$$\text{Ⓐ} \begin{cases} \dot{C}_1(t) = \dfrac{i\beta E_0}{2\hbar} e^{i\Delta t} C_2(t) \\ \dot{C}_2(t) = \dfrac{i\beta E_0}{2\hbar} e^{-i\Delta t} C_1(t) \end{cases}$$

$$\boxed{\Delta\Sigma \ \mu\varepsilon\tau\alpha \ \tau\eta\nu \ RWA}$$

$$\Omega_R := \frac{\beta E_0}{\hbar}$$

$$\Delta : \omega - \Omega$$

$$\text{Ⓐ'} \begin{cases} \dot{C}_1(t) = \dfrac{i\Omega_R}{2} e^{i\Delta t} C_2(t) \\ \dot{C}_2(t) = \dfrac{i\Omega_R}{2} e^{-i\Delta t} C_1(t) \end{cases}$$

$$\ddot{C}_1(t) = \frac{i\Omega_R}{2} i\Delta e^{i\Delta t} C_2(t) + \frac{i\Omega_R}{2} e^{i\Delta t} \dot{C}_2(t)$$

$$= \underbrace{\frac{i\Omega_R}{2} i\Delta e^{i\Delta t} \frac{\dot{C}_1(t)}{\frac{i\Omega_R}{2} e^{i\Delta t}}} + \frac{i\Omega_R}{2} e^{i\Delta t} \frac{i\Omega_R}{2} e^{-i\Delta t} C_1(t) \Rightarrow$$

$$\ddot{C}_1(t) = i\Delta \dot{C}_1(t) - \frac{\Omega_R^2}{4} C_1(t) \Rightarrow \boxed{\ddot{C}_1(t) - i\Delta \dot{C}_1(t) + \frac{\Omega_R^2}{4} C_1(t) = 0} \quad ①$$

$$\ddot{C}_2(t) = \frac{i\Omega_R}{2} (-i\Delta) e^{-i\Delta t} C_1(t) + \frac{i\Omega_R}{2} e^{-i\Delta t} \dot{C}_1(t)$$

$$= \underbrace{\frac{i\Omega_R}{2} (-i\Delta) e^{-i\Delta t} \frac{\dot{C}_2(t)}{\frac{i\Omega_R}{2} e^{-i\Delta t}}} + \frac{i\Omega_R}{2} e^{-i\Delta t} \frac{i\Omega_R}{2} e^{i\Delta t} C_2(t) \Rightarrow$$

$$\ddot{C}_2(t) = (-i\Delta) \dot{C}_2(t) - \frac{\Omega_R^2}{4} C_2(t) \Rightarrow \boxed{\ddot{C}_2(t) + i\Delta \dot{C}_2(t) + \frac{\Omega_R^2}{4} C_2(t) = 0} \quad ②$$

$$\Delta\Lambda M \quad C_k(t) = U_k e^{i\mu_k t} \qquad \Delta\Lambda M = \Delta o\kappa\iota\mu\alpha\zeta\omega \ \Lambda\upsilon\sigma\varepsilon\iota\varsigma \ \tau\eta\varsigma \ Mo\rho\phi\eta\varsigma$$

$$\dot{C}_k(t) = U_k \, i\mu_k \, e^{i\mu_k t}$$

$$\ddot{C}_k(t) = U_k (i\mu_k)^2 e^{i\mu_k t}$$

$$① \Rightarrow U_1 (i\mu_1)^2 e^{i\mu_1 t} - i\Delta U_1 i\mu_1 e^{i\mu_1 t} + \frac{\Omega_R^2}{4} U_1 e^{i\mu_1 t} = 0 \Rightarrow$$

$$\boxed{-\mu_1^2 + \Delta\mu_1 + \frac{\Omega_R^2}{4} = 0} \quad \text{(XE1)} \qquad \nexists \ \sigma\upsilon\nu\theta\eta\kappa\eta \ \gamma\iota\alpha \ \tau o \ U_1$$

$$② \Rightarrow U_2 (i\mu_2)^2 e^{i\mu_2 t} + i\Delta U_2 i\mu_2 e^{i\mu_2 t} + \frac{\Omega_R^2}{4} U_2 e^{i\mu_2 t} = 0 \Rightarrow$$

$$\boxed{-\mu_2^2 - \Delta\mu_2 + \frac{\Omega_R^2}{4} = 0} \quad \text{(XE2)} \qquad \nexists \ \sigma\upsilon\nu\theta\eta\kappa\eta \ \gamma\iota\alpha \ \tau o \ U_2$$

$$\Delta_1 = \Delta^2 - 4(-1)\frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2 \qquad\qquad \lambda_{2,1} := \pm\frac{\sqrt{\Delta^2 + \Omega_R^2}}{2} = \pm\lambda \quad \textcircled{2}$$

$$\mu_1 = \frac{-\Delta \pm \sqrt{\Delta^2 + \Omega_R^2}}{-2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2 + \Omega_R^2}}{2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \lambda$$

$$\Delta_2 = (-\Delta)^2 - 4(-1)\frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2$$

$$\mu_2 = \frac{+\Delta \pm \sqrt{\Delta^2 + \Omega_R^2}}{-2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2 + \Omega_R^2}}{2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \lambda$$

"Άρα, οί λύσεις θα είναι

$$\begin{cases} C_1(t) = a\, e^{i\frac{\Delta}{2}t}\, e^{i\lambda t} + \beta\, e^{i\frac{\Delta}{2}t}\, e^{-i\lambda t} \\ C_2(t) = \gamma\, e^{-i\frac{\Delta}{2}t}\, e^{i\lambda t} + \delta\, e^{-i\frac{\Delta}{2}t}\, e^{i\lambda t} \end{cases}$$

$$\begin{cases} C_1(t) = e^{i\frac{\Delta}{2}t}\left[a\, e^{i\lambda t} + \beta\, e^{-i\lambda t}\right] \\ C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\gamma\, e^{i\lambda t} + \delta\, e^{-i\lambda t}\right] \end{cases} \xrightarrow{\Delta = 0} \begin{cases} C_1(t) = a\, e^{i\lambda t} + \beta\, e^{-i\lambda t} \\ C_2(t) = \gamma\, e^{i\lambda t} + \delta\, e^{i\lambda t} \end{cases}$$

Τίς δοκιμάζουμε τώρα στις Ⓐ' γία $\Delta = 0$, οπότε $\lambda = \frac{\Omega_R}{2}$

$$a\, i\lambda\, e^{i\lambda t} + \beta(-i\lambda)\, e^{-i\lambda t} = \frac{i\Omega_R}{2}\left(\gamma\, e^{i\lambda t} + \delta\, e^{-i\lambda t}\right) \Rightarrow \gamma = a \quad \beta = -\delta$$

$$\gamma\, i\lambda\, e^{i\lambda t} + \delta(-i\lambda)\, e^{-i\lambda t} = \frac{i\Omega_R}{2}\left(a\, e^{i\lambda t} + \delta\, e^{-i\lambda t}\right) \Rightarrow \gamma = a \quad \beta = -\delta$$

$$\begin{cases} C_1(t) = a\, e^{i\frac{\Omega_R}{2}t} + \beta\, e^{-i\frac{\Omega_R}{2}t} \\ C_2(t) = a\, e^{i\frac{\Omega_R}{2}t} - \beta\, e^{-i\frac{\Omega_R}{2}t} \end{cases}$$

"Εδωσαν οί αρχικές συνθήκες (Α.Σ.) $C_1(0) = 1$, $C_2(0) = 0$

$$\left.\begin{array}{l} 1 = a + \beta \\ 0 = a - \beta \Rightarrow \beta = a \end{array}\right\} \quad 2a = 1 \Rightarrow a = \frac{1}{2}$$

$$C_1(t) = \frac{1}{2}\, e^{i\frac{\Omega_R}{2}t} + \frac{1}{2}\, e^{-i\frac{\Omega_R}{2}t} \Rightarrow \boxed{C_1(t) = \cos\left(\frac{\Omega_R}{2}t\right)}$$

$$C_2(t) = \frac{1}{2}\, e^{i\frac{\Omega_R}{2}t} - \frac{1}{2}\, e^{-i\frac{\Omega_R}{2}t} \Rightarrow \boxed{C_2(t) = i\sin\left(\frac{\Omega_R}{2}t\right)}$$

$i\dfrac{\Delta}{2} e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] + e^{i\frac{\Delta}{2}t}\left[\alpha i\lambda e^{i\lambda t} + \beta(-i\lambda)e^{-i\lambda t}\right] =$

$i\dfrac{\Omega_R}{2} e^{i\Delta t} e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] \Rightarrow$

$\dfrac{\Delta}{2}\alpha + \alpha\lambda = \dfrac{\Omega_R}{2}\gamma$ και $\dfrac{\Delta}{2}\beta - \beta\lambda = \dfrac{\Omega_R}{2}\delta$

$\Rightarrow \dfrac{\Delta\alpha + 2\alpha\lambda}{2} = \dfrac{\Omega_R}{2}\gamma$ και $\dfrac{\Delta\beta - 2\beta\lambda}{2} = \dfrac{\Omega_R}{2}\delta$

$\boxed{\gamma = \dfrac{\Delta + 2\lambda}{\Omega_R}\alpha}$ και $\boxed{\delta = \dfrac{\Delta - 2\lambda}{\Omega_R}\beta}$

$-i\dfrac{\Delta}{2} e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] + e^{-i\frac{\Delta}{2}t}\left[\gamma(i\lambda)e^{i\lambda t} + \delta(-i\lambda)e^{-i\lambda t}\right] =$

$i\dfrac{\Omega_R}{2} e^{-i\Delta t} e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \Rightarrow$

$-\dfrac{\Delta}{2}\gamma + \gamma\lambda = \dfrac{\Omega_R}{2}\alpha$ και $-\dfrac{\Delta}{2}\delta - \delta\lambda = \dfrac{\Omega_R}{2}\beta$

$\dfrac{-\Delta\gamma + 2\gamma\lambda}{2} = \dfrac{\Omega_R}{2}\alpha$ και $\dfrac{-\Delta\delta - 2\delta\lambda}{2} = \dfrac{\Omega_R}{2}\beta$

$\boxed{\alpha = \dfrac{-\Delta + 2\lambda}{\Omega_R}\gamma}$ και $\boxed{\beta = \dfrac{-\Delta - 2\lambda}{\Omega_R}\delta}$

$\gamma = \dfrac{(2\lambda + \Delta)}{\Omega_R} \cdot \dfrac{(2\lambda - \Delta)}{\Omega_R}\gamma \Rightarrow \dfrac{4\lambda^2 - \Delta^2}{\Omega_R^2} = 1$ τό οποίον ισχύει

$\delta = \dfrac{(\Delta - 2\lambda)}{\Omega_R} \cdot \dfrac{-(\Delta + 2\lambda)}{\Omega_R}\delta \Rightarrow -\dfrac{\Delta^2 - 4\lambda^2}{\Omega_R^2} = 1$ τό οποίον ισχύει

$\begin{cases} C_1(t) = e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \\[2mm] C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\alpha\dfrac{\Delta + 2\lambda}{\Omega_R}e^{i\lambda t} + \dfrac{\Delta - 2\lambda}{\Omega_R}\beta e^{-i\lambda t}\right] \end{cases}$

Έστωσαν οί άρχικές συνθήκες (Α·Σ) $C_1(0)=1$, $C_2(0)=0$

$1 = a + \beta$

$0 = a \dfrac{\Delta + 2\lambda}{\Omega_R} + \beta \dfrac{\Delta - 2\lambda}{\Omega_R} \Rightarrow a(\Delta + 2\lambda) = \beta(2\lambda - \Delta) \Rightarrow \Big\} \Rightarrow$

$\beta = \dfrac{2\lambda + \Delta}{2\lambda - \Delta} a$

$1 = a + \dfrac{2\lambda + \Delta}{2\lambda - \Delta} a = a \dfrac{2\lambda - \Delta + 2\lambda + \Delta}{2\lambda - \Delta} \Rightarrow (2\lambda - \Delta) = a \, 4\lambda$

$\Rightarrow \boxed{a = \dfrac{2\lambda - \Delta}{4\lambda}}$ $\beta = \dfrac{2\lambda + \Delta}{2\lambda - \Delta} \dfrac{2\lambda - \Delta}{4\lambda} \Rightarrow \boxed{\beta = \dfrac{2\lambda + \Delta}{4\lambda}}$

$C_1(t) = e^{i\frac{\Delta}{2}t} \left[\dfrac{2\lambda - \Delta}{4\lambda} e^{i\lambda t} + \dfrac{2\lambda + \Delta}{4\lambda} e^{-i\lambda t} \right]$

$C_2(t) = e^{-i\frac{\Delta}{2}t} \left[\dfrac{\Delta + 2\lambda}{\Omega_R} \dfrac{2\lambda - \Delta}{4\lambda} e^{i\lambda t} + \dfrac{\Delta - 2\lambda}{\Omega_R} \dfrac{2\lambda + \Delta}{4\lambda} e^{-i\lambda t} \right]$

$\dfrac{4\lambda^2 - \Delta^2}{4\Omega_R \lambda} = \dfrac{\Delta^2 + \Omega_R^2 - \Delta^2}{4\Omega_R \lambda} = \dfrac{\Omega_R}{4\lambda}$

$\dfrac{\Delta^2 - 4\lambda^2}{4\Omega_R \lambda} = \dfrac{\Delta^2 - \Delta^2 - \Omega_R^2}{4\Omega_R \lambda} = \dfrac{-\Omega_R}{4\lambda}$

$\cos + i\sin$
$-\cos + i\sin$

$C_2(t) = e^{-i\frac{\Delta}{2}t} \left[\dfrac{\Omega_R}{4\lambda} e^{i\lambda t} - \dfrac{\Omega_R}{4\lambda} e^{-i\lambda t} \right] \Rightarrow C_2(t) = e^{-i\frac{\Delta}{2}t} \dfrac{\Omega_R}{4\lambda} 2i\sin(\lambda t)$

Άρα $P_2(t) = |C_2(t)|^2 = \dfrac{\Omega_R^2}{\Delta^2 + \Omega_R^2} \sin^2(\lambda t)$

$P_1(t) = |C_1(t)|^2 = 1 - \dfrac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2(\lambda t)$