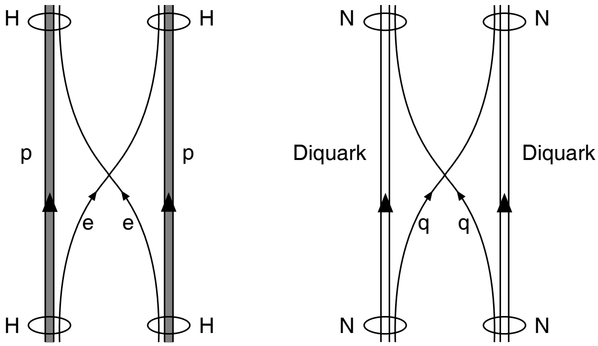

Fig. 16.9. Symbolic representation of the covalent bonds in a hydrogen molecule (*left*) and in a two nucleon system (*right*). The time axis runs vertically upwards. The electron exchange of the hydrogen molecule is replaced by quark exchange in the nucleonic system.

third of the experimental value [Ro94]. In fact quark exchange is less effective than its atomic counterpart of electron exchange. This is partly because to be exchanged the quarks must have the same colour, and there is only a 1/3 probability of this. The contribution of direct quark exchange sinks still further if one takes the part of the nucleon wave function into account where the diquarks have spin 1 and isospin 1. Thus the covalent bond concept, if it is directly transferred from molecules to nuclei, does not give us a good quantitative description of what is going on in nuclei. It should be noted that this is not a consequence of confinement, but rather of direct quark exchange being suppressed as a result of the quarks having three different colour charges.

Meson exchange. Up to now we have neglected the fact that as well as the three constituent quarks in the nucleon there are additional quark-antiquark pairs (sea quarks) which are continually being created from gluons and annihilated back into them again. We may interpret this admixture of quark-antiquark pairs as a relativistic effect, which, due to the size of the strong coupling constant α_s, we would be wrong to neglect. An effective quark-quark exchange may be produced by colour neutral quark-antiquark pairs, as is shown in Fig. 16.10a.

This quark-antiquark exchange actually plays a larger role in the nucleon-nucleon interaction than does the simple swapping of two quarks. It must be stressed that this exchange of colour neutral quark-antiquark pairs does not only dominate at great separations where confinement only allows the exchange of colour neutral objects but also at relatively short distances. One may thus understand the nuclear force as a relativistic generalisation of the covalent strong force via which the nucleons finally exchange quarks.

Ever since Yukawa in 1935 first postulated the existence of the pion [Yu35, Br65], there have been attempts to describe the inter-nuclear forces in terms of mesonic exchange. The exchange of mesons with mass m leads to

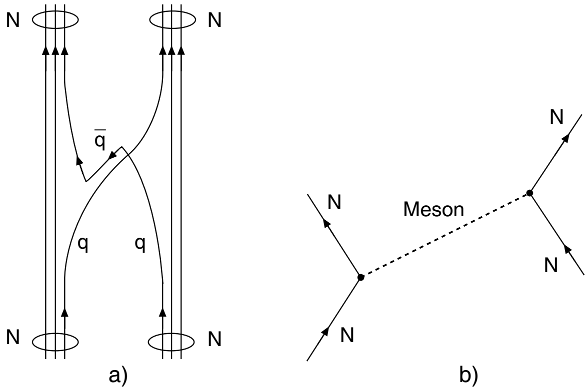

Fig. 16.10. (**a**) Representation of quark exchange between nucleons via the exchange of a quark-antiquark pair. Antiquarks are here depicted as quarks moving backwards in time. (**b**) The exchange of a meson is rather similar to this.

a potential of the form

$$V = g \cdot \frac{e^{-\frac{mc}{\hbar}r}}{r}\,, \tag{16.14}$$

where g is a charge-like constant. This is known as the *Yukawa potential.*

■ To derive the Yukawa potential we first assume that the nucleon acts as a source of virtual mesons in the same way as an electric charge may be viewed as a source of virtual photons.

We start with the wave equation of a free, relativistic particle with mass m. If we replace the energy E and momentum $\boldsymbol{p}$ in the energy momentum relationship $E^2 = \boldsymbol{p}^2c^2 + m^2c^4$ by the operators $i\hbar\partial/\partial t$ and $-i\hbar\boldsymbol{\nabla}$, as is done in the Schrödinger equation, we obtain the *Klein-Gordon equation*:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi(\boldsymbol{x},t) = \left(\boldsymbol{\nabla}^2 - \mu^2\right)\Psi(\boldsymbol{x},t) \qquad \text{where} \quad \mu = \frac{mc}{\hbar}\,. \tag{16.15}$$

For a massless particle ($\mu = 0$) this equation describes a wave travelling at the speed of light. If we replace Ψ by the electromagnetic four-potential $A = (\phi/c, \boldsymbol{A})$ we obtain the equation for electromagnetic waves in vacuo at a great distance from the source. One may thus interpret $\Psi(\boldsymbol{x},t)$ as the wave function of the photon.

Consider now the static field limit where (16.15) reduces to

$$\left(\boldsymbol{\nabla}^2 - \mu^2\right)\psi(\boldsymbol{x}) = 0\,. \tag{16.16}$$

If we demand a spherically symmetric solution, i.e., one that solely depends upon $r = |\boldsymbol{x}|$ we find

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\psi(r)}{\mathrm{d}r}\right) - \mu^2\psi(r) = 0\,. \tag{16.17}$$

A particularly simple ansatz for the potential V that results from exchanging the particle is $V(r) = g \cdot \psi(r)$, where g is an arbitrary constant. It is clear that this ansatz can make sense if we consider the electromagnetic case: in the limit $\mu \to 0$ we obtain the Poisson equation for a space without charges from (16.16) and we obtain from (16.17) the Coulomb potential $V_C \propto 1/r$, i.e., the potential of a charged particle at a great separation where the charge density is zero. If we now solve (16.17) for the massive case, we obtain the Yukawa potential (16.14). This potential initially decreases roughly as $1/r$ and then much more rapidly. The range is of the order of $1/\mu = \hbar/mc$, which is also what one would expect from the uncertainty relation [Wi38]. The interaction due to pion exchange has a range of about 1.4 fm.

The above remarks are somewhat naive and not an exact derivation. We have ignored the spin of the particle: the Klein-Gordon equation holds for spinless particles (luckily this is true of the pion). Additionally a virtual meson does not automatically have the rest mass of a free particle. Furthermore these interactions take place in the immediate vicinity of the nucleons and the mesons can strongly interact with them. The wave equation of a free particle can at best be an approximation.

Since the range of this potential decreases as the meson mass m increases, the most important exchange particles apart from the pion itself are the lightest vector mesons, the ϱ and the ω. The central potential of the nuclear force can be understood in this framework as a consequence of two pion exchange, where the pions combine to $J^P(I) = 0^+(0)$. The spin and isospin dependence of the nuclear force comes from 1 meson exchange and in particular because both pseudoscalar and vector mesons are exchanged. The trading of pions between the nucleons is especially important since the pion mass is so small that they can be exchanged at relatively large distances (> 2 fm). In these models one neglects the internal structure of nucleons and mesons and assumes that they are point particles. The meson-nucleon coupling constants that emerge from experiment must be slightly adapted to take this into account.

Since mesons are really colour neutral quark-antiquark pairs their exchange and that of colour neutral $q\overline{q}$ pairs give us, in principle, two equivalent ways of describing the nucleon-nucleon interaction (Fig. 16.10b). At shorter distances, where the structure of the nucleons must definitely play a part, a description in terms of meson exchange is inadequate. The coupling constant for the exchange of ω mesons, which is responsible for the repulsive part of the potential, has to be given an unrealistically high value – about two or three times the size one would accept from a comparison with the other meson-nucleon couplings. The repulsive part of the potential is better described in a quark picture. On the other hand one pion exchange models give an excellent fit to the data at larger separations. At intermediate distances various parameters need to be fitted by hand in both types of model.

In this way we see that it could be possible to trace back the nuclear force to the fundamental constituents of matter. This is very satisfying for our theoretical understanding of the nuclear force, but a quantitative description of the nuclear force is not made any easier by this transition from a mesonic to a quark picture. To describe the forces emanating from meson exchange inside a quark picture we would need to know the probability with which the quark-antiquark pairs in the nucleus can turn into mesons. These calculations are intractable since the strong coupling constant α_s is very large at small momenta. For this reason phenomenological meson exchange models are still today the best way to quantitatively describe the nuclear force.

Problems

1. **The nuclear force**
   The nuclear force is transmitted by exchanging mesons. What are the ranges of the forces generated by exchanging the following: a π, two π's, a ϱ, an ω? Which properties of the nuclear force are determined by the exchange particles?

2. **Neutron-proton scattering**
   How large would the total cross-section for neutron-proton scattering be if only the short range repulsion (range, $b = 0.7\,\text{fm}$) contributed? Consider the energy regime in which $\ell = 0$ dominates.