

Exposing Nuclear Magic

Calculations show how the mysterious “magic numbers” that stabilize nuclear structures emerge naturally from nuclear forces—once these are described with appropriate spatial resolution.

By **Vittorio Somà**

Atomic nuclei have been studied for over a century, yet some of nuclear physics’ most basic questions remain unanswered: How many bound combinations of protons and neutrons, or isotopes, can exist? Where do the limits of nuclear existence lie? How are chemical elements synthesized in the Universe? Clues to solving these puzzles lie in the vast phenomenology of nuclear structure—the measured properties of tens of thousands of nuclear states, their decays, and their reactions. In this bedlam of information, patterns and irregularities in data provide crucial hints. One such irregularity was spotted as early as 1934 [1]: Nuclei containing specific numbers of protons and neutrons (2, 8, 20, 28, 50, 82...) are unexpectedly stable. These “magic numbers” (Fig. 1), as Eugene Wigner later called them, resisted a microscopic, first-principles explanation for decades and were only accounted for

phenomenologically through the nuclear-shell model [2]. Now Chenrong Ding of Sun Yat-sen University in China and his collaborators have shown how magic numbers emerge from the underlying interactions between protons and neutrons, provided that such interactions are described at the appropriate resolution [3]. The results offer an important bridge between phenomenological descriptions and first-principles models.

Explaining how simplicity emerges from underlying complexity is, as Leonardo da Vinci put it, the ultimate form of sophistication. Atomic nuclei—where all fundamental forces but gravity come into play in a delicate balance—are the archetypical example of complex systems. Their constituents, protons and neutrons (collectively dubbed nucleons), interact with each other via strong and electromagnetic forces, with the weak interaction governing some of the nuclear-decay channels. This physics can be cast as a quantum many-body problem. But in practice, solving it poses enormous challenges, formally and computationally. To make progress, nuclear theorists have developed over the years a plethora of effective models to describe these tricky objects.

The nuclear-shell model has arguably had the biggest impact on the field. In this model, nucleons are described as independent particles in an average binding potential. This potential generates discrete energy levels, or shells, that the nucleons fill sequentially from low to high energy. Magic numbers are at the core of this framework. They emerge as large energy gaps separating groups of proton or neutron levels (Fig. 2, leftmost diagram). These gaps hinder nucleon excitations and confer enhanced stability to configurations in which a set of shells is completely full—much like closed electron shells stabilize the noble gases in the atomic-shell model. Despite its conceptual simplicity, the nuclear-shell



Figure 1: Nuclear “magic numbers” (2, 8, 20, 28, 50, 82...) correspond to particularly stable configurations of atomic nuclei. New work connects their traditional, phenomenological shell-model interpretation to a microscopic explanation rooted in the underlying interactions between nucleons.

Credit: APS/Carin Cain

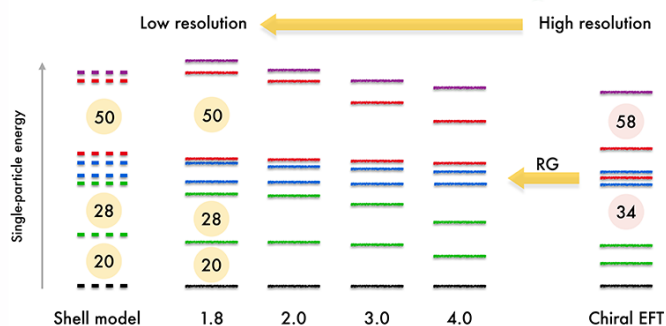


Figure 2: Single-particle energy levels for protons in tin-132 as a function of the resolution scale with which nuclear forces are described. On the right, the single-particle spectrum obtained from chiral effective-field-theory (EFT) interactions is displayed. In the middle, spectra are shown for decreasing resolutions (expressed in fm^{-1}). The leftmost spectrum is the single-particle spectrum from the shell model, displaying the standard magic numbers (20, 28, and 50), and matching the spectrum obtained at the smallest resolution.

Credit: Adapted from C.R. Ding *et al.* [3]

model has proved remarkably successful, accounting for a wide range of nuclear phenomena—from explaining natural abundances of stable isotopes and trends in the properties of excited states to predicting reaction observables like neutron-capture cross sections.

When the actual interactions between nucleons are considered, however, this framework becomes very difficult to justify. The bare nucleon–nucleon force is strong and extends to about 2 femtometers (fm), which corresponds to the mean separation between nucleons within a nucleus. Moreover, forces involving the simultaneous interaction of three nucleons are far from negligible. In other words, it looks like protons and neutrons interact strongly and frequently, seemingly at odds with an independent-particle description. What’s more, if one derives average potentials directly from “realistic” nuclear forces, they fail to produce the large energy gaps associated with the magic numbers (Fig. 2). Why so? At first glance, the success of the shell model does indeed appear almost magical.

An important piece of the puzzle is that, in quantum mechanics, neither the interaction potential nor the single-particle energy levels are themselves observable quantities [4]. As a

consequence, neither the bare potential nor the shell-model potential can be regarded as the “true” nuclear interactions. The apparent tension between these two descriptions therefore does not, in principle, invalidate either one. How, then, can the two contrasting views be reconciled? The answer lies in the freedom to modify the quantum-mechanical potential, provided that the corresponding many-body wave function changes appropriately. Specifically, these variations should leave physical observables (total energies, cross sections, and the like) unchanged. Recently, this freedom has been exploited to construct nuclear interactions [5] better suited for first-principles, or *ab initio*, calculations [6]. These developments were achieved by borrowing mathematical techniques from the renormalization group, which allows tracking how a physical system changes when viewed at different scales of resolution.

Ding and co-workers build on these advances by starting with the modern description of realistic nuclear forces: two- and three-nucleon interactions modeled with chiral effective field theory (EFT) [7]. They then apply renormalization-group techniques to drive this interaction model to progressively lower resolutions, generating a nuclear potential for each resolution step (with resolution ranging from infinite to 1.8 fm^{-1}). Employing state-of-the-art *ab initio* methods [8], they solved—for each resolution—the many-body Schrödinger equation for the heavy nucleus tin-132 (an ideal test case as a doubly magic nucleus comprising 50 protons and 82 neutrons). Finally, they extracted the corresponding single-nucleon shell structure from the computed ground-state wave functions.

Figure 2 illustrates how the resolution of the nuclear interaction reconciles the shell-model and chiral-EFT descriptions. At high resolution—where the nuclear interaction is nonperturbative and induces strong correlations in the many-nucleon wave function—the single-particle spectrum displays only modest gaps, appearing in correspondence of proton numbers 34 and 58. As the resolution scale is lowered, the single-particle spectrum changes significantly. The high-resolution gaps disappear, and other gaps start to emerge. At sufficiently low resolution, the resulting level structure becomes very similar to that generated by the shell-model potential. Remarkably, the familiar magic numbers then emerge.

Ding and collaborators’ work thus reconciles two seemingly

opposing views of the atomic nucleus: an empirical model that has long guided our understanding of nuclear phenomenology and a reductionist approach that seeks to derive nuclear structure from the fundamental theory of the strong force. Bridging these two views is a milestone for the field and opens up exciting perspectives for the study of the poorly understood frontiers of the nuclear chart. There, traditional magic numbers are known to disappear and new ones to emerge [9]. Developing a unified, first-principles description may ultimately lead researchers to solving nuclear physics' longest-standing questions.

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