

είχαμε δείξει πως για την κατάσταση

$$|\Psi_A(t)\rangle = c_1(t)|\downarrow n\rangle + c_2(t)|\uparrow n-1\rangle = |A\rangle$$

$$\left. \begin{aligned} \langle \hat{a}^\dagger \hat{a} \rangle &= n - |c_2(t)|^2 \\ \langle \hat{a} \hat{a}^\dagger \rangle &= n + |c_1(t)|^2 \end{aligned} \right\} \Rightarrow \langle \hat{a} \hat{a}^\dagger \rangle - \langle \hat{a}^\dagger \hat{a} \rangle = 1$$

$$\left. \begin{aligned} \langle \hat{S}_+ \hat{S}_- \rangle &= |c_2(t)|^2 \\ \langle \hat{S}_- \hat{S}_+ \rangle &= |c_1(t)|^2 \end{aligned} \right\} \Rightarrow \langle \hat{S}_+ \hat{S}_- \rangle + \langle \hat{S}_- \hat{S}_+ \rangle = 1$$

$$\langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{S}_+ \hat{S}_- \rangle = n$$

$$\langle \hat{S}_+ \hat{a} \rangle = c_2^*(t) c_1(t) \sqrt{n}$$

$$\langle \hat{S}_+ \hat{a}^\dagger \rangle = 0$$

$$\langle \hat{S}_- \hat{a}^\dagger \rangle = c_1^*(t) c_2(t) \sqrt{n}$$

$$\langle \hat{S}_- \hat{a} \rangle = 0$$

Μεταφράζοντας δηλαδή την $|\Psi_A(t)\rangle = c_1(t)|\downarrow n\rangle + c_2(t)|\uparrow n-1\rangle = |A\rangle$

• $\hat{a}^\dagger \hat{a}$ μετρά τον αριθμό των φωτονίων, άρα

$$\begin{aligned} \langle \hat{a}^\dagger \hat{a} \rangle_{|A\rangle} &= |c_1(t)|^2 n + |c_2(t)|^2 (n-1) = n (|c_1(t)|^2 + |c_2(t)|^2) - |c_2(t)|^2 \\ &= n - |c_2(t)|^2 \end{aligned}$$

• $\hat{a} \hat{a}^\dagger$ μετρά τον αριθμό των φωτονίων και προσθέτει 1, άρα

$$\begin{aligned} \langle \hat{a} \hat{a}^\dagger \rangle_{|A\rangle} &= |c_1(t)|^2 (n+1) + |c_2(t)|^2 n = n (|c_1(t)|^2 + |c_2(t)|^2) + |c_1(t)|^2 \\ &= n + |c_1(t)|^2 \end{aligned}$$

• $\hat{S}_+ \hat{S}_-$ μετρά τον αριθμό των ηλεκτρονίων στην άνω στάθμη, άρα

$$\langle \hat{S}_+ \hat{S}_- \rangle_{|A\rangle} = |c_2(t)|^2$$

• $\hat{S}_- \hat{S}_+$ μετρά τον αριθμό των ηλεκτρονίων στην κάτω στάθμη, άρα

$$\langle \hat{S}_- \hat{S}_+ \rangle_{|A\rangle} = |c_1(t)|^2$$

$$|\psi_A(t)\rangle = c_1(t) |\downarrow n\rangle + c_2(t) |\uparrow n-1\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi_A(t)\rangle = \hat{H} |\psi_A(t)\rangle$$

$$\hat{H} = \hat{H}_{JCM} = \hbar \omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar \Omega \hat{S}_+ \hat{S}_- + \hbar g_m (\hat{S}_+ \hat{a}_m + \hat{S}_- \hat{a}_m^\dagger)$$

A.Σ. $c_1(0) = 1, c_2(0) = 0$

$$A' = i\hbar \frac{\partial}{\partial t} |\psi_A(t)\rangle = (\dot{c}_1(t) |\downarrow n\rangle + \dot{c}_2(t) |\uparrow n-1\rangle) i\hbar$$

$$\begin{aligned} \Delta' &= (\hbar \omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar \Omega \hat{S}_+ \hat{S}_- + \hbar g_m \hat{S}_+ \hat{a}_m + \hbar g_m \hat{S}_- \hat{a}_m^\dagger) (c_1(t) |\downarrow n\rangle + c_2(t) |\uparrow n-1\rangle) = \\ &= \hbar \omega_m c_1(t) n |\downarrow n\rangle + \hbar \omega_m c_2(t) (n-1) |\uparrow n-1\rangle + \\ &\quad \hbar \Omega c_1(t) |0 n\rangle + \hbar \Omega c_2(t) |\uparrow n-1\rangle + \\ &\quad \hbar g_m c_1(t) \sqrt{n} |\uparrow n-1\rangle + \hbar g_m c_2(t) \sqrt{n-1} |0 n-2\rangle + \\ &\quad \hbar g_m c_1(t) \sqrt{n+1} |0 n+1\rangle + \hbar g_m c_2(t) \sqrt{n} |\downarrow n\rangle = \end{aligned}$$

$\langle \downarrow n |$ $A' = i\hbar \dot{c}_1(t)$
 $\Delta' = \hbar \omega_m c_1(t) n + \hbar g_m c_2(t) \sqrt{n}$ } $i\hbar \dot{c}_1(t) = \hbar \omega_m c_1(t) + \hbar g_m \sqrt{n} c_2(t)$

$\langle \uparrow n-1 |$ $A' = i\hbar \dot{c}_2(t)$
 $\Delta' = \hbar \omega_m c_2(t) (n-1) + \hbar \Omega c_2(t) + \hbar g_m \sqrt{n} c_1(t)$ } \Rightarrow
 $i\hbar \dot{c}_2(t) = \hbar g_m \sqrt{n} c_1(t) + [\hbar \omega_m (n-1) + \hbar \Omega] c_2(t)$

$$i \begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \end{bmatrix} = \begin{bmatrix} n \omega_m & \sqrt{n} g_m \\ \sqrt{n} g_m & \Omega + (n-1) \omega_m \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix}$$

δεν δουλεύει και για διπλάσια
 $\omega = \omega_m$ "ξεχνάμε",
 $g = g_m$ το δεικνύει
 $n = n_m$

Ορίζουμε $\Omega_n := \left[\left(\frac{\omega - \Omega}{2} \right)^2 + g^2 n \right]^{1/2} = \sqrt{\left(\frac{\Delta}{2} \right)^2 + g^2 n}$

και παραλείπουμε τις ηρέσεις αποκλίσεων

$$c_1(t) = \exp \left[-i \left(n \omega + \frac{\Omega - \omega}{2} \right) t \right] \left\{ \cos(\Omega_n t) + i \frac{\Omega - \omega}{2 \Omega_n} \sin(\Omega_n t) \right\}$$

$$c_2(t) = \exp \left[-i \left(n \omega + \frac{\Omega - \omega}{2} \right) t \right] \left\{ -i \frac{g \sqrt{n}}{\Omega_n} \sin(\Omega_n t) \right\}$$

"Αρα $|C_2(t)|^2 = \frac{ng^2}{\Omega_n^2} \sin^2(\Omega_n t)$

$|C_1(t)|^2 = 1 - |C_2(t)|^2 = \dots$

(8)

$\langle \hat{a}^\dagger \hat{a} \rangle_{\oplus} = n - \frac{ng^2}{\Omega_n^2} \sin^2(\Omega_n t)$

$\langle \hat{S}_+ \hat{S}_- \rangle_{\oplus} = \frac{ng^2}{\Omega_n^2} \sin^2(\Omega_n t)$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$

$\langle \hat{a}^\dagger \hat{a} \rangle_{\oplus} = n - \frac{ng^2}{2\Omega_n^2} + \frac{ng^2}{2\Omega_n^2} \cos(2\Omega_n t)$

$\langle \hat{S}_+ \hat{S}_- \rangle_{\oplus} = \frac{ng^2}{2\Omega_n^2} - \frac{ng^2}{2\Omega_n^2} \cos(2\Omega_n t)$

maximum transfer percentage
μέγιστο ποσοστό μεταβίβασης

$\alpha_R = \frac{ng^2}{\Omega_n^2} = \frac{ng^2}{\frac{\Delta^2}{4} + ng^2} = \frac{4ng^2}{4ng^2 + \Delta^2}$

περίοδος ταλαντώσεων

$T_R = \frac{2\pi}{2\Omega_n} = \frac{\pi}{\Omega_n} = \frac{2\pi}{2\sqrt{\frac{\Delta^2}{4} + g^2 n}} = \frac{2\pi}{\sqrt{\Delta^2 + 4ng^2}}$

$\Omega_n = \sqrt{\frac{\Delta^2}{4} + g^2 n}$

$2\Omega_n = \sqrt{\Delta^2 + 4ng^2}$

$2\Omega_n = \sqrt{\Delta^2 + \Omega_R^2}$

$\Omega_R := 2\sqrt{ng}$ (κωκλιική) συχνότητα Rabi

$\alpha_R = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}$

$T_R = \frac{2\pi}{\sqrt{\Delta^2 + \Omega_R^2}} = \frac{1}{f_R}$

Οηλαδή η (κωκλιική) συχνότητα Rabi Ω_R

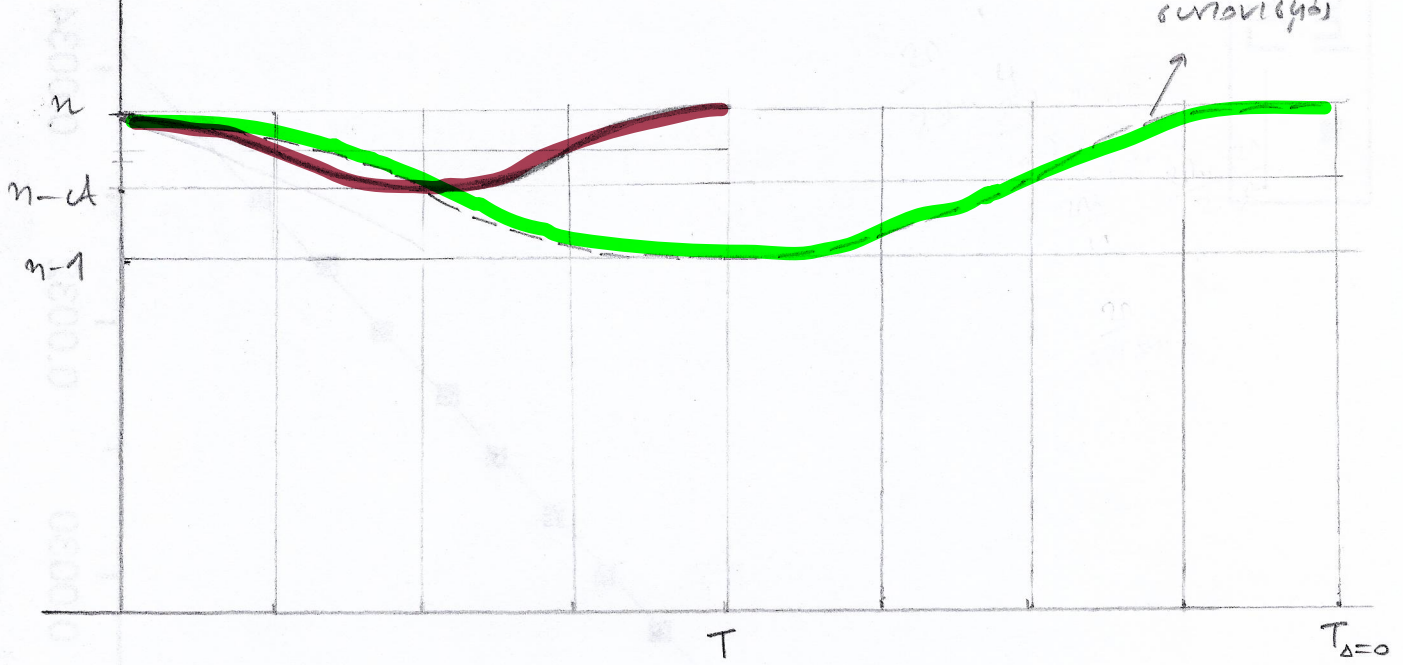
και ο ανισορρονητισμός Δ καθορίζουν

την περίοδο και το μέγιστο ποσοστό μεταβίβασης

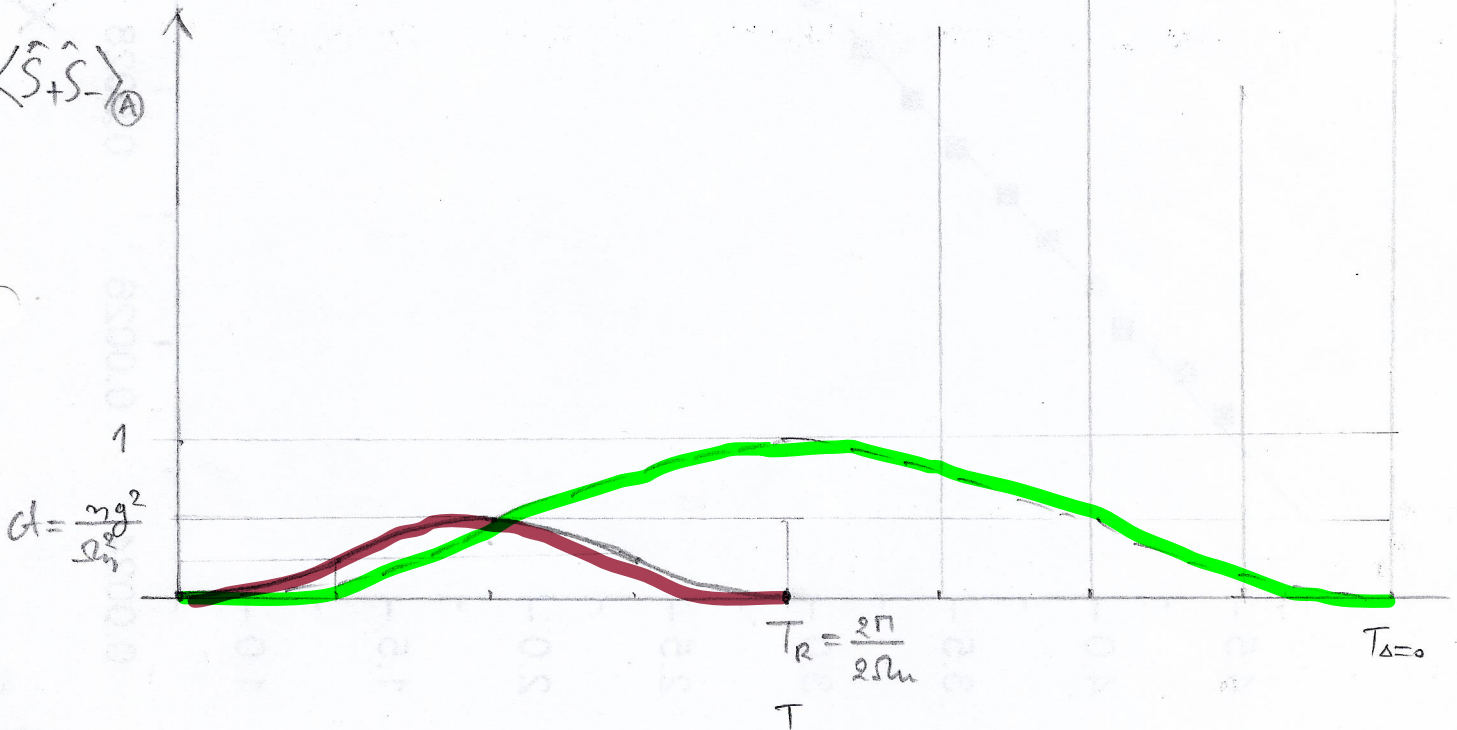
"Αν $\Delta=0 \Rightarrow \alpha_R=1$

$T_R = \frac{2\pi}{\Omega_R}$

$$\langle \hat{a}^\dagger \hat{a} \rangle_A$$



$$\langle \hat{S}_+ \hat{S}_- \rangle_A$$



$$i \begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{pmatrix}}_A \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \Rightarrow \dot{\vec{x}}(t) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \Rightarrow \dot{\vec{x}}(t) = \begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \end{pmatrix} \quad (5)$$

$$i \dot{\vec{x}}(t) = A \vec{x}(t) \Rightarrow \boxed{\dot{\vec{x}}(t) = -i A \vec{x}(t)}$$

$$\Delta \Lambda M \quad \vec{x}(t) = \vec{u} e^{-i\lambda t} \Rightarrow \dot{\vec{x}}(t) = \dot{\vec{u}}(-i\lambda) e^{-i\lambda t} \Rightarrow \vec{u}(-i\lambda) e^{-i\lambda t} = -i A \vec{u} e^{-i\lambda t}$$

$$\Rightarrow \boxed{A \vec{u} = \lambda \vec{u}} \quad (\Rightarrow A \vec{u} = \lambda I \vec{u} \Leftrightarrow (A - \lambda I) \vec{u} = 0)$$

πρόβλημα ιδιοτιμών

πρέπει $\det(A - \lambda I) = 0$

$$\begin{vmatrix} n\omega - \lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega - \lambda \end{vmatrix} = 0 \Rightarrow (n\omega - \lambda) [\Omega + (n-1)\omega - \lambda] - n g^2 = 0$$

$$\lambda^2 - [\Omega + (n-1)\omega + n\omega] \lambda + n\omega [\Omega + (n-1)\omega] - n g^2 = 0$$

Πίεση $n=1$
 ένα φωτόνιο στην κοιλότητα

$$A = \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix} \text{ και } \det(A - \lambda I) = 0 \Rightarrow$$

$$\begin{vmatrix} \omega - \lambda & g \\ g & \Omega - \lambda \end{vmatrix} = 0 \Rightarrow$$

$$\begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{pmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \lambda \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} n\omega - \lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega - \lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(\omega - \lambda)(\Omega - \lambda) - g^2 = 0$$

$$\lambda^2 - (\omega + \Omega)\lambda + \omega\Omega - g^2 = 0$$

$$\Delta' = (\omega + \Omega)^2 - 4(\omega\Omega - g^2) =$$

$$= \omega^2 + \Omega^2 + 2\omega\Omega - 4\omega\Omega + 4g^2 =$$

$$= (\omega - \Omega)^2 + 4g^2 \Rightarrow$$

$$\Delta' = \Delta^2 + 4g^2$$

$$\lambda_{2,1} = \frac{(\omega + \Omega) \pm \sqrt{\Delta^2 + 4g^2}}{2}$$

$$\lambda_{2,1} = \frac{\omega + \Omega}{2} \pm \sqrt{\left(\frac{\Delta}{2}\right)^2 + g^2}$$

$$\boxed{\lambda_{2,1} = H_1 \pm \Omega_1}$$

$$H_1 = \frac{\omega + \Omega}{2}$$

$$\Omega_1 = \sqrt{\left(\frac{\Delta}{2}\right)^2 + g^2}$$

$$\lambda_1 = H_1 - \Omega_1$$

$$A \vec{U}_1 = \lambda_1 \vec{U}_1 \Rightarrow \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix} \begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix} = (H_1 - \Omega_1) \begin{pmatrix} U_{11} \\ U_{21} \end{pmatrix}$$

$$\Rightarrow \omega U_{11} + g U_{21} = (H_1 - \Omega_1) U_{11} \Rightarrow g U_{21} = (H_1 - \Omega_1 - \omega) U_{11}$$
$$g U_{11} + \Omega U_{21} = (H_1 - \Omega_1) U_{21} \Rightarrow g U_{11} = (H_1 - \Omega_1 - \Omega) U_{21}$$

$$g U_{21} = \frac{(H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) U_{21}}{g}$$

$$g^2 U_{21} = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) U_{21}$$

$$\text{Αρα } U_{21} = 0 \Rightarrow U_{11} = 0$$

$$\text{η } g^2 = (H_1 - \Omega_1 - \omega)(H_1 - \Omega_1 - \Omega) \Rightarrow$$

$$g^2 = \left(\frac{\Omega - \omega}{2} - \Omega_1 \right) \left(\frac{\omega - \Omega}{2} - \Omega_1 \right) \Rightarrow$$

$$g^2 = - \left(\frac{\Delta}{2} + \Omega_1 \right) \left(\frac{\Delta}{2} - \Omega_1 \right) \Rightarrow g^2 = - \left[\frac{\Delta^2}{4} - \Omega_1^2 \right] \Rightarrow$$

$$g^2 + \frac{\Delta^2}{4} = \Omega_1^2 \text{ που ισχύει εφ' όσονως τής } \Omega_1$$

δηλαδή το U_{21} μπορεί να είναι οποδήποτε π.χ. $U_{21} = 1 \Rightarrow$

$$g U_{11} = (H_1 - \Omega_1 - \Omega) \cdot 1 = \frac{\Delta}{2} - \Omega_1 \Rightarrow U_{11} = \frac{\frac{\Delta}{2} - \Omega_1}{g} \Rightarrow$$

$$U_{11} = \frac{\Delta - 2\Omega_1}{2g}$$

$$\vec{U}_1 = \begin{bmatrix} \frac{\Delta - 2\Omega_1}{2g} \\ 1 \end{bmatrix}$$

$$\lambda_2 = H_1 + \Omega_1$$

\Rightarrow ηράφεις \Rightarrow

$$\vec{U}_2 = \begin{bmatrix} \frac{\Delta + 2\Omega_1}{2g} \\ 1 \end{bmatrix}$$

$$\lambda_2 = H_1 + \Omega_1$$

$$A \vec{U}_2 = \lambda_2 \vec{U}_2 \Rightarrow \begin{pmatrix} \omega & g \\ g & \Omega \end{pmatrix} \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} = (H_1 + \Omega_1) \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} \quad (3')$$

$$\Rightarrow \omega U_{12} + g U_{22} = (H_1 + \Omega_1) U_{12} \Rightarrow g U_{22} = (H_1 + \Omega_1 - \omega) U_{12}$$

$$g U_{12} + \Omega U_{22} = (H_1 + \Omega_1) U_{22} \Rightarrow g U_{12} = (H_1 + \Omega_1 - \Omega) U_{22}$$

$$g U_{22} = \frac{(H_1 + \Omega_1 - \omega)(H_1 + \Omega_1 - \Omega)}{g} U_{22}$$

$$\text{"Αρα } U_{22} = 0 \quad (\Rightarrow U_{12} = 0)$$

$$\frac{?}{?} \quad g^2 = (H_1 + \Omega_1 - \omega)(H_1 + \Omega_1 - \Omega)$$

$$g^2 = \left(\frac{\omega + \Omega - 2\omega}{2} + \Omega_1 \right) \left(\frac{\omega + \Omega - 2\Omega}{2} + \Omega_1 \right)$$

$$g^2 = \left(\frac{\Omega - \omega}{2} + \Omega_1 \right) \left(\frac{\omega - \Omega}{2} + \Omega_1 \right)$$

$$g^2 = \left(\Omega_1 - \frac{\omega - \Omega}{2} \right) \left(\Omega_1 + \frac{\omega - \Omega}{2} \right) = \left(\Omega_1 - \frac{\Delta}{2} \right) \left(\Omega_1 + \frac{\Delta}{2} \right)$$

$$g^2 = \left(\Omega_1 - \frac{\Delta}{2} \right)^2 \Rightarrow \Omega_1^2 = \left(\frac{\Delta}{2} \right)^2 + g^2$$

το οποίο ισχύει 'εφ' διαχωρίσει την Ω_1

δηλαδή το U_{22} μπορεί να είναι οτιδήποτε ο.κ. $U_{22} = 1$

$$g U_{12} = (H_1 + \Omega_1 - \Omega) \cdot 1 = \frac{\omega + \Omega - 2\Omega}{2} + \Omega_1 = \frac{\omega - \Omega}{2} + \Omega_1$$

$$U_{12} = \frac{\Delta + 2\Omega_1}{2g}$$

$$\vec{U}_2 = \begin{bmatrix} \frac{\Delta + 2\Omega_1}{2g} \\ 1 \end{bmatrix}$$

Η γενική λύση είναι

$$\vec{x}(t) = \sigma_1 \vec{u}_1 e^{-i\omega_1 t} + \sigma_2 \vec{u}_2 e^{-i\omega_2 t}$$

$$\vec{x}(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} \sigma_1 \frac{\Delta - 2\Omega_1}{2g} e^{-i(\omega_1 - \Omega_1)t} + \sigma_2 \frac{\Delta + 2\Omega_1}{2g} e^{-i(\omega_1 + \Omega_1)t} \\ \sigma_1 \cdot 1 \cdot e^{-i(\omega_1 - \Omega_1)t} + \sigma_2 \cdot 1 \cdot e^{-i(\omega_1 + \Omega_1)t} \end{bmatrix}$$

ΑΡΧΙΚΕΣ ΣΥΝΘΗΚΕΣ $c_1(0) = 1$ $c_2(0) = 0 \Rightarrow$

$$1 = \sigma_1 \frac{\Delta - 2\Omega_1}{2g} + \sigma_2 \frac{\Delta + 2\Omega_1}{2g} \Rightarrow 2g = \sigma_1(\Delta - 2\Omega_1) - \sigma_2(\Delta + 2\Omega_1)$$

$$0 = \sigma_1 + \sigma_2 \Rightarrow \sigma_2 = -\sigma_1 \Rightarrow 2g = -2\Omega_1\sigma_1 - 2\Omega_1\sigma_1$$

$$g = -2\Omega_1\sigma_1 \Rightarrow \sigma_1 = -\frac{g}{2\Omega_1} = -\sigma_2$$

"Αρα $c_2(t) = -\frac{g}{2\Omega_1} e^{-i(\omega_1 - \Omega_1)t} + \frac{g}{2\Omega_1} e^{-i(\omega_1 + \Omega_1)t} \Rightarrow$

$$c_2(t) = -\frac{g}{2\Omega_1} e^{-i\omega_1 t} e^{i\Omega_1 t} + \frac{g}{2\Omega_1} e^{-i\omega_1 t} e^{-i\Omega_1 t}$$

$$c_2(t) = \frac{g}{2\Omega_1} e^{-i\omega_1 t} \left\{ -\cos(\Omega_1 t) - i\sin(\Omega_1 t) + \cos(\Omega_1 t) - i\sin(\Omega_1 t) \right\}$$

$$c_2(t) = \frac{g}{2\Omega_1} e^{-i\omega_1 t} (-2i) \sin(\Omega_1 t) = e^{-i\frac{\omega_1 + \Omega_1}{2} t} \left\{ -i \frac{g}{\Omega_1} \sin(\Omega_1 t) \right\}$$

$$|c_2(t)|^2 = \frac{g^2}{\Omega_1^2} \sin^2(\Omega_1 t)$$

$$|c_1(t)|^2 = 1 - |c_2(t)|^2 = 1 - \frac{g^2}{\Omega_1^2} (1 - \cos^2(\Omega_1 t))$$

$$= \left(1 - \frac{g^2}{\Omega_1^2}\right) + \frac{g^2}{\Omega_1^2} \cos^2(\Omega_1 t)$$

$$\Omega_1^2 = \frac{\Delta^2}{4} + g^2 \quad \Omega_1^2 - g^2 = \frac{\Delta^2}{4}$$

$$|c_1(t)|^2 = \frac{\left(\frac{\Delta^2}{4}\right)}{\Omega_1^2} + \frac{g^2}{\Omega_1^2} \cos^2(\Omega_1 t)$$

Έστω η φωνήνια στην κοιλότητα

(θ')

$$A = \begin{bmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{bmatrix}$$

και $\det(A - \lambda I) = 0 \Rightarrow$

$$\begin{vmatrix} n\omega - \lambda & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (n\omega - \lambda) [\Omega + (n-1)\omega - \lambda] - ng^2 = 0$$

$$\lambda^2 - \lambda [\Omega + (n-1)\omega + n\omega] + n\omega [\Omega + (n-1)\omega] - ng^2 = 0$$

$$\Delta' = [\Omega + (n-1)\omega + n\omega]^2 - 4(n\omega [\Omega + (n-1)\omega] - ng^2)$$

$$\Delta' = [\underbrace{\Omega + (n-1)\omega + n\omega}]^2 - 4\underbrace{n\omega [\Omega + (n-1)\omega]} + 4ng^2$$

$$\Delta' = [\underbrace{\Omega + (n-1)\omega - n\omega}]^2 + 4ng^2 > 0$$

$$\lambda_{2,1} = \frac{[\Omega + (n-1)\omega + n\omega] \pm \sqrt{[\Omega + (n-1)\omega - n\omega]^2 + 4ng^2}}{2}$$

$$\lambda_{2,1} = \frac{\Omega + (n-1)\omega + n\omega}{2} \pm \sqrt{\left(\frac{\Omega + (n-1)\omega - n\omega}{2}\right)^2 + ng^2}$$

$$\lambda_{2,1} = \frac{\Omega + (n-1)\omega + n\omega}{2} \pm \sqrt{\left(\frac{\omega - \Omega}{2}\right)^2 + ng^2}$$

$$\boxed{\lambda_{2,1} = H_n \pm \Omega_n}$$

$$H_n = \frac{\Omega + (n-1)\omega + n\omega}{2}$$

$$\Omega_n = \sqrt{\left(\frac{\Delta}{2}\right)^2 + ng^2}$$

$$\lambda_1 = H_n - \Omega_n$$

$$A \vec{u}_1 = \lambda_1 \vec{u}_1 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{pmatrix} \begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = (H_n - \Omega_n) \begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} \quad (1)$$

$$n\omega u_{11} + g\sqrt{n} u_{21} = (H_n - \Omega_n) u_{11}$$

$$g\sqrt{n} u_{11} + [\Omega + (n-1)\omega] u_{21} = (H_n - \Omega_n) u_{21}$$

$$g\sqrt{n} u_{21} = (H_n - \Omega_n - n\omega) u_{11}$$

$$g\sqrt{n} u_{11} = (H_n - \Omega_n - [\Omega + (n-1)\omega]) u_{21}$$

$$g\sqrt{n} u_{21} = \frac{(H_n - \Omega_n - n\omega)(H_n - \Omega_n - [\Omega + (n-1)\omega])}{g\sqrt{n}} u_{21}$$

$$\text{"Ass } u_{21} = 0 \quad \Rightarrow \quad g^2 n = (H_n - \Omega_n - n\omega)(H_n - \Omega_n - [\Omega + (n-1)\omega])$$

$$\Downarrow \\ (u_{11} = 0)$$

$$H_n - \Omega_n - n\omega = \frac{\Omega + (n-1)\omega + n\omega - 2n\omega}{2} - \Omega_n \\ = \frac{\Omega - \omega}{2} - \Omega_n = -\frac{\Delta}{2} - \Omega_n$$

$$H_n - \Omega_n - [\Omega + (n-1)\omega] = \frac{\Omega + (n-1)\omega + n\omega - 2\Omega - 2(n-1)\omega}{2} - \Omega_n \\ = \frac{\omega - \Omega}{2} - \Omega_n = \frac{\Delta}{2} - \Omega_n$$

$$\Rightarrow g^2 n = -\left(\frac{\Delta}{2} + \Omega_n\right) \cdot \left(\frac{\Delta}{2} - \Omega_n\right) = -\left(\frac{\Delta}{2}\right)^2 + \Omega_n^2$$

$$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + ng^2} \quad \text{το ελάχιστο ποσότος 'εί' σπρίγος τής } \Omega_n$$

$$\text{επιλέγει το } u_{21} \text{ υποστήριξη είναι συνδύναμη } n \times u_{21} = 1$$

$$g\sqrt{n} u_{11} = H_n - \Omega_n - [\Omega + (n-1)\omega] = \frac{\Delta}{2} - \Omega_n$$

$$\boxed{u_{11} = \frac{\Delta - 2\Omega_n}{2g\sqrt{n}}}$$

$$\vec{u}_1 = \begin{bmatrix} \frac{\Delta - 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix}$$

$$\lambda_2 = H_n + \Omega_n$$

$$A \vec{U}_2 = \lambda_2 \vec{U}_2 \Rightarrow \begin{pmatrix} n\omega & g\sqrt{n} \\ g\sqrt{n} & \Omega + (n-1)\omega \end{pmatrix} \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} = (H_n + \Omega_n) \begin{pmatrix} U_{12} \\ U_{22} \end{pmatrix} \quad (\text{Lid})$$

$$n\omega U_{12} + g\sqrt{n} U_{22} = (H_n + \Omega_n) U_{12}$$

$$g\sqrt{n} U_{12} + [\Omega + (n-1)\omega] U_{22} = (H_n + \Omega_n) U_{22}$$

$$g\sqrt{n} U_{22} = (H_n + \Omega_n - n\omega) U_{12}$$

$$g\sqrt{n} U_{12} = (H_n + \Omega_n - [\Omega + (n-1)\omega]) U_{22}$$

$$g\sqrt{n} U_{22} = \frac{(H_n + \Omega_n - n\omega)(H_n + \Omega_n - [\Omega + (n-1)\omega])}{g\sqrt{n}} U_{22}$$

$$U_{22} = 0 \quad \vee \quad g^2 n = (H_n + \Omega_n - n\omega)(H_n + \Omega_n - [\Omega + (n-1)\omega])$$

$$\Downarrow \\ U_{12} = 0$$

$$H_n + \Omega_n - n\omega = \frac{\Omega + (n-1)\omega + n\omega - 2n\omega}{2} + \Omega_n$$

$$= \frac{\Omega - \omega}{2} + \Omega_n = -\frac{\Delta}{2} + \Omega_n$$

$$H_n + \Omega_n - [\Omega + (n-1)\omega] = \frac{\Omega + (n-1)\omega + n\omega - 2\Omega - 2(n-1)\omega}{2} + \Omega_n$$

$$= \frac{\omega - \Omega}{2} + \Omega_n = \frac{\Delta}{2} + \Omega_n$$

$$g^2 n = \left(\Omega_n + \frac{\Delta}{2}\right) \left(\Omega_n - \frac{\Delta}{2}\right) = \Omega_n^2 - \left(\frac{\Delta}{2}\right)^2 \Rightarrow$$

$$\boxed{\Omega_n^2 = \left(\frac{\Delta}{2}\right)^2 + n g^2} \quad \text{το ερώτημα ίσως είναι εφ' όσον αφορά το } \Omega_n$$

Επιπλέον το U_{22} μπορεί να είναι ο,τιδήποτε ο.κ. $U_{22} = 1$

$$g\sqrt{n} U_{12} = H_n + \Omega_n - [\Omega + (n-1)\omega] = \frac{\Delta}{2} + \Omega_n \Rightarrow$$

$$\boxed{U_{12} = \frac{\Delta + 2\Omega_n}{2g\sqrt{n}}}$$

$$\vec{U}_2 = \begin{bmatrix} \frac{\Delta + 2\Omega_n}{2g\sqrt{n}} \\ 1 \end{bmatrix}$$

(H) γενική λύση είναι $\vec{x}(t) = \sigma_1 \vec{u}_1 e^{-i\lambda_1 t} + \sigma_2 \vec{u}_2 e^{-i\lambda_2 t}$

(4β)

$$\vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \sigma_1 \begin{bmatrix} \frac{\Delta - 2\Omega_n}{2g\sqrt{h}} \\ 1 \end{bmatrix} e^{-i(H_n - \Omega_n)t} + \sigma_2 \begin{bmatrix} \frac{\Delta + 2\Omega_n}{2g\sqrt{h}} \\ 1 \end{bmatrix} e^{-i(H_n + \Omega_n)t}$$

Αρχικές συνθήκες $C_1(0) = 1$ $C_2(0) = 0$

$$\sigma_1 \frac{\Delta - 2\Omega_n}{2g\sqrt{h}} + \sigma_2 \frac{\Delta + 2\Omega_n}{2g\sqrt{h}} = 1 \quad \Rightarrow \quad \sigma_1 \frac{\Delta - 2\Omega_n - \Delta - 2\Omega_n}{2g\sqrt{h}} = 1$$

$$\sigma_1 + \sigma_2 = 0 \quad \Rightarrow \quad \sigma_2 = -\sigma_1$$

$$\Rightarrow \sigma_1 \frac{-4\Omega_n}{2g\sqrt{h}} = 1 \quad \Rightarrow \quad \sigma_1 = \frac{-g\sqrt{h}}{2\Omega_n} = -\sigma_2$$

$$C_2(t) = \frac{-g\sqrt{h}}{2\Omega_n} e^{-i(H_n - \Omega_n)t} + \frac{g\sqrt{h}}{2\Omega_n} e^{-i(H_n + \Omega_n)t}$$

$$C_2(t) = \frac{g\sqrt{h}}{2\Omega_n} e^{-iH_n t} \begin{bmatrix} e^{-i\Omega_n t} & -i\sin(\Omega_n t) \\ e^{i\Omega_n t} & -e^{i\Omega_n t} \end{bmatrix} \Rightarrow C_2(t) = \frac{g\sqrt{h}}{2\Omega_n} e^{-iH_n t} (-2i) \sin(\Omega_n t)$$

$$C_2(t) = -i \frac{g\sqrt{h}}{\Omega_n} e^{-iH_n t} \sin(\Omega_n t)$$

$$|C_2(t)|^2 = \frac{mg^2}{\Omega_n^2} \sin^2(\Omega_n t)$$