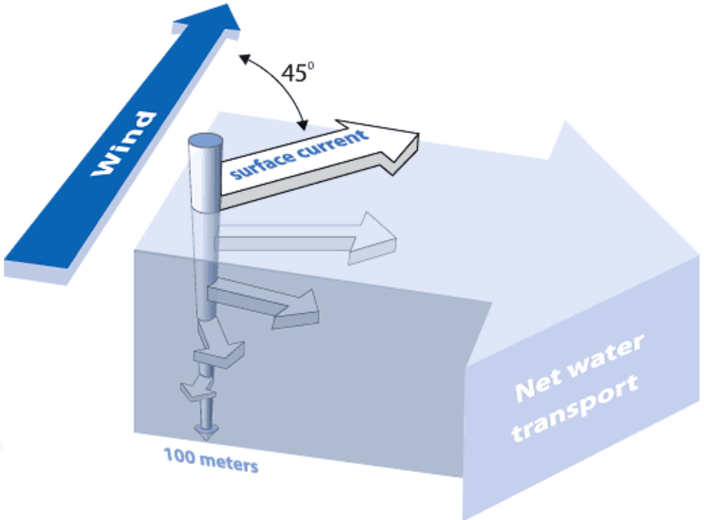
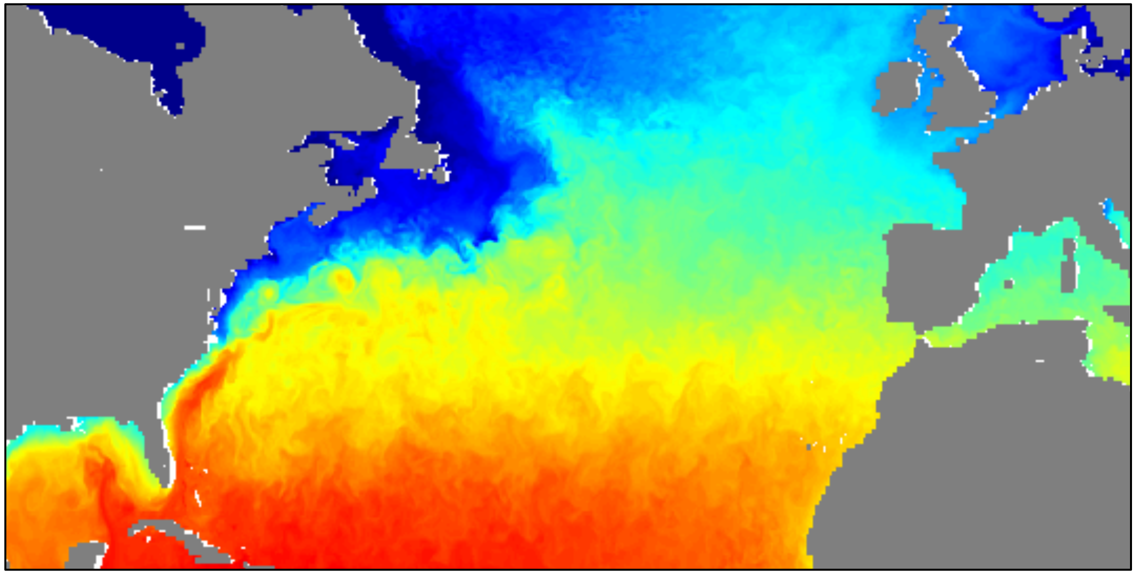
Department of Physics
University of Athens



5. Wind driven circulation

5. Ανεμογενής κυκλοφορία



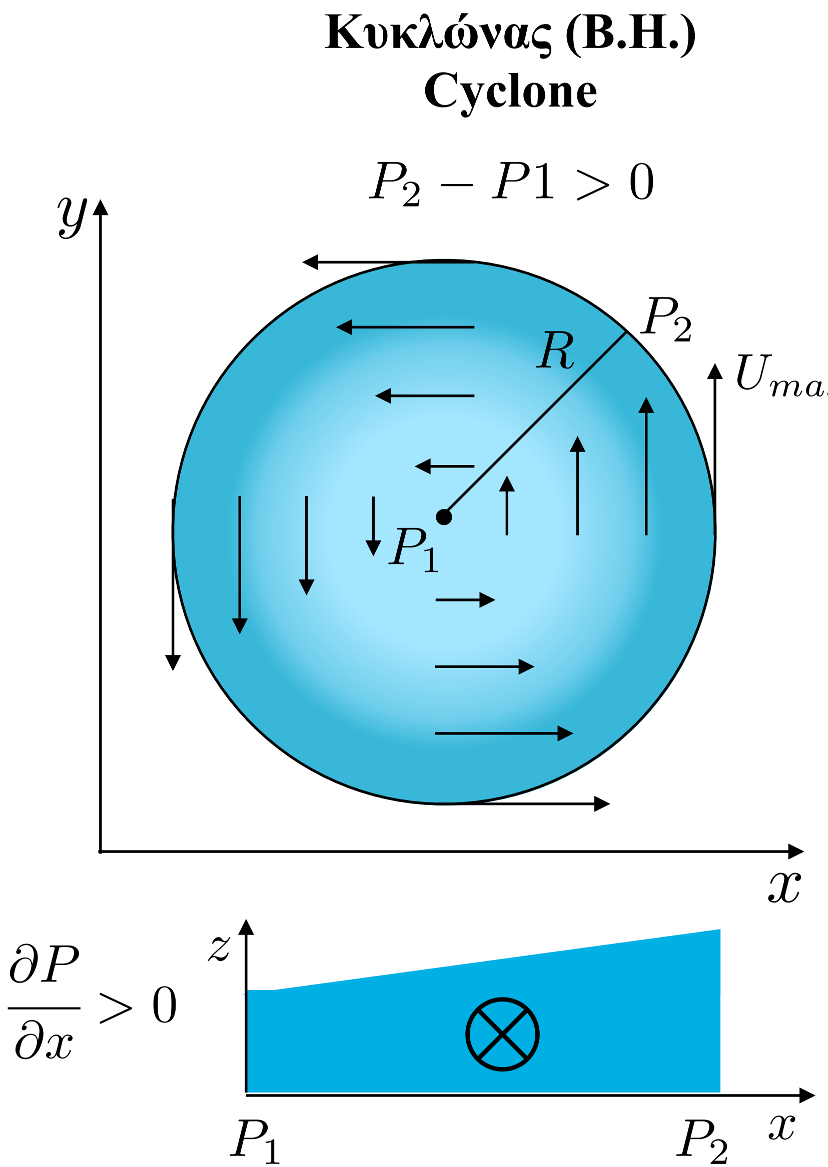
- Εισαγωγικές έννοιες
- Η θεωρία του Ekman
- Η θεωρία του Sverdrup για την ανεμογενή κυκλοφορία
- Το δυτικό όριο και οι προσεγγίσεις των Stommel καιMunk

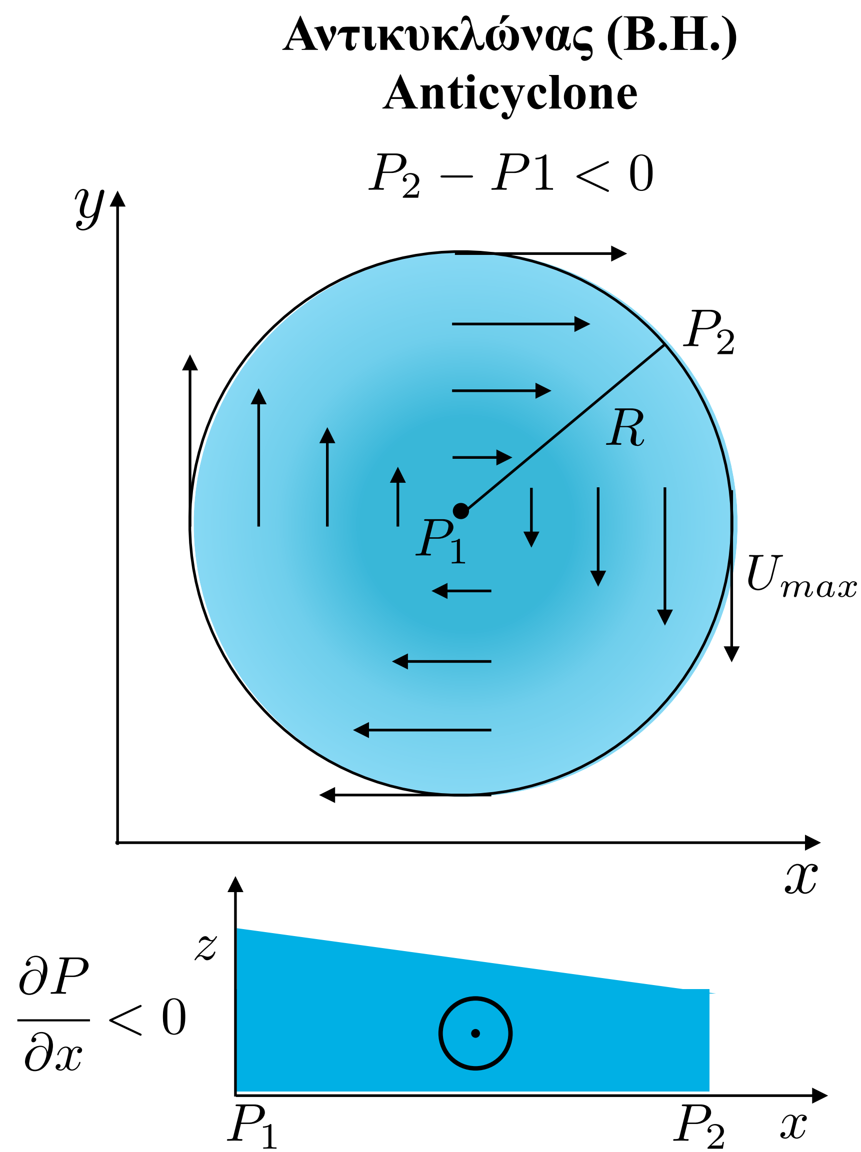
Σ. Σοφιανός & Β. Βερβάτης

The geostrophic – hydrostatic limit

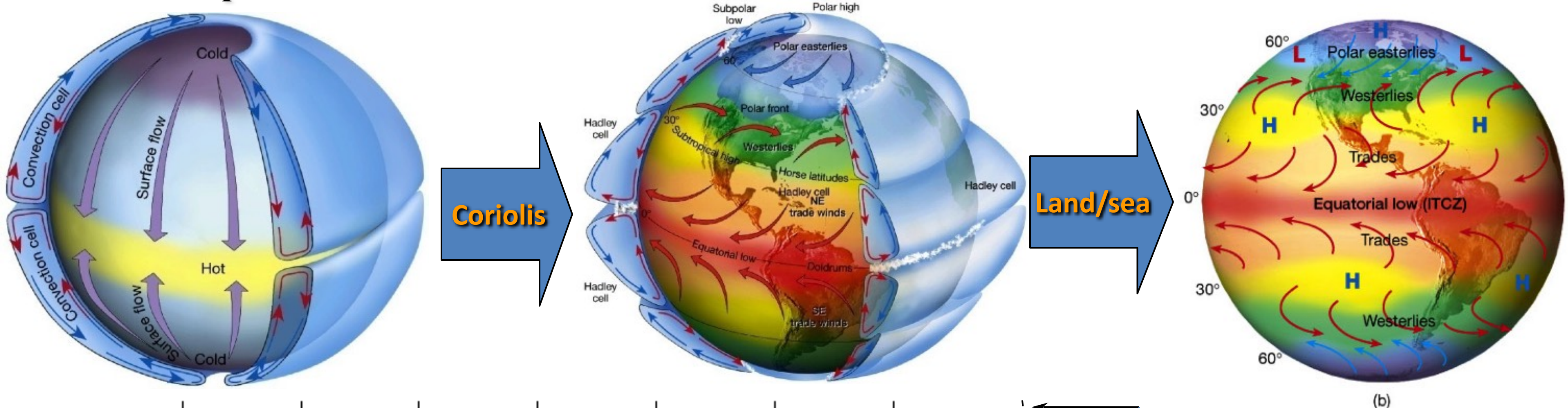
Large L

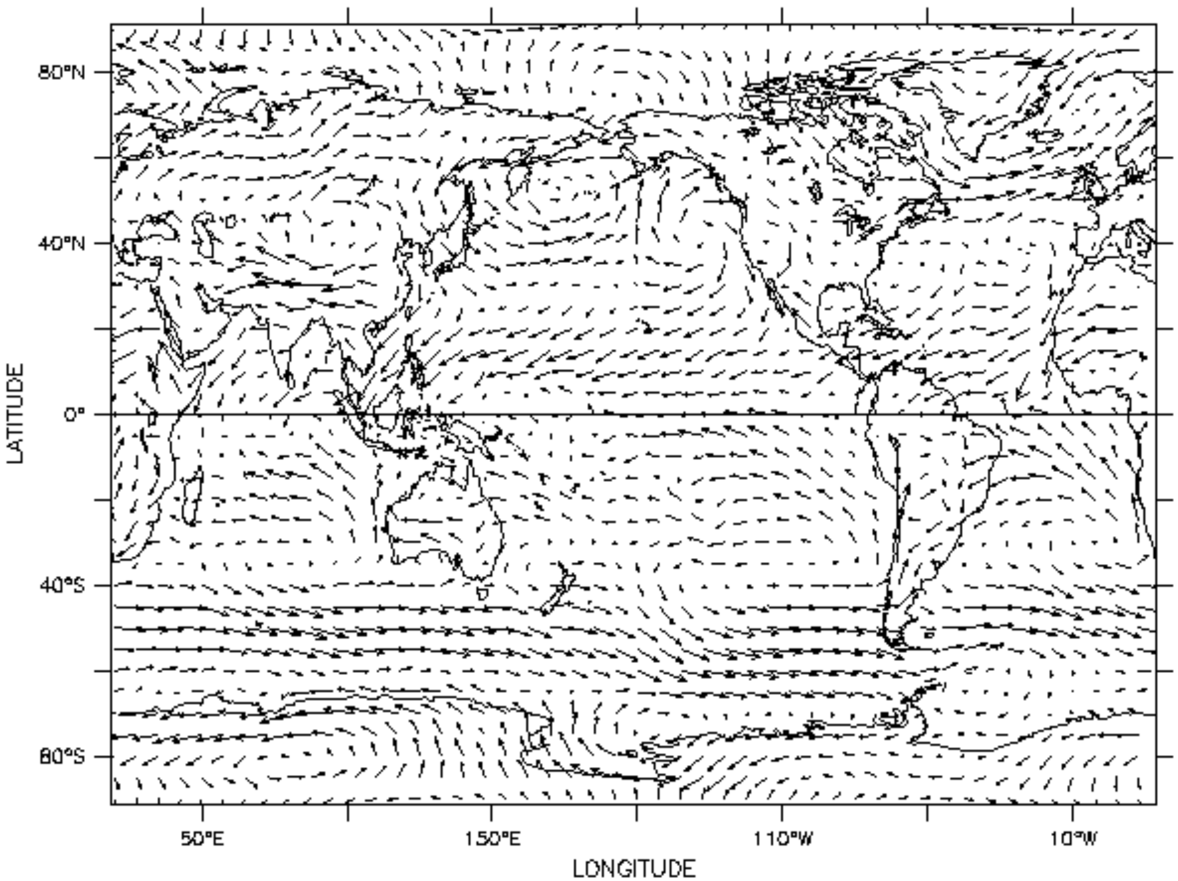
$$fv = \frac{1}{\rho_0}\frac{\partial p}{\partial x}, \quad (1) \quad fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}, \quad (2) \quad \frac{1}{\rho_0}\frac{\partial p}{\partial z} = -g \quad (3)$$

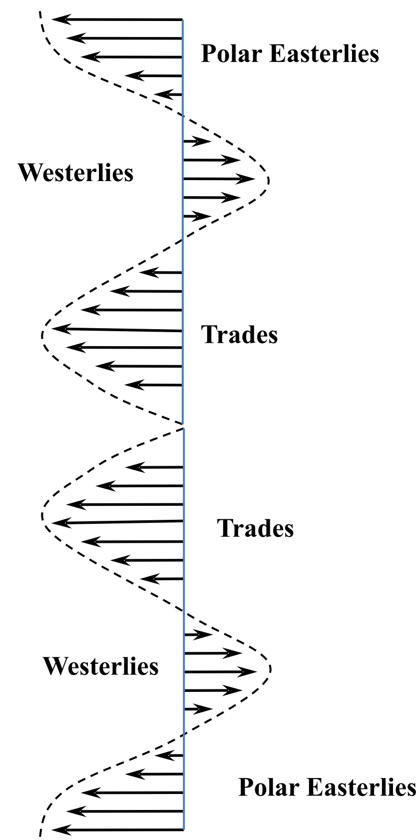




Global atmospheric circulation







Κατανομή του ανέμου με το γεωγραφικό πλάτος

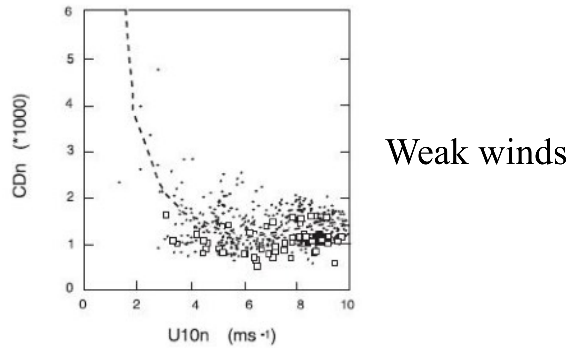
Wind stress on the sea

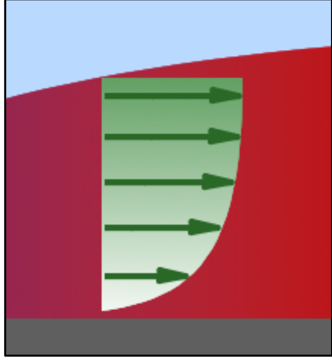
$$\vec{\tau} = c_D \rho_a |u_{10}| \vec{u}_{10}$$

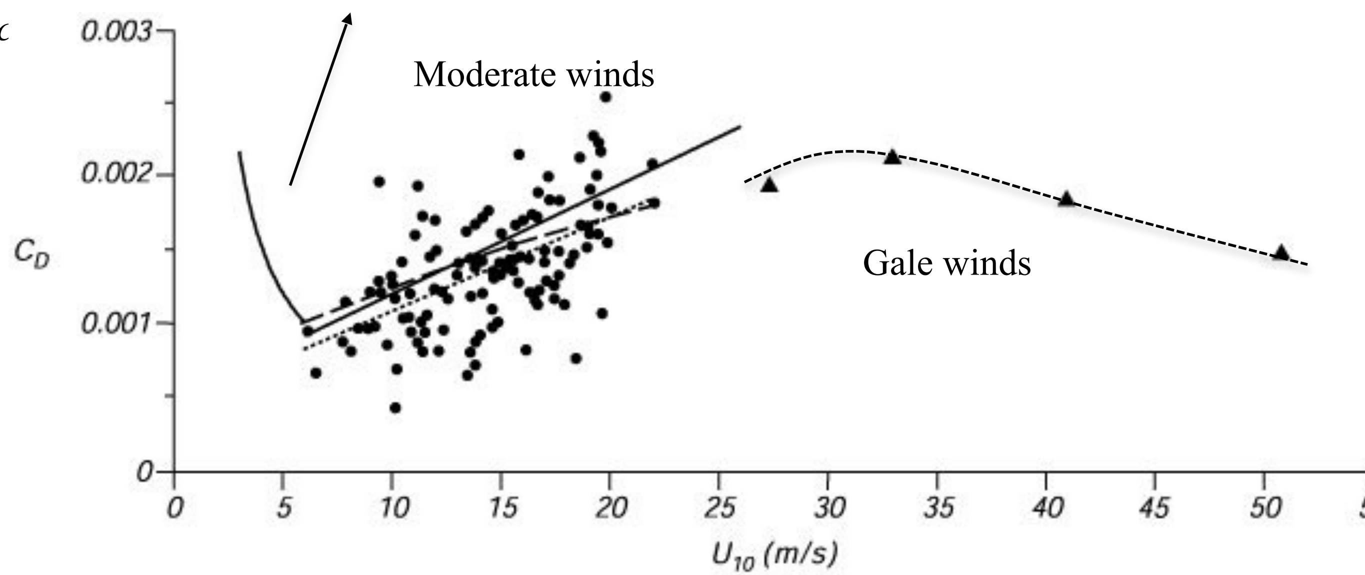
where:

ρ_α air density (1.3 kg/m^3)
u_{10} wind speed at10 m (m/sec
c_D drag coefficient (unitless)

Measurements showed that c_D is related to the wind speed (not a constant). Nevertheless, we usually use a constant value for simplicity.







Examples of c_D used:

$$c_D = 1.4\times10^{-3}$$

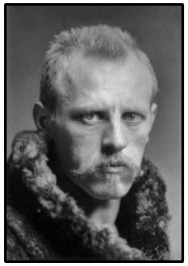
$c_D = 1.1\times10^{-3}$ ασθενείς άνεμοι

$c_D = (0.61 + 0.063u)\times10^{-3}$ ισχυροί άνεμοι

$c_D = (0.29 + 3.1u_{10}^{-1} + 7.7u_{10}^{-2})10^{-3}$
for $3m/\sec \le u_{10} \le 6m/\sec$

$c_D = (0.60 + 0.070u_{10)})10^{-3}$
for $6m/\sec \le u_{10} \le 26m/\sec$

Evolution of wind-driven circulation theory:



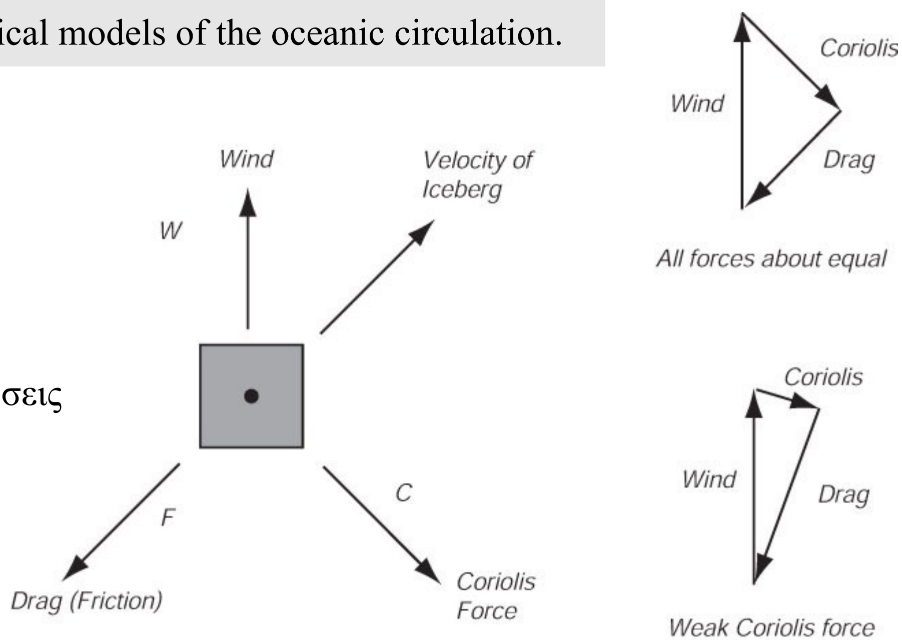
Fridtjof Nansen
1861-1930

Fridtjof Nansen	(1898)	Qualitative theory, currents transport water at an angle to the wind.
Vagn Walfrid Ekman	(1902)	Quantitative theory for wind-driven transport at the sea surface.
Harald Sverdrup	(1947)	Theory for wind-driven circulation in the eastern Pacific.
Henry Stommel	(1948)	Theory for westward intensification of wind-driven circulation (western boundary currents).
Walter Munk	(1950)	Quantitative theory for main features of the wind-driven circulation
Kirk Bryan	(1963)	Numerical models of the oceanic circulation.

The surface wind effect (Nansen theory):

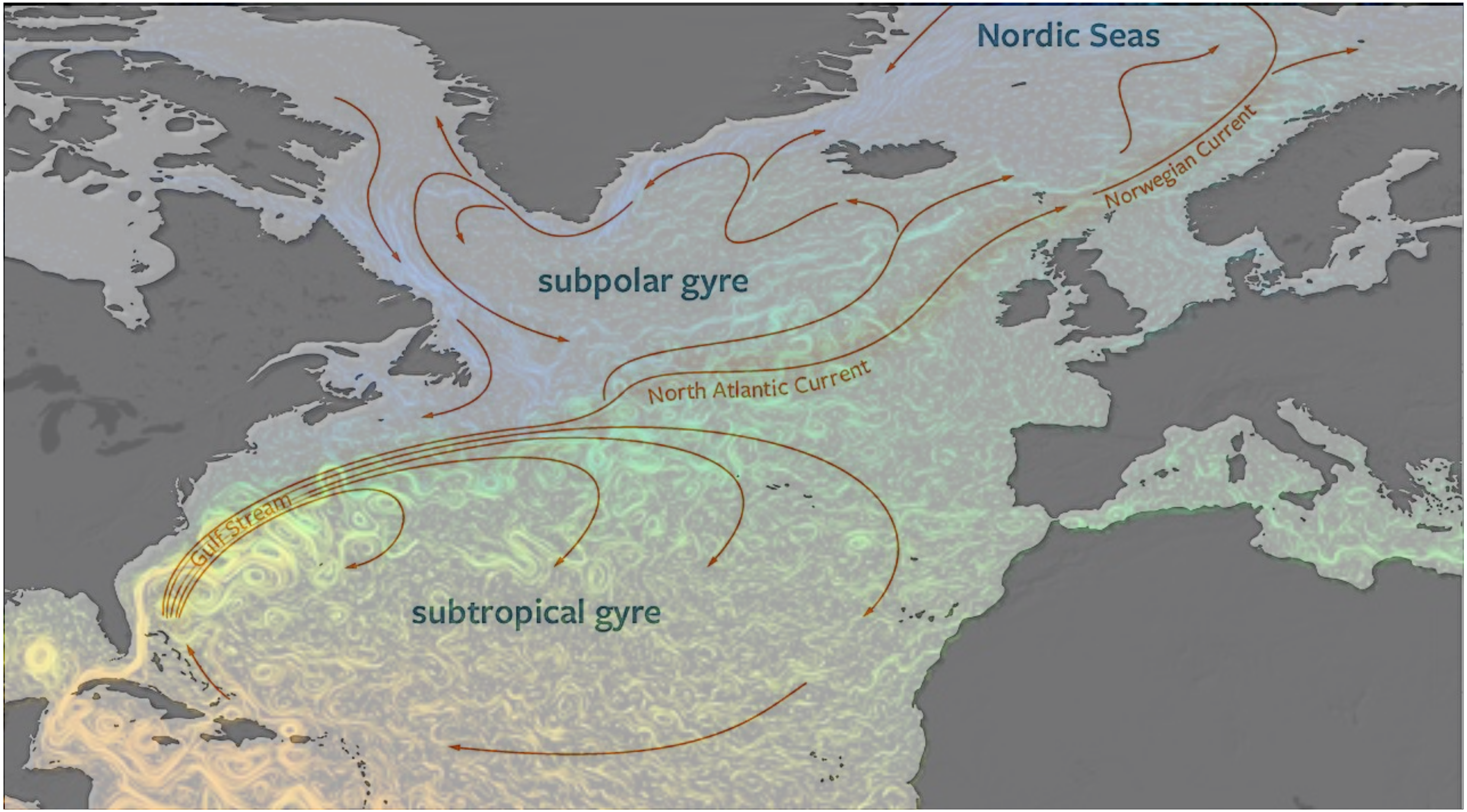
Βασισμένος σε παρατηρήσεις
της κίνησης παγόβουνων

$$W + F + C = 0$$



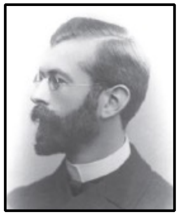
From Stewart, 2008: Introduction to Physical Oceanography

Η περιοχή μελέτης



Πως εξηγείται η δομή της κυκλοφορίας (με τα ίδια χαρακτηριστικά στους τρεις ωκεανούς); Αυτά θα μελετηθούν με τις θεωρίες Ekman, Sverdrup, Stommel και Munk.

Ekman Theory



Vagn Walfrid Ekman
1874-1954

Εξισώσεις εργασίας:

Conservation of momentum equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + f\upsilon + A_H\frac{\partial^2 u}{\partial x^2} + A_H\frac{\partial^2 u}{\partial y^2} + A_V\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial \upsilon}{\partial t} + u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} + w\frac{\partial \upsilon}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} - fu + A_H\frac{\partial^2 \upsilon}{\partial x^2} + A_H\frac{\partial^2 \upsilon}{\partial y^2} + A_V\frac{\partial^2 \upsilon}{\partial z^2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + \upsilon\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - g + A_H\frac{\partial^2 w}{\partial x^2} + A_H\frac{\partial^2 w}{\partial y^2} + A_V\frac{\partial^2 w}{\partial z^2}$$

Conservation of mass equation:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Υποθέσεις:

Κινήσεις μεγάλης (οριζόντιας) κλίμακας: very large L

Για απλούστευση των λύσεων θα αγνοήσουμε μεταβολές στην πυκνότητα: $\rho = \rho_0$

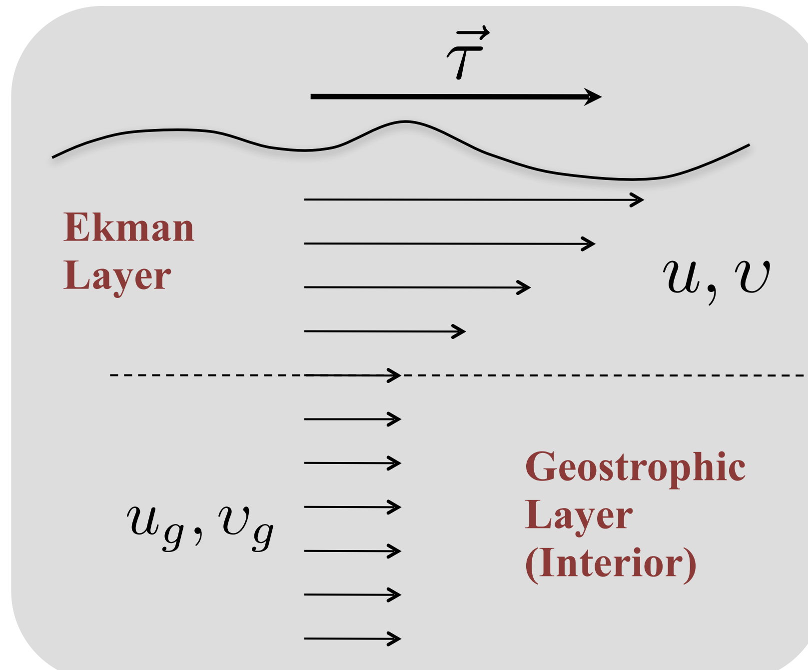
$$E_k = \frac{\text{Viscosity terms}}{\text{Coriolis terms}} \begin{cases} A_H \dfrac{U}{L^2}\dfrac{1}{fU} = \dfrac{A_H}{fL^2} \ll 1 \\[2ex] A_V \dfrac{U}{H^2}\dfrac{1}{fU} = \dfrac{A_V}{fH^2} \sim 1 \end{cases} \qquad R_o = \frac{\text{Non-linear terms}}{\text{Coriolis terms}} = \frac{U}{fL} \ll 1$$

Θεωρώντας ότι το H είναι το βάθος επίδρασης του ανέμου << του ολικού βάθους

$$A_H \nabla_H^2 \vec{u} \ll A_V \frac{\partial^2 \vec{u}}{\partial z^2}$$

Και οι εξισώσεις κίνησης γίνονται:

$$\left.\begin{aligned} -fv &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + A_V \frac{\partial^2 u}{\partial z^2} \\ fu &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + A_V \frac{\partial^2 v}{\partial z^2} \end{aligned}\right\} \quad \text{(A)}$$



Η γεωστροφική ισορροπία συνεχίζει να ισχύει (ο άνεμος δημιουργεί βαθμίδες πίεσης):

$$\left.\begin{aligned} -fv_g &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \\ fu_g &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \end{aligned}\right\} \quad \text{(B)}$$

(A) - (B) $\longrightarrow$

$$\boxed{\begin{aligned} -f(v - v_g) &= A_V \frac{\partial^2 u}{\partial z^2} \\ f(u - u_g) &= A_V \frac{\partial^2 v}{\partial z^2} \end{aligned}}$$

Ekman Equations

$$\boxed{\begin{aligned} -f\left(\upsilon-\upsilon_g\right) &= A_V\frac{\partial^2 u}{\partial z^2}\ (1) \\ f\left(u-u_g\right) &= A_V\frac{\partial^2 \upsilon}{\partial z^2}\ (2) \end{aligned}}$$

για απλούστευση θεωρούμε μόνο ζωνικό άνεμο: $\tau^y = 0$

Οριακές συνθήκες: $\begin{cases} \text{at } z=0 \dashrightarrow \rho_0 A_V\dfrac{\partial u}{\partial z} = \tau^x; \rho_0 A_V\dfrac{\partial \upsilon}{\partial z} = 0 \\ \text{at } z\to\infty \dashrightarrow u = u_g; \upsilon = \upsilon_g \end{cases}$

Πολ/ζουμε (2) με $i=\sqrt{-1}$ και προσθέτουμε την (1):

$$\frac{d^2V}{dz^2} - \frac{if}{A_V}V = 0\ (3),\ \text{where } V = u - u_g + i\left(\upsilon-\upsilon_g\right)$$

remember: $u_g, \upsilon_g \not\propto z$

Η λύση είναι: $V = Ae^{(1+i)z/\delta} + Be^{-(1+i)z/\delta}$

where $\delta = \sqrt{\dfrac{2A_V}{f}}$

B=0, αφού η λύση δεν μπορεί να απειρίζεται για $z\to-\infty$

Χρησιμοποιώντας τις δύο οριακές συνθήκες: $A = \dfrac{\tau^x\delta(1-i)}{2\rho_0 A_V}$

Αντικαθιστώντας στην V:

$$u-u_g+i\left(\upsilon-\upsilon_g\right) = \frac{\tau^x\delta(1-i)}{2\rho_0A_V}\,e^{\frac{(1+i)z}{\delta}} = (1-i)\,\frac{\tau^x/\rho_0}{\sqrt{fA_V}}\frac{\sqrt{2}}{2}\,e^{z/\delta}\left(\cos\frac{z}{\delta}+i\sin\frac{z}{\delta}\right) =$$

$$= \frac{\tau^x/\rho_0}{\sqrt{fA_V}}e^{z/\delta}\left(\frac{\sqrt{2}}{2}\cos\frac{z}{\delta}+\frac{\sqrt{2}}{2}\sin\frac{z}{\delta}\right) - i\frac{\tau^x/\rho_0}{\sqrt{fA_V}}e^{z/\delta}\left(\frac{\sqrt{2}}{2}\cos\frac{z}{\delta}-\frac{\sqrt{2}}{2}\sin\frac{z}{\delta}\right)$$

$$\cos\frac{\pi}{4}\cos\frac{z}{\delta}+\sin\frac{\pi}{4}\sin\frac{z}{\delta} = \cos\left(-\frac{z}{\delta}+\frac{\pi}{4}\right) \qquad \sin\frac{\pi}{4}\cos\frac{z}{\delta}-\cos\frac{\pi}{4}\sin\frac{z}{\delta} = \sin\left(-\frac{z}{\delta}+\frac{\pi}{4}\right)$$

$$\boxed{u = u_g + \frac{\tau^x/\rho_0}{\sqrt{fA_V}}\,e^{z/\delta}\cos\left(-\frac{z}{\delta}+\frac{\pi}{4}\right) \qquad \upsilon = \upsilon_g - \frac{\tau^x/\rho_0}{\sqrt{fA_V}}\,e^{z/\delta}\sin\left(-\frac{z}{\delta}+\frac{\pi}{4}\right)}$$

$$u = u_g + \frac{\tau^x/\rho_0}{\sqrt{fA_V}}\, e^{z/\delta} \cos\left(-\frac{z}{\delta} + \frac{\pi}{4}\right) \qquad \upsilon = \upsilon_g - \frac{\tau^x/\rho_0}{\sqrt{fA_V}}\, e^{z/\delta} \sin\left(-\frac{z}{\delta} + \frac{\pi}{4}\right)$$

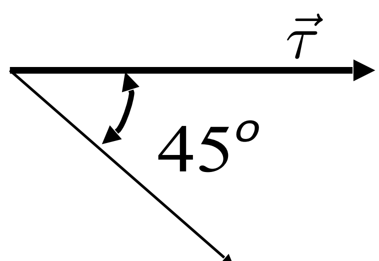
II. Επιφανειακή ταχύτητα Ekman:

at $z = 0$

$$u_{surface} = u_g + \frac{\tau^x/\rho_0}{\sqrt{fA_V}} \cos\left(\frac{\pi}{4}\right)$$

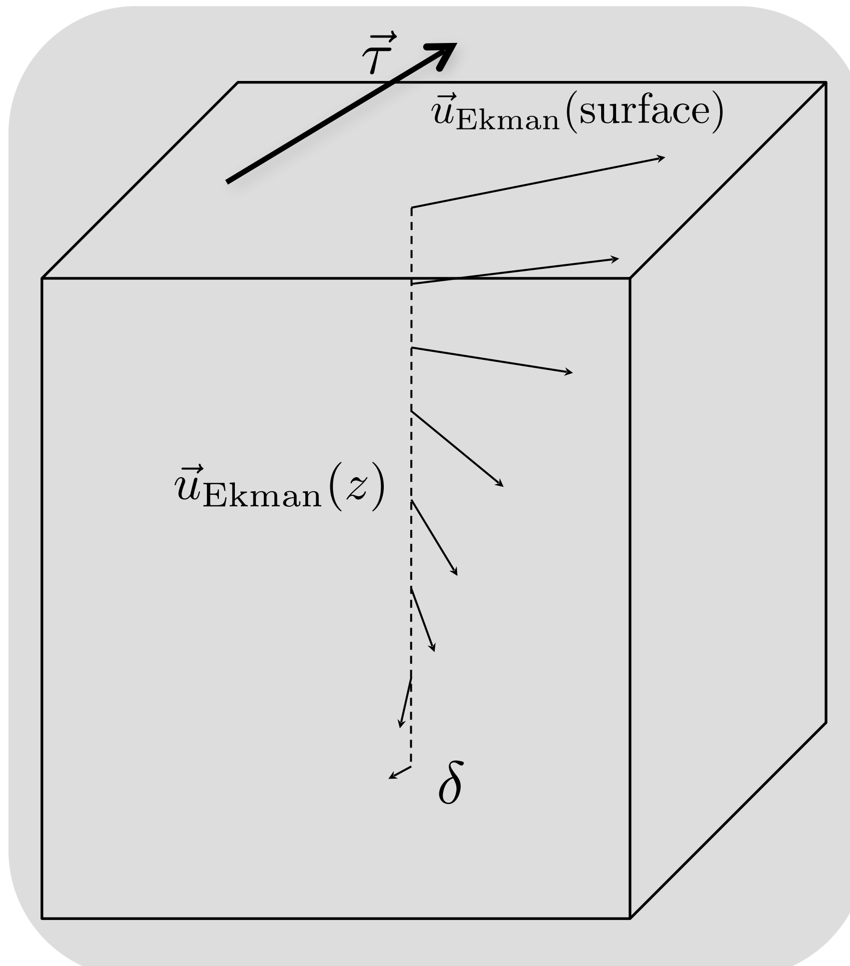
$$\upsilon_{surface} = \upsilon_g - \frac{\tau^x/\rho_0}{\sqrt{fA_V}} \sin\left(\frac{\pi}{4}\right)$$

at $z = 0$



$$\vec{u}_{\text{Ekman}} = \vec{u} - \vec{u}_g$$

II. Η κατακόρυφη κατανομή της ταχύτητας Ekman:



Ekman Spiral (Η σπείρα του Ekman)

Το βάθος του στρώματος Ekman (**Ekman layer**) ορίζεται ως το βάθος όπου η ταχύτητα μειώνεται κατά ένα e (e-folding scale)

$$\delta = \sqrt{\frac{2A_V}{f}}$$

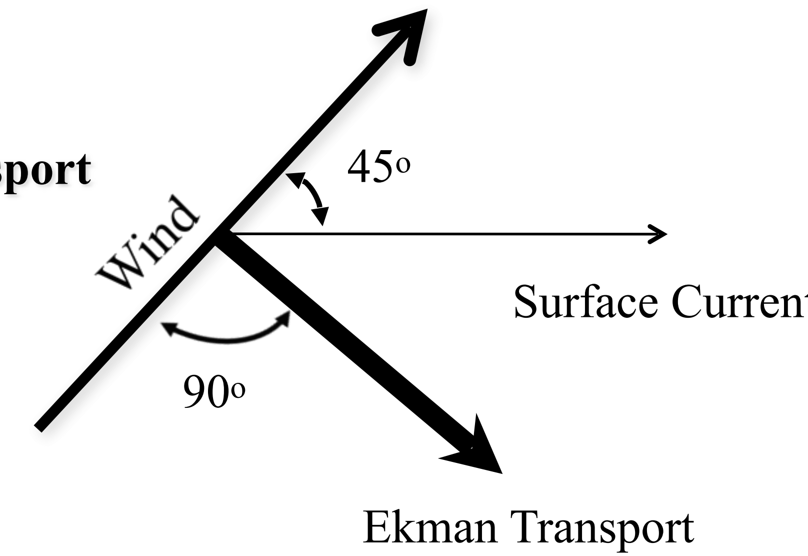
III. Η κατά βάθος ολοκληρωμένη μεταφορά Ekman (Ekman transport):

Ολοκληρώνοντας την ταχύτητα λόγω επίδρασης του ανέμου $(\vec{u} - \vec{u}_g)$ από την επιφάνεια μέχρι το εσωτερικό του ωκεανού:

$$U_E = \int_{-\infty}^{0} (u - u_g)\, dz = \frac{1}{\rho_0 f}\tau^y$$

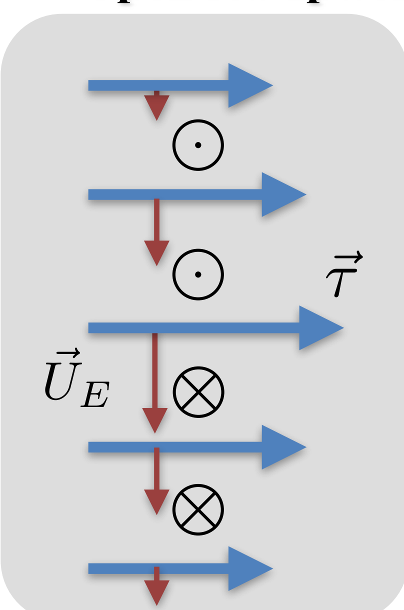
$$V_E = \int_{-\infty}^{0} (v - v_g)\, dz = -\frac{1}{\rho_0 f}\tau^x$$

Ekman Transport



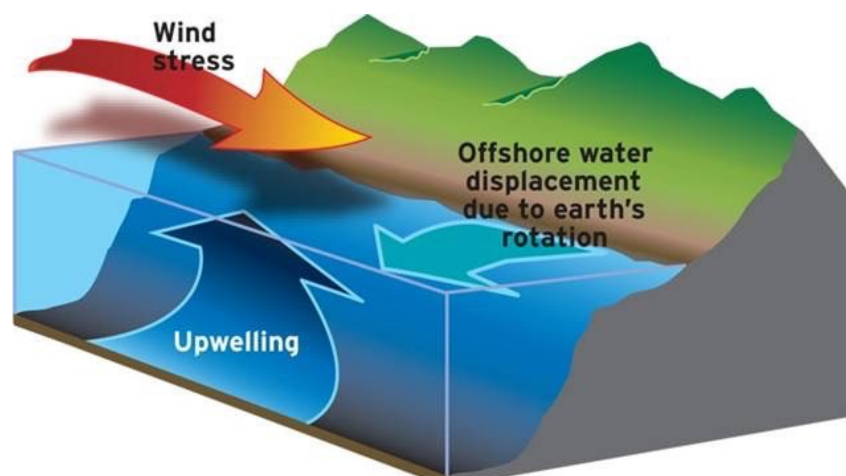
Συνέπειες:

Open sea upwelling/downwelling

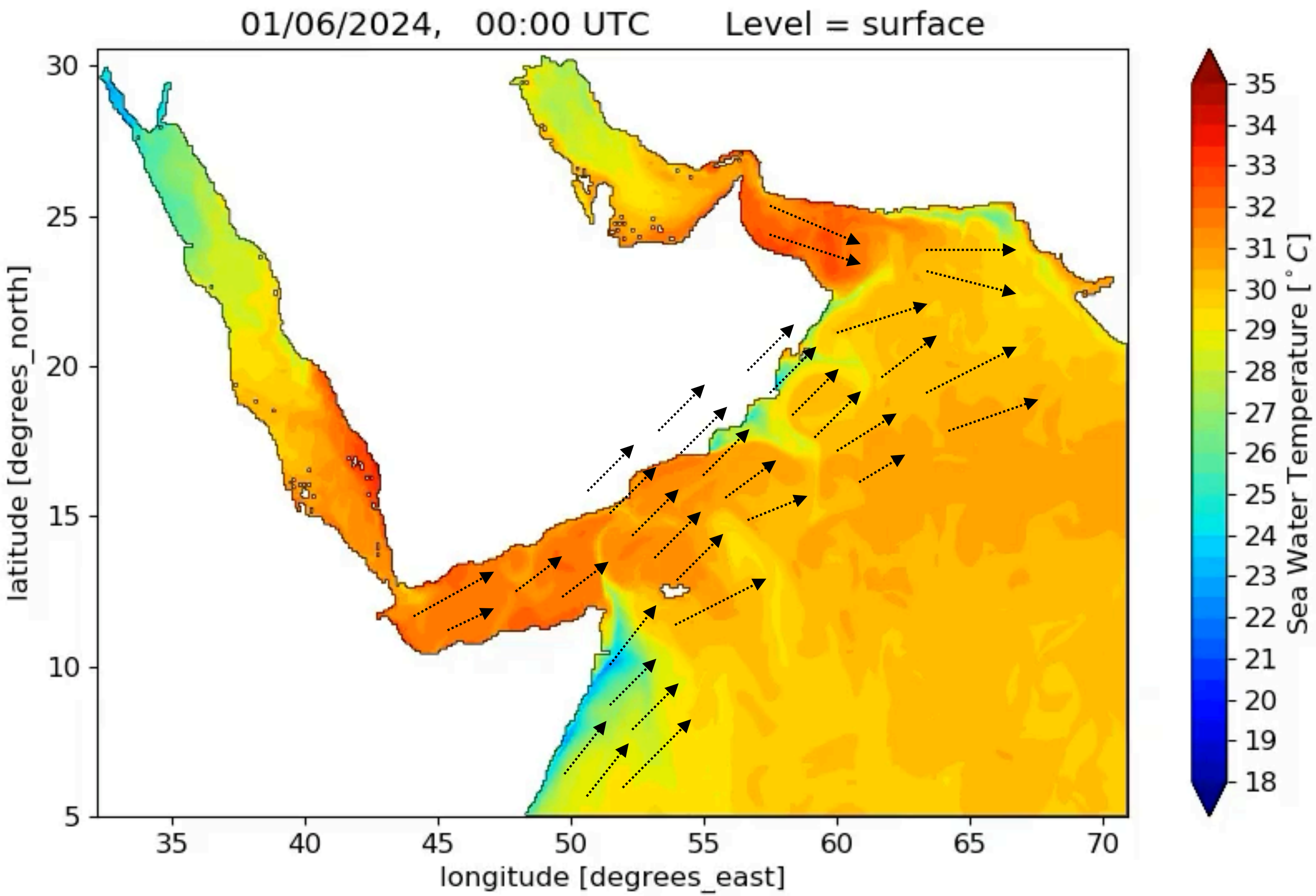


Η σύγκλιση/απόκλιση μεταφοράς Ekman δημιουργεί καταβύθιση/ανάβλιση, αντίστοιχα

Coastal upwelling

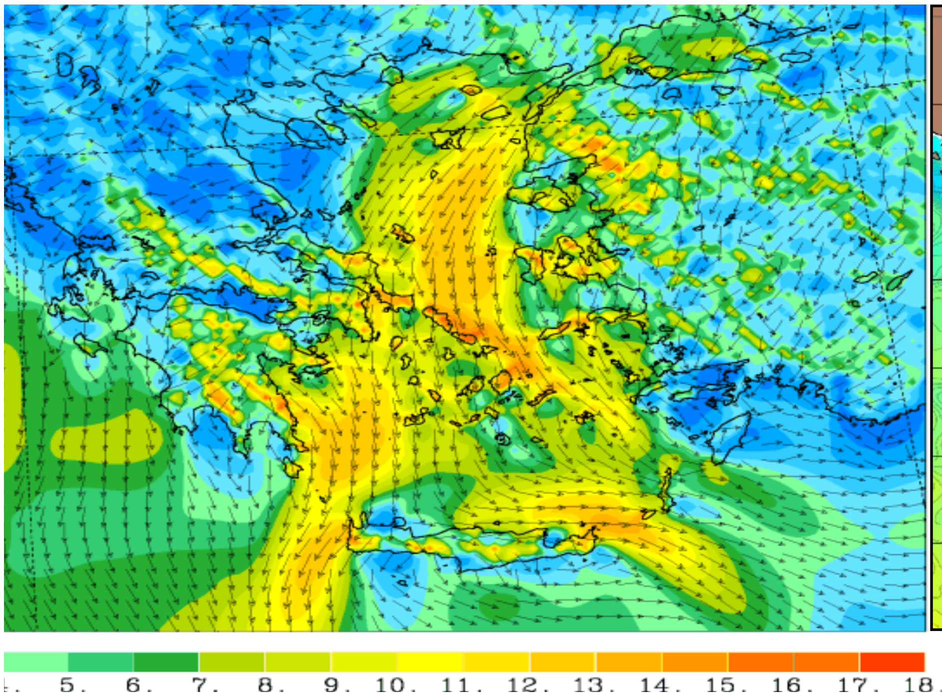


Coastal upwelling induced by the summer monsoon wind forcing



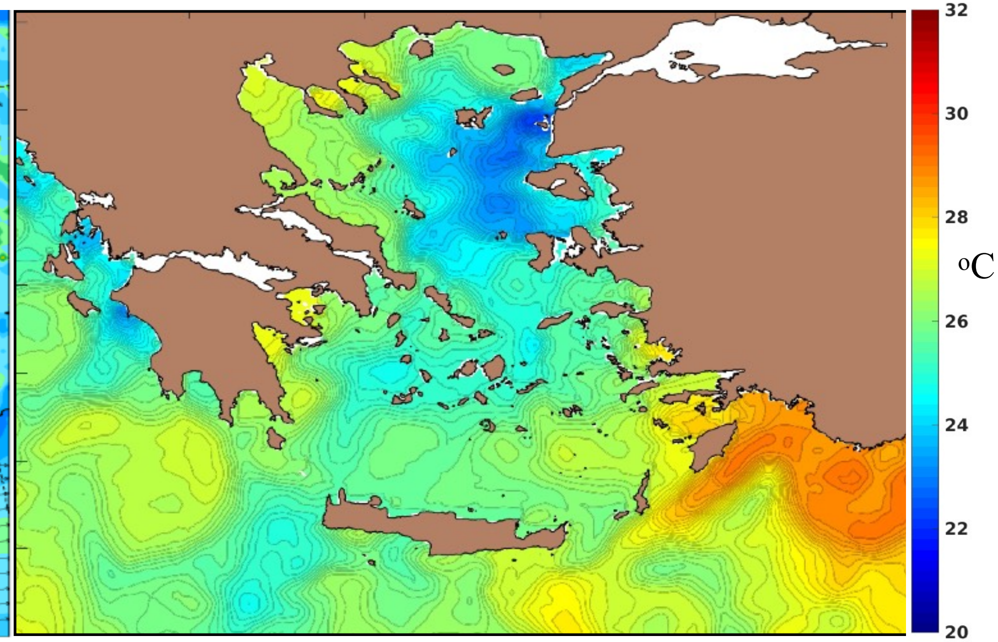
August 9, 2025

Winds at 10 m (AM&WFG)

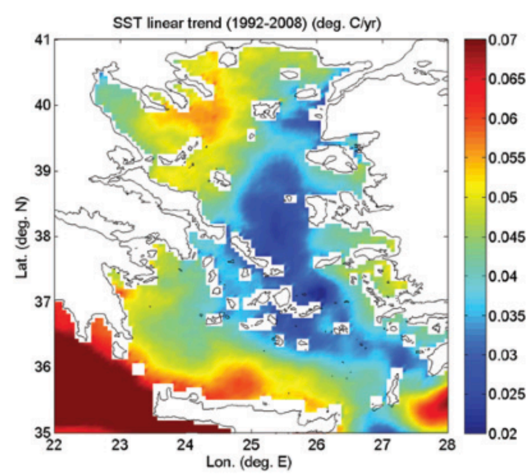


m/s

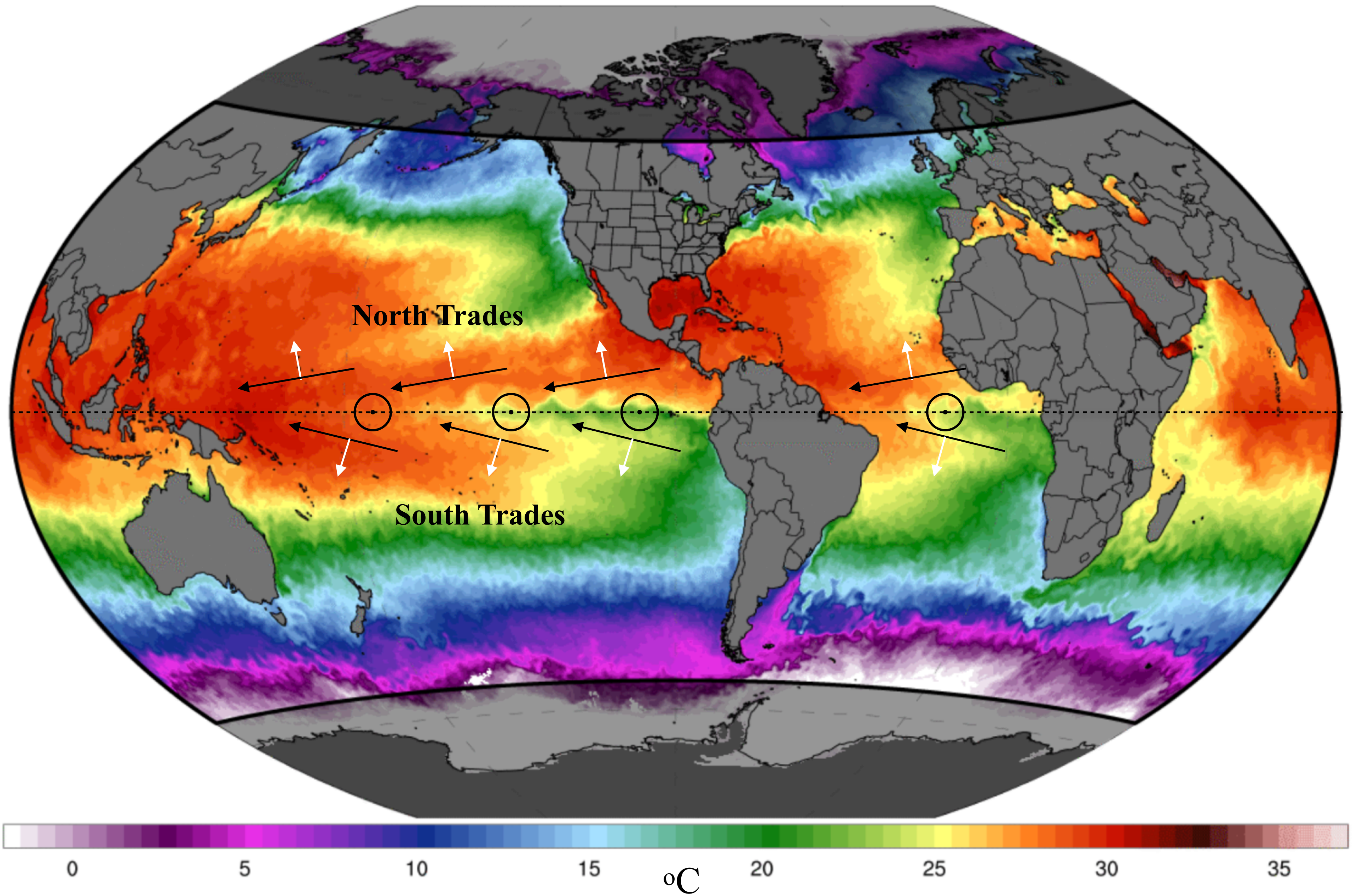
Sea Surface temperature (AVHRR - OPAM)



και η επίδραση στις κλιματικές τάσεις →



Open sea upwelling/downwelling

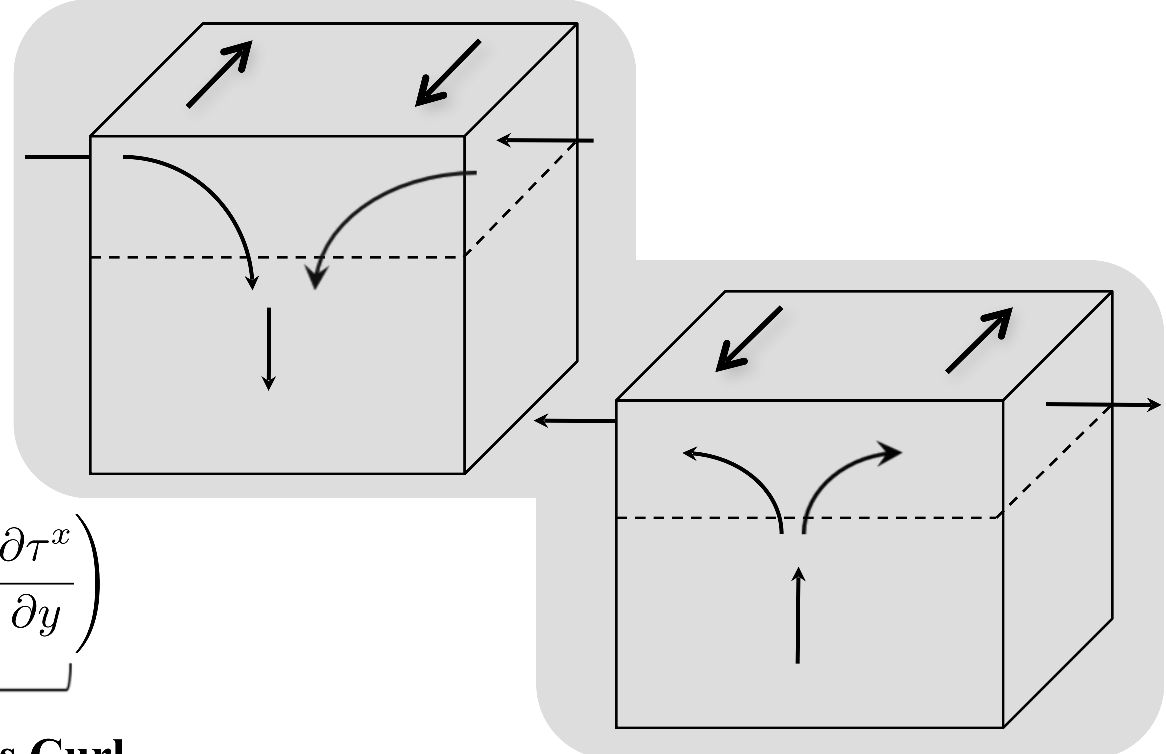


IV. Ekman pumping

Από την εξίσωση συνέχειας: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$-\int_{-\infty}^{0} dw=\int_{-\infty}^{0}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dz$$

$$\Rightarrow -\cancel{w(0)}+w(-\infty)=\frac{1}{\rho_0 f}\underbrace{\left(\frac{\partial \tau^y}{\partial x}-\frac{\partial \tau^x}{\partial y}\right)}_{\textbf{Wind Stress Curl}}$$



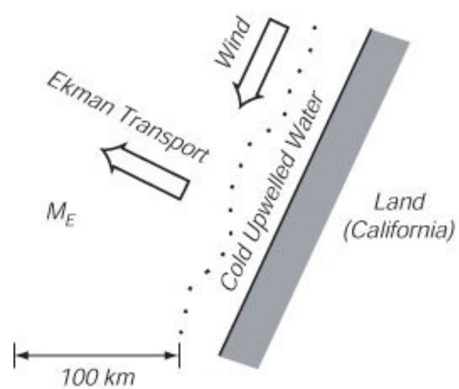
Coastal Ekman pumping

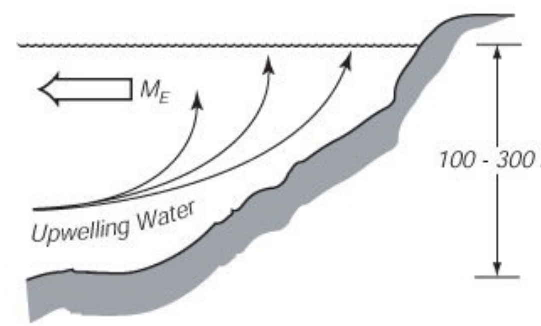
Στο εσωτερικό (κάτω από το στρώμα Ekman):

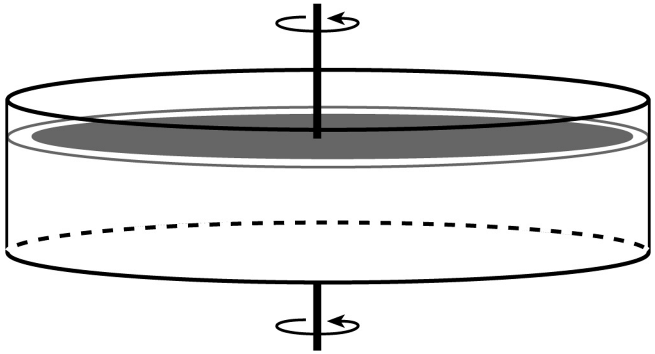
$$\boxed{w_{Ek}=\frac{1}{\rho_0 f}\left(\frac{\partial \tau^y}{\partial x}-\frac{\partial \tau^x}{\partial y}\right)}$$

$$\boxed{w_{Ek}=\frac{U_E}{L}=\frac{\tau^{x,y}}{L\rho_0 f}}$$

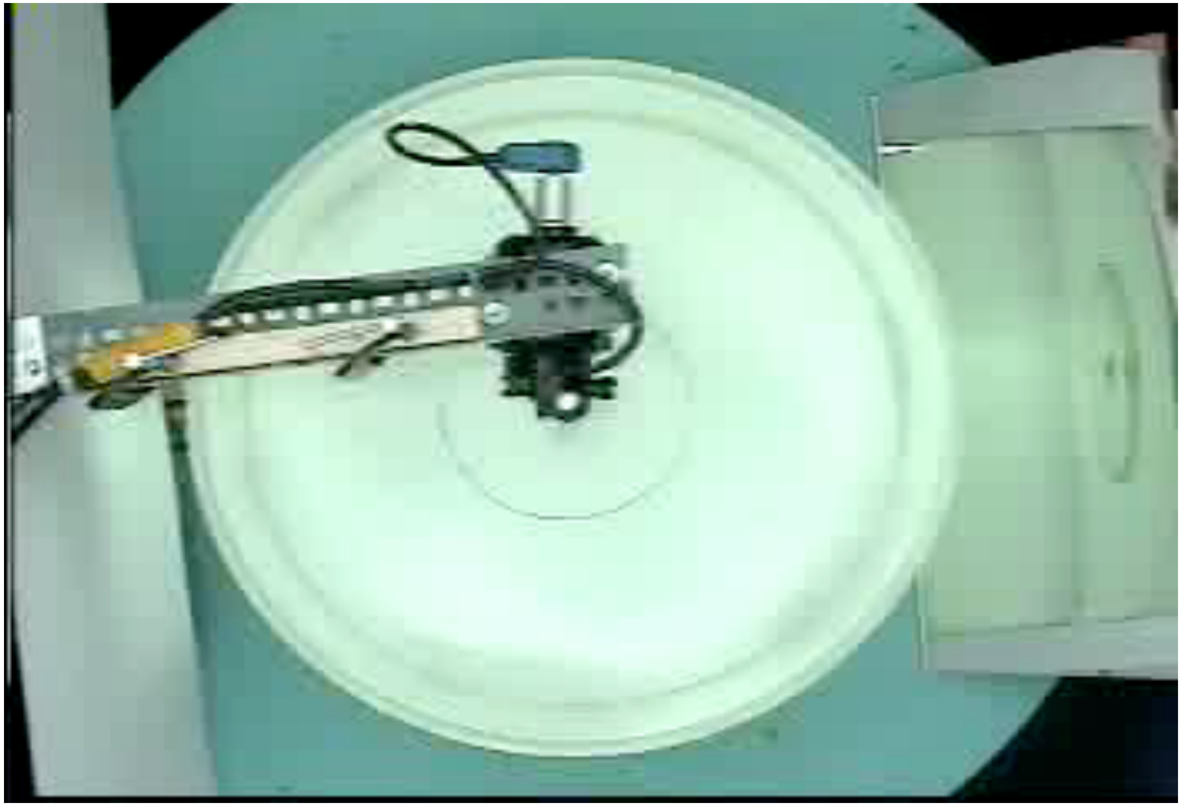
Επειδή η παράγωγος δεν ορίζεται στην παράκτια περιοχή (η τάση είναι ασυνεχής συνάρτηση)







Ekman dynamics in a tank



Sverdrup's theory for the wind-driven circulation

Harald Ulrik Sverdrup
1888-1957

Ξεκινώντας από τις εξισώσεις κίνησης παρουσία (κατακόρυφης) τριβής:

$$\left.\begin{aligned} -f\upsilon &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + A_V\frac{\partial^2 u}{\partial z^2} \quad (1) \\ fu &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + A_V\frac{\partial^2 \upsilon}{\partial z^2} \quad (2) \end{aligned}\right\} \qquad \frac{\partial}{\partial x}(2) - \frac{\partial}{\partial y}(1)$$

$\boldsymbol{\beta}$

$$\cancel{u\frac{\partial f}{\partial x}} + f\frac{\partial u}{\partial x} + \upsilon\frac{\partial f}{\partial y} + f\frac{\partial \upsilon}{\partial y} = \frac{1}{\rho_0}\cancel{\frac{\partial^2 p}{\partial x\partial y}} - \frac{1}{\rho_0}\cancel{\frac{\partial^2 p}{\partial x\partial y}} + A_V\frac{\partial^3 \upsilon}{\partial x\partial z^2} - A_V\frac{\partial^3 u}{\partial y\partial z^2}$$

$$\Rightarrow f\left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y}\right) + \beta\upsilon = A_V\frac{\partial^2}{\partial z^2}\left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right)$$

at $z = 0 \dashrightarrow \rho_0 A_V\frac{\partial u}{\partial z} = \tau^x$

$\rho_0 A_V\frac{\partial \upsilon}{\partial z} = \tau^y$

at $z = -H \dashrightarrow \rho_0 A_V\frac{\partial u}{\partial z} = 0$

$\rho_0 A_V\frac{\partial \upsilon}{\partial z} = 0$

Ολοκληρώνοντας από το βυθό (-*H*) μέχρι την επιφάνεια της θάλασσας:

$$\Rightarrow \rho_0 f\int_{-H}^{0}\cancel{\left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y}\right)}dz + \beta\int_{-H}^{0}\rho_0\upsilon dz = \frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}$$

Using continuity and the fact that w(0)=w(-H)=0

Meridional mass transport M_y

Wind stress curl

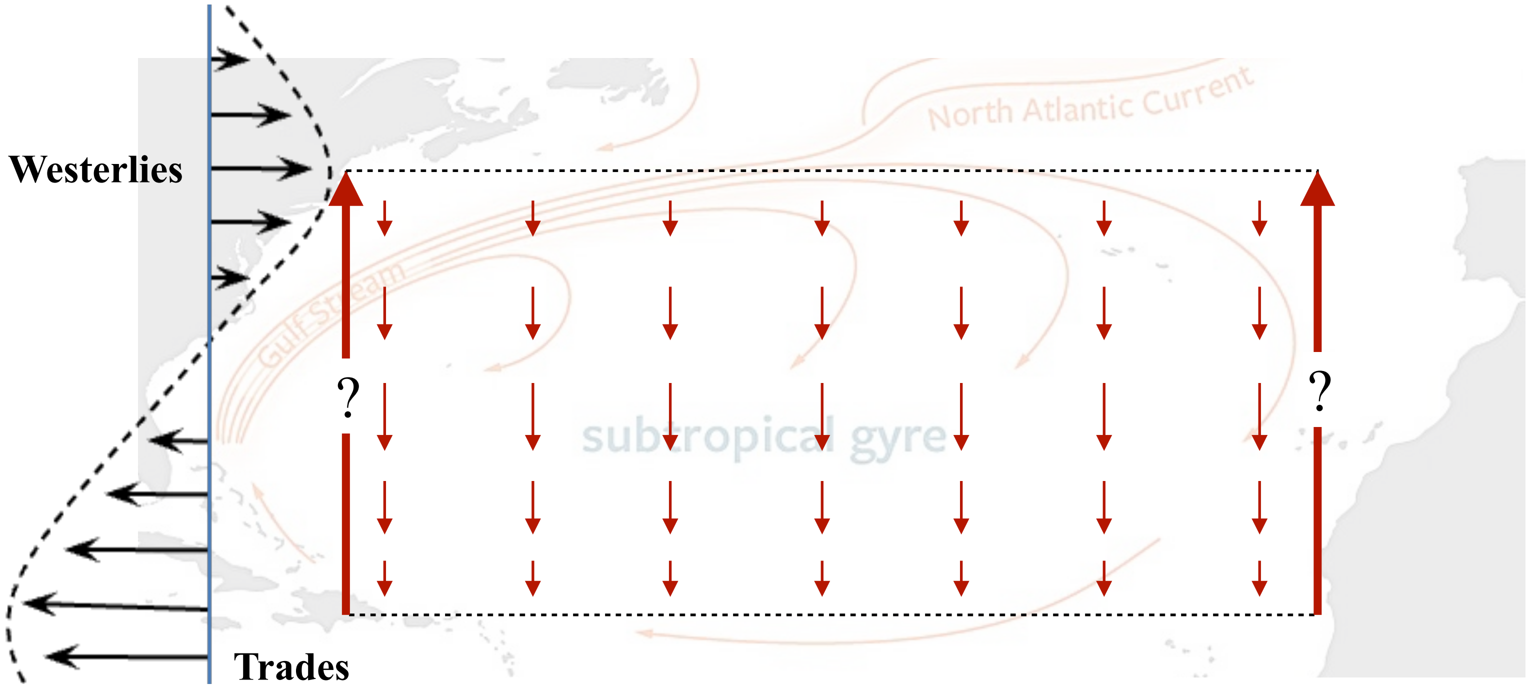
Depth integrated vorticity equation for the wind-driven circulation

and for $\tau_y = 0$

$$\beta M_y = \frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}$$ **(wind-stress curl)**

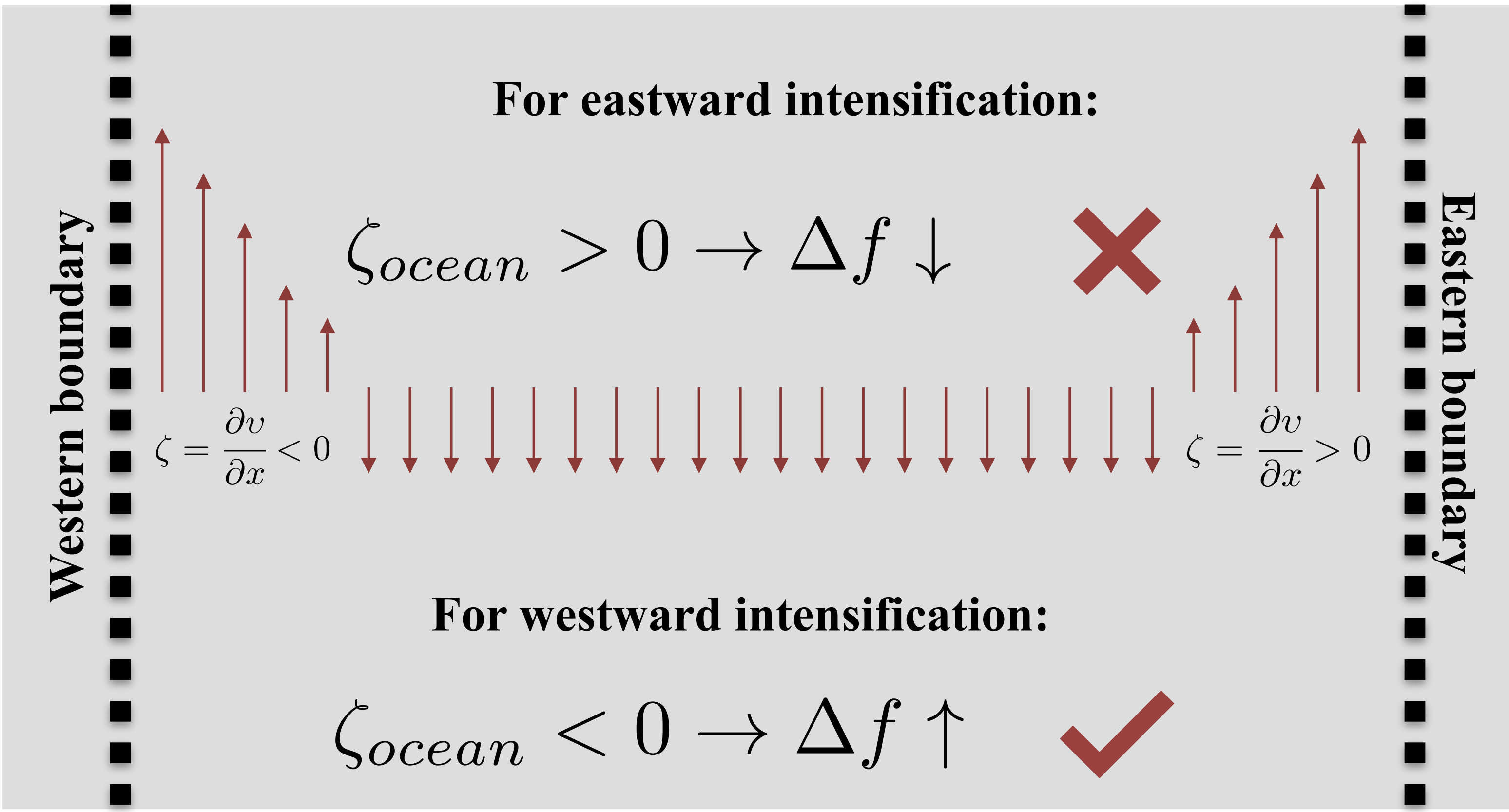
$$\boxed{M_y = -\frac{1}{\beta}\frac{\partial \tau^x}{\partial y}}$$

Meridional Sverdrup Tansport



M_y

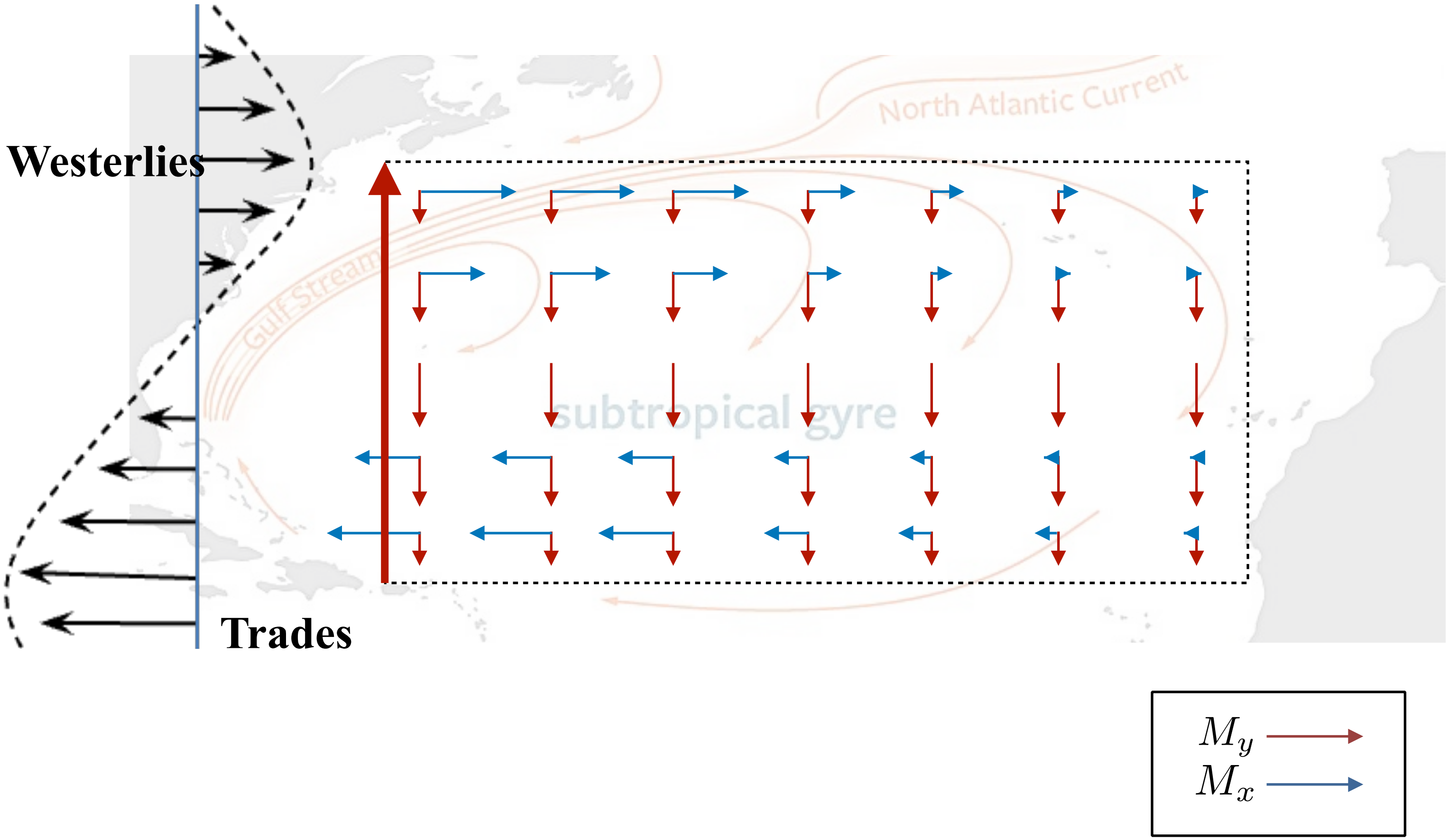
Vorticity considerations $f + \zeta = \mathrm{ct}$



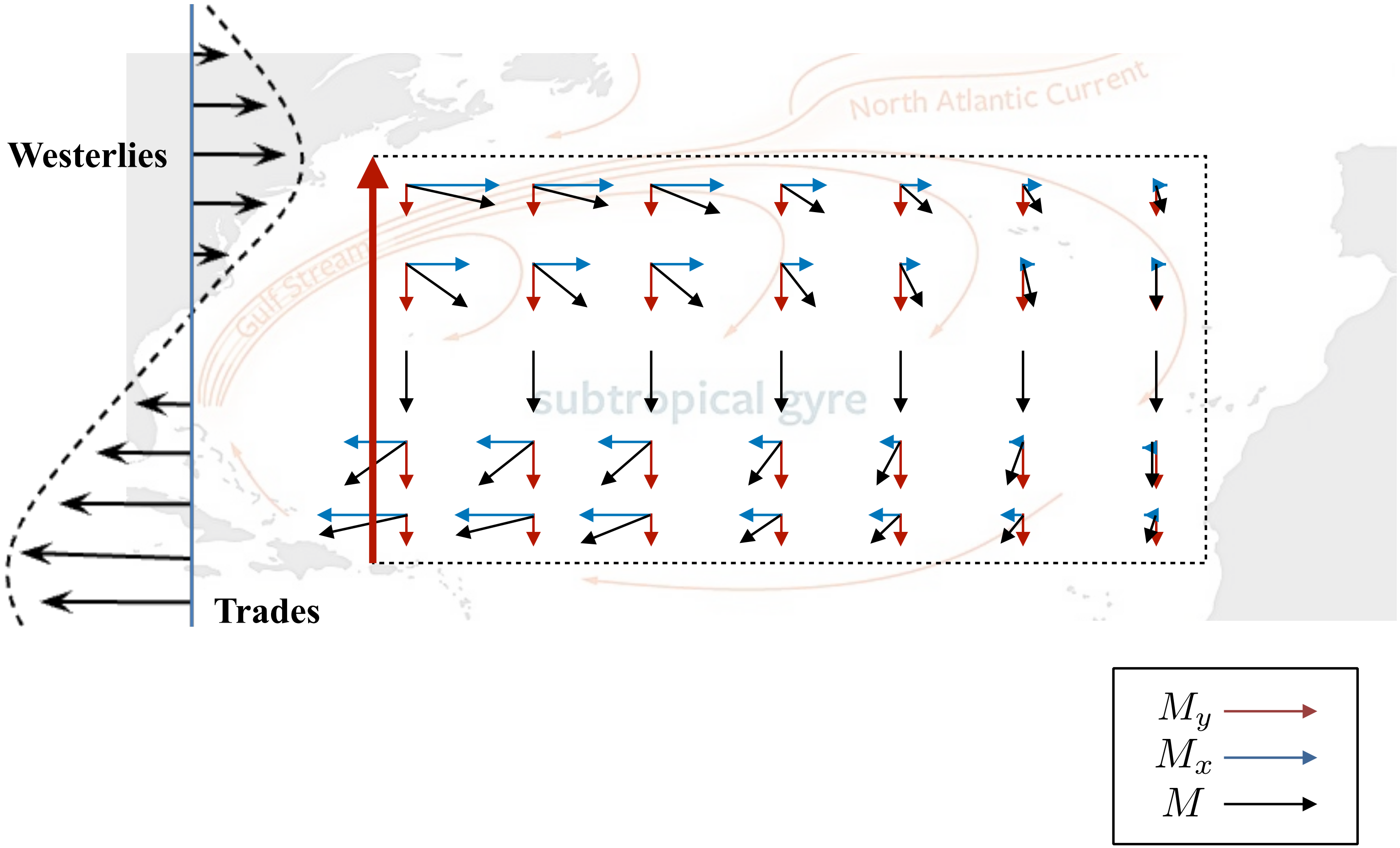
The zonal Sverdrup transport

Using the depth integrated mass conservation

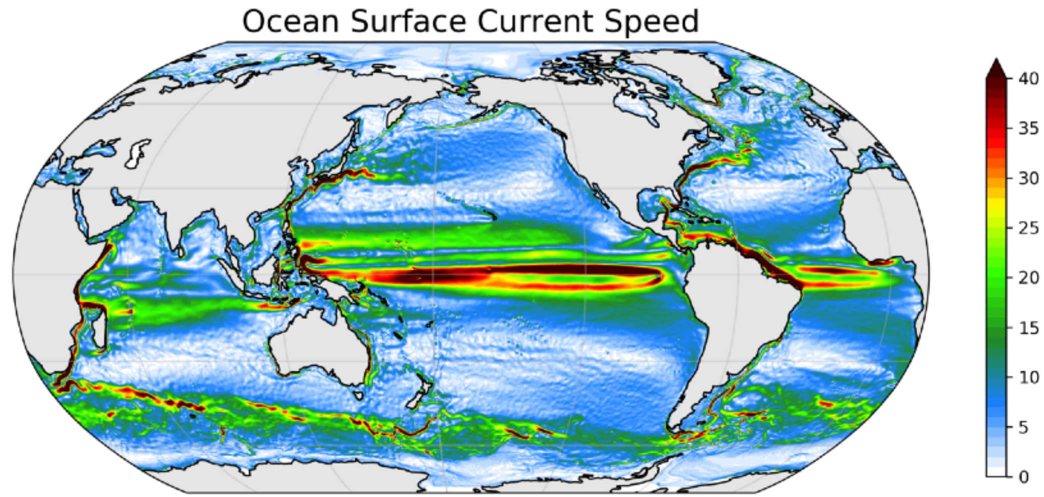
$$\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0 \Rightarrow M_x(x) = \frac{\partial}{\partial y}\int_x^{x_{East}} -M_y \, dx = -\frac{1}{\beta}\frac{\partial^2 \tau^x}{\partial y^2}\left(x_{East} - x\right)$$

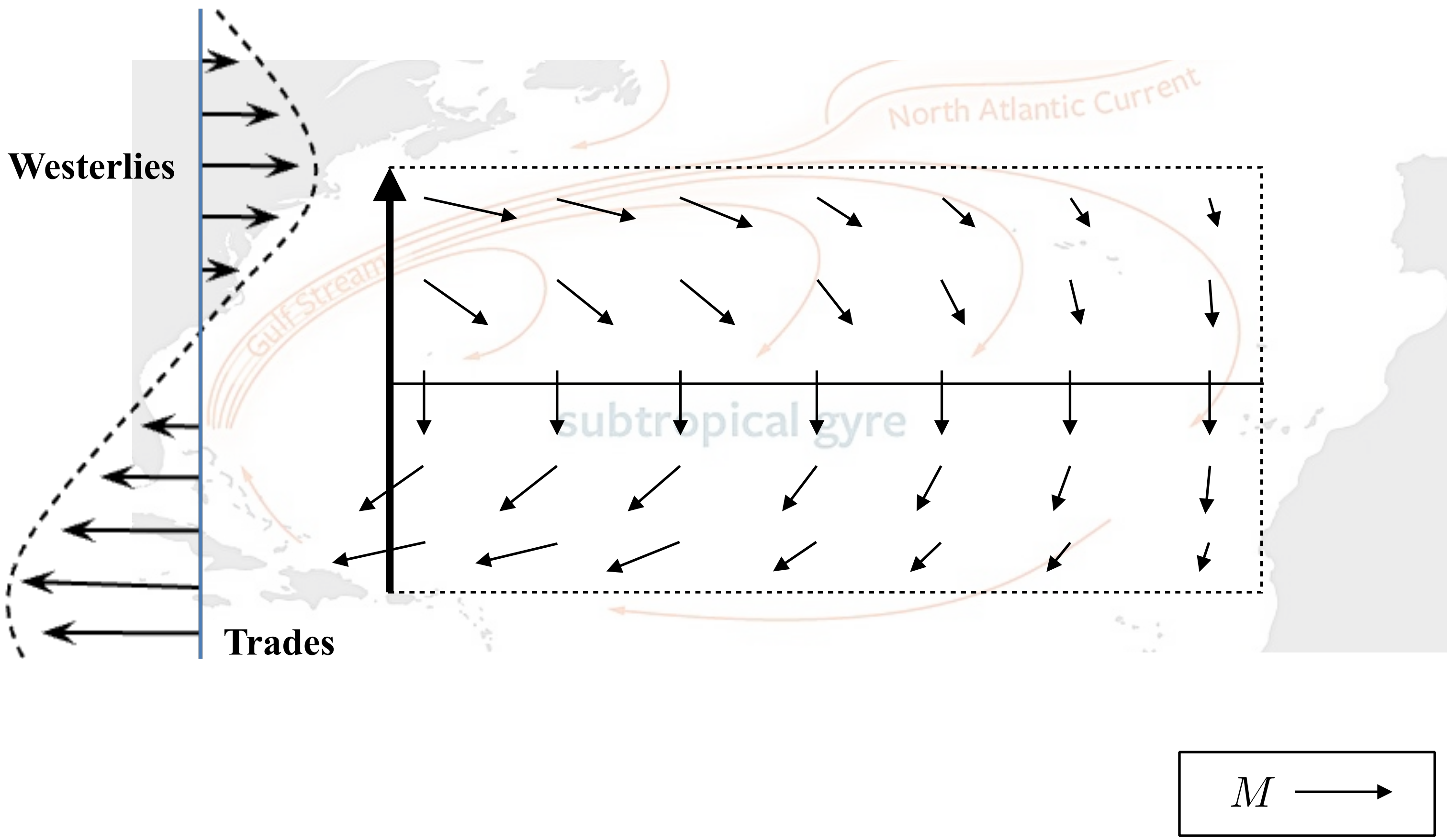


Combining M_x and M_y to estimate the Sverdrup Circulation



Sverdrup Circulation for the Subtropical Gyre





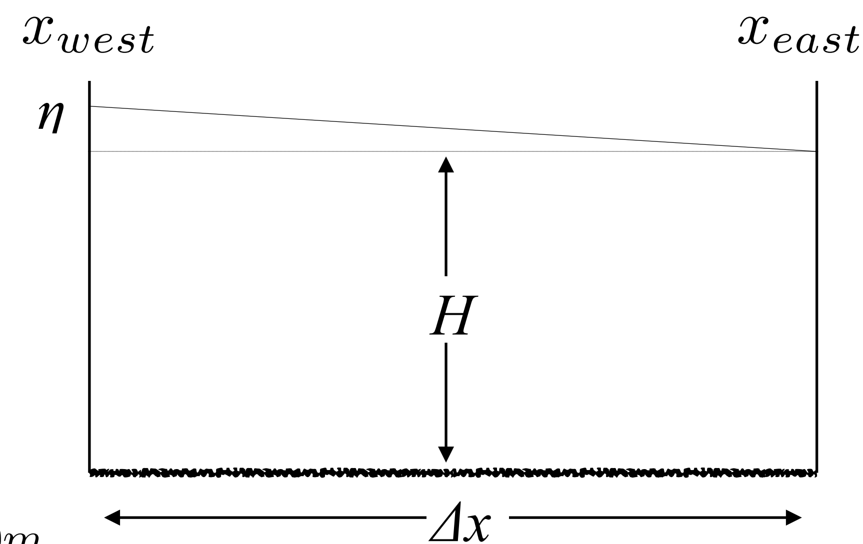
The Sverdrup Circulation and the zonal sea level variability

$$v = \frac{U_{Sv}}{H} = -\frac{1}{H\beta\rho_0}\left(\frac{\partial \tau^x}{\partial y}\right)_{max} = v_g = \frac{g}{f}\frac{\partial \eta}{\partial x} \Rightarrow$$

$$\frac{\partial \eta}{\partial x} = -\frac{f}{gH\beta\rho_0}\left(\frac{\partial \tau^x}{\partial y}\right)_{max} \Rightarrow$$

$$\eta(x) = -\frac{f}{gH\beta\rho_0}\left(\frac{\partial \tau^x}{\partial y}\right)_{max}\int_x^{x_{east}} dx$$

$$\eta(x) = \frac{f}{gH\beta\rho_0}\left(\frac{\partial \tau^x}{\partial y}\right)_{max}(x_{east} - x)$$

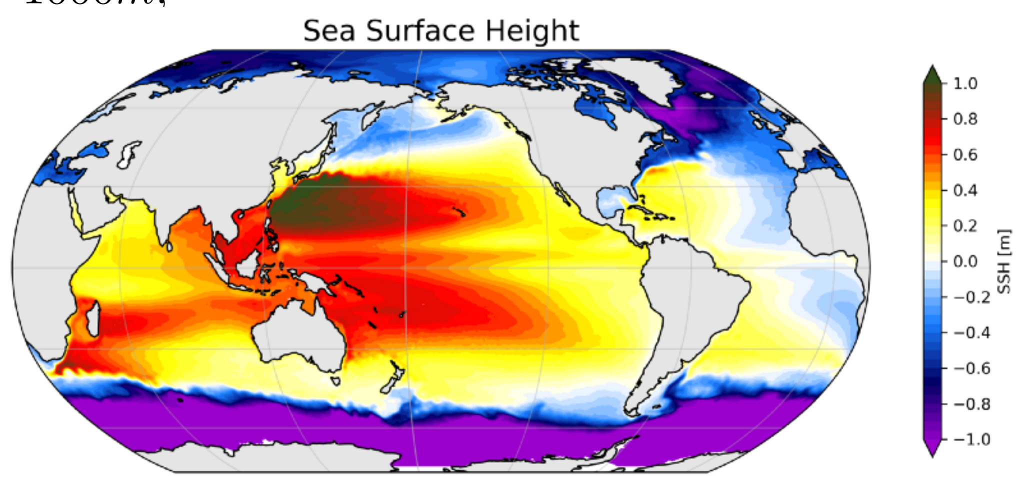


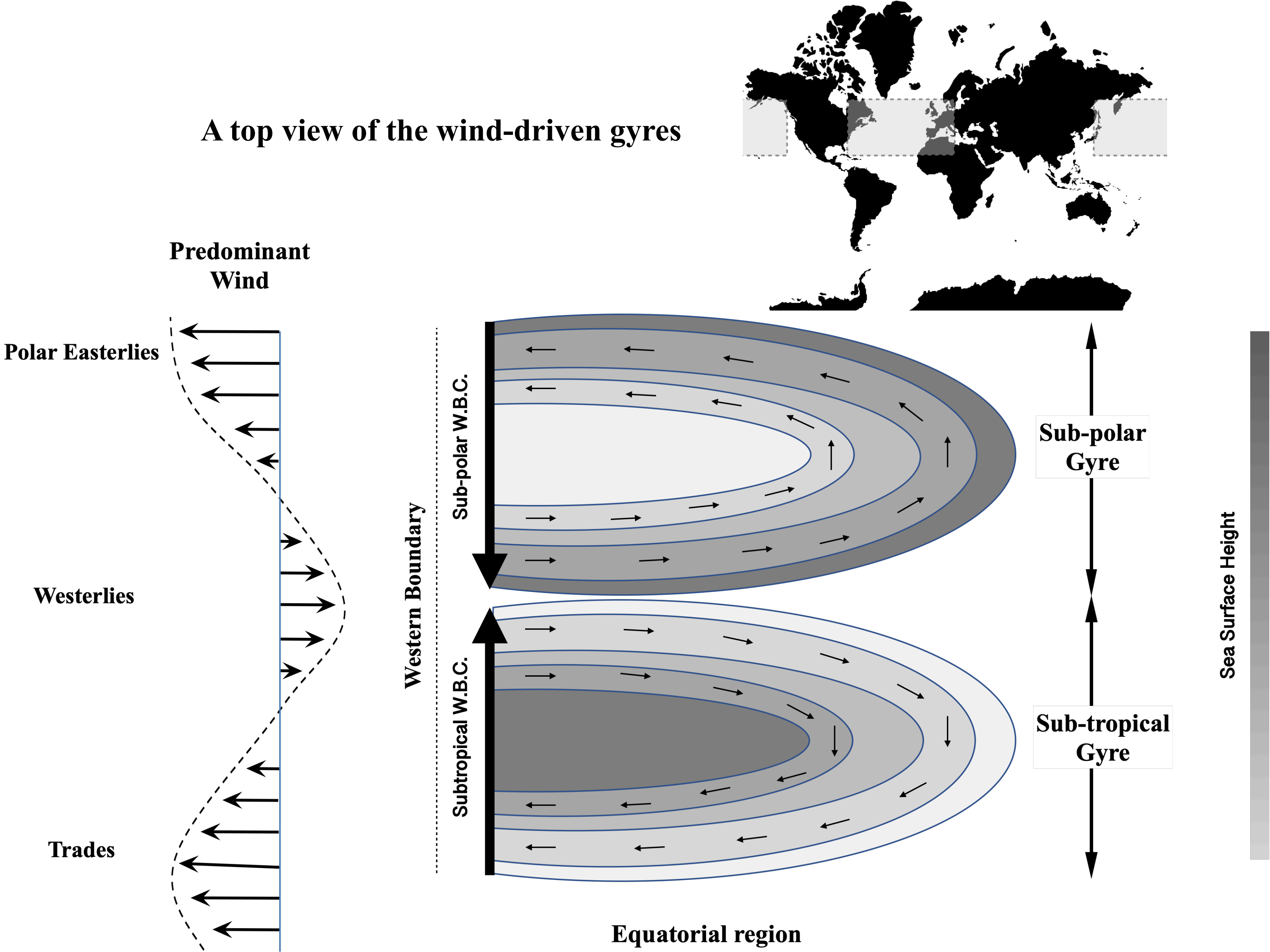
and η_{max} (Western boundary) $= 1$m

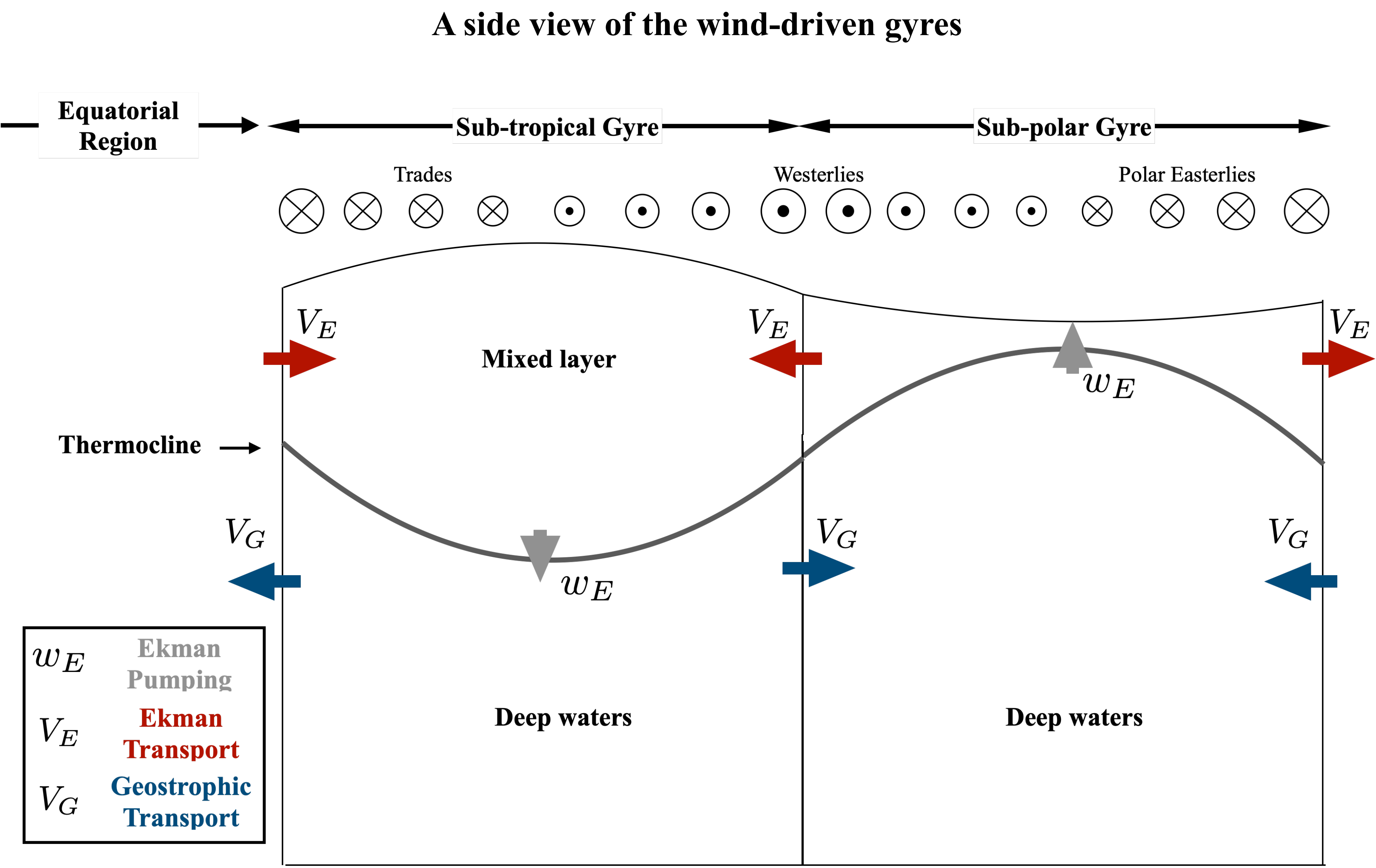
for: $f = 10^{-4}s^{-1}, \beta = 10^{-11}m^{-1}s^{-1}, H = 1000m,$

$g = 10ms^{-1}, \tau_{max} = 10^{-1}Pa, \Delta x = 10^7 m$

Meridional Gyre span: 30^o

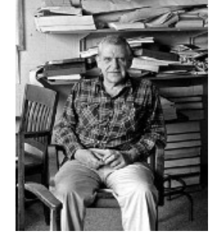



A top view of the wind-driven gyres
Predominant Wind
Polar Easterlies
Westerlies
Trades
Western Boundary
Sub-polar W.B.C.
Subtropical W.B.C.
Sub-polar Gyre
Sub-tropical Gyre
Sea Surface Height
Equatorial region



A side view of the wind-driven gyres
Equatorial Region
Sub-tropical Gyre
Sub-polar Gyre
Trades
Westerlies
Polar Easterlies
V_E
V_E
V_E
Mixed layer
w_E
Thermocline
V_G
V_G
V_G
w_E
w_E Ekman Pumping
V_E Ekman Transport
V_G Geostrophic Transport
Deep waters
Deep waters

Ο **Stommel** πρότεινε μια πιο πλήρη σχέση για τη διατήρηση του στροβιλισμού, που περιλαμβάνει και όρο απομάκρυνσης του σχετικού στροφιλισμού *ζ*, της μορφής -*Rζ*



Henry Stommel
1920-1992

Walter Munk
1917-

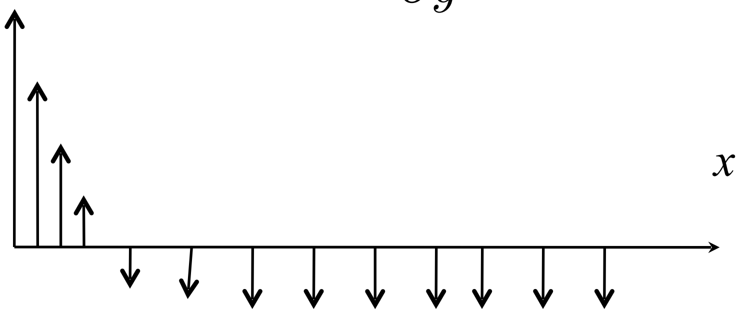
Interior

$$\beta U + \frac{\partial \tau^x}{\partial y} = 0$$

$$\beta U - R\zeta + \frac{\partial \tau^x}{\partial y} = 0$$

WBL

$$\beta U - R\zeta = 0$$



Ο **Munk** πρότεινε μια πιο πλήρη διατύπωση για τον όρο απομάκρυνσης του σχετικού στροφιλισμού *ζ*, της μορφής: $A_H \nabla_H^2 \zeta$

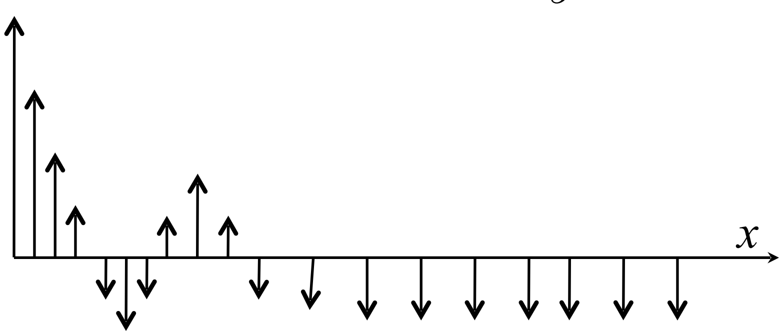
Interior

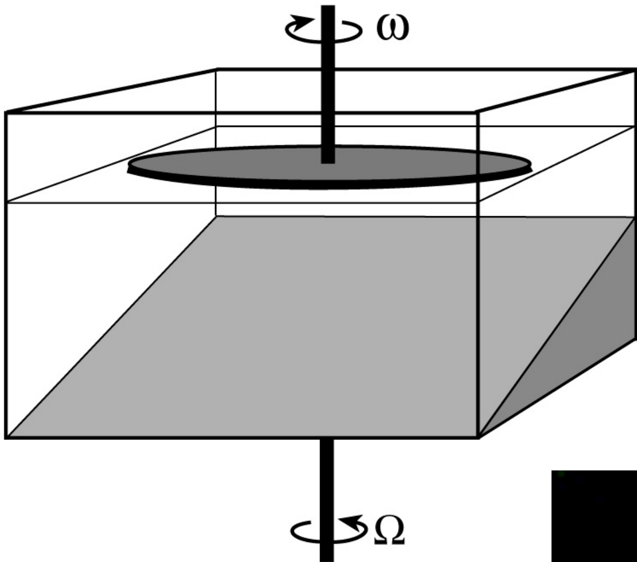
$$\beta U + \frac{\partial \tau^x}{\partial y} = 0$$

$$\beta U - A_H \nabla_H^2 \zeta + \frac{\partial \tau^x}{\partial y} = 0$$

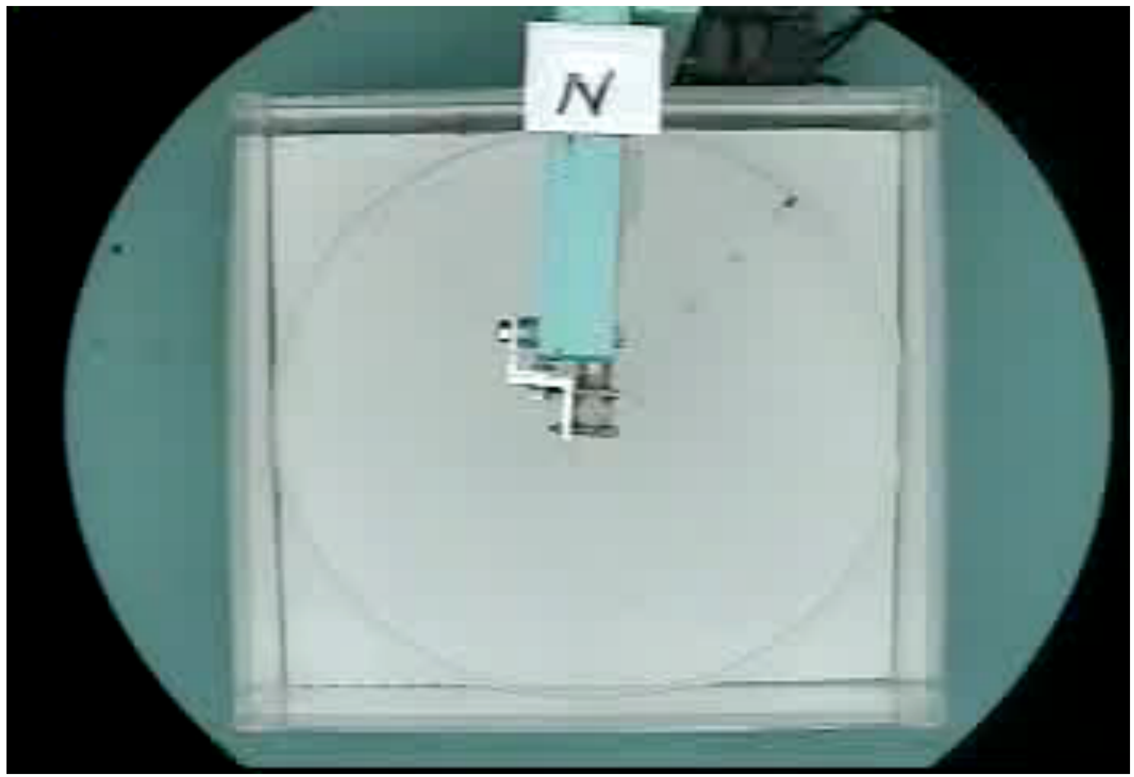
WBL

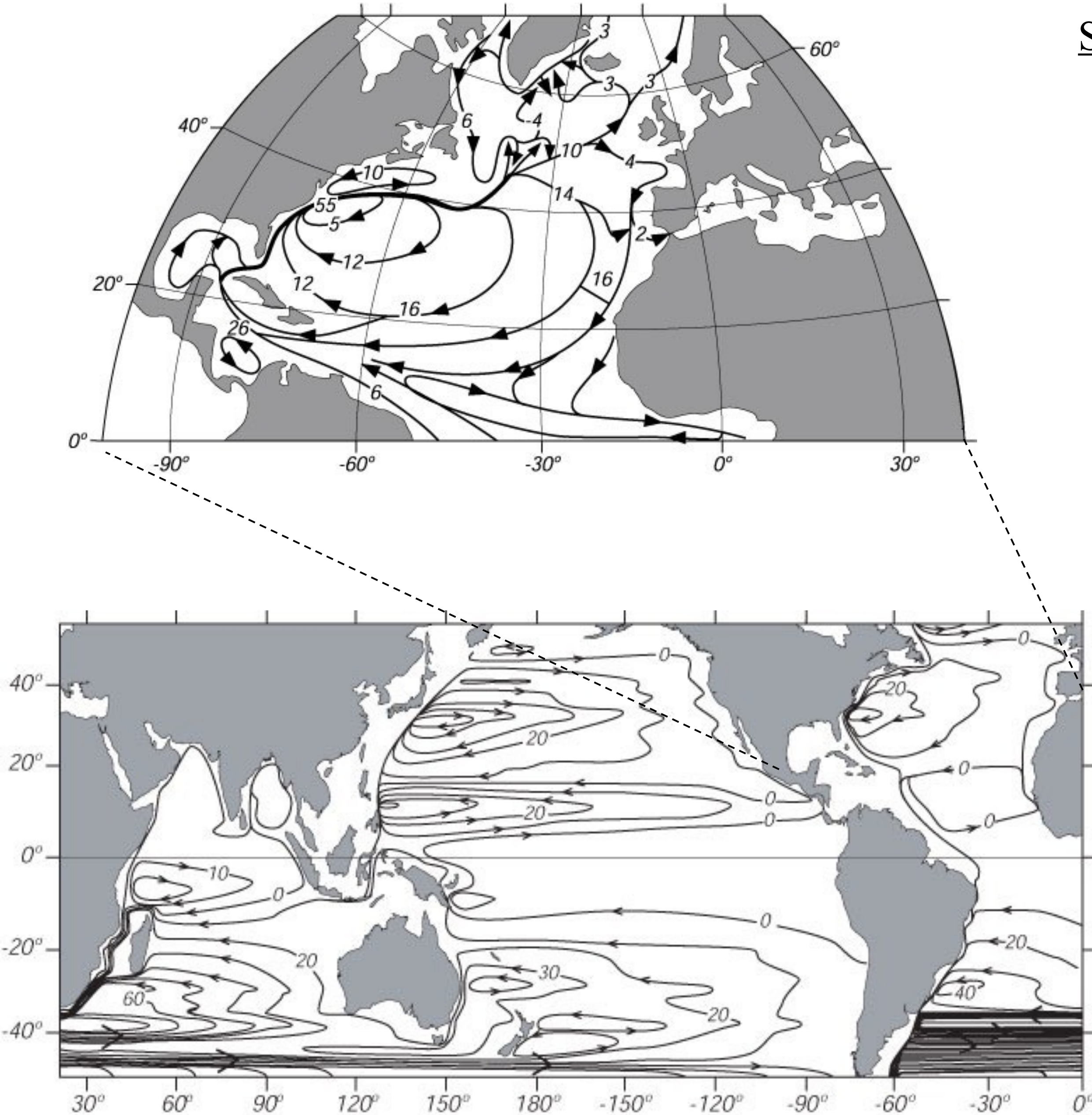
$$\beta U - A_H \nabla_H^2 \zeta = 0$$





Wind-driven gyre dynamics (the "Stommel Gyre")





Stommel Model:

$$R\zeta \simeq \beta U$$

$$\Rightarrow R\frac{U}{L} \simeq \beta U$$

$$\Rightarrow L = O\left(\frac{R}{\beta}\right)$$

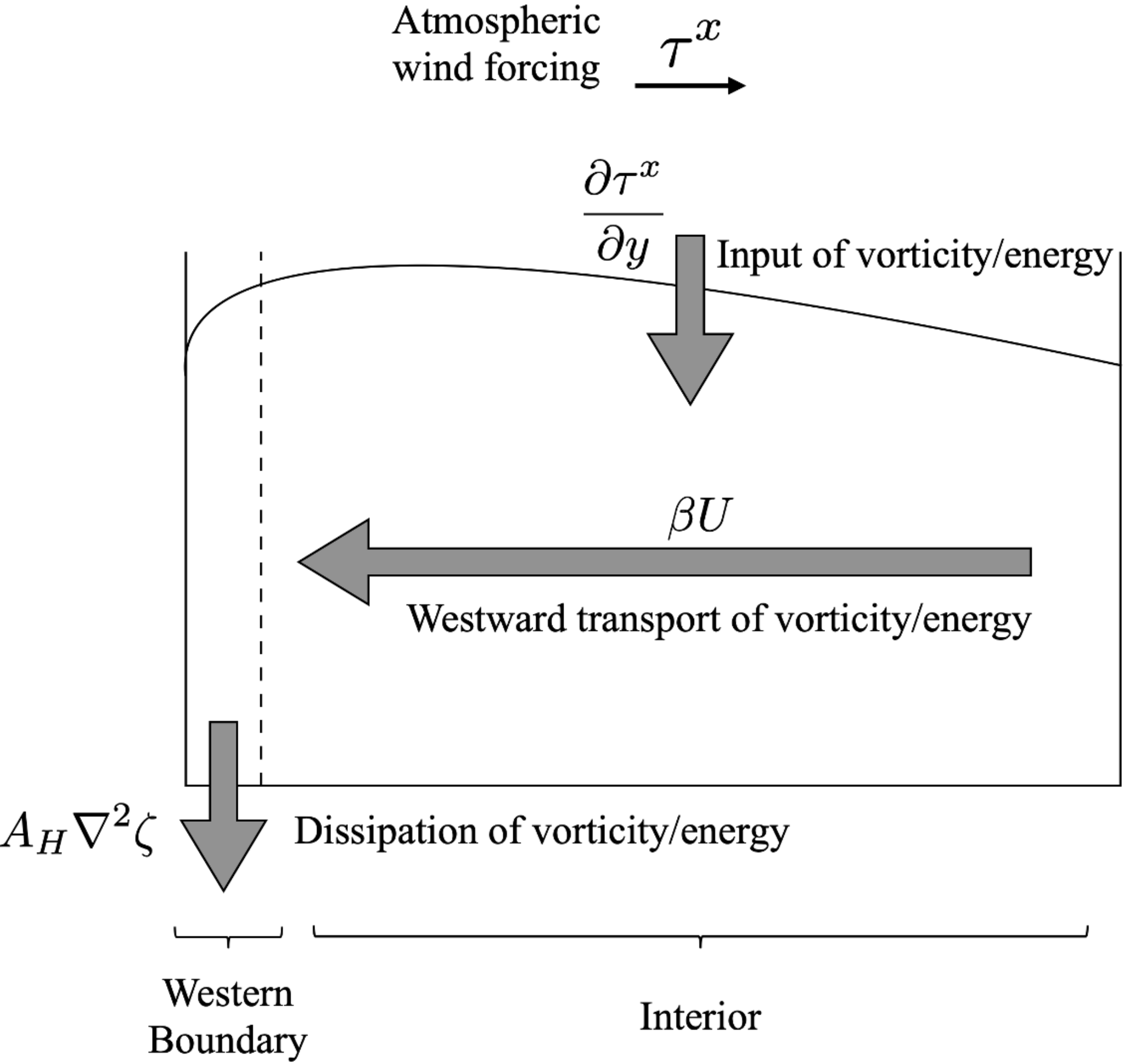
Munk Model:

$$A_H \nabla_H^2 \zeta \simeq \beta U$$

$$\Rightarrow A_H \frac{U}{L^3} \simeq \beta U$$

$$\Rightarrow L = O\left(\sqrt[3]{\frac{A_H}{\beta}}\right)$$

We can estimate L or R/A_H.



Which mechanism is tranposrting vorticity/energy **westwards**? (see large-scale waves)