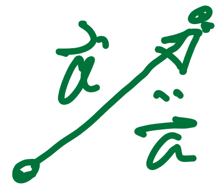
Τρίτη 14 Οκτωβρίου

$$\vec{L} = \vec{L}_{KM} + M\,\vec{X}_{KM} \wedge \dot{\vec{X}}_{KM}$$

Ιδιοστροφορμή

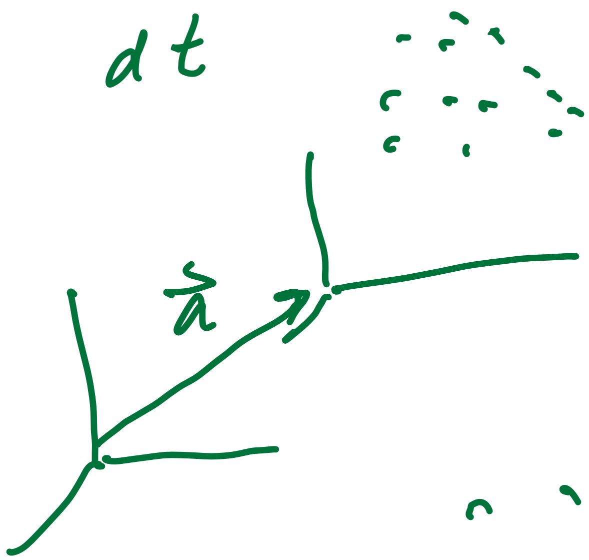
τι



$\ddot{\vec{a}} \parallel \vec{a}$

$$\frac{d\vec{L}}{dt} = \vec{\tau} + M(\vec{a} - \vec{X}_{KM}) \wedge \ddot{\vec{a}}$$

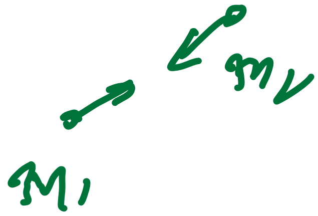
α) $\vec{a} = \vec{X}_{KM}$



$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\frac{d\vec{L}^{*}_{KM}}{dt} = 0$$

Εάν έχουμε ΝΕ η στροφορμή



$$\vec{L} = M \vec{X} \wedge \dot{\vec{X}} + \mu \vec{x} \wedge \dot{\vec{x}}$$

B

$$\frac{d\vec{L}}{dt} = ?$$

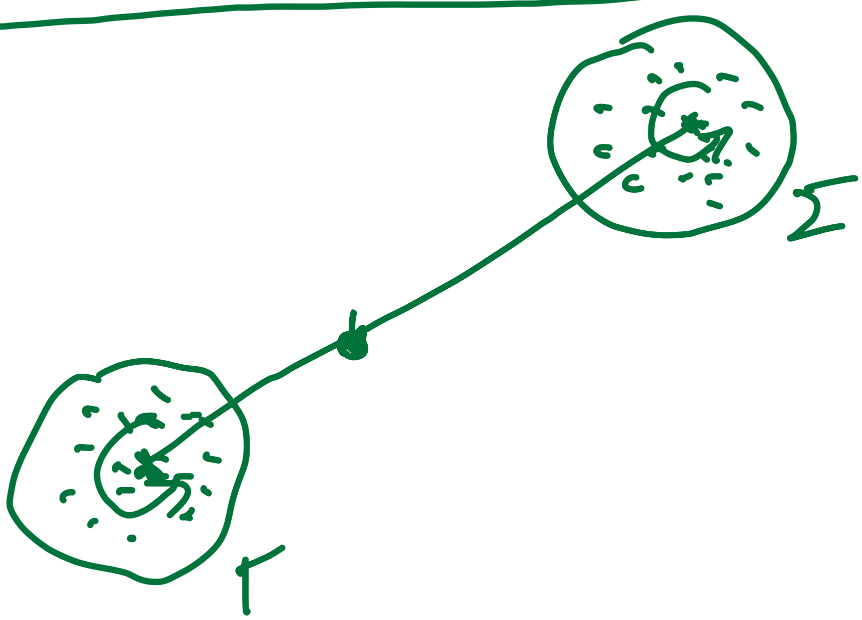
A

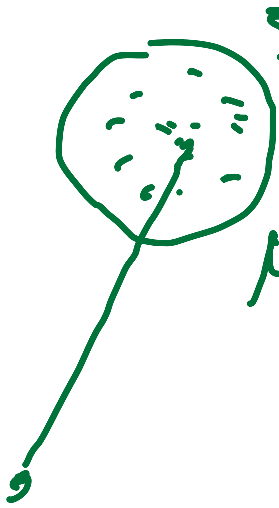
$$L = m_B (\vec{x}_B - \vec{x}_A) \wedge (\dot{\vec{x}}_B - \dot{\vec{x}}_A)$$

$$\frac{d\vec{L}}{dt} = m_B \underline{(\vec{x}_B - \vec{x}_A)} \wedge (\ddot{\vec{x}}_B - \ddot{\vec{x}}_A)$$

$$= 0$$

Σελήνη – Γη



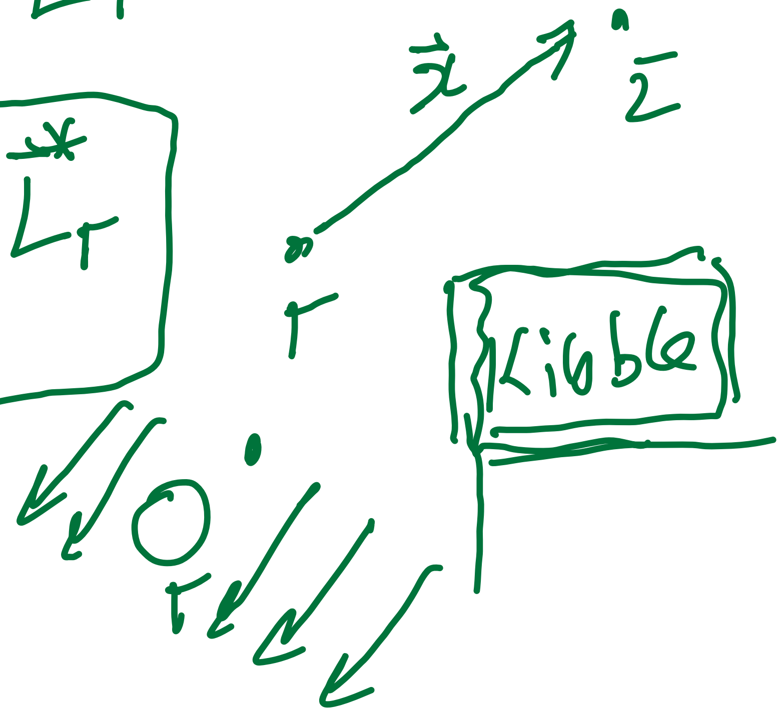


$$\overset{*}{L}_\Sigma = I \cdot \Omega_\Sigma$$

$$M_\Sigma \vec{x}_\Sigma \wedge \dot{\vec{x}}_\Sigma + \overset{*}{L}_\Sigma$$

$$M_\Sigma \vec{x}_\Sigma \wedge \dot{\vec{x}}_\Sigma + \overset{*}{L}_\Sigma + M_\Gamma \vec{x}_\Gamma \wedge \dot{\vec{x}}_\Gamma + \overset{*}{L}_\Gamma$$

$$\boxed{\vec{L}_{\kappa\iota\nu} = \mu \vec{x} \wedge \dot{\vec{x}} + \overset{*}{\vec{L}}_\Sigma + \overset{*}{\vec{L}}_\Gamma}$$



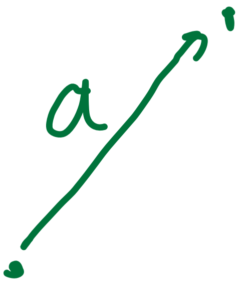
~ $\vec{L}_{\kappa\iota\nu}$ διατηρείται

Ήλιος

$$\|L_\Sigma\| \ll \|L_\Gamma\|$$

$$\vec{L}_{KM} = \mu \vec{x} \wedge \dot{\vec{x}} + \vec{L}_{\Gamma}$$

① $$L_{KM} = \mu a^2 \Omega + \frac{2}{5} m r_\Gamma^2 \omega$$

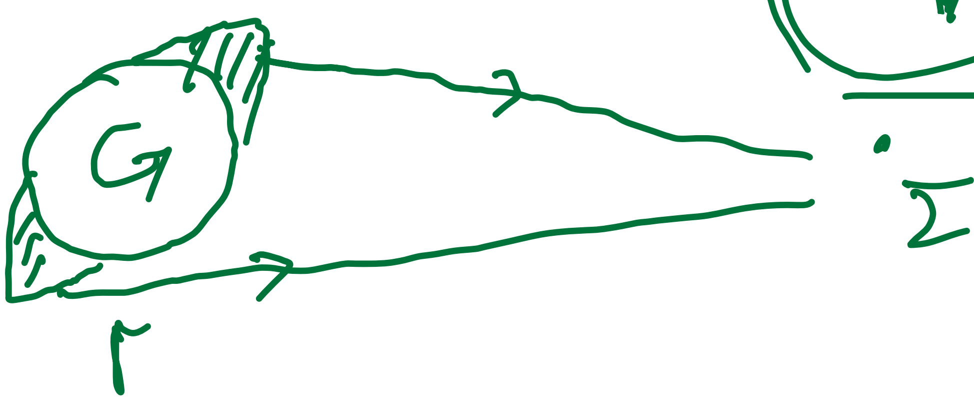


$$\Omega^2 a^3 = K \qquad ②$$

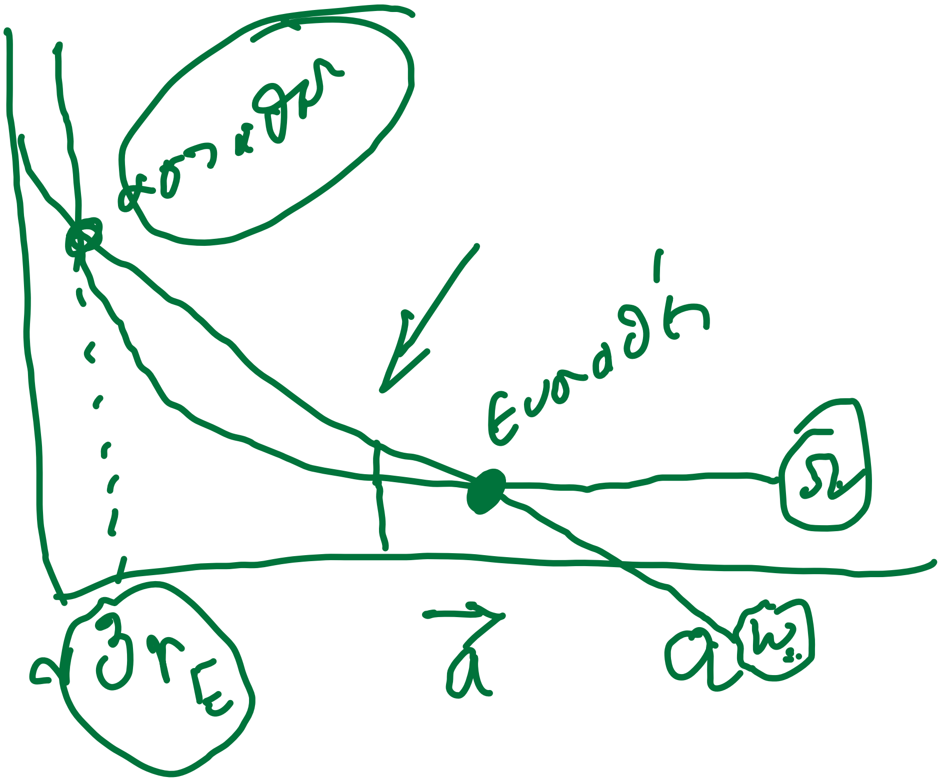
Παλιρροϊκή τριβή

$\omega \gg \Omega$

$$\boxed{\omega \downarrow \to a \uparrow}$$



~ 4 cm/year



Βαρυτική ενέργεια

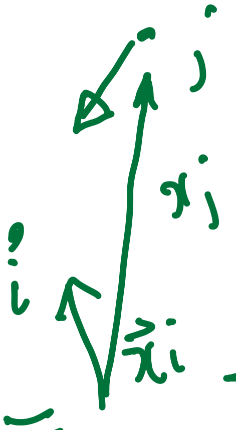
$$\vec F_i = -\nabla_i \underbrace{\left[-\sum_{\substack{i,j\\ i<j}} \frac{G m_i m_j}{|\vec x_i - \vec x_j|} \right]}_{V}$$

$$E = \frac{1}{2}\sum_{i=1}^{N} m_i |\dot{\vec x}_i|^2 - \sum_{i<j} \frac{G m_i m_j}{|\vec x_i - \vec x_j|}$$

$$V = \frac{1}{2}\sum\sum_{i\neq j}$$

$$\vec{F}_i = \sum_{j\neq i} \frac{G m_i m_j}{|\vec{x}_i - \vec{x}_j|^3}(\vec{x}_j - \vec{x}_i)$$

η δυναμη που ασκειται στο i-σωματιδιο



$$-\nabla_i V = -\sum_{i<j} \frac{\partial}{\partial x_{i,\alpha}}\left(-\frac{G m_i m_j}{\sqrt{\underbrace{(x_{i,\beta}-x_{j,\beta})(x_{i,\beta}-x_{j,\beta})}_{r_{ij}}}}\right)$$

$$-\nabla_i V = -\sum_{j\neq i}\frac{1}{2}\frac{G m_i m_j}{r_{ij}^3}\frac{\partial}{\partial x_{i,\alpha}}(x_{i,\beta}-x_{j,\beta})(x_{i,\beta}-x_{j,\beta})$$

$$= -\sum_{j\neq i}\frac{1}{2}\frac{G m_i m_j}{r_{ij}^3}\,2\delta_{\beta\alpha}[x_{i,\beta}-x_{j,\beta}]$$

$$= -\sum_{\substack{j\neq i\\ j=1}}^{N}\frac{G m_i m_j}{r_{ij}^3}(x_{i,\alpha}-x_{j,\alpha})$$

$$\vec{F}_i = -\nabla_i V = \sum_{j\neq i}^{N}\frac{G m_i m_j}{r_{ij}^3}(\vec{x}_j - \vec{x}_i)$$

$j=1 \qquad i)$

$$m_i \ddot{\vec{x}}_i = \boxed{\vec{F}_i = -\nabla_i V} \rightarrow$$

$$m_i \dot{\vec{x}}_i \cdot \ddot{\vec{x}}_i = - \dot{\vec{x}}_i \cdot \nabla_i V$$

$$\frac{d}{dt}\frac{m_i |\dot{\vec{x}}_i|^2}{2} = - \dot{\vec{x}}_i \cdot \left(\nabla_i V(\vec{x}_1, \dots, \vec{x}_N)\right)$$

$$\nabla_i = \nabla_{x_i}$$

αθροίζω για τα i όλα

$$\frac{d}{dt}\underbrace{\sum \frac{m_i |\dot{\vec{x}}_i|^2}{2}}_{K} = - \sum_{i=1}^{N} \dot{\vec{x}}_i \cdot \nabla_{\vec{x}_i} V(\vec{x}_1, \dots, \vec{x}_N)$$

$$= - \frac{d}{dt} V(\vec{x}_1, \dots, \vec{x}_N)$$

$$\boxed{\frac{dK}{dt} = - \frac{dV}{dt}} \qquad \frac{d}{dt}(\underbrace{K+V}_{E}) = 0$$

Βαρυτικά

$$E = \sum \frac{1}{2} m |\dot{\vec{x}}_i|^2 - \sum_{i<j} \frac{G m_i m_j}{r_{ij}}$$

$$r_{ij} = |\vec{x}_i - \vec{x}_j|$$

$$V = - \sum_{i<j} \frac{G m_i m_j}{r_{ij}}$$

$E \geq 0$ $\boxed{\text{δεν είναι ευσταθές}}$

$\boxed{E < 0}$ $\boxed{\text{Πρόβλημα 2 σωμάτων}}$

$$I = \sum m_i |\vec{x}_i|^2$$ ως προς το ΚΜ

$$\boxed{I \to 0 \longrightarrow V \to -\infty}$$

$$\frac{\dot I}{2} = \sum m_i \vec{x}_i \cdot \dot{\vec{x}}_i = \sum \vec{x}_i \cdot \vec{p}_i = G$$

$$\boxed{\text{virial}} = G$$

$$\dot G \equiv \frac{\ddot I}{2} = \sum m_i |\dot{x}_i|^2 + \sum m_i \vec{x}_i \cdot \ddot{\vec{x}}_i$$

$$= 2K + \sum_{i=1}^{N} \vec{x}_i \cdot \vec{F}_i$$

$$-\sum_{i=1}^{N} \vec{x}_i \cdot \nabla_i V(\vec{x}_1, \dots, \vec{x}_N)$$

Τεχνικό

$$\boxed{V(\lambda \vec{x}_1, \dots, \lambda \vec{x}_N) = \lambda^n V(\vec{x}_1, \dots, \vec{x}_N)}$$

Π.χ. βαρύτη $\qquad n = -1$

$$\frac{1}{|\vec{x}|} \qquad \frac{1}{|\lambda \vec{x}|} = \frac{1}{\lambda} \frac{1}{|\vec{x}|}$$

Euler

$$n \lambda^{n-1} V(\vec{x}_1, \dots, \vec{x}_N) = \sum_i \left(\nabla_{\lambda \vec{x}_i} V\right) \cdot \vec{x}_i$$

$$\lambda = 1$$

$$n V(\vec{x}_1, \dots, \vec{x}_N) = \sum_i \vec{x}_i \cdot \nabla_i V$$

$$= - \sum \vec{x}_i \cdot \vec{F}_i$$

$$n V = - \sum \vec{x}_i \cdot \vec{F}_i$$

$$\dot{G} = \frac{\ddot{I}}{2} = 2K - nV$$

<u>Βαρύτη</u> $n = -1$

$$\frac{\ddot{I}}{2} = 2K + V \,, \quad E = K + V$$

$$= 2E - V \quad (\text{Lagrange})$$

α) $E > 0$, $V < 0$ $\quad V = - \sum_{i<j} \frac{m_i m_j}{r_{ij}}$

$$\ddot{I} > 0 \quad \forall t$$

$$I \geq \alpha + \beta t + \frac{\gamma}{2} t^2$$

Θα $I \to \infty$ <u>$t \to \infty$</u>

β) $E=0$ $\quad \frac{\ddot{I}}{2} = -V$

πάλι θα πάει στο άπειρο

εάν δεν γίνεται αυτό $\quad r_{ij} < K$

$$V = -\sum_{i<j} \frac{m_i m_j}{r_{ij}} < -\frac{\sum m_i m_j}{K} = \alpha$$

>0

$-V > 0 \qquad -V > 0$ δείχνει

$\ddot{I} > 0 \qquad I \to \infty$

σημαντικά

γ) $E < 0$ $\quad \longleftrightarrow$

$\frac{\ddot{I}}{2} = 2E - V \qquad E = K + V < 0$

$$\boxed{V \le E < 0}$$

δεν μπορεί να έχει διαφυγή σε άπειρο

$r_{ij} > d$

$$V = -\sum_{i<j} \frac{m_i m_j}{r_{ij}}$$

εάν θεωρήσουμε τώρα.

ότι $G = \frac{\dot{I}}{2}$ virial φραγμένο

$$G = \sum_{i=1} \vec{x}_i \cdot \vec{p}_i$$

εδώ αφού

$G < K$

$$\frac{dG}{dt} = \frac{\ddot{I}}{2} = 2E - V = 2K + V \quad (1)$$

$$\bar{A} = \lim_{T\to\infty} \frac{\int_{-T}^{T} A\,dt}{2T}$$

lim

$$\overline{\frac{dG}{dt}} = \lim_{T\to\infty} \frac{\int_{-T}^{T} \frac{dG}{dt}\,dt}{2T} = \lim_{T\to\infty}\frac{G(T)-G(-T)}{2T} \to 0$$

Εάν η G είναι φραγμένη

Τότε

$$0 = \overline{\frac{dG}{dt}} = 2E - \overline{V} = 2\overline{K} + \overline{V}$$

$$\boxed{2E = \overline{V}}$$

Βαρυτικό

$\overline{V} < 0$

φραγμένη $G \Rightarrow$ $\boxed{E < 0}$

$$\overline{K} = -\frac{\overline{V}}{2} \qquad E = \frac{\overline{V}}{2} = -\overline{K}$$

$$\dot{G} = 2K + \sum \vec{x}_i \cdot \vec{F}_i$$

$\overset{\parallel}{\dfrac{\ddot{I}}{2}}$

$$F_i = -\nabla_i V \quad , \quad V(\lambda \vec{x}) = \lambda^n V(\vec{x})$$

$$-(\vec{x}_i \cdot \nabla_i V) = -nV$$

$$\dot{G} = 2K - nV$$

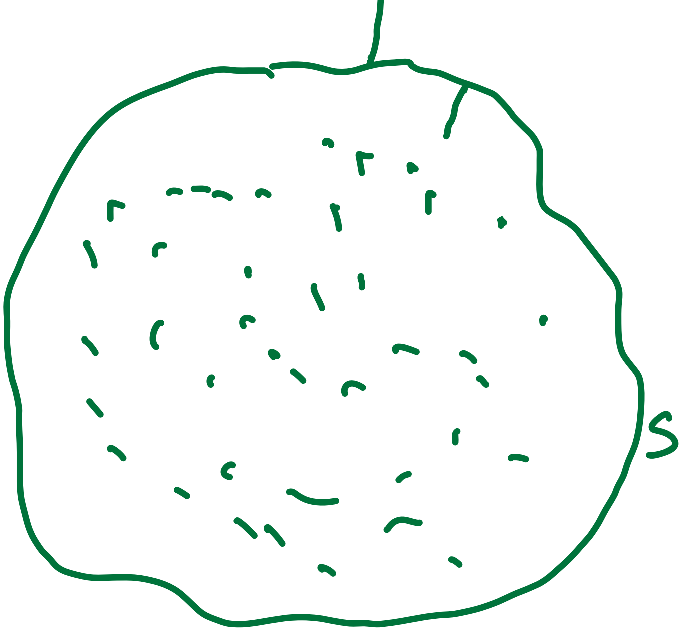
εάν G φραγμένο

$$\boxed{2\bar{K} = n\bar{V}}$$

Π.χ. αρμ. $V = \frac{1}{2}\alpha |\vec{x}|^2 \quad n = 2$

$$\bar{K} = \bar{V}$$

$1\vec{n}$



$$\overline{K} = \left(\frac{3}{2}k_B T\right) nN$$

$$\vec{F} = -P\vec{n}$$

$$2\overline{K} + \overline{\sum \vec{x}_i \cdot \vec{F}_i} = 0$$

$$\parallel$$

$$-\int_S dS\, P\, \vec{n}\cdot\vec{x}$$

$$\parallel$$

$$-P\int dV\, \underbrace{\nabla\cdot\vec{x}}_{3}$$

$$-3PV$$

$$2\left(\frac{3}{2}k_B T\right) nN = 3PV$$

$$PV = n\,(\overbrace{k_B N}^{R})\,T$$

$$PV = nRT$$

Βαρυτικά

$I \to 0$, αυτό θα γίνει σε πεπερασμένο χρόνο

$I \to 0$ $\quad$ $|V| \to \infty$

$$\frac{\ddot{I}}{2} = 2E - V$$

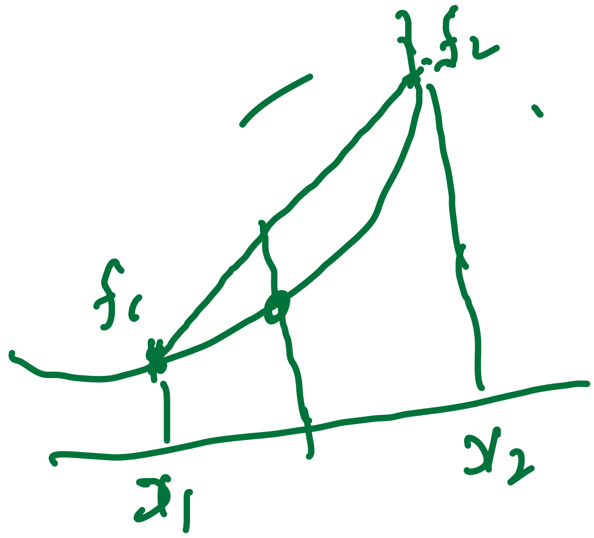
$$\frac{\ddot{I}}{2} > 0 \quad \to \quad t > t_0 \quad \boxed{I \to \infty}$$

$\ddot{I} > 0$ $\quad$ $\boxed{I = 0}$

$I \to 0$ $\quad$ $|V| \to \infty$

$$|V| = \sum_{i<j} \frac{m_i m_j}{r_{ij}}$$

Jensen $\quad$ κυρτή $\quad$ καμπύλη



$\lambda \in [0,1]$

<u>Ikupzik</u>

$f(\lambda x_1 + (1-\lambda) x_2)$

$\leq \lambda f(x_1) + (1-\lambda) f(x_2)$

<u>Jensen</u>

$$f\left(\frac{\sum m_i x_i}{\sum m_i}\right) \leq \frac{\sum m_i f(x_i)}{\sum m_i}$$

$\frac{1}{x}$