$$\hat{H}_{\Delta\Sigma} = \begin{pmatrix} E_2 & 0 \\ 0 & E_1 \end{pmatrix} = E_2 \hat{S}_+ \hat{S}_- + E_1 \hat{S}_- \hat{S}_+$$
$$= E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$
$$= E_2 \hat{a}_{12}^\dagger \hat{a}_{12} + E_1 \hat{a}_{11}^\dagger \hat{a}_{11}$$
$$:= E_2 \hat{c}_2^\dagger \hat{c}_2 + E_1 \hat{c}_1^\dagger \hat{c}_1$$

$$|x\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad \langle x| = (\alpha^* \ \beta^*)$$

$$|\ \rangle = |0\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \langle\ | = \langle 0| = (0\ 0)$$
$$|\downarrow\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \langle\downarrow| = \langle 1| = (0\ 1)$$
$$|\uparrow\rangle = |2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \langle\uparrow| = \langle 2| = (1\ 0)$$

$$\hat{a}_{\mu\nu} = |\mu\rangle\langle\nu|$$
$$\hat{a}_{\mu\nu}^\dagger = |\nu\rangle\langle\mu|$$

Στοιχειώδεις Διεγέρσεις από τη Θεμελιώδη Κατάσταση

elementary excitations from the ground state

$$\hat{c}_i = \hat{a}_{1i} = |1\rangle\langle i|$$
$$\hat{c}_i^\dagger = \hat{a}_{1i}^\dagger = |i\rangle\langle 1|$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix}(0\ 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\hat{a}_{11}^\dagger = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix}(0\ 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}(1\ 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\hat{a}_{22}^\dagger = |2\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}(1\ 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\hat{a}_{12} = |1\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \end{pmatrix}(1\ 0) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \hat{S}_- = \hat{a}_{21}^\dagger$$
$$\hat{a}_{12}^\dagger = |2\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}(0\ 1) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \hat{S}_+ = \hat{a}_{21}$$

Πού ∃ 1 στον πίνακα;

$$\begin{bmatrix} \hat{a}_{22} = \hat{a}_{22}^\dagger & \hat{a}_{21} = \hat{a}_{12}^\dagger \\ \hat{a}_{12} = \hat{a}_{21}^\dagger & \hat{a}_{11} = \hat{a}_{11}^\dagger \end{bmatrix}$$

$$\hat{H}_{T\Sigma} = \begin{pmatrix} E_3 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{pmatrix} = E_3 |3\rangle\langle 3| + E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$
$$= E_3 \hat{a}^{+}_{13} \hat{a}_{13} + E_2 \hat{a}^{+}_{12} \hat{a}_{12} + E_1 \hat{a}^{+}_{11} \hat{a}_{11}$$
$$:= E_3 \hat{c}^{+}_3 \hat{c}_3 + E_2 \hat{c}^{+}_2 \hat{c}_2 + E_1 \hat{c}^{+}_1 \hat{c}_1$$

$$|x\rangle = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \qquad \langle x| = (\alpha^* \ \beta^* \ \gamma^*)$$

$$|\ \rangle = |0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \langle\ | = \langle 0| = (0\ 0\ 0)$$

$$|1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \langle 1| = (0\ 0\ 1)$$

$$|2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \langle 2| = (0\ 1\ 0)$$

$$|3\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \langle 3| = (1\ 0\ 0)$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (0\ 0\ 1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \hat{a}^{+}_{11}$$

$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (0\ 1\ 0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}^{+}_{22}$$

$$\hat{a}_{33} = |3\rangle\langle 3| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (1\ 0\ 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}^{+}_{33}$$

$$\hat{a}_{12} = |1\rangle\langle 2| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (0\ 1\ 0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \hat{a}^{+}_{21}$$

$$\hat{a}^{+}_{12} = |2\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (0\ 0\ 1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{21}$$

$$\hat{a}_{13} = |1\rangle\langle 3| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (1\ 0\ 0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \hat{a}^{+}_{31}$$

$$\hat{a}^{+}_{13} = |3\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (0\ 0\ 1) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{31}$$

$$\hat{a}_{23} = |2\rangle\langle 3| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}^{\dagger}_{32}$$

$$\hat{a}^{\dagger}_{23} = |3\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{32}$$

πού $\exists$ 1 στον πίνακα;

$$\begin{bmatrix} \hat{a}_{33} = \hat{a}^{\dagger}_{33} & \hat{a}_{32} = \hat{a}^{\dagger}_{23} & \hat{a}_{31} = \hat{a}^{\dagger}_{13} \\ \hat{a}_{23} = \hat{a}^{\dagger}_{32} & \hat{a}_{22} = \hat{a}^{\dagger}_{22} & \hat{a}_{21} = \hat{a}^{\dagger}_{12} \\ \hat{a}_{13} = \hat{a}^{\dagger}_{31} & \hat{a}_{12} = \hat{a}^{\dagger}_{21} & \hat{a}_{11} = \hat{a}^{\dagger}_{11} \end{bmatrix}$$

$$\hat{H}_{\Delta\Sigma} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} = E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$
$$= E_2 \hat{a}_{12}^{+} \hat{a}_{12} + E_1 \hat{a}_{11}^{+} \hat{a}_{11}$$
$$= E_2 \hat{c}_2^{+} \hat{c}_2 + E_1 \hat{c}_1^{+} \hat{c}_1$$

όπου εδώ τώρα
...

$$\hat{a}_{\mu\nu} = |\mu\rangle\langle\nu|$$
$$\hat{a}_{\mu\nu}^{+} = |\nu\rangle\langle\mu|$$

Στοιχειώδεις Διεγέρσεις
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$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \;\; 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \hat{a}_{11}^{+}$$

$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0 \;\; 1) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \hat{a}_{22}^{+}$$

$$\hat{a}_{12} = |1\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (0 \;\; 1) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \hat{a}_{21}^{+}$$

$$\hat{a}_{21} = |2\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \end{pmatrix} (1 \;\; 0) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \hat{a}_{12}^{+}$$

ΕΝΑΛΛΑΚΤΙΚΗ ΠΕΡΙΓΡΑΦΗ

$$\hat{c}_i = \hat{a}_{1i} = |1\rangle\langle i|$$
$$\hat{c}_i^{+} = \hat{a}_{1i}^{+} = |i\rangle\langle 1|$$

πού ∃ 1 στον πίνακα;

$$\begin{bmatrix} \hat{a}_{11} = \hat{a}_{11}^{+} & \hat{a}_{12} = \hat{a}_{21}^{+} \\ \hat{a}_{21} = \hat{a}_{12}^{+} & \hat{a}_{22} = \hat{a}_{22}^{+} \end{bmatrix}$$

$$\hat{H}_{T\Sigma} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} = E_3 |3\rangle\langle 3| + E_2 |2\rangle\langle 2| + E_1 |1\rangle\langle 1|$$
$$= E_3 \hat{a}_{13}^{+} \hat{a}_{13} + E_2 \hat{a}_{12}^{+} \hat{a}_{12} + E_1 \hat{a}_{11}^{+} \hat{a}_{11}$$
$$= E_3 \hat{c}_3^{+} \hat{c}_3 + E_2 \hat{c}_2^{+} c_2 + E_1 \hat{c}_1^{+} \hat{c}_1$$

όπου εδώ τώρα

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad |2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad |3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\hat{a}_{11} = |1\rangle\langle 1| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (1 \;\; 0 \;\; 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{11}^{+}$$

$$\hat{a}_{22} = |2\rangle\langle 2| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (0 \;\; 1 \;\; 0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{22}^{+}$$

$$\hat{a}_{33} = |3\rangle\langle 3| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \hat{a}_{33}^{+}$$

$$\hat{a}_{12} = |1\rangle\langle 2| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{21}^{+}$$

$$\hat{a}_{12}^{+} = |2\rangle\langle 1| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{21}$$

$$\hat{a}_{13} = |1\rangle\langle 3| = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{31}^{+}$$

$$\hat{a}_{13}^{+} = |3\rangle\langle 1| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \hat{a}_{31}$$

$$\hat{a}_{23} = |2\rangle\langle 3| = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \hat{a}_{32}^{+}$$

$$\hat{a}_{23}^{+} = |3\rangle\langle 2| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \hat{a}_{32}$$

πού ∃ 1 στον πίνακα;

$$\begin{bmatrix} \hat{a}_{11} = \hat{a}_{11}^{+} & \hat{a}_{12} = \hat{a}_{21}^{+} & \hat{a}_{13} = \hat{a}_{31}^{+} \\ \hat{a}_{21} = \hat{a}_{12}^{+} & \hat{a}_{22} = \hat{a}_{22}^{+} & \hat{a}_{23} = \hat{a}_{32}^{+} \\ \hat{a}_{31} = \hat{a}_{13}^{+} & \hat{a}_{32} = \hat{a}_{23}^{+} & \hat{a}_{33} = \hat{a}_{33}^{+} \end{bmatrix}$$