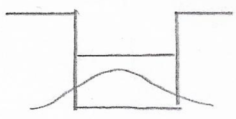
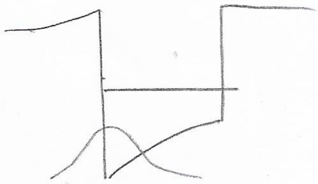
⊛ Ας υποθέσουμε 3Δ κβαντικό εγκλωβισμό (κβαντική τελεία) π.χ με τα χαρακτηριστικά

①



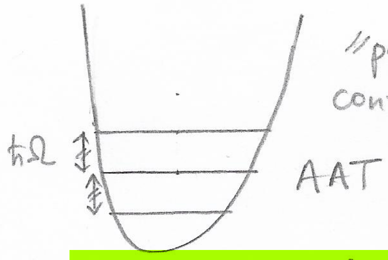
$U_1(x)$

π.χ. χωράει μια στάθμη



$U_2(y)$

π.χ. χωράει μια στάθμη



"parabolic confinement"

$$U_3(z) = \frac{m\Omega^2}{2} z^2$$

π.χ. στρωματική επιταξία
$Al_x Ga_{1-x} As$
μεταβλητού x

$$E_n = \hbar\Omega\left(n+\frac{1}{2}\right)$$

άν, λοιπόν, η δυναμική ενέργεια μπορεί να γραφεί ως άθροισμα

$$U(\vec{r}) = \underbrace{U_1(x)}_{\text{χωρά 1 μόνο στάθμη}} + \underbrace{U_2(y)}_{\text{χωρά 1 μόνο στάθμη}} + \underbrace{U_3(z)}_{\text{AAT} \dots}$$

Τότε μπορούμε να χωρίσουμε μεταβλητές

$$\Phi_k(\vec{r}) = X_1(x)\, Y_1(y)\, Z_n(z) \qquad E_k = E_{1x} + E_{1y} + E_n$$

ΤΡΙΣΤΑΘΜΙΚΟ ΜΕ ΑΑΤ

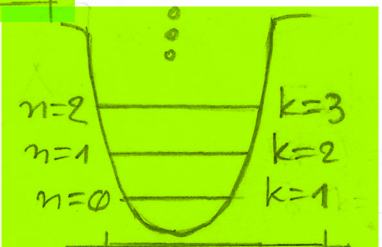
$$\dot{C}_{k'} = -\frac{i}{\hbar}\sum_k C_k(t)\, e^{i(\Omega_{k'}-\Omega_k)t}\, U_{\varepsilon k'k}(t)$$

ιδιοενέργειες 1Δ ΑΑΤ

$$\boxed{E_n = \hbar\Omega\left(n+\frac{1}{2}\right)}$$

$$E_{n+1}-E_n = \hbar\Omega$$

$$E_0 = \frac{\hbar\Omega}{2},\quad E_1 = \frac{3\hbar\Omega}{2}\quad E_2 = \frac{5\hbar\Omega}{2}$$



$$\boxed{k=\{1,1,n\}}$$

$$\boxed{\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2}\hat{z}^2}$$

✻ ΔΕΙΤΕ ⓪ $\vec{3\Delta}$

Υποτίθεται πώς

$$\Phi_n(\vec{r}) = X_1(x)Y_1(y)Z_n(z)$$

Ας θεωρήσουμε ότι μπορούμε να περιοριστούμε στις 3 κατώτερε
στάθμες του ΑΑΤ

$$U_{\varepsilon k'k}(t) = e\mathcal{E}_0\cos\omega t\, z_{k'k}$$

$$z_{k'k} = \int d^3r\, \Phi^*_{k'}(\vec{r})\, z\, \Phi_k(\vec{r})$$

$$z_{kk} = \int d^3r\, \underbrace{\underbrace{|\Phi_k(\vec{r})|^2}_{(Α)}\; \underbrace{z}_{(Π)}}_{(Π)} = 0$$

$$z_{12} = \int d^3r\, \underset{Α}{\Phi^*_1(\vec{r})}\, \underset{Π}{z}\, \underset{Π}{\Phi_2(\vec{r})} \neq 0$$

$$\|$$

$$z_{21} = \int d^3r\, \Phi^*_2(\vec{r})\, z\, \Phi_1(\vec{r}) \neq 0$$

$$z_{13} = \int d^3r\, \underbrace{\underset{Α}{\Phi^*_1(\vec{r})}\, \underset{Π}{z}\, \underset{Α}{\Phi_3(\vec{r})}}_{Π} = 0$$

$$\|$$

$$z_{31} = \;\cdots\; = 0$$

$$z_{23} = \int d^3r\, \underbrace{\underset{Π}{\Phi^*_2(\vec{r})}\, \underset{Π}{z}\, \underset{Α}{\Phi_3(\vec{r})}}_{Α} \neq 0$$

$$\|$$

$$z_{32} = z_{23}$$

ιδιοσυναρτήσεις 1Δ ΑΑΤ

$$a = \left(\frac{\hbar}{m\Omega}\right)^{1/2}$$

$$Z_n(z) = u_n(z)\exp\left(-\frac{m\Omega z^2}{2\hbar}\right)$$

k		n	$u_n(z)$
1	(Α)	0	$(1/a\sqrt{\pi})^{1/2}$
2	(Π)	1	$(1/2a\sqrt{\pi})^{1/2}\, 2\,\frac{z}{a}$
3	(Α)	2	$(1/8a\sqrt{\pi})^{1/2}\left[4\left(\frac{z}{a}\right)^2-2\right]$
4	(Π)	3	$(1/48a\sqrt{\pi})^{1/2}\left[8\left(\frac{z}{a}\right)^3-12\left(\frac{z}{a}\right)\right]$
k		n	$(1/n!\,2^n a\sqrt{\pi})^{1/2} H_n\left(\frac{z}{a}\right)$

↑ πολυώνυμα Hermite

μάλλον...

$$z_{12} = z_{21} \neq z_{23} = z_{32}$$

$$\boxed{\begin{aligned} -e z_{12} &= -e z_{21} \\ \wp_{z12} = \wp_{z21} &:= \wp \end{aligned}}$$

$$U_{\varepsilon 12}(t) = e\mathcal{E}_0\cos\omega t\, z_{12}$$

$$= -\wp_{z12}\cdot\mathcal{E}_0\cos\omega t$$

$$U_{\varepsilon 21}(t) = e\mathcal{E}_0\cos\omega t\, z_{21}$$

$$= -\wp_{z21}\,\mathcal{E}_0\cos\omega t$$

$$\boxed{\begin{aligned}&\wp_{z23} = \wp_{z32} := \wp' \\ &= -e z_{23} = -e z_{32}\end{aligned}} \longrightarrow \text{ὁμοίως...}$$

②

$$\dot{c}_1(t) = -\frac{i}{\hbar} c_1(t) e^{i(\Omega_1-\Omega_1)t} \cancelto{0}{U_{\mathcal{E}11}(t)} - \frac{i}{\hbar} c_2(t) e^{i(\Omega_1-\Omega_2)t} U_{\mathcal{E}12}(t)$$

$$-\frac{i}{\hbar} c_3(t) e^{i(\Omega_1-\Omega_3)t} \cancelto{0}{U_{\mathcal{E}13}(t)}$$

$$\boxed{\dot{c}_1(t) = +\frac{i}{\hbar} c_2(t) e^{-i\Omega t} \wp \mathcal{E}_0 \cos\omega t} \quad ①$$

$$\dot{c}_2(t) = -\frac{i}{\hbar} c_1(t) e^{i(\Omega_2-\Omega_1)t} U_{\mathcal{E}21}(t) - \frac{i}{\hbar} c_2(t) e^{i(\Omega_2-\Omega_2)t} \cancelto{0}{U_{\mathcal{E}22}(t)}$$

$$-\frac{i}{\hbar} c_3(t) e^{i(\Omega_2-\Omega_3)t} U_{\mathcal{E}23}(t)$$

$$\boxed{\dot{c}_2(t) = +\frac{i}{\hbar} c_1(t) e^{i\Omega t} \wp \mathcal{E}_0 \cos\omega t + \frac{i}{\hbar} c_3(t) e^{-i\Omega t} \wp' \mathcal{E}_0 \cos\omega t} \quad ②$$

$$\dot{c}_3(t) = -\frac{i}{\hbar} c_1(t) e^{i(\Omega_3-\Omega_1)t} \cancelto{0}{U_{\mathcal{E}31}(t)} - \frac{i}{\hbar} c_2(t) e^{i(\Omega_3-\Omega_2)t} U_{\mathcal{E}32}(t)$$

$$-\frac{i}{\hbar} c_3(t) e^{i(\Omega_3-\Omega_3)t} \cancelto{0}{U_{\mathcal{E}33}(t)}$$

$$\boxed{\dot{c}_3(t) = +\frac{i}{\hbar} c_2(t) e^{i\Omega t} \wp' \mathcal{E}_0 \cos\omega t} \quad ③$$

μετασχηματισμός Ⓜ
$$\begin{cases} c_1(t) = \mathbb{C}_1(t)\, e^{\frac{i\Delta t}{2}} \\ c_2(t) = \mathbb{C}_2(t)\, e^{-\frac{i\Delta t}{2}} \\ c_3(t) = \mathbb{C}_3(t)\, e^{\frac{\underline{3}i\Delta t}{2}} \end{cases}$$

$$\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

ώστε να κάνουμε RWA

ΛΥΣΗ ✿ ②'

$$Z_{12} = \int d^3r\, \Phi_1^*(\vec r)\, z\, \Phi_2(\vec r) \qquad p_{z12} = -eZ_{12} \qquad U_{\varepsilon 12} = e\cdot\mathcal{E}_0\cdot\cos\omega t\cdot Z_{12}$$

$$= -p_{z12}\cdot\mathcal{E}_0\cdot\cos\omega t$$

$$Z_{23} = \int d^3r\, \Phi_2^*(\vec r)\, z\, \Phi_3(\vec r) \qquad p_{z23} = -eZ_{23} \qquad U_{\varepsilon 23} = e\mathcal{E}_0\cos\omega t\cdot Z_{23}$$

$$= -p_{z23}\,\mathcal{E}_0\cos\omega t$$

$$\Phi_1(\vec r) = X_1(x)\cdot Y_1(y)\cdot Z_1(z) \qquad p := p_{z12} \quad \Omega_R = \frac{p\mathcal{E}_0}{\hbar}$$

$$\Phi_2(\vec r) = X_1(z)\cdot Y_1(y)\cdot Z_2(z) \qquad p' := p_{z23} \quad \Omega_R' = \frac{p'\mathcal{E}_0}{\hbar}$$

$$\Phi_3(\vec r) = X_1(z)\cdot Y_1(z)\cdot Z_3(z)$$

✿ ΑΣΚΗΣΗ

Νά ὑπολογιστεῖ ὁ λόγος $\frac{p}{p'} = \frac{\Omega_R}{\Omega_R'}$ σε αὐτό το σύστημα

$$Z_{12} = \underbrace{\int dx\,|X_1(x)|^2 \int dy\,|Y_1(y)|^2}\int dz\, Z_1(z)\, z\, Z_2(z)$$

$$Z_{23} = \underbrace{\int dx\,|X_1(x)|^2 \int dy\,|Y_1(y)|^2}_{K}\int dz\, Z_2(z)\, z\, Z_3(z)$$

κι ἀν οἱ $X_1(x), Y_1(y)$ εἶναι κανονικοποιημένες $\sim = 1$

$$Z_{12} = \int dz\left(\frac{1}{a\sqrt{\pi}}\right)^{1/2}\exp\left(\frac{-m\Omega z^2}{2\hbar}\right)\cdot z\cdot\left(\frac{1}{a\sqrt{\pi}\,2}\right)^{1/2} 2\,\frac{z}{a}\,\exp\left(\frac{-m\Omega z^2}{2\hbar}\right)$$

$$= \frac{2a}{\sqrt{\pi}\sqrt{2}}\int \frac{dz}{a}\,\exp\left(-\frac{z^2}{2a^2}\right)\frac{z}{a}\cdot\frac{z}{a}\,\exp\left(-\frac{z^2}{2a^2}\right) \qquad \frac{z}{a} := \theta$$

$$= \frac{2a}{\sqrt{\pi}\sqrt{2}}\int d\theta\,\exp(-\theta^2)\cdot\theta^2 = \frac{2a}{\sqrt{\pi}\sqrt{2}}\cdot\frac{\sqrt{\pi}}{2} = \frac{a}{\sqrt{2}}$$

$$Z_{23} = \int dz\left(\frac{1}{2a\sqrt{\pi}}\right)^{1/2} 2\,\frac{z}{a}\,\underbrace{\exp\left(\frac{-z^2}{2a^2}\right)}\cdot z\cdot\underbrace{\left(\frac{1}{8a\sqrt{\pi}}\right)^{1/2}}\left[4\left(\frac{z}{a}\right)^2 - 2\right]\exp\left(-\frac{z^2}{2a^2}\right)$$

$$\frac{2a^2}{\sqrt{\pi}\sqrt{2}\sqrt{8}}\frac{1}{a}\int \frac{dz}{a}\left(\frac{z}{a}\right)^2\exp\left(-\frac{z^2}{a^2}\right)\left[4\left(\frac{z}{a}\right)^2 - 2\right]$$

$$\frac{a}{2\sqrt{\pi}}\int d\theta\cdot\exp(-\theta^2)\cdot\theta^2\left[4\theta^2 - 2\right] = \frac{a}{2\sqrt{\pi}}\cdot(+2)\sqrt{\pi} = +a$$

Eisberg $\psi_n(\xi) = \left(\frac{1}{\sqrt{\pi}\,2^n n!}\right)^{1/2} e^{-\xi^2/2}\cdot H_n(\xi),\ n\in\mathbb{N} \quad \xi = \frac{z}{a} \quad d\xi = \frac{dz}{a}$

①⇒ $$\dot{C}_1(t) = \frac{i}{\hbar}\wp\mathcal{E}_0\, C_2(t)\, e^{-i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2}$$

③

$$\boxed{\Delta := \omega - \Omega}$$

$$\Omega_R = \frac{\wp\mathcal{E}_0}{\hbar} > 0 \text{ για } \wp > 0$$

...

$$\boxed{\text{RWA}}$$

$$\dot{C}_1(t) = \frac{i}{2\hbar}\wp\mathcal{E}_0\, C_2(t)\, e^{i(\omega-\Omega)t} \Rightarrow \boxed{\dot{C}_1(t) = \frac{i}{2}\Omega_R\, C_2(t)\, e^{i\Delta t}} \quad (1')$$

②⇒ $$\dot{C}_2(t) = \frac{i}{\hbar}\wp\mathcal{E}_0\, C_1(t)\, e^{i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2} + \frac{i}{\hbar}\wp'\mathcal{E}_0\, C_3(t)\, e^{-i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2}$$

για $\wp < 0$

$$\Omega_R = \frac{-\wp\mathcal{E}_0}{\hbar}$$

μπλε

$$\Omega_R' = \frac{-\wp'\mathcal{E}_0}{\hbar}$$

κλπ...

$$\boxed{\text{RWA}}$$

$$\dot{C}_2(t) = \frac{i\wp\mathcal{E}_0}{2\hbar}\, C_1(t)\, e^{-i(\omega-\Omega)t} + \frac{i}{2\hbar}\wp'\mathcal{E}_0\, C_3(t)\, e^{i(\omega-\Omega)t}$$

$$\boxed{\Omega_R' = \frac{\wp'\mathcal{E}_0}{\hbar} > 0}$$

για $\wp' > 0$

...

$$\boxed{\dot{C}_2(t) = \frac{i}{2}\Omega_R\, C_1(t)\, e^{-i\Delta t} + \frac{i}{2}\Omega_R'\, C_3(t)\, e^{i\Delta t}} \quad (2')$$

Η 2 είναι ο διαμεσολαβητής.

αν ορίζουμε τις Ω_R, Ω_R' θετικές

③⇒ $$\dot{C}_3(t) = \frac{i\wp'\mathcal{E}_0}{\hbar}\, C_2(t)\, e^{i\Omega t}\, \frac{e^{i\omega t}+e^{-i\omega t}}{2}$$

$$\boxed{\text{RWA}}$$

$$\boxed{\dot{C}_3(t) = \frac{i\wp'\mathcal{E}_0}{2\hbar}\, C_2(t)\, e^{-i\Delta t}} \quad (3')$$

(1')(2')(3')

οι παράγωγοι των συναρτήσεων σχετίζονται με τις συναρτήσεις με χρονικώς εξαρτώμενους συντελεστές

$$\left.\begin{aligned}
\dot{c}_1(t) &= \dot{C}_1(t)\, e^{\frac{i\Delta t}{2}} + C_1(t)\,\frac{i\Delta}{2}\, e^{\frac{i\Delta t}{2}} \\
\dot{c}_2(t) &= \dot{C}_2(t)\, e^{-\frac{i\Delta t}{2}} + C_2(t)\left(-\frac{i\Delta}{2}\right) e^{-\frac{i\Delta t}{2}} \\
\dot{c}_3(t) &= \dot{C}_3(t)\, e^{-3\frac{i\Delta t}{2}} + C_3(t)\left(-3\frac{i\Delta}{2}\right) e^{-3\frac{i\Delta t}{2}}
\end{aligned}\right\} \Longleftarrow \text{Ⓜ}$$

(1')Ⓜ

$$\dot{C}_1(t)\,\cancel{e^{\frac{i\Delta t}{2}}} + C_1(t)\,\frac{i\Delta}{2}\,\cancel{e^{\frac{i\Delta t}{2}}} = \frac{i}{2}\Omega_R\, C_2(t)\,\cancel{e^{-\frac{i\Delta t}{2}}}\;\cancel{e^{i\Delta t}}$$

$$\boxed{\dot{C}_1(t) = -i\frac{\Delta}{2}\, C_1(t) \overset{-}{+} i\frac{\Omega_R}{2}\, C_2(t)} \quad (1'')$$

(2')Ⓜ

$$\dot{C}_2(t)\,\cancel{e^{-\frac{i\Delta t}{2}}} + C_2(t)\left(-\frac{i\Delta}{2}\right)\cancel{e^{-\frac{i\Delta t}{2}}} = i\frac{\Omega_R}{2}\, C_1(t)\,\cancel{e^{\frac{i\Delta t}{2}}}\;\cancel{e^{-i\Delta t}} + \frac{i}{2}\Omega_R'\, C_3(t)\,\cancel{e^{-3\frac{i\Delta t}{2}}}\;\cancel{e^{i\Delta t}}$$

$$\boxed{\dot{C}_2(t) = \overset{-}{\vphantom{i}} i\frac{\Omega_R}{2}\, C_1(t) + i\frac{\Delta}{2}\, C_2(t) \overset{-}{+} i\frac{\Omega_R'}{2}\, C_3(t)} \quad (2'')$$

(3')Ⓜ

$$\dot{C}_3(t)\,\cancel{e^{-\frac{3i\Delta t}{2}}} + C_3(t)\left(-\frac{3i\Delta}{2}\right)\cancel{e^{-\frac{3i\Delta t}{2}}} = \frac{i}{2}\Omega_R'\, C_2(t)\,\cancel{e^{-\frac{i\Delta t}{2}}}\;\cancel{e^{-i\Delta t}}$$

$$\boxed{\dot{C}_3(t) = \overset{-}{\vphantom{i}} i\frac{\Omega_R'}{2}\, C_2(t) + i\frac{3\Delta}{2}\, C_3(t)} \quad (3'')$$

(1'')(2'')(3'')
οι παράγωγοι των συναρτήσεων
σχετίζονται με τις συναρτήσεις
με χρονικώς ανεξάρτητους συντελεστές

$$\begin{bmatrix}\dot{c}_1(t)\\ \dot{c}_2(t)\\ \dot{c}_3(t)\end{bmatrix} = \begin{bmatrix} -i\frac{\Delta}{2} & -i\frac{\Omega_R}{2} & 0\\ -i\frac{\Omega_R}{2} & i\frac{\Delta}{2} & -i\frac{\Omega_R'}{2}\\ 0 & -i\frac{\Omega_R'}{2} & i\frac{3\Delta}{2}\end{bmatrix}\begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix}$$

$$\dot{\vec{x}}(t) \qquad \tilde{A} \qquad \vec{x}(t)$$

$$\boxed{\tilde{A} = -iA}$$

ΔΛΜ $\boxed{\vec{x}(t) = \vec{u}\, e^{\tilde{\lambda} t}}$

$$\dot{\vec{x}}(t) = \tilde{A}\vec{x}(t)$$

$$\vec{u}\tilde{\lambda}\cancel{e^{\tilde{\lambda}t}} = \tilde{A}\vec{u}\cancel{e^{\tilde{\lambda}t}}$$

$$\tilde{A}\vec{u} = \tilde{\lambda}\vec{u}$$

$$\boxed{\tilde{\lambda} = -i\lambda}$$

$$\boxed{A\vec{u} = \lambda\vec{u}}$$

$$A = \begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} & 0\\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} & +\frac{\Omega_R'}{2}\\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2}\end{bmatrix}$$

$$(A - \lambda I)\vec{u} = \vec{0}$$

$$\det\begin{bmatrix} \frac{\Delta}{2}-\lambda & +\frac{\Omega_R}{2} & 0\\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2}-\lambda & +\frac{\Omega_R'}{2}\\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2}-\lambda\end{bmatrix} \cdot \begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

ΛΥΣΗ για $\boxed{\Delta = 0}$

$$\begin{bmatrix} -\lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\lambda \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow -\lambda \begin{vmatrix} -\lambda & +\frac{\Omega_R'}{2} \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} \mp \frac{\Omega_R}{2} \begin{vmatrix} +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} = 0$$

$$-\lambda \left[\lambda^2 - \frac{\Omega_R'^2}{4} \right] + \frac{\Omega_R}{2} \frac{\Omega_R}{2} \lambda = 0$$

ἀπό ἐδῶ ϗ πέρα ἴδιο...

$$-\lambda^3 + \lambda \frac{\Omega_R'^2}{4} + \lambda \frac{\Omega_R^2}{4} = 0 \Rightarrow \lambda \left[-\lambda^2 + \frac{\Omega_R'^2}{4} + \frac{\Omega_R^2}{4} \right] = 0$$

$\boxed{\lambda = 0}$ ή $\lambda^2 = \frac{\Omega_R^2 + \Omega_R'^2}{4}$

$$\boxed{\lambda = \pm \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}}$$

$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$ $\quad \lambda_2 = 0$ $\quad \lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$

$\lambda_1 = -\Lambda < 0$ $\quad \lambda_2 = 0$ $\quad \lambda_3 = \Lambda > 0$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2} = -\Lambda < 0$$

$$\begin{bmatrix} \Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & \Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & \Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow \Lambda U_1 = \frac{\Omega_R}{2} U_2 \Rightarrow U_1 = \frac{\Omega_R}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} U_1 + \Lambda U_2 - \frac{\Omega_R'}{2} U_3 = 0$$

$$-\frac{\Omega_R'}{2} U_2 + \Lambda U_3 = 0 \Rightarrow \Lambda U_3 = \frac{\Omega_R'}{2} U_2 \Rightarrow U_3 = \frac{\Omega_R'}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2}\frac{\Omega_R}{2\Lambda} U_2 + \Lambda U_2 - \frac{\Omega_R'}{2}\frac{\Omega_R'}{2\Lambda} U_2 = 0$$

$$\Rightarrow U_2\left[-\frac{\Omega_R^2}{4\Lambda} + \Lambda - \frac{\Omega_R'^2}{4\Lambda}\right] = 0$$

$$\Lambda = \frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2}$$

$$\Rightarrow U_2\left[\underbrace{\frac{-\Omega_R^2+4\Lambda^2-\Omega_R'^2}{4\Lambda}}_{\text{μηδέν}}\right] = 0$$

$$\Lambda^2 = \frac{\Omega_R^2+\Omega_R'^2}{4}$$

(αν $U_2 = 0$ $\Rightarrow U_1 = U_2 = U_3 = 0$) $\Rightarrow U_2$ ό,τι θέλουμε (μη μηδενικό)

π.χ. $U_2 = 1$

$$\vec{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_1 = \beta \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R}{2\Lambda} \end{bmatrix} \qquad \vec{V}_1 \cdot \vec{V}_1 = 1 \Rightarrow |\beta|^2\left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_R'^2}{4\Lambda^2}\right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + 4\Lambda^2 + \Omega_R'^2}{4\Lambda^2} = 1$$

$$|\beta|^2 \frac{2}{1} = 1 \Rightarrow \text{π.χ.}\ \beta = \frac{1}{\sqrt{2}}$$

- $\lambda_2 = 0$

$$\begin{bmatrix} 0 & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & 0 & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$-\frac{\Omega_R}{2} u_2 = 0 \qquad \boxed{u_2 = 0}$$

$$-\frac{\Omega_R}{2} u_1 - \frac{\Omega_R'}{2} u_3 = 0 \qquad -\frac{\Omega_R}{\cancel{2}} u_1 = \frac{\Omega_R'}{\cancel{2}} u_3 \Rightarrow \boxed{u_3 = -\frac{\Omega_R}{\Omega_R'} u_1}$$

$$-\frac{\Omega_R'}{2} u_2 = 0 \qquad \boxed{u_2 = 0}$$

$$\vec{V}_2 = \beta \begin{bmatrix} 1 \\ 0 \\ -\frac{\Omega_R}{\Omega_R'} \end{bmatrix} \qquad \vec{V}_2 \cdot \vec{V}_2 = 1 \Rightarrow |\beta|^2 \left(1 + \frac{\Omega_R^2}{\Omega_R'^2}\right) = 1 \Rightarrow |\beta|^2 \frac{\Omega_R^2 + \Omega_R'^2}{\Omega_R'^2} = 1$$

$$|\beta|^2 \frac{4\Lambda^2}{\Omega_R'^2} = 1 \Rightarrow \text{π.χ.} \ \beta = \frac{\Omega_R' \ 2}{2\sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

$$\vec{V}_2 = \begin{bmatrix} \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \\ 0 \\ -\frac{\Omega_R}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \end{bmatrix} = \boxed{\begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix}}$$

- $\lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} = \Lambda > 0$

$$\begin{bmatrix} -\Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & -\Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & -\Lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$-\Lambda u_1 - \frac{\Omega_R}{2} u_2 = 0 \Rightarrow -\frac{\Omega_R}{2} u_2 = \Lambda u_1 \Rightarrow u_1 = -\frac{\Omega_R}{2\Lambda} u_2$$

$$-\frac{\Omega_R}{2} u_1 - \Lambda u_2 - \frac{\Omega_R'}{2} u_3 = 0$$

$$-\frac{\Omega_R'}{2} u_2 - \Lambda u_3 = 0 \Rightarrow -\frac{\Omega_R'}{2} u_2 = \Lambda u_3 \Rightarrow u_3 = -\frac{\Omega_R'}{2\Lambda} u_2$$

$$+\frac{\Omega_R}{2} \frac{\Omega_R}{2\Lambda} u_2 - \Lambda u_2 + \frac{\Omega_R'}{2} \frac{\Omega_R'}{2\Lambda} u_2 = 0 \Rightarrow u_2 \left[\frac{\Omega_R^2}{4\Lambda} - \frac{4\Lambda^2}{4\Lambda} + \frac{\Omega_R'^2}{4\Lambda}\right] = 0$$

$$u_2 \underbrace{\left[\frac{\Omega_R^2 + \Omega_R'^2 - 4\Lambda^2}{4\Lambda}\right]}_{\text{μηδέν}} = 0 \Rightarrow u_2 \ \text{ό,τι θέλουμε (μη μηδενικό)}$$

$$\text{π.χ.} \ u_2 = 1$$

$$\vec{V}_3 = \beta \begin{bmatrix} -\frac{\Omega_R}{2\Lambda} \\ 1 \\ -\frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_3 \cdot \vec{V}_3 = 1 \Longrightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_R'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + \Omega_R'^2 + 4\Lambda^2}{4\Lambda^2} = 1 \Longrightarrow |\beta|^2 \cdot 2 = 1 \Longrightarrow$$

$$\pi.\chi. \quad \beta = -\frac{1}{\sqrt{2}}$$

$$\vec{V}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

γενική λύση

$$\vec{x}(t) = \sum_{k=1}^{3} \sigma_k \vec{v}_k e^{-i\lambda_k t}$$

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2}$$

$$4\Lambda^2 = (\Omega_R^2 + \Omega_R'^2)$$

$$16\Lambda^4 = (\Omega_R^2 + \Omega_R'^2)^2$$

$C_1(0)=1$ $C_2(0)=0$ $C_3(0)=0$ ίσχυσαν αρχ. συνθήκες  ⑩

$$\vec{x}(t)=\begin{bmatrix} C_1(t)\,e^{-i\frac{\Delta t}{2}} \\ C_2(t)\,e^{i\frac{\Delta t}{2}} \\ C_3(t)\,e^{3i\frac{\Delta t}{2}}\end{bmatrix}=\frac{\sigma_1}{\sqrt{2}}\begin{bmatrix}\frac{\Omega_R}{2\Lambda}\\ 1 \\ \frac{\Omega_R'}{2\Lambda}\end{bmatrix}e^{-i\lambda_1 t}+\sigma_2\begin{bmatrix}\frac{\Omega_R'}{2\Lambda}\\ 0 \\ -\frac{\Omega_R}{2\Lambda}\end{bmatrix}e^{-i\lambda_2 t}+\frac{\sigma_3}{\sqrt{2}}\begin{bmatrix}\frac{\Omega_R}{2\Lambda}\\ -1 \\ \frac{\Omega_R'}{2\Lambda}\end{bmatrix}e^{-i\lambda_3 t}$$

$$1=\frac{\sigma_1}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}+\sigma_2\frac{\Omega_R'}{2\Lambda}+\frac{\sigma_3}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}$$

$$0=\frac{\sigma_1}{\sqrt{2}}-\frac{\sigma_3}{\sqrt{2}}\Rightarrow\boxed{\sigma_1=\sigma_3=\sigma}$$

$$0=\frac{\sigma_1}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}-\sigma_2\frac{\Omega_R}{2\Lambda}+\frac{\sigma_3}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}$$

$$0=\frac{\sigma}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}-\frac{\sigma_2\Omega_R}{2\Lambda}+\frac{\sigma}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}\Rightarrow 0=\frac{\sigma\Omega_R'}{\sqrt{2}\Lambda}-\frac{\sigma_2\Omega_R}{2\Lambda}\Rightarrow$$

$$\frac{\sigma_2\Omega_R}{2\cancel{\Lambda}}=\frac{\sigma\Omega_R'}{\sqrt{2}\cancel{\Lambda}}\Rightarrow\boxed{\sigma_2=\sigma\sqrt{2}\frac{\Omega_R'}{\Omega_R}}$$

$$1=\frac{\sigma}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}+\sigma\sqrt{2}\frac{\Omega_R'}{\Omega_R}\frac{\Omega_R'}{2\Lambda}+\frac{\sigma}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}$$

$$2\Lambda=\sigma\left(\frac{\Omega_R}{\sqrt{2}}+\sqrt{2}\frac{\Omega_R'^2}{\Omega_R}+\frac{\Omega_R}{\sqrt{2}}\right)=\sigma\frac{\Omega_R^2+2\Omega_R'^2+\Omega_R^2}{\sqrt{2}\,\Omega_R}=\sigma\frac{2}{\sqrt{2}}\frac{\Omega_R^2+\Omega_R'^2}{\Omega_R}$$

$$\sigma=\frac{\sqrt{\Omega_R^2+\Omega_R'^2}\,\sqrt{2}\,\Omega_R}{2(\Omega_R^2+\Omega_R'^2)}\Rightarrow\boxed{\sigma=\frac{\Omega_R}{\sqrt{2}\sqrt{\Omega_R^2+\Omega_R'^2}}=\frac{\Omega_R}{\sqrt{2}\,2\Lambda}}$$

$$\sigma=\frac{\Omega_R}{\sqrt{2}\,2\Lambda}$$

$$C_1(t)e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R}{2\sqrt{\underbrace{\Omega_R^2+\Omega_R'^2}_{2\Lambda}}}\;\frac{\Omega_R\,\cancel{2}}{\cancel{2}\sqrt{\underbrace{\Omega_R^2+\Omega_R'^2}_{2\Lambda}}}\,e^{+i\Lambda t}$$

$$+\frac{\cancel{\Omega_R}}{\cancel{\sqrt{2}}\sqrt{\underbrace{\qquad}_{2\Lambda}}}\cdot\frac{\cancel{\sqrt{2}}\,\Omega_R'}{\cancel{\Omega_R}}\;\frac{\Omega_R'}{\sqrt{\underbrace{\qquad}_{2\Lambda}}}\,e^{-i0}$$

$$+\frac{\Omega_R}{\sqrt{2}\sqrt{\underbrace{\qquad}_{2\Lambda}}}\cdot\frac{\Omega_R\,\cancel{2}}{\cancel{2}\sqrt{\underbrace{\qquad}_{2\Lambda}}}\,e^{-i\Lambda t}$$

$$C_1(t)e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2(\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2})}\cdot e^{i\Lambda t}+\frac{\Omega_R'^2}{\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2}}+\frac{\Omega_R^2}{2(\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2})}\cdot e^{-i\Lambda t}$$ ←

$$C_1(t)e^{-\frac{i\Delta t}{2}} = \frac{\cancel{2}\,\Omega_R^2}{\cancel{2}(\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2})}\cos\Lambda t+\frac{\Omega_R'^2}{\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2}}$$ ←

$$C_2(t)e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{\qquad}}\,1\cdot e^{+i\Lambda t}-\frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{\qquad}}\cdot e^{-i\Lambda t}$$

$$=\frac{\Omega_R}{2\sqrt{\qquad}}\left(e^{+i\Lambda t}-e^{-i\Lambda t}\right)$$

$\cancel{\cos}+i\sin$

$-\cancel{\cos}+i\sin$

$$C_2(t)e^{i\frac{\Delta t}{2}} = \frac{\Omega_R}{\cancel{2}\sqrt{\qquad}}\,\cancel{2}i\sin\Lambda t$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2+\Omega_R'^2}\cdot\sin^2(\Lambda t) = \frac{\Omega_R^2}{\Omega_R^2+\Omega_R'^2}\cdot\left(\frac{1}{2}-\frac{\cos(2\Lambda t)}{2}\right)$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

$$|c_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2} \cdot \left(\frac{1}{2} - \frac{\cos(2\Lambda t)}{2} \right)$$

$$= \frac{\Omega_R^2}{2\left(\Omega_R^2 + \Omega_R'^2\right)} - \frac{\Omega_R^2}{2\left(\Omega_R^2 + \Omega_R'^2\right)} \cdot \cos(2\Lambda t)$$

$$\boxed{\alpha_2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2}}$$

maximum transfer percentage
μέγιστο ποσοστό μεταβιβάσεως

$$\boxed{T_2} = \frac{2\pi}{2\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} = \boxed{\frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}}}$$

$\underbrace{\text{για } \Omega_R = \Omega_R'}$ $\Rightarrow \boxed{\alpha_2 = \frac{1}{2}}$

$$T_2 = \frac{2\pi}{\sqrt{2}\,\Omega_R} = \frac{1}{\sqrt{2}} \boxed{\frac{2\pi}{\Omega_R}}$$

↑
περίοδος αντιστοίχου δισταθμικού

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2}$$
$$= \sqrt{2}\,\Omega_R$$

$$|C_2(t)|^2 = \frac{1}{2}\left(\frac{1}{2} - \frac{\cos(\sqrt{2}\,\Omega_R t)}{2} \right)$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cdot \cos(\underbrace{\sqrt{2}\,\Omega_R}_{2\omega_1 = \omega_2} t)$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4}\cos(\omega_2 t)$$

$$C_1(t)\,e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2\cdot 4\Lambda^2}\,e^{+i\Lambda t} + \frac{\Omega_R'^2}{4\Lambda^2} + \frac{\Omega_R^2}{2\cdot 4\Lambda^2}\,e^{-i\Lambda t}$$

$$C_1(t)\,e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{4\Lambda^2}\cos(\Lambda t) + \frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + 2\,\frac{\Omega_R^2}{4\Lambda^2}\cdot\cos(\Lambda t)\cdot\frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\,\Omega_R^2\,\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos(2\Lambda t)+1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\,\Omega_R^2\,\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$T_A = \frac{2\pi}{2\Lambda}$ $\quad T_B = \frac{2\pi}{\Lambda}$ $\quad \frac{T_B}{T_A} = 2 \Rightarrow$ περιοδική κίνηση

με περίοδο $\boxed{T_1 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Omega_R'^2}}\cdot 2 = 2T_2}$

$$\left|C_1\left(\frac{2\pi}{\Lambda}\right)\right|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos\left(2\cancel{\Lambda}\frac{2\pi}{\cancel{\Lambda}}\right)+1}{2} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\,\Omega_R^2\,\Omega_R'^2}{16\Lambda^4}\cdot\cos\left(\cancel{\Lambda}\frac{2\pi}{\cancel{\Lambda}}\right) =$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\Omega_R^2\,\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2+\Omega_R'^2)^2}{16\Lambda^4} = 1$$

$$\left|C_1\left(\frac{2\pi}{2\Lambda}\right)\right|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos\left(2\cancel{\Lambda}\frac{2\pi}{\cancel{2\Lambda}}\right)+1}{2} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\,\Omega_R^2\,\Omega_R'^2}{16\Lambda^4}\cdot\cos\left(\cancel{\Lambda}\frac{2\pi}{2\cancel{\Lambda}}\right)$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\,\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

ἡ τιμή τῆς $|C_1(t)|^2$ στο ἥμισυ τῆς περιόδου

$$\frac{d}{dt}|C_1(t)|^2 = \frac{\Omega_R^4}{\boxed{16\Lambda^4}}\cdot\boxed{2}\cos(\Lambda t)\cdot\underline{(-\Lambda)}\,\underline{\sin(\Lambda t)} + \frac{\boxed{2}\Omega_R^2\,\Omega_R'^2}{\boxed{16\Lambda^4}}\underline{(-\Lambda)}\,\underline{\sin(\Lambda t)}$$

$$\frac{d}{dt}|C_1(t)|^2 = \frac{(-2\Lambda)\,\Omega_R^2}{16\Lambda^4}\cdot\sin(\Lambda t)\cdot\left[\Omega_R^2\cdot\cos(\Lambda t) + \Omega_R'^2\right]$$

$$\frac{d^2}{dt^2}|C_1(t)|^2 = \frac{(-2\Lambda)\,\Omega_R^2}{16\Lambda^4}\cdot\Lambda\cdot\cos(\Lambda t)\cdot\left[\Omega_R^2\cos(\Lambda t) + \Omega_R'^2\right] + \frac{(-2\Lambda)\,\Omega_R^2}{16\Lambda^4}\sin(\Lambda t)\cdot\Omega_R^2(-\Lambda)\cdot\sin(\Lambda t)$$

$$= \frac{(-2\Lambda)\,\Omega_R^2}{16\Lambda^4}\cdot\Lambda\cdot\left\{\Omega_R^2\cdot\cos^2(\Lambda t) + \Omega_R'^2\cos(\Lambda t) - \Omega_R^2\sin^2(\Lambda t)\right\}$$

$$\frac{d|G_1(t)|^2}{dt} = \frac{(-2\Lambda)\cdot\Omega_R^2}{16\Lambda^4}\cdot\sin(\Lambda t)\cdot\left[\Omega_R^2\cdot\cos(\Lambda t) + \Omega_R'^2\right]$$

$$\frac{d^2|G_1(t)|^2}{dt^2} = \frac{(-2\Lambda^2\cdot\Omega_R^2}{16\Lambda^4}\left[\Omega_R^2\sin^2(\Lambda t) - \Omega_R^2\cos^2(\Lambda t) - \Omega_R'^2\cos(\Lambda t)\right]$$

$$\frac{d|G_1(t)|^2}{dt} = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$$

$$\Downarrow$$

$$\Lambda t = n\pi, n\in\mathbb{Z}$$

$\boxed{\Pi 1}$

Προσοχή, πρέπει $\Omega_R'^2 \leq \Omega_R^2$

$\boxed{\Pi 2}$

$\boxed{\Pi 2}$

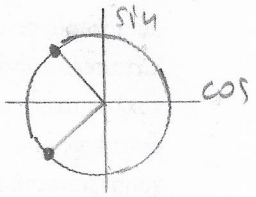
$$\frac{d^2}{dt^2}|G_1(t)|^2 = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\cdot\left[\Omega_R^2\cdot\sin^2(\Lambda t) - \Omega_R^2\cdot\frac{\Omega_R'^4}{\Omega_R^4} + \Omega_R'^2\frac{\Omega_R'^2}{\Omega_R^2}\right]$$

$$= \frac{2\Lambda^2}{16\Lambda^4}\left[\Omega_R^4\cdot\sin^2(\Lambda t) - \Omega_R'^4 + \Omega_R'^4\right] = \frac{2\Lambda^2}{16\Lambda^4}\cdot\Omega_R^4\cdot\sin^2(\Lambda t) > 0$$

Δηλ. στην $\boxed{\Pi 2}$ έχουμε ελάχιστα, με τιμή μηδενική διότι τότε:

$$|G_1(t)|^2 = \frac{\cancel{\Omega_R^4}}{16\Lambda^4}\cdot\frac{\Omega_R'^4}{\cancel{\Omega_R^4}} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\cancel{\Omega_R^2}\Omega_R'^2}{16\Lambda^4}(-1)\frac{\Omega_R'^2}{\cancel{\Omega_R^2}} = \frac{2\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R'^4}{16\Lambda^4} = 0$$

Μάλιστα σε μία περίοδο $T_1 = \frac{2\pi}{\Lambda}$, η οποία είναι και η περίοδος του όρου $\cos(\Lambda t)$, $\exists$ 2 φορές όπου έχουμε $\cos(\Lambda t) = \frac{-\Omega_R'^2}{\Omega_R^2}$ δηλαδή θα έχουμε 2 μηδενισμούς σε μία περίοδο T_1.



Οπότε τότε, $d_1 = 1$ μέγιστο ποσοστό μεταβιβάσεως
maximum transfer percentage

$\boxed{\Pi 1}$

$$\frac{d^2}{dt^2}|G_1(t)|^2 = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left[-\Omega_R^2\cdot\cos^2(\Lambda t) - \Omega_R'^2\cos(\Lambda t)\right]$$

$t = 0, T_1, 2T_1, \ldots$

$\sin(\Lambda t) = 0 \Rightarrow \cos(\Lambda t) = \pm 1$

$\Pi 1\alpha$ $\Lambda t = 0, 2\pi, 4\pi, \ldots$ $\cos(\Lambda t) = 1$

$\Pi 1\beta$ $\Lambda t = \pi, 3\pi, 5\pi, \cdots$ $\cos(\Lambda t) = -1$

$t = \frac{T_1}{2}, \frac{3T_1}{2}, \frac{5T_1}{2}, \ldots$

Π1α $$\frac{d^2|C_1(t)|^2}{dt^2} = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left(-\Omega_R^2 - \Omega_R'^2\right) < 0 \Rightarrow$$ τοπικό μέγιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot 1 + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 + \Omega_R'^2)^2}{16\Lambda^4} = 1$$

δηλ. είναι ολικό μέγιστο

Π1β $$\frac{d^2|C_1(t)|^2}{dt^2} = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left(-\Omega_R^2 + \Omega_R'^2\right) > 0 \Rightarrow$$ τοπικό ελάχιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

η οποία είναι η τιμή της $|C_1(t)|^2$ στο ήμισυ της περιόδου

$$A_1 = 1 - \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2} = \frac{4\cdot\Omega_R^2\cdot\Omega_R'^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

για $\Omega_R = \Omega_R'$ $$T_1 = \frac{2\pi}{\sqrt{2}\,\Omega_R}\cdot 2 = \sqrt{2}\,\frac{2\pi}{\Omega_R}$$

↓ περίοδος αντιστοίχου διστάθμικού

Π2 $\Rightarrow$ $A_1 = 1$ $\cos(\Lambda t) = -1 \Rightarrow \Lambda t = +\pi, 3\pi, \ldots$ $t = \frac{\pi}{\Lambda} = \frac{2\pi}{2\Lambda} =$ το ήμισυ της περιόδου

Π1 $\Rightarrow$ $A_1 = 1$ στο ήμισυ της περιόδου

1 φορά μηδενισμός...

δηλ. για $\Omega_R = \Omega_R'$ οι Π1, Π2 ταυτίζονται

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2}\,\Omega_R$$

$$4\Lambda^2 = 2\Omega_R^2$$

$$16\Lambda^4 = 4\Omega_R^4$$

π.χ. όπως στο GG, GGG

$T_{GG} \simeq 20.6783$ fs

$T_{GGG} = 29.2436$ fs $\simeq \sqrt{2}\,T_{GG}$

$T_{31} = 14.6218$ fs $= \frac{T_{GG}}{\sqrt{2}}$

$T_{32} = 29.2436$ fs $\simeq T_{GGG} = \sqrt{2}\,T_{GG}$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4}\cdot\frac{\cos(\sqrt{2}\,\Omega_R t) + 1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} + \frac{2\Omega_R^4}{4\Omega_R^4}\cdot\cos\left(\frac{\sqrt{2}}{2}\Omega_R t\right)$$

$$|c_1(t)|^2 = \frac{1}{4}\left(\frac{1}{2}\cdot\cos(\underbrace{\sqrt{2}\,\Omega_R}_{2\omega_1=\omega_2} t) + \frac{1}{2}\right) + \frac{1}{4} + \frac{1}{2}\cos\left(\underbrace{\frac{\sqrt{2}}{2}\Omega_R}_{\omega_1} t\right)$$

$$\frac{1}{8}\cos(2\omega_1 t) + \frac{1}{8} + \frac{1}{4} + \frac{1}{2}\cos(\omega_1 t) =$$

$$\frac{1}{8}\left(\underbrace{\cos(2\omega_1 t) + 1}_{2\cos^2(\omega_1 t)} + 2 + 4\cos(\omega_1 t)\right) = \frac{1}{8}\left(2\cos^2(\omega_1 t) + 2 + 4\cos(\omega_1 t)\right)$$

$$= \frac{1}{4}\left(\cos^2(\omega_1 t) + 1 + 2\cos(\omega_1 t)\right) = \frac{1}{4}\left(\underbrace{\cos(\omega_1 t) + 1}_{2\cos^2\left(\frac{\omega_1 t}{2}\right)}\right)^2 = \cos^4\left(\frac{\omega_1 t}{2}\right)$$

$$C_3(t)\,e^{3i\frac{\Delta t}{2}} = \frac{\Omega_R}{4\Lambda}\cdot\frac{\Omega_R'}{2\Lambda}e^{+i\Lambda t} + \frac{\cancel{\Omega_R}}{2\Lambda}\frac{\Omega_R'}{\cancel{\Omega_R}}(-1)\frac{\Omega_R}{2\Lambda} + \frac{\Omega_R}{4\Lambda}\cdot\frac{\Omega_R'}{2\Lambda}e^{-i\Lambda t} \qquad ⑭$$

$$C_3(t)\,e^{3i\frac{\Delta}{2}t} = \frac{\Omega_R\Omega_R'}{4\Lambda^2}\cos(\Lambda t) - \frac{\Omega_R\Omega_R'}{4\Lambda^2}$$

$$|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos^2(\Lambda t) + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - 2\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,\frac{\cos(2\Lambda t)+1}{2} + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \qquad T_B = \frac{2\pi}{\Lambda} \qquad \frac{T_B}{T_A} = 2$$

⇒ η κίνηση είναι περιοδική !!

με περίοδο

$$T_3 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Omega_R'^2}}\,2 = 2T_2 = T_1$$

$$\frac{d}{dt}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2\cos(\Lambda t)\,(-\Lambda)\sin(\Lambda t) - 2\,\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}(-\Lambda)\sin(\Lambda t)$$

$$\frac{d}{dt}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2(-\Lambda)\left[\cos(\Lambda t)\cdot\sin(\Lambda t) - \sin(\Lambda t)\right]$$

$$\frac{d}{dt}|C_3(t)|^2 = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = 1$$

$$\nearrow t = T_3,\ 2T_3, \dots$$

(Π1) $\Lambda t = 0, 2\pi, 4\pi, \dots \Rightarrow \sin(\Lambda t) = 0$ και $\cos(\Lambda t) = 1$

(Π2) $\Lambda t = \pi, 3\pi, 5\pi, \dots \Rightarrow \sin(\Lambda t) = 0$ και $\cos(\Lambda t) = -1$

$$\searrow t = \frac{T_3}{2},\ \frac{3T_3}{2}, \dots$$

(Π1)
(Π2) $|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot 1 + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}(\pm 1) = \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} \mp \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}$

(Π1) $\Rightarrow |C_3(t)|^2 = 0$ ελάχιστο

(Π2) $\Rightarrow |C_3(t)|^2 = \frac{4\Omega_R^2\Omega_R'^2}{16\Lambda^4}$ μέγιστο διότι...

$$\frac{d^2}{dt^2}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2(-\Lambda)\left[\frac{1}{\cancel{2}}\cancel{2}\Lambda\cos(2\Lambda t) - \Lambda\cos(\Lambda t)\right] = \underbrace{\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2\Lambda^2}_{\vartheta}\left(\cos(\Lambda t) - \cos(2\Lambda t)\right)$$

(Π1) $\Rightarrow (\dots) = \cos(2\pi) - \cos(4\pi) = 0$

(Π2) $\Rightarrow (\dots) = \cos(\pi) - \cos(2\pi) = -1-1 = -2 \Rightarrow \frac{d^2}{dt^2}|C_3(t)|^2 < 0$ μέγιστο

Ἀν τα πράγματα εἶναι ἔτσι, τότε

$$\frac{d^3|C_3(t)|^2}{dt^3} = \vartheta\cdot\left(\Lambda(-1)\cdot\sin(\Lambda t) + 2\Lambda\sin(2\Lambda t)\right)$$

$$= \vartheta\Lambda\left(2\sin(2\Lambda t) - \sin(\Lambda t)\right)$$

καὶ γιὰ $\Lambda t = 2\pi \overset{\Pi 1}{\vee} = \vartheta\Lambda\left(2\sin(4\pi) - \sin(2\pi)\right) = 0$

$$\frac{d^4|C_3(t)|^2}{dt^4} = \vartheta\Lambda\left(2\cdot 2\Lambda\cos(2\Lambda t) - \Lambda\cdot\cos(\Lambda t)\right)$$

$$= \vartheta\Lambda^2\left(4\cdot\cos(2\Lambda t) - \cos(\Lambda t)\right)$$

καὶ γιὰ $\Lambda t = 2\pi \overset{\Pi 1}{\vee} = \vartheta\Lambda^2\left(4\cdot\underbrace{\cos(4\pi)}_{1} - \underbrace{\cos(2\pi)}_{1}\right) > 0$

ΞΑΝΑ $\frac{d^2}{dt^2}|C_3(t)|^2 = \vartheta\left(\underbrace{\cos(\Lambda t)}_{1} - \underbrace{\cos(2\Lambda t)}_{1}\right) = 0$

γιὰ $\Lambda t = 2\pi \overset{\Pi 1}{\vee}$

$$\frac{d^3}{dt^3}|C_3(t)|^2 = \vartheta\left(-\underbrace{\Lambda\sin(\Lambda t)}_{0} + \underbrace{2\Lambda\sin(2\Lambda t)}_{0}\right) = 0$$

γιὰ $\Lambda t = 2\pi \overset{\Pi 1}{\vee}$

$$\frac{d^4}{dt^4}|C_3(t)|^2 = \vartheta\left(-\Lambda^2\cos(\Lambda t) + 2\Lambda\cdot 2\Lambda\cos(2\Lambda t)\right) = \vartheta\Lambda^2\left(4\underbrace{\cos(2\Lambda t)}_{1} - \underbrace{\cos(\Lambda t)}_{1}\right)$$

γιὰ $\Lambda t = 2\pi \overset{\Pi 1}{\vee}$ $= \vartheta\Lambda^2\cdot 3 > 0$

δηλαδή στα ἐλάχιστα μηδενίζονται ἡ $|C_3(t)|^2$, καθὼς καὶ ἡ 1η, 2η καὶ 3η παράγωγός της!

ἐνῶ, ἡ 4η παράγωγός της εἶναι θετική...

μοιάζει κάπως μὲ ἐπίπεδη συνάρτηση
(flat function)
[ὅπου μηδενίζονται σὲ κάποιο σημεῖο
ὅλες οἱ παράγωγοι]

Άρα $$\boxed{\lambda_3 = \frac{4\Omega_R^2\,\Omega_R'^2}{(\Omega_R^2+\Omega_R'^2)^2}}$$ μέγιστο ποσοστό μεταβιβάσεως
maximum transfer percentage

για $\Omega_R' = \Omega_R$

$$T_3 = \frac{2\pi}{\sqrt{2}\,\Omega_R}\cdot 2 = \sqrt{2}\,\boxed{\frac{2\pi}{\Omega_R}} \qquad 2\Lambda = \sqrt{2\Omega_R^2} = \sqrt{2}\,\Omega_R$$

↓περίοδος αντίστοιχου δισταθμικού

$$|c_3(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4}\,\frac{\cos(\sqrt{2}\,\Omega_R t)+1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} - \frac{2\Omega_R^4}{4\Omega_R^4}\cdot\cos\left(\frac{\sqrt{2}\,\Omega_R t}{2}\right)$$

$$|c_3(t)|^2 = \frac{1}{4}\left(\frac{1}{2}\cos(\underbrace{\sqrt{2}\,\Omega_R}_{2\omega_3 = 2\omega_1 = \omega_2} t) + \frac{1}{2} + \frac{1}{4} - \frac{1}{2}\cos\left(\underbrace{\frac{\sqrt{2}\,\Omega_R}{2}}_{\omega_1 = \omega_3} t\right)\right)$$

$$= \frac{1}{8}\cos(2\omega_3 t) + \frac{1}{8} + \frac{1}{4} - \frac{1}{2}\cos(\omega_3 t)$$

$$= \frac{1}{8}\left(\underline{\cos(2\omega_3 t)+1} + 2 - 4\cos(\omega_3 t)\right)$$

$$= \frac{1}{8}\left(2\cos^2(\omega_3 t) + 2 - 4\cos(\omega_3 t)\right)$$

$$= \frac{1}{4}\left(\cos^2(\omega_3 t) + 1 - 2\cos(\omega_3 t)\right)$$

$$= \frac{1}{4}\left(1-\cos(\omega_3 t)\right)^2$$

$$= \frac{1}{4}\left(2\sin^2\left(\frac{\omega_3 t}{2}\right)\right)^2 = \sin^4\left(\frac{\omega_3 t}{2}\right)$$

Άρα $$A_3 = \frac{4\Omega_R^2 \Omega_{R'}^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

μέγιστος ρυθμός μεταβιβάσεως
maximum transfer rate 1→3

$$:= \frac{A_3}{T_3} = \frac{A_3}{T_3} = \frac{4\Omega_R^2\Omega_{R'}^2}{(\Omega_R^2+\Omega_R'^2)^2}\,\frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2\pi\cdot 2}$$

Μέσες πιθανότητες παρουσίας του ηλεκτρονίου σε κάθε στάθμη

$$\langle |c_3(t)|^2\rangle = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,\frac{1}{2} + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{3\Omega_R^2\Omega_R'^2}{2\cdot 16\Lambda^4}$$

$$\langle |c_1(t)|^2\rangle = \frac{\Omega_R^4}{16\Lambda^4}\,\frac{1}{2} + \frac{\Omega_{R'}^4}{16\Lambda^4} = \frac{\Omega_R^4 + 2\Omega_R'^4}{2\cdot 16\Lambda^4}$$

$$\langle |c_2(t)|^2\rangle = \frac{\Omega_R^2}{\Omega_R^2+\Omega_R'^2}\,\frac{1}{2} = \frac{\Omega_R^2}{4\Lambda^2}\,\frac{1}{2} = \frac{4\Omega_R^2\Lambda^2}{2\cdot 16\Lambda^4}.$$

$$\langle |c_1(t)|^2\rangle + \langle |c_2(t)|^2\rangle + \langle |c_3(t)|^2\rangle = \frac{3\Omega_R^2\Omega_R'^2 + \Omega_R^4 + 2\Omega_R'^4 + 4\Omega_R^2\Lambda^2}{2\cdot 16\Lambda^4}$$

$$= \frac{3\Omega_R^2\Omega_R'^2 + \Omega_R^4 + 2\Omega_R'^4 + \Omega_R^2(\Omega_R^2+\Omega_R'^2)}{2\cdot 16\Lambda^4} = \frac{4\Omega_R^2\Omega_R'^2 + 2\Omega_R^4 + 2\Omega_R'^4}{2\cdot 16\Lambda^4} =$$

$$\frac{\Omega_R^4 + 2\Omega_R^2\Omega_R'^2 + \Omega_R'^4}{(\Omega_R^2+\Omega_R'^2)^2} = 1$$

ορισμός t_{3mean}

$$\frac{3\cancel{\Omega_R^2\Omega_R'^2}}{2\cdot\cancel{16\Lambda^4}} = \frac{\cancel{\Omega_R^2\Omega_R'^2}}{\cancel{16\Lambda^4}}\cos^2(\Lambda t_{3mean}) + \frac{\cancel{\Omega_R^2\Omega_R'^2}}{\cancel{16\Lambda^4}} - \frac{2\cancel{\Omega_R^2\Omega_R'^2}}{\cancel{16\Lambda^4}}\cos(\Lambda t_{3mean})$$

ο απαιτούμενος χρόνος για να πιάσει τη μέση τιμή

$$\frac{3}{2} = \cos^2(\Lambda t_{3mean}) + 1 - 2\cos(\Lambda t_{3mean})$$

$$\frac{1}{2} = \cos^2(\Lambda t_{3mean}) - 2\cdot\cos(\Lambda t_{3mean})$$

$$1 = 2\cos^2 x - 4\cos x$$

$$2\psi^2 - 4\psi - 1 = 0 \qquad \psi = \cos x$$

$$\delta = 16 + 4\cdot 2 = 24$$

$$\frac{4 \pm \sqrt{24}}{2\cdot 2} = 1 \pm \frac{\sqrt{24}}{4} = 1 \pm \sqrt{\frac{24}{16}} = 1 \pm \sqrt{\frac{3\cdot\cancel{8}}{2\cdot\cancel{8}}}$$

$$= 1 \pm \frac{\sqrt{3}}{\sqrt{2}} \qquad 1 - \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}} \qquad 1 + \frac{\sqrt{3}}{\sqrt{2}} > 1$$

απορρίπτεται

$$\cos(\Lambda t_{3mean}) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\cos\left(\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}\, t_{3mean}\right) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}\, t_{3mean} = 1.797477\ldots$$

$$t_{3mean} = \frac{2\cdot 1.797477}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

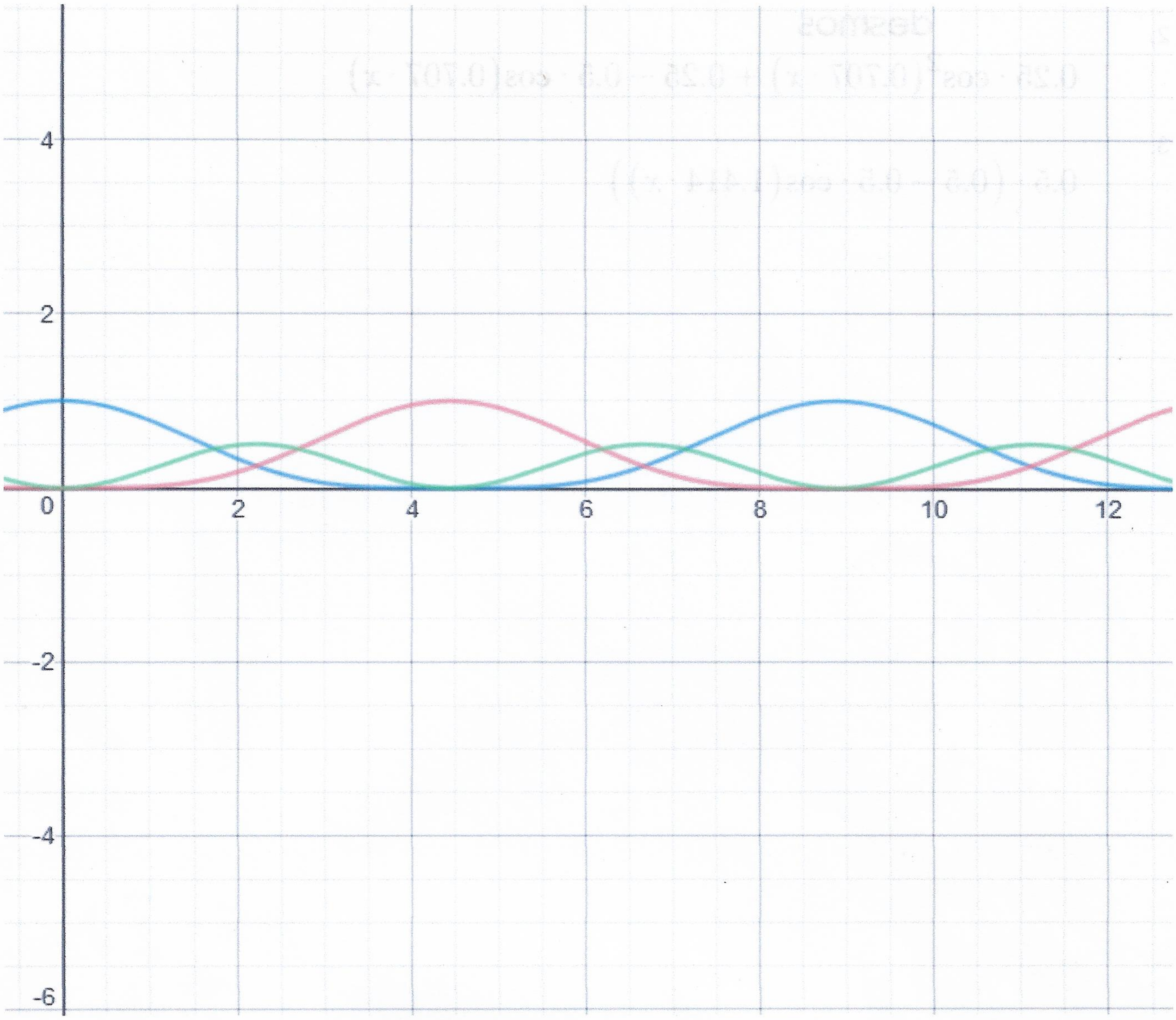
mean tranfer rate

μέσος ρυθμός μεταβιβάσεως

$$k := \frac{\langle |c_3(t)|^2 \rangle}{t_{3mean}} = \frac{3\,\Omega_R^2\,\Omega_R'^2}{2\cdot 16\Lambda^4}\;\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2\cdot 1.797477}$$

$$\frac{k}{\frac{cA_3}{T_3}} = \frac{\textcircled{3}\,\cancel{\Omega_R^2\,\Omega_R'^2}}{\textcircled{2}\,\cancel{(\Omega_R^2+\Omega_R'^2)^2}}\;\frac{\cancel{\sqrt{\Omega_R^2+\Omega_R'^2}}}{\textcircled{2}\cdot 1.797477}\cdot\frac{\cancel{(\Omega_R^2+\Omega_R'^2)^2}\;\textcircled{2\pi}\;\textcircled{2}}{\textcircled{4}\,\cancel{\Omega_R^2\,\Omega_R^3}\,\cancel{\sqrt{\Omega_R^2+\Omega_R'^2}}}$$

$$= \frac{3\cdot\pi}{4\cdot 1.797477} \simeq 1.31083\ldots$$



— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 + 0.5 \cdot \cos(0.707 \cdot x)$

— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 - 0.5 \cdot \cos(0.707 \cdot x)$

— $0.5 \cdot (0.5 - 0.5 \cdot \cos(1.414 \cdot x))$

πράσινο

για

$\Omega_R = \Omega_R' = 1$

$2\Lambda = \sqrt{1^2+1^2} = \sqrt{2} \Rightarrow \Lambda = \frac{\sqrt{2}}{2}$

ΑΣΚΗΣΗ (matlab)

Χρησιμοποιώντας το πρόγραμμα Oscillations.m να γίνει η γραφική παράσταση των ταλαντώσεων ΔΣ με $|\Delta| = \sqrt{3}\,\Omega_R$ $\left(\Rightarrow \alpha = \frac{1}{4},\ T = \frac{1}{2}\frac{2\pi}{\Omega_R}\right)$

ΑΣΚΗΣΗ (matlab)

Να φτιαχθεί ανάλογο πρόγραμμα για Τριστάθμικό Σύστημα και να γίνει η γραφική παράσταση για $\Omega_R = \Omega_c = 1$ $\quad \Delta = 0$

Θέμα: Άσκηση με matlab 2
Χρησιμοποιώντας τα προγραμματάκια, τα οποία υπάρχουν στην η-τάξη (έγγραφα)
callnoRWA.m, noRWA.m, setGlobalOmegaROmegaomega.m
δοκιμάστε να συγκρίνετε και να σχολιάσετε
τη λύση με RWA (rotating wave approximation, προσέγγιση περιστρεφομένου κύματος)
με την αριθμητική λύση χωρίς προσέγγιση περιστρεφομένου κύματος (noRWA).
Περιπτώσεις (μονάδες, ας πούμε fs):

α) $\Omega R = 1, \omega = 1, \Omega = 0.9$
$\Delta = 0.1$ ή 1 ή 10

β) $\Delta = 1, \omega = 10, \Omega = 9$
$\Omega R = 0.1$ ή 1 ή 10

γ) $\Omega R = 1, \Delta = 1,$
$\omega = 10, \Omega = 9$
$\omega =100, \Omega = 99$
$\omega = 1000, \Omega = 999$

Θέμα: Άσκηση με matlab 1
Χρησιμοποιώντας το προγραμματάκι, το οποίο υπάρχει στην η-τάξη (έγγραφα)
Oscillations.m
δοκιμάστε να συγκρίνετε και να σχολιάσετε
τη λύση με RWA (rotating wave approximation, προσέγγιση περιστρεφομένου κύματος)
σε συντονισμό και χωρίς συντονισμό για τέσσερεις περιπτώσεις, δικής σας επιλογής, π.χ.
$\Delta = \Omega R$, $\Delta = 3\ \Omega R$, ...

ΛΥΣΗ

$$Z_{12} = \int d^3r\, \Phi_1^*(\vec{r})\, z\, \Phi_2(\vec{r}) \qquad p_{z12} = -eZ_{12} \qquad U_{\varepsilon 12} = e\cdot\mathcal{E}_0\cdot\cos\omega t\cdot Z_{12}$$

$$= -p_{z12}\cdot\mathcal{E}_0\cdot\cos\omega t$$

$$Z_{23} = \int d^3r\, \Phi_2^*(\vec{r})\, z\, \Phi_3(\vec{r}) \qquad p_{z23} = -eZ_{23} \qquad U_{\varepsilon 23} = e\,\mathcal{E}_0\,\cos\omega t\cdot Z_{23}$$

$$= -p_{z23}\cdot\mathcal{E}_0\cos\omega t$$

$$\Phi_1(\vec{r}) = X_1(x)\cdot Y_1(y)\cdot Z_1(z)$$

$$\Phi_2(\vec{r}) = X_1(x)\cdot Y_1(y)\cdot Z_2(z)$$

$$\Phi_3(\vec{r}) = X_1(x)\cdot Y_1(y)\cdot Z_3(z)$$

$$p := p_{z12} \qquad \Omega_R = \frac{p\mathcal{E}_0}{\hbar}$$

$$p' := p_{z23} \qquad \Omega_R' = \frac{p'\mathcal{E}_0}{\hbar}$$

ΑΣΚΗΣΗ

Νὰ ὑπολογιστεῖ ὁ λόγος $\frac{p}{p'} = \frac{\Omega_R}{\Omega_R'}$ σε αὐτό το σύστημα

$$Z_{12} = \underbrace{\int dx\,|X_1(x)|^2 \int dy\,|Y_1(y)|^2}\int dz\, Z_1(z)\, z\, Z_2(z)$$

$$Z_{23} = \underbrace{\int dx\,|X_1(x)|^2 \int dy\,|Y_1(y)|^2}_{K}\int dz\, Z_2(z)\, z\, Z_3(z)$$

κι ἂν οἱ $X_1(x), Y_1(y)$ εἶναι κανονικοποιημένες $\sim\sim = 1$

$$Z_{12} = \int dz \left(\frac{1}{\alpha\sqrt{\pi}}\right)^{1/2} \exp\left(\frac{-m\Omega z^2}{2\hbar}\right)\cdot z \left(\frac{1}{\alpha\sqrt{\pi}\,2}\right)^{1/2} 2\,\frac{z}{\alpha}\,\exp\left(\frac{-m\Omega z^2}{2\hbar}\right)$$

$$= \frac{2\alpha}{\sqrt{\pi}\sqrt{2}}\int \frac{dz}{\alpha}\, \exp\left(-\frac{z^2}{2a^2}\right)\frac{z}{\alpha}\cdot\frac{z}{\alpha}\,\exp\left(-\frac{z^2}{2a^2}\right) \qquad \frac{z}{\alpha} := \theta$$

$$= \frac{2\alpha}{\sqrt{\pi}\sqrt{2}}\int d\theta\, \exp(-\theta^2)\cdot\theta^2 = \frac{2a}{\sqrt{\pi}\sqrt{2}}\,\frac{\sqrt{\pi}}{2} = \frac{\alpha}{\sqrt{2}}$$

$$Z_{23} = \int dz \left(\frac{1}{2a\sqrt{\pi}}\right)^{1/2} 2\,\frac{z}{\alpha}\,\exp\left(\frac{-z^2}{2a^2}\right)\cdot z\cdot\left(\frac{1}{8a\sqrt{\pi}}\right)^{1/2}\left[4\left(\frac{z}{a}\right)^2 - 2\right]\exp\left(-\frac{z^2}{2a^2}\right)$$

$$\frac{2\alpha^2}{\sqrt{\pi}\sqrt{2}\sqrt{8}}\,\frac{1}{\alpha} \int \frac{dz}{a}\left(\frac{z}{a}\right)^2 \exp\left(-\frac{z^2}{a^2}\right)\left[4\left(\frac{z}{a}\right)^2 - 2\right]$$

$$\frac{\alpha}{2\sqrt{\pi}} \int d\theta\cdot\exp(-\theta^2)\cdot\theta^2\left[4\theta^2 - 2\right] = \frac{\alpha}{2\sqrt{\pi}}\cdot(+2)\sqrt{\pi} = +\alpha$$

$$\text{Eisberg} \quad \psi_n(\xi) = \left(\frac{1}{\sqrt{\pi}\,2^n n!}\right)^{1/2} e^{-\xi^2/2}\cdot H_n(\xi),\ n\in\mathbb{N} \qquad \xi = \frac{z}{a} \qquad d\xi = \frac{dz}{a}$$