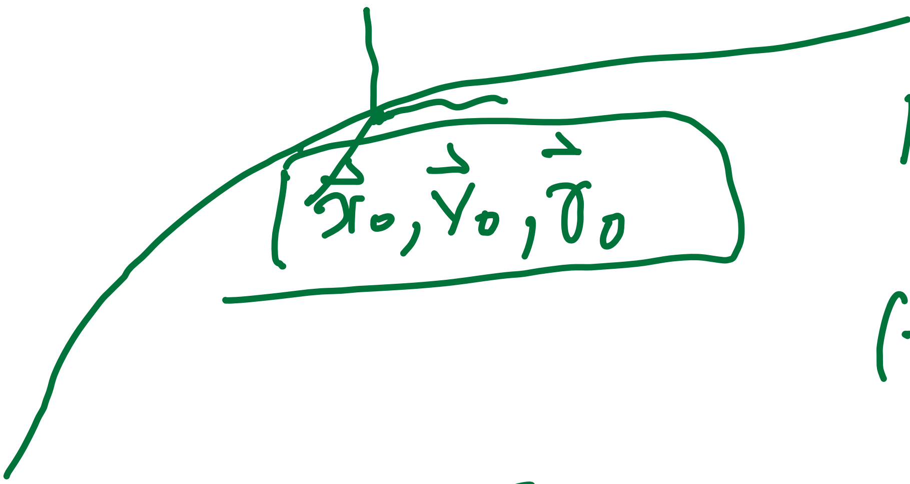
Δευτέρα 13/10/2025



$F = \mu_m \gamma$

$(t - t_0)^3$

$\vec x_0, \vec v_0, \vec \gamma_0$

$$\vec x' = \vec x_0 + \vec v_0 t + \frac{1}{2} \vec \gamma_0 t^2$$

$$\vec x(t) - \vec x'$$

Θέση
στη ένι
ταχυνόμετρο
σάλμα

$$\boxed{\ddot{\vec x} = -\vec g} \quad -g$$

$$\boxed{\vec x(t) = \vec x_0 + \vec v_0 t + \ddot{\vec x}\big|_0 \frac{t^2}{2!}}$$

$$+ \cancel{\dddot{x}\Big|_0 t^3\Big|_3} + \cdots$$

$$\dddot{x}\Big|_0 = \cdots\cdots \vec{x}_0^{(k)} \Rightarrow$$

$$\cancel{\vec{x}' = \frac{1}{2}\vec{\gamma}_0 t}$$

$$\Rightarrow \vec{x}' = -\frac{1}{2}\vec{g}\,t^2$$

$$\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t - \frac{1}{2}\vec{g}\,t^2$$

$$\boxed{\vec{x}(t) - \vec{x}' = \vec{x}_0 + \vec{v}_0 t}$$

$$\vec{V}_0 \rightarrow \vec{t} = \frac{\vec{v}_0}{|\vec{v}_0|}$$

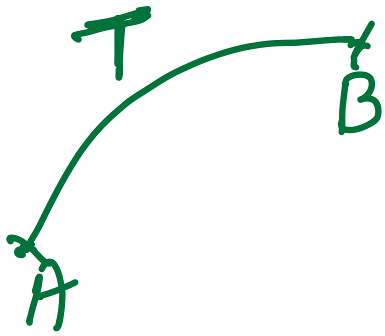
$$\vec{\gamma} = \frac{d^2 s}{dt^2}\,\vec{t} + \frac{v^2}{\rho}\,\hat{n}$$

$$\vec{\gamma} \wedge \hat{t} = \frac{v_0^2}{\rho}\,\underbrace{\vec{t} \wedge \hat{n}}_{\vec{1}}$$

b

$$|\vec{\gamma} \wedge \vec{t}| = \frac{v_1^2}{\rho}$$

$$\boxed{\rho_0 = \frac{v_0^3}{|\vec{\gamma}_0 \wedge \vec{v}_0|}}$$



$$\begin{cases} m\dfrac{d^2\vec{x}}{dt^2} = \vec{F}(\vec{x}) \\ m' = \lambda m \end{cases}$$

$$t' = \lambda^n t$$

$$\lambda^{2n} \frac{m'}{\lambda} \frac{d^2\vec{x}}{dt'^2} = \vec{F}$$

$\lambda^{(2n-1)}$ $\quad m' \dfrac{d^2 \vec{x}'}{dt'^2} = \vec{F}$

$m' = \lambda m$

$t' = \lambda^{1/2} t$

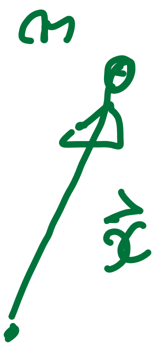
$2n-1=0 \qquad n = 1/2$

$$m' \frac{d^2 x'}{dt'^2} = \vec{F}$$

4

2

$$\cancel{m} \frac{d^2 \vec{x}}{dt^2} = - \frac{\cancel{m} K}{|\vec{x}|^2} \hat{x}$$



$$\frac{d^2 \vec{x}}{dt^2} = - \frac{K}{|\vec{x}|^{\alpha}} \hat{x}$$

$$\boxed{\begin{aligned} \vec{x}' &= \lambda \vec{x} \\ t' &= \lambda^{n} t \end{aligned}}$$

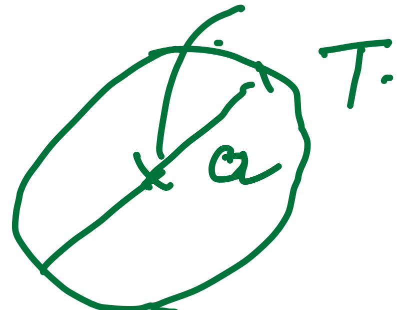
$\boxed{\text{Kλi fraka Scaling}}$

$$\frac{\lambda^{2n}}{\lambda}\,\frac{d^2\vec{x}'}{dt'^2} = -\,\frac{k\lambda^{\alpha}}{|\vec{x}'|^{\alpha}}\,\hat{x}'$$

$$\lambda^{2n-1-\alpha} = 1 \qquad \boxed{n = \frac{\alpha+1}{2}}$$

βαρύτητα, Coulomb δυνάμεις

$\alpha = +2 \qquad n = 3/2$



Kepler

$$\frac{a^3}{T^2}$$

$$\vec{x}' = \lambda\,\vec{x}$$
$$t' = \lambda^{3/2}\,t$$

$\vec{x}'^2$

$$t'^2 = \lambda^3\,t$$

είναι ανάλογα

$$\cancel{\frac{a}{T^{3/2}}} \qquad \cancel{\frac{a^2}{T^3}}$$

$$\frac{|x'|}{|x|} = \lambda$$

$$\frac{t'}{t} = \lambda^{3/2}$$

$$\cancel{t}\left(\frac{t'}{t}\right)^2 = \left(\frac{x'}{x}\right)^3$$

$\frac{a^3}{T^2}$ σταθερό Kepler

$n = \frac{\alpha+1}{2}$ $\boxed{\alpha = -1}$

$n = 0$

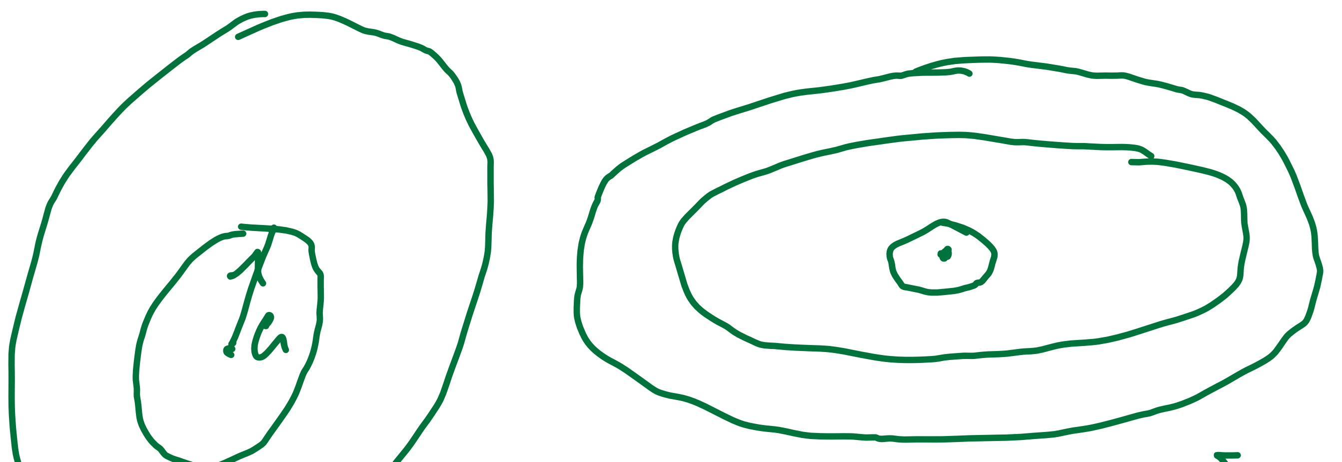
$$\boxed{\begin{aligned} \vec{x}' &= \lambda \vec{x} \\ t' &= \lambda^0 t. \end{aligned}}$$

αρμονική ισόχρονη κίνηση

$$m\ddot{\vec{x}} = -K|\vec{x}|\hat{x} = -K\vec{x}$$

ισότροπη αρμονική ταλαντωτής

μέθοδοι ομοιότητας.



Περίοδος
είναι
ίδια!

Kepler

$$\frac{a^3}{T^2} \text{ σταθ.}$$

α/ ταχύτητα
ταχύτητα
ν/ταχύτητα

$\vec{v}(\vec{x},t)$

$\rho(\vec{x},t)$

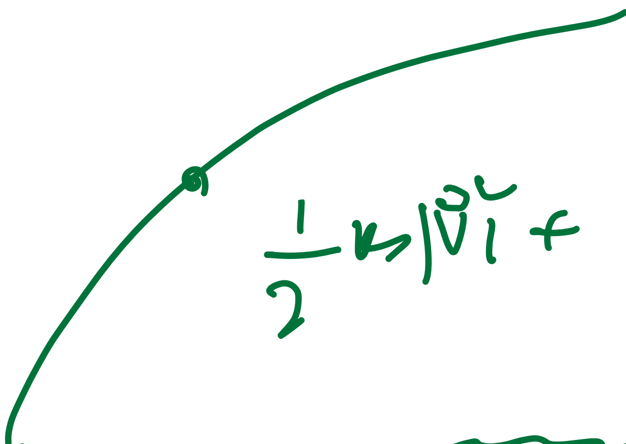
$\partial\rho$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

Εξίσωση
διατήρησης
της μάζας

local
τοπικά

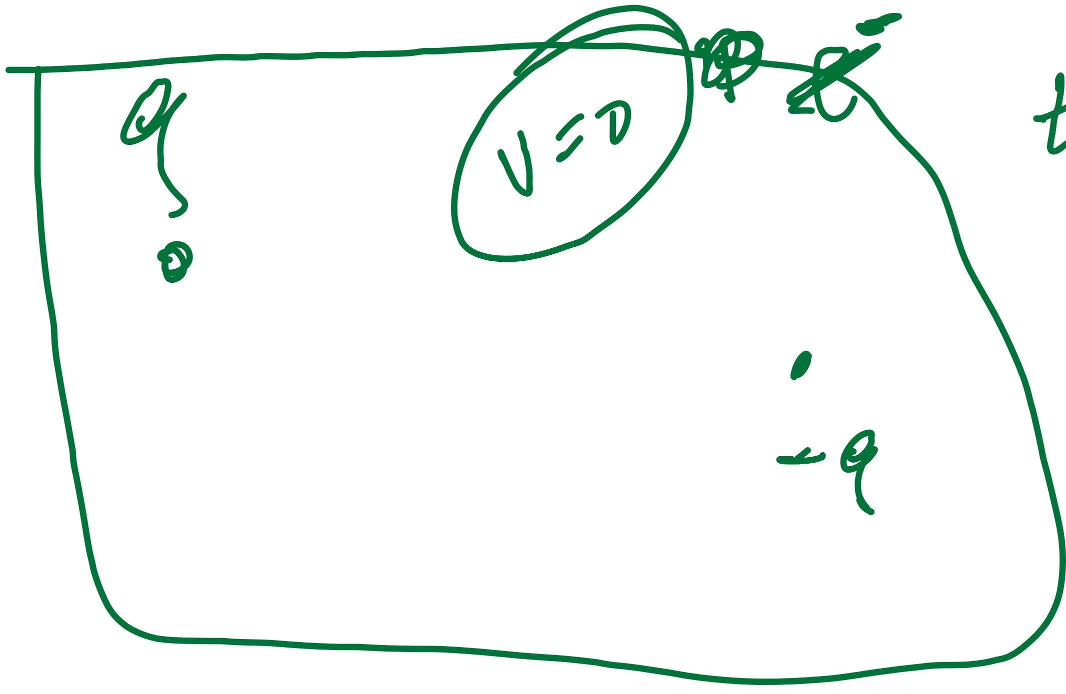
$E(t)$



$$\frac{1}{2}m|\vec{v}|^2 + V(\vec{x}) = E(t)$$

$$\boxed{\frac{dE}{dt} = 0}$$

$\cancel{x}$



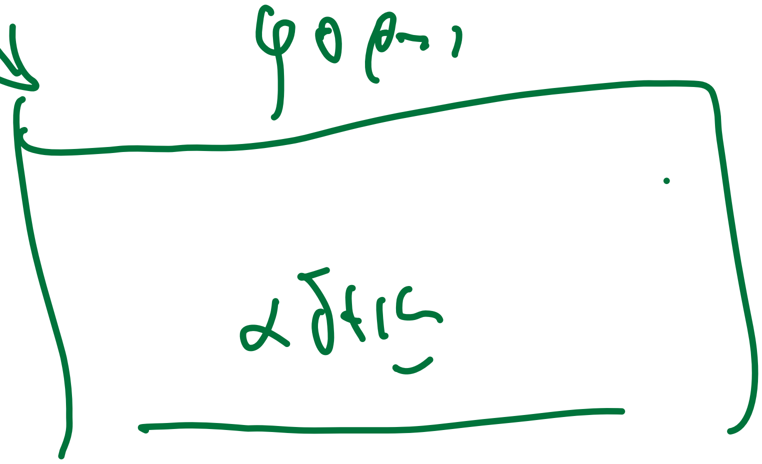
t=0

$\oint \sum q(t)$

t

$t = 0.1$

καυτό φορτίο
φορτί,



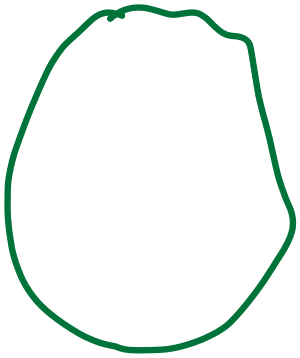
$t = 0.1$

Τοπική $d\dot{\alpha}$ $\leftrightarrow$ globalr

$$\frac{\partial}{\partial t}\int \rho\, d^3\vec{x} = -\int \nabla\cdot(\rho\vec{v})\, d^3x = -\oint \rho\, \dot{\vec{x}}\cdot d\vec{S} \rightarrow 0$$

Ένα

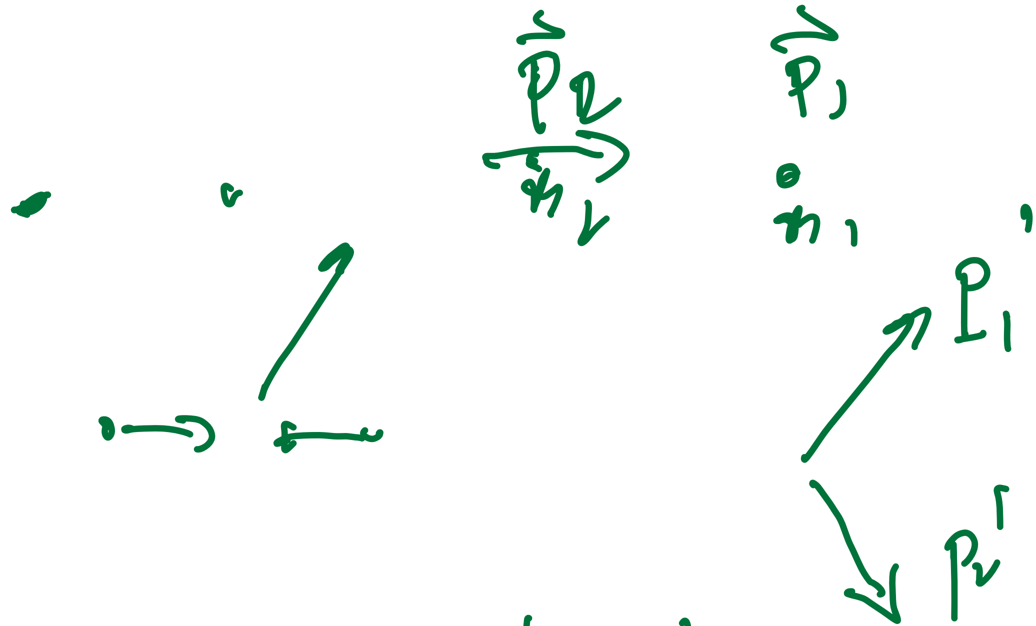
Σύστημα



α) $$\vec{P} = \sum \underbrace{m_i \dot{\vec{x}}_i}_{\vec{p}_i} = M \dot{\vec{X}}_{KM}$$

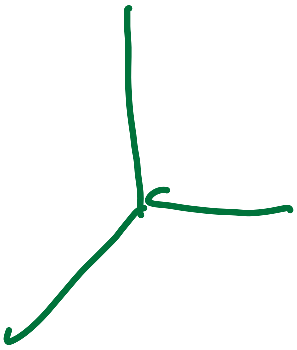
$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}^{\varepsilon\xi}}$$

$\vec{F}^{\varepsilon\xi} = 0 \rightarrow$ $\boxed{\vec{P} = \text{σταθερό}}$



$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

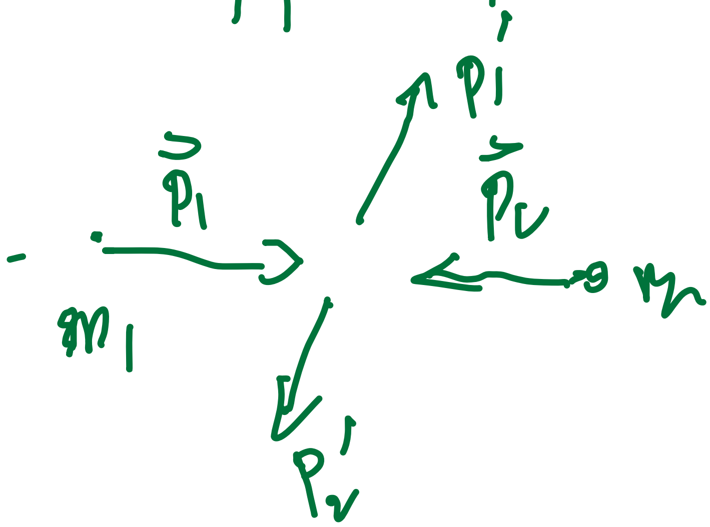
Oi KM



$$\vec{P} = 0$$

2 ou

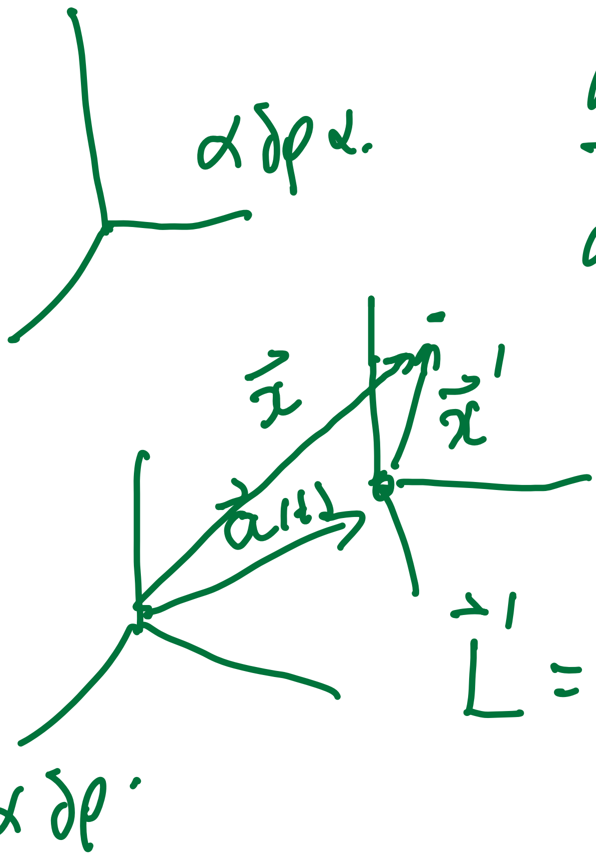
$$\vec{p}_1 = -\vec{p}_2$$



β)

$$\left.\begin{aligned}\vec{L} &= \sum m_i \, \vec{x}_i \wedge \dot{\vec{x}}_i \\ \vec{\tau} &= \sum \vec{x}_i \wedge \vec{F}_i^{\varepsilon\xi}\end{aligned}\right\}$$

εσωτερικές
δυνάμεις
δεν συμβάλλουν



$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\vec{x}' = \vec{x} - \vec{a}$$

$$\boxed{\vec{\Omega} \wedge \vec{x}'_i}$$

$$\vec{L}' = \sum m_i \, \vec{x}'_i \wedge \dot{\vec{x}}'_i$$

$$= \sum m_i (\vec{x}_i - \vec{a}) \wedge (\dot{\vec{x}}_i - \dot{\vec{a}})$$

$$= \underbrace{\sum m_i \, \vec{x}_i \wedge \dot{\vec{x}}_i}_{\vec{L}} + M \vec{a} \wedge \dot{\vec{a}} - \vec{a} \wedge \sum m_i \dot{\vec{x}}_i - \left(\sum m_i \vec{x}_i\right) \wedge \dot{\vec{a}}$$

$$\boxed{\vec{L}' = \vec{L} + M\,\vec{a}\wedge\dot{\vec{a}} - \vec{a}\wedge\vec{P} - M\vec{X}_{KM}\wedge\dot{\vec{a}}}$$

$$\frac{d\vec{L}'}{dt} = \frac{d\vec{L}}{dt} + M\,\vec{a}\wedge\ddot{\vec{a}} - \vec{a}\wedge\dot{\vec{P}} - \cancel{\underline{\dot{\vec{a}}\wedge\vec{P}}} - \cancel{\underline{\vec{P}\wedge\dot{\vec{a}}}} - M\vec{X}_{KM}\wedge\ddot{\vec{a}}$$

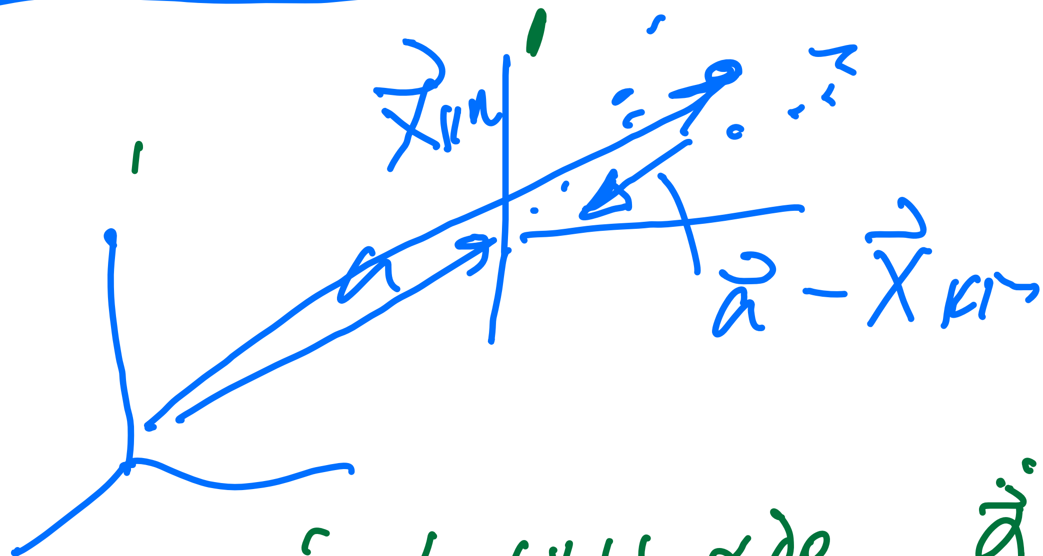
$$= \frac{d\vec{L}}{dt} + M(\vec{a} - \vec{X}_{KM})\wedge\ddot{\vec{a}} - \vec{a}\wedge\underset{\vec{F}_{\varepsilon\xi}}{\overset{}{\underset{\shortparallel}{\dot{\vec{P}}}}}$$

$$= \underbrace{\vec{\tau} - \vec{a}\wedge\vec{F}^{\varepsilon\xi}}_{\vec{\tau}'} + M(\vec{a} - \vec{X}_{KM})\wedge\ddot{\vec{a}}$$

$$\vec{\tau}' = \sum \vec{x}_i'\wedge\vec{F}_i = \sum(\vec{x}_i - \vec{a})\wedge\vec{F}_i$$

$$= \vec{\tau} - \vec{a}\wedge\sum\vec{F}_i = \tau - \vec{a}\wedge\vec{F}_{\varepsilon\xi}$$

$$\boxed{\frac{d\vec{L}'}{dt} = \vec{\tau}' + M(\vec{a} - \vec{X}_{KM}) \wedge \ddot{\vec{a}}}$$



α) αν το ' είναι αδρ $\qquad \underline{\ddot{\vec{a}} = 0}$

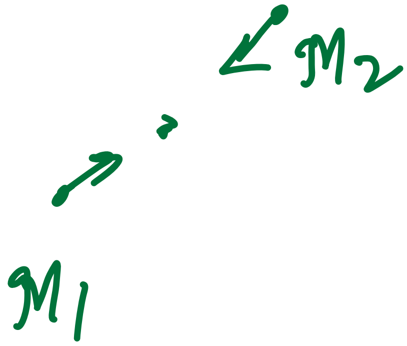
$$\frac{d\vec{L}'}{dt} = \vec{\tau}'$$

β) $\vec{a} = \vec{X}_{KM}$

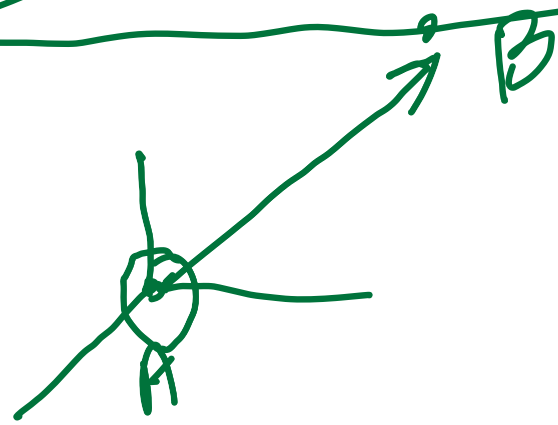
$$\boxed{\frac{d\vec{L}'}{dt} = \vec{\tau}'}$$

ως προς

το KM



$$\frac{d\vec{L}_{KM}'}{dt} = 0$$



$$\frac{d\vec{L}_B}{dt} \ ?$$

$$\vec{L} = (\vec{X}_B - \vec{X}_A) \wedge (\dot{\vec{x}}_B - \dot{\vec{x}}_A)$$

$$\boxed{\begin{aligned} m_1 \frac{d^2\vec{x}_1}{dt^2} &= \vec{F} \\ m_2 \frac{d^2\vec{x}_2}{dt^2} &= -\vec{F} \end{aligned}} \quad \vec{F}(\vec{x}_1, \vec{x}_2) \quad (\vec{x}_1, \vec{x}_2)$$

$$\vec{X} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{M}$$

$$M = m_1 + m_2$$

M

$$\boxed{\vec{x} = \vec{x}_1 - \vec{x}_2}$$

αθροίζοντας $\quad M\ddot{\vec{X}} = 0 \Rightarrow \vec{P}$ σταθερή

$$\frac{d^2\vec{x}}{dt^2} = \vec{F}\underbrace{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}_{1/\mu} \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\boxed{\begin{aligned} \mu \frac{d^2\vec{x}}{dt^2} &= \vec{F} \\ M\ddot{\vec{X}} &= 0 \end{aligned}} \xrightarrow[\mu]{M}$$

$2 \to 1$

$$\cancel{\vec{F}} + \cancel{\left(\vec{F}_1(\vec{X}_1) - \vec{F}_2(\vec{x}_2)\right)}$$

$$\cancel{F(\vec{X}_1)} + F(\vec{x}_2) \qquad F(\vec{x})$$

Χρειάζεται απομονωμένο
σύστημα για να είναι ο
διαχωρισμός!

$$\vec{x}_1 = \vec{X} + \frac{m_2}{M}\vec{x} \qquad \vec{x}_1 - \vec{x}_2 = \vec{x}$$

$$\vec{x}_2 = \vec{X} - \frac{m_1}{M}\vec{x}$$

$$\vec{p} = \mu\dot{\vec{x}} = \mu(\dot{\vec{x}}_1 - \dot{\vec{x}}_2) =$$

$$= \frac{m_2}{M}\vec{p}_1 - \frac{m_1}{M}\vec{p}_2$$

$$\boxed{\begin{aligned}\vec{p} &= \frac{m_2}{M}\vec{p}_1 - \frac{m_1}{M}\vec{p}_2 \\ \underline{\vec{P}} &= \vec{p}_1 + \vec{p}_2\end{aligned}} \qquad \boxed{\begin{aligned}\vec{p}_1 &= \vec{p} + \frac{m_1}{M}\underline{\vec{P}} \\ \vec{p}_2 &= -\vec{p} + \frac{m_2}{M}\underline{\vec{P}}\end{aligned}}$$

$$\vec{L} = \vec{x}_1 \wedge \vec{p}_1 + \vec{x}_2 \wedge \vec{p}_2$$

$$=$$

$$\boxed{\begin{aligned} \frac{m_2}{M}\vec{p}_1 &= \frac{m_2}{M}\vec{p} + \mu \vec{P} \\ \frac{m_1}{M}\vec{p}_1 &= -\frac{m_1}{M}\vec{p} + \mu \vec{P} \end{aligned}}$$

$$= \left(\vec{X} + \frac{m_2}{M}\vec{x}\right) \wedge \left(\vec{p} + \frac{m_1}{M}\vec{P}\right) +$$

$$+ \left(\vec{X} - \frac{m_1}{M}\vec{x}\right) \wedge \left(-\vec{p} + \frac{m_2}{M}\vec{P}\right)$$

$$= \vec{X} \wedge \vec{P} + \vec{x} \wedge \vec{p}$$

$$\vec{L} = \vec{x}_1 \wedge \vec{p}_1 + \vec{x}_2 \wedge \vec{p}_2$$

$$= \underbrace{\vec{X} \wedge \vec{P}}_{\text{σταθερό}} + \underline{\vec{x} \wedge \vec{p}}$$ διατηρείται

σύνολα α

$\underline{KM}$ $\qquad \vec{p}_1^{\,\prime} = -\vec{p}_2^{\,\prime} \qquad \vec{p}_1 + \vec{p}_2 = 0$

$$\vec{\ell}_1^{\,\prime} = (\vec{x}_1 - \vec{X}) \wedge \vec{p}_1^{\,\prime} = \frac{m_2 \vec{x}}{M} \wedge \vec{p}_1^{\,\prime}$$

$$\vec{\ell}_2^{\,\prime} = (\vec{x}_2 - \vec{X}) \wedge \vec{p}_2^{\,\prime} = -\frac{m_1}{M} \vec{x} \wedge \vec{p}_2^{\,\prime}$$

$$\boxed{m_1 \vec{\ell}_1^{\,\prime} = m_2 \vec{\ell}_2^{\,\prime}}$$

$\vec{L}^{\prime} = \vec{\ell}_1^{\,\prime} + \vec{\ell}_2^{\,\prime}$ διατηρείται

$\Rightarrow$ $\vec{\ell}_1^{\,\prime}$ διατηρείται

$\qquad \vec{\ell}_2^{\,\prime}$ διατηρείται

αδρανειακό σ.	$\vec{p}_1 + \vec{p}_2$ δεν $\vec{p}_1, \vec{p}_2$		$\vec{L}$ δεν ℓ_1, ℓ_2	
KM	p		$\vec{\ell}_1^{\,\prime}$	

$\vec{p}\,\delta r_i$

e_2'