$$\dot{C}_1(t) = C_2(t)\,\frac{i\Omega_R}{2}\left[e^{i\Delta t} + e^{-i\Sigma t}\right] \quad \boxed{\Gamma}$$

$$\dot{C}_2(t) = C_1(t)\,\frac{i\Omega_R}{2}\left[e^{-i\Delta t} + e^{i\Sigma t}\right]$$

Α΄ ΤΡΟΠΟΣ

Ας υποθέτουμε πως $e^{\cdots} \sim 1$ δηλαδή $\Delta \sim 0$, $\Sigma \sim 0$

δηλαδή πως υπερτερεί συντριπτικά ο όρος $\frac{i\Omega_R}{2}$ έξω από την τετράγωνη παρένθεση.

external ext

Τότε οι εξισώσεις γίνονται:

$$\dot{C}_1(t) = C_2(t)\,\frac{i\Omega_R}{2}\cdot 2 = i\Omega_R\,C_2(t) \quad \boxed{\Omega_R}$$

$$\dot{C}_2(t) = C_1(t)\,\frac{i\Omega_R}{2}\cdot 2 = i\Omega_R\,C_1(t)$$

$$\ddot{C}_1(t) = i\Omega_R\,\dot{C}_2(t) = (i\Omega_R)^2\,C_1(t)$$

$$\ddot{C}_2(t) = i\Omega_R\,\dot{C}_1(t) = (i\Omega_R)^2\,C_2(t)$$

$$\varnothing\lambda M \quad C_k(t) = U_k\,e^{i\lambda_k t} \Rightarrow \lambda_k^2 = \Omega_R^2$$

$$\lambda_k = \pm\,\Omega_R$$

$$\boxed{\Omega_R}\boxed{\text{M}}\boxed{\Lambda 2} \Rightarrow$$

$$A\,i\Omega_R\,e^{i\Omega_R t} + B\,(-i\Omega_R)\,e^{-i\Omega_R t} =$$
$$i\Omega_R\left(\Gamma\,e^{i\Omega_R t} + \Delta\,e^{-i\Omega_R t}\right) \xRightarrow{t=0}$$

$$A - B = \Gamma + \Delta \quad \boxed{3}$$

$$\Gamma\,i\Omega_R\,e^{i\Omega_R t} + \Delta\,(-i\Omega_R)\,e^{-i\Omega_R t} =$$
$$i\Omega_R\left(A\,e^{i\Omega_R t} + B\,e^{-i\Omega_R t}\right) \xRightarrow{t=0}$$

$$\Gamma - \Delta = A + B \quad \boxed{4}$$

$$C_1(t) = A\,e^{i\Omega_R t} + B\,e^{-i\Omega_R t} \quad \boxed{\text{M}}$$

$$C_2(t) = \Gamma\,e^{i\Omega_R t} + \Delta\,e^{-i\Omega_R t} \quad \boxed{\Lambda 2}$$

$$\text{Α.Σ.}\quad C_1(0) = 1,\quad C_2(0) = 0$$

$$1 = A + B \quad \boxed{1}$$

$$0 = \Gamma + \Delta \Rightarrow \Delta = -\Gamma \quad \boxed{2}$$

$$\boxed{1}\boxed{2}\boxed{3}\boxed{4} \qquad \Delta = -\Gamma \quad A = B \quad 2\Gamma = 2A \quad \Gamma = A \quad 1 = A + A = 2A$$

$$A = B = \Gamma = \frac{1}{2} = -\Delta \quad \boxed{5}$$

$$C_1(t) = \frac{e^{i\Omega_R t} + e^{-i\Omega_R t}}{2} \Rightarrow C_1(t) = \cos(\Omega_R t) \Rightarrow P_1(t) = \cos^2(\Omega_R t) = \frac{1}{2} + \frac{1}{2}\cos(2\Omega_R t)$$

$$C_2(t) = \frac{e^{i\Omega_R t} - e^{-i\Omega_R t}}{2} \Rightarrow C_2(t) = i\sin(\Omega_R t) \Rightarrow P_2(t) = \sin^2(\Omega_R t) = \frac{1}{2} - \frac{1}{2}\cos(2\Omega_R t)$$

$$\frac{1}{f} = T = \frac{2\Pi}{2\Omega_R} = \frac{\Pi}{\Omega_R} \qquad A = 1$$

$$\dot{C_1}(t) = i\Omega_R C_2(t)$$

$$\dot{C_2}(t) = i\Omega_R C_1(t)$$

$$\begin{bmatrix} \dot{C_1}(t) \\ \dot{C_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & i\Omega_R \\ i\Omega_R & 0 \end{bmatrix} \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix}$$

$$\underbrace{}_{\vec{X}(t)} \qquad \underbrace{}_{iA} \qquad \underbrace{}_{\vec{X}(t)}$$

$$\boxed{\partial \lambda M} \quad \vec{X}(t) = \vec{U} e^{i\lambda t}$$

$$\dot{\vec{X}}(t) = \vec{U} i\lambda e^{i\lambda t}$$

$$\dot{\vec{X}}(t) = iA\vec{X}(t) \Rightarrow \vec{U}i\lambda e^{i\lambda t} = iA\vec{U}e^{i\lambda t} \Rightarrow A\vec{U} = \lambda\vec{U}$$

$$\boxed{(A - \lambda I)\vec{U} = \vec{0}}$$

$$\det(A - \lambda I) = \vec{0} \Rightarrow \boxed{\begin{vmatrix} -\lambda & \Omega_R \\ \Omega_R & -\lambda \end{vmatrix} = 0} \Rightarrow \lambda^2 = \Omega_R^2 \Rightarrow \boxed{\lambda = \pm\Omega_R}$$

$$\boxed{ⓐ \; \lambda_1 = -\Omega_R} \quad \begin{bmatrix} 0 & \Omega_R \\ \Omega_R & 0 \end{bmatrix} \begin{bmatrix} U_{11} \\ U_{21} \end{bmatrix} = -\Omega_R \begin{bmatrix} U_{11} \\ U_{21} \end{bmatrix} \Rightarrow \left. \begin{array}{l} \Omega_R U_{21} = -\Omega_R U_{11} \\ \Omega_R U_{11} = -\Omega_R U_{21} \end{array} \right\} \Rightarrow$$

$$U_{21} = -U_{11}$$

$$\vec{U_1} = \begin{bmatrix} C \\ -C \end{bmatrix} \quad |\vec{U_1}|^2 = 1 \Rightarrow 2|C|^2 = 1 \Rightarrow |C|^2 = \frac{1}{2}$$

$$\pi.\chi. \; C = \frac{1}{\sqrt{2}}$$

$$\boxed{\vec{U_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$\boxed{ⓑ \; \lambda_2 = \Omega_R} \quad \begin{bmatrix} 0 & \Omega_R \\ \Omega_R & 0 \end{bmatrix} \begin{bmatrix} U_{12} \\ U_{22} \end{bmatrix} = \Omega_R \begin{bmatrix} U_{12} \\ U_{22} \end{bmatrix} \Rightarrow \left. \begin{array}{l} \Omega_R U_{22} = \Omega_R U_{12} \\ \Omega_R U_{12} = \Omega_R U_{22} \end{array} \right\} \Rightarrow U_{12} = U_{22}$$

$$\vec{U_2} = \begin{bmatrix} C \\ C \end{bmatrix} \quad |\vec{U_2}|^2 = 1 \Rightarrow 2|C|^2 = 1 \Rightarrow |C|^2 = \frac{1}{2} \Rightarrow \pi.\chi. \; C = \frac{1}{\sqrt{2}}$$

$$\boxed{\vec{U_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

Γενική λύση

$$\boxed{\vec{X}(t) = \sigma_1 \vec{U_1} e^{i\lambda_1 t} + \sigma_2 \vec{U_2} e^{i\lambda_2 t}}$$

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \sigma_1 \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-i\Omega_R t} + \sigma_2 \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{i\Omega_R t}$$

Α.Σ. $C_1(0) = 1$ $C_2(0) = 0$

$$1 = \frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}}$$

$$0 = -\frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}} \Rightarrow \boxed{\sigma_1 = \sigma_2 := \sigma}$$

$$\left. \begin{array}{c} \\ \\ \end{array} \right\} \quad 1 = 2\frac{\sigma}{\sqrt{2}} \Rightarrow \boxed{\sigma = \frac{\sqrt{2}}{2}}$$

$$C_1(t) = \frac{1}{2} e^{-i\Omega_R t} + \frac{1}{2} e^{i\Omega_R t} \Rightarrow C_1(t) = \frac{e^{i\Omega_R t} + e^{-i\Omega_R t}}{2} \Rightarrow$$

$$C_2(t) = -\frac{1}{2} e^{-i\Omega_R t} + \frac{1}{2} e^{i\Omega_R t} \Rightarrow C_2(t) = \frac{e^{i\Omega_R t} - e^{-i\Omega_R t}}{2} \Rightarrow$$

$$\boxed{C_1(t) = \cos(\Omega_R t)} \Rightarrow P_1(t) = \cos^2(\Omega_R t) = \frac{1}{2} + \frac{1}{2}\cos(2\Omega_R t)$$

$$\boxed{C_2(t) = i\sin(\Omega_R t)} \Rightarrow P_2(t) = \sin^2(\Omega_R t) = \frac{1}{2} - \frac{1}{2}\cos(2\Omega_R t)$$

$$\frac{1}{f} = T = \frac{2\pi}{2\Omega_R} = \frac{\pi}{\Omega_R} \qquad A = 1$$