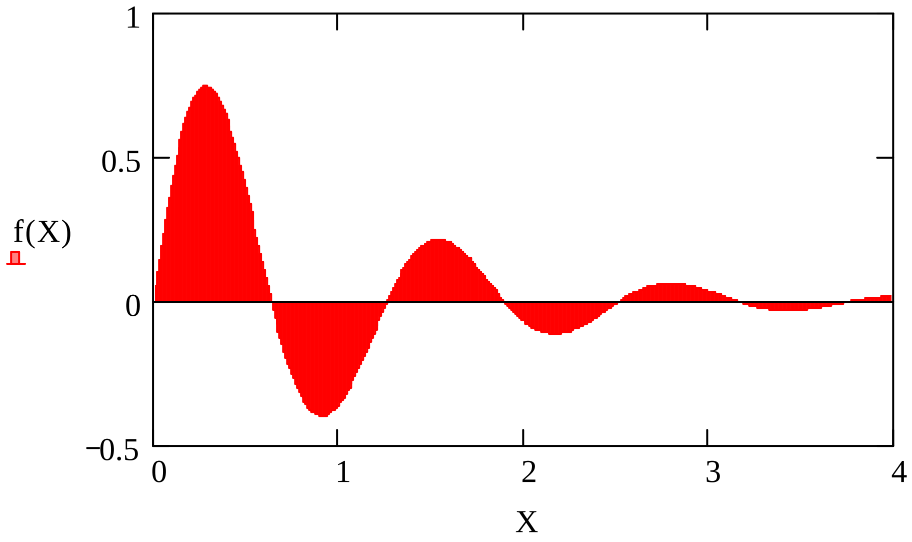
Simple Programming

$f(x) := \sin(5x)\cdot e^{-x}$ $X := 0, 0.01 .. 4$



$$\int_0^{10} f(x)\,dx = 0.1923$$

Integration: Simpson's Rule

$$INTG(F, A, B, N) := \left| \begin{array}{l} NN \leftarrow 2^N \\ Step \leftarrow \dfrac{(B - A)}{NN} \\ Sum \leftarrow 0.0 \\ \text{for } i \in 1 .. NN - 1 \\ \quad \left| \begin{array}{l} xx \leftarrow A + i\cdot Step \\ Sum \leftarrow Sum + F(xx) \end{array} \right. \\ Sum \leftarrow Sum + \dfrac{F(A) + F(B)}{2} \\ INTG \leftarrow Sum\cdot Step \\ \text{return } INTG \end{array} \right.$$

$$\int_0^{10} f(x)\,dx = 0.19229973$$

$$INTG(f, 0, 10, 12) = 0.19229724$$

Symbolic Evaluation

$$\int \sin(5x)\cdot e^{-x}\,dx \rightarrow \frac{-5}{26}\cdot \exp(-x)\cdot \cos(5\cdot x) - \frac{1}{26}\cdot \sin(5\cdot x)\cdot \exp(-x)$$

Differential Equations

Harmonic Oszillator: Solutions through Runge-Kutta Integration

The differential equation has the form: $\frac{d^2x}{dt^2} = -kx$

If $x(t) = X_0$ and $dx/dt = X_1$ then the differential equation is equivalent to the following system:

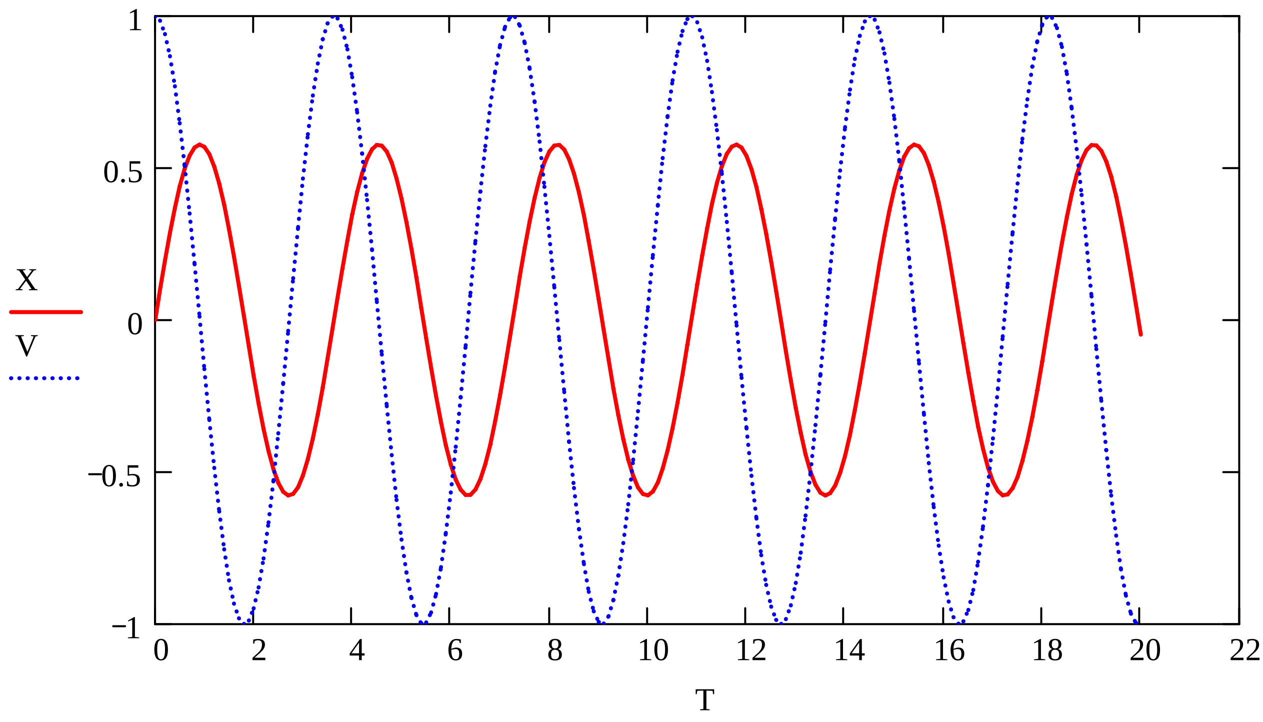
$$dX_0/dt = X_1$$
$$dX_1/dt = -kX_0$$

Using a fixed step Runge-Kutta method:

$t0 := 0.0$ $t1 := 20$ $N := 200$ $k := 3$

$D(t,X) := \begin{pmatrix} X_1 \\ -k \cdot X_0 \end{pmatrix}$ $ic := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $S := rkfixed(ic, t0, t1, N, D)$

$V := S^{\langle 2 \rangle}$ $X := S^{\langle 1 \rangle}$ $T := S^{\langle 0 \rangle}$



Harmonic Oszillator with Friction

$t0 := 0.0$ $t1 := 20$ $N := 200$ $k := 3$ $c := 0.2$

$$D(t,X) := \begin{pmatrix} X_1 \\ -k \cdot X_0 - c \cdot X_1 \end{pmatrix}$$

$$ic := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S := \text{rkfixed}(ic, t0, t1, N, D)$$

$V := S^{\langle 2 \rangle}$ $X := S^{\langle 1 \rangle}$ $T := S^{\langle 0 \rangle}$