

$\Delta\Sigma \quad \Delta = 0$ άρχικες συνθήκες $C_1(0) = 1, C_2(0) = 0$ $\Omega_R = \omega$

απουσία αλληλεπίδρασης

$$P_1(t) = |C_1(t)|^2 = \cos^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} + \frac{1}{2} \cos(\Omega_R t)$$

$$\Delta := \omega - \Omega$$

$$P_2(t) = |C_2(t)|^2 = \sin^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} - \frac{1}{2} \cos(\Omega_R t)$$

$$\Omega_R := \frac{\beta E_0}{\hbar} \quad (\beta > 0)$$

συχνότητα Rabi
ορίζεται θετική

$$\Omega_R := \frac{-\beta E_0}{\hbar} \quad (\beta < 0)$$

Ω_R : εκφράζει την ταχύτητα διαταραχής

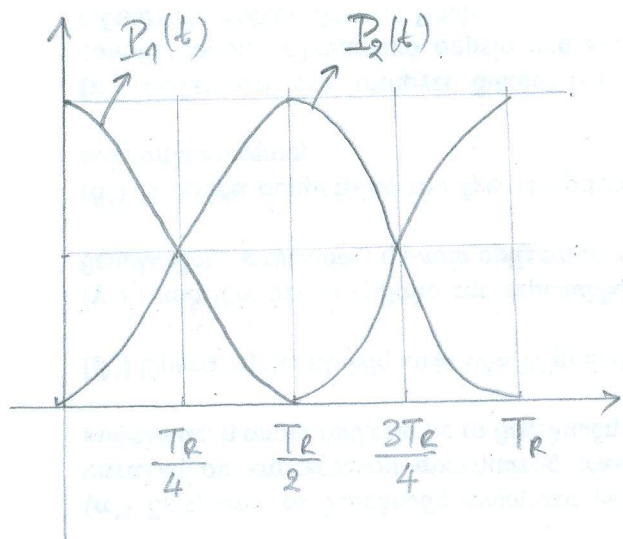
Δ : εκφράζει την απόσταση των ω (ΗΜ πεδίο) η Ω ($\Delta\Sigma$)

περίοδος (period)

$$T_R = \frac{2\pi}{\Omega_R}$$

$$A_R = 1$$

μέγιστο ποσοστό μεταβίβασης
(maximum transfer percentage)



$$\langle P_1(t) \rangle = \langle |C_1(t)|^2 \rangle = \frac{1}{2}$$

μέση πιθανότητα παρουσία στη στάση 1

$$\langle P_2(t) \rangle = \langle |C_2(t)|^2 \rangle = \frac{1}{2}$$

μέση πιθανότητα παρουσία στη στάση 2

μέγιστος ρυθμός μεταβίβασης
(maximum transfer rate)

$$\frac{A_R}{T_R} = \frac{1}{\frac{2\pi}{\Omega_R}} = \frac{\Omega_R}{2\pi}$$

$t_{2\text{mean}}$:= ο χρόνος, ο οποίος απαιτείται ώστε η $P_2(t)$ να πάρει τη φάση της $\langle P_2(t) \rangle$

$$\Rightarrow \frac{1}{2} - \frac{1}{2} \cos(\Omega_R t_{2\text{mean}}) = \frac{1}{2} \Rightarrow \cos(\Omega_R t_{2\text{mean}}) = 0 \Rightarrow$$

$$\Omega_R t_{2\text{mean}} = \frac{\pi}{2} \Rightarrow t_{2\text{mean}} = \frac{\pi}{2\Omega_R}$$

μέσος ρυθμός μεταβίβασης
(mean transfer rate)

$$k := \frac{\langle |C_2(t)|^2 \rangle}{t_{2\text{mean}}} = \frac{\frac{1}{2}}{\frac{\pi}{2\Omega_R}} = \frac{\Omega_R}{\pi} \Rightarrow k = 2 \frac{A_R}{T_R}$$

Είχαμε βρή, για $\Delta = 0$

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} \frac{\sigma_1}{\sqrt{2}} e^{i\frac{\Omega t}{2}} + \frac{\sigma_2}{\sqrt{2}} e^{-i\frac{\Omega t}{2}} \\ \frac{\sigma_1}{\sqrt{2}} e^{i\frac{\Omega t}{2}} - \frac{\sigma_2}{\sqrt{2}} e^{-i\frac{\Omega t}{2}} \end{bmatrix}$$



ας βάλουμε αρχικές συνθήκες

$$C_1(0) = \frac{1}{\sqrt{2}} e^{i\theta} \quad C_2(0) = \frac{1}{\sqrt{2}} e^{i\varphi} \Rightarrow$$

$$|C_1(0)|^2 = \frac{1}{2} = |C_2(0)|^2$$

$$\begin{cases} \frac{1}{\sqrt{2}} e^{i\theta} = \frac{\sigma_1}{\sqrt{2}} + \frac{\sigma_2}{\sqrt{2}} \Rightarrow \sigma_1 + \sigma_2 = e^{i\theta} \\ \frac{1}{\sqrt{2}} e^{i\varphi} = \frac{\sigma_1}{\sqrt{2}} - \frac{\sigma_2}{\sqrt{2}} \Rightarrow \sigma_1 - \sigma_2 = e^{i\varphi} \end{cases} \Rightarrow \begin{cases} \sigma_1 = \frac{e^{i\theta} + e^{i\varphi}}{2} \\ \sigma_2 = \frac{e^{i\theta} - e^{i\varphi}}{2} \end{cases}$$

$$\begin{aligned} \oplus 2\sigma_1 &= e^{i\theta} + e^{i\varphi} \\ \ominus 2\sigma_2 &= e^{i\theta} - e^{i\varphi} \end{aligned}$$

$$\begin{cases} C_1(t) = \frac{e^{i\theta} + e^{i\varphi}}{2\sqrt{2}} e^{i\frac{\Omega t}{2}} + \frac{e^{i\theta} - e^{i\varphi}}{2\sqrt{2}} e^{-i\frac{\Omega t}{2}} \\ C_2(t) = \frac{e^{i\theta} + e^{i\varphi}}{2\sqrt{2}} e^{i\frac{\Omega t}{2}} - \frac{e^{i\theta} - e^{i\varphi}}{2\sqrt{2}} e^{-i\frac{\Omega t}{2}} \end{cases} \Rightarrow$$

$$\begin{cases} 2\sqrt{2} C_1(t) = e^{i\theta} e^{i\frac{\Omega t}{2}} + e^{i\varphi} e^{i\frac{\Omega t}{2}} + e^{i\theta} e^{-i\frac{\Omega t}{2}} - e^{i\varphi} e^{-i\frac{\Omega t}{2}} \\ 2\sqrt{2} C_2(t) = e^{i\theta} e^{i\frac{\Omega t}{2}} + e^{i\varphi} e^{i\frac{\Omega t}{2}} - e^{i\theta} e^{-i\frac{\Omega t}{2}} + e^{i\varphi} e^{-i\frac{\Omega t}{2}} \end{cases} \Rightarrow$$

$$\begin{cases} 2\sqrt{2} C_1(t) = e^{i\theta} 2 \cos\left(\frac{\Omega t}{2}\right) + e^{i\varphi} 2i \sin\left(\frac{\Omega t}{2}\right) \\ 2\sqrt{2} C_2(t) = e^{i\theta} 2i \sin\left(\frac{\Omega t}{2}\right) + e^{i\varphi} 2 \cos\left(\frac{\Omega t}{2}\right) \end{cases} \Rightarrow$$

$$8 |C_1(t)|^2 = 4 \cos^2\left(\frac{\Omega t}{2}\right) + 4 \sin^2\left(\frac{\Omega t}{2}\right) + e^{i\theta} 2 \cos\left(\frac{\Omega t}{2}\right) e^{-i\varphi} 2(-i) \sin\left(\frac{\Omega t}{2}\right) + e^{i\varphi} 2i \sin\left(\frac{\Omega t}{2}\right) 2 e^{-i\theta} \cos\left(\frac{\Omega t}{2}\right) \Rightarrow$$

$$2 |C_1(t)|^2 = \cos^2\left(\frac{\Omega t}{2}\right) + \sin^2\left(\frac{\Omega t}{2}\right) - i e^{i\theta} e^{-i\varphi} \cos\left(\frac{\Omega t}{2}\right) \cdot \sin\left(\frac{\Omega t}{2}\right) + i e^{i\varphi} e^{-i\theta} \cos\left(\frac{\Omega t}{2}\right) \cdot \sin\left(\frac{\Omega t}{2}\right)$$

$$\frac{1}{2} \sin(\Omega t) i \left\{ e^{i(\varphi-\theta)} - e^{-i(\varphi-\theta)} \right\} = \frac{i}{2} \sin(\Omega t) 2i \sin\psi = -\sin(\Omega t) \sin\psi$$

$\psi := \varphi - \theta$

$$\begin{aligned} &\cos\psi \quad i \sin\psi \\ &-\cos\psi + i \sin\psi \end{aligned} \qquad = \sin(\Omega t) \cdot \sin(\theta - \varphi)$$

$$|C_1(t)|^2 = \frac{1}{2} + \frac{1}{2} \sin(\Omega t) \sin(\theta - \varphi)$$

γενικώς, ∃ ταξίτητων

av $\theta = \varphi \Rightarrow |C_1(t)|^2 = \frac{1}{2}$ \forall ταξίτητων

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Av $\theta \neq \varphi$ $\frac{1}{2} + \frac{1}{2} \sin(\Omega t) \sin(\theta - \varphi) = \frac{1}{2} + \frac{1}{2} \cos(\Omega t + \frac{\pi}{2}) \Rightarrow$
 $\sin(\Omega t) \sin(\theta - \varphi) = -\sin(\Omega t) \Rightarrow$
 $\sin(\theta - \varphi) = -1 \Rightarrow \theta - \varphi = -\frac{\pi}{2} \Rightarrow \theta = \varphi - \frac{\pi}{2}$

$8 |C_2(t)|^2 = 4 \sin^2(\frac{\Omega t}{2}) + 4 \cos^2(\frac{\Omega t}{2}) + e^{i\theta} 2i \sin(\frac{\Omega t}{2}) \cdot e^{-i\varphi} 2 \cos(\frac{\Omega t}{2})$
 $e^{i\varphi} 2 \cos(\frac{\Omega t}{2}) \cdot e^{-i\theta} 2(-i) \sin(\frac{\Omega t}{2}) \Rightarrow$

$$2 |C_2(t)|^2 = 1 + \frac{1}{2} \sin(\Omega t) i \left\{ e^{i\theta} e^{-i\varphi} - e^{i\varphi} e^{-i\theta} \right\}$$

$$e^{i(\theta - \varphi)} - e^{-i(\theta - \varphi)}$$

$\psi' = \theta - \varphi$ $\psi' = -\varphi$

$$e^{i\psi'} - e^{-i\psi'}$$

$$\cos \psi' \quad i \sin \psi'$$

$$- \cos \psi' + i \sin \psi'$$

$$2 |C_2(t)|^2 = 1 + \frac{1}{2} \sin(\Omega t) i 2i \sin \psi'$$

$$|C_2(t)|^2 = \frac{1}{2} - \frac{1}{2} \sin(\Omega t) \sin(\theta - \varphi)$$

γενικώς, ∃ ταξίτητων

av $\theta = \varphi \Rightarrow |C_2(t)|^2 = \frac{1}{2}$ \forall ταξίτητων

Av $\theta \neq \varphi$ $\frac{1}{2} - \frac{1}{2} \sin(\Omega t) \sin(\theta - \varphi) = \frac{1}{2} - \frac{1}{2} \cos(\Omega t + \frac{\pi}{2}) \Rightarrow$
 $\sin(\Omega t) \sin(\theta - \varphi) = -\sin(\Omega t) \Rightarrow$
 $\sin(\theta - \varphi) = -1 \Rightarrow \theta - \varphi = -\frac{\pi}{2} \Rightarrow \theta = \varphi - \frac{\pi}{2}$

Να λύσει το πρόβλημα $\Delta=0$ με αρχικές συνθήκες $G_1(0)=0$, $G_2(0)=1$

δίν. το ήλεκτρονικό βολόμετρο αρχικώς
στην ΑΝΩ ΣΤΑΣΗ

ΛΥΣΗ

Είχαμε βρει
για $\Delta=0$

$$G_1(t) = \frac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} + \frac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t}$$

$$G_2(t) = \frac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} - \frac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t}$$

με αρχικές συνθήκες $G_1(0)=0$, $G_2(0)=1 \Rightarrow$

$$0 = \frac{C_1 + C_2}{\sqrt{2}} \Rightarrow C_2 = -C_1 := -C$$

$$1 = \frac{C_1 - C_2}{\sqrt{2}} \Rightarrow \sqrt{2} = C + C \Rightarrow 2C = \sqrt{2} \Rightarrow C = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

"Άρα $G_1(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} - \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} = i \sin\left(\frac{\Omega_R}{2}t\right)$

$$G_2(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} + \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} = \frac{1}{2} 2 \cos\left(\frac{\Omega_R}{2}t\right) = \cos\left(\frac{\Omega_R}{2}t\right)$$

Να λυθεί το πρόβλημα $\Delta=0$ και αρχική συνθήκη

$$C_1(0) = \frac{1}{\sqrt{2}} = C_2(0)$$

$$\Rightarrow |C_1(0)|^2 = \frac{1}{2} = |C_2(0)|^2$$

Συν. το ψ εκφράζονται βρίσκονται έξ' ἴσου

στις δύο στάθμες τη χρονική στιγμή 0.

ΛΥΣΗ

Είχαμε βρει
για $\Delta=0$

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} \frac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} + \frac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \\ \frac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} - \frac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \end{bmatrix}$$

με αρχική συνθήκη $C_1(0) = \frac{1}{\sqrt{2}} = C_2(0) \Rightarrow$

$$\frac{1}{\sqrt{2}} = \frac{C_1}{\sqrt{2}} + \frac{C_2}{\sqrt{2}} \Rightarrow 1 = C_1 + C_2$$

$$\frac{1}{\sqrt{2}} = \frac{C_1}{\sqrt{2}} - \frac{C_2}{\sqrt{2}} \Rightarrow 1 = C_1 - C_2$$

$$2 = 2C_1 \Rightarrow C_1 = 1$$

$$C_2 = 0$$

$$C_1(t) = \frac{1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} \Rightarrow |C_1(t)|^2 = \frac{1}{2} = \text{σταθερό}$$

$$C_2(t) = \frac{1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} \Rightarrow |C_2(t)|^2 = \frac{1}{2} = \text{σταθερό}$$

Δηλαδή δεν υπάρχει ταλάντωση φορτίου...

• ΛΥΣΗ για $\Delta \neq 0$

για $\beta < 0$

A

$$\begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \lambda \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\lambda_{2,1} = \pm \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} = \pm \lambda$$

$$\lambda > 0$$

$$\vec{U}_1 = \begin{bmatrix} 1 \\ \frac{\alpha}{\sqrt{1+\alpha^2}} \\ \frac{\alpha}{\sqrt{1+\alpha^2}} \end{bmatrix} \quad \alpha = \frac{\frac{\Delta}{2} + \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}}{\frac{\Omega_R}{2}}$$

οι πράξεις υπάρχουν στο βιβλίο

$$\vec{U}_2 = \begin{bmatrix} 1 \\ \frac{1}{\sqrt{1+\alpha'^2}} \\ \frac{\alpha'}{\sqrt{1+\alpha'^2}} \end{bmatrix} \quad \alpha' = \frac{\frac{\Delta}{2} - \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}}{\frac{\Omega_R}{2}}$$

Γενική λύση

$$\begin{aligned} \vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} &= \begin{bmatrix} C_1(t) e^{-i\frac{\Delta}{2}t} \\ C_2(t) e^{i\frac{\Delta}{2}t} \end{bmatrix} = \sum_{k=1}^2 c_k \vec{U}_k e^{-i\lambda_k t} = c_1 \vec{U}_1 e^{-i\lambda_1 t} + c_2 \vec{U}_2 e^{-i\lambda_2 t} \\ &= c_1 \begin{bmatrix} 1 \\ \frac{\alpha}{\sqrt{1+\alpha^2}} \\ \frac{\alpha}{\sqrt{1+\alpha^2}} \end{bmatrix} e^{-i\lambda_1 t} + c_2 \begin{bmatrix} 1 \\ \frac{1}{\sqrt{1+\alpha'^2}} \\ \frac{\alpha'}{\sqrt{1+\alpha'^2}} \end{bmatrix} e^{-i\lambda_2 t} \end{aligned}$$

"Εστώς"

Αρχικές συνθήκες $C_1(0)=1$ $C_2(0)=0$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{c_1}{\sqrt{1+\alpha^2}} + \frac{c_2}{\sqrt{1+\alpha'^2}} \\ \frac{c_1 \alpha}{\sqrt{1+\alpha^2}} + \frac{c_2 \alpha'}{\sqrt{1+\alpha'^2}} \end{bmatrix} \Rightarrow \dots \quad \begin{aligned} c_1 &= \frac{\alpha' \sqrt{1+\alpha'^2}}{\alpha' - \alpha} \\ c_2 &= -\frac{\alpha \sqrt{1+\alpha'^2}}{\alpha' - \alpha} \end{aligned} \quad \text{'Αρα...}$$

$$\begin{bmatrix} C_1(t) e^{-i\frac{\Delta}{2}t} \\ C_2(t) e^{i\frac{\Delta}{2}t} \end{bmatrix} = \frac{\alpha' \sqrt{1+\alpha'^2}}{\alpha' - \alpha} \begin{bmatrix} 1 \\ \frac{\alpha}{\sqrt{1+\alpha^2}} \\ \frac{\alpha}{\sqrt{1+\alpha^2}} \end{bmatrix} e^{-i\lambda_1 t} - \frac{\alpha \sqrt{1+\alpha'^2}}{\alpha' - \alpha} \begin{bmatrix} 1 \\ \frac{1}{\sqrt{1+\alpha'^2}} \\ \frac{\alpha'}{\sqrt{1+\alpha'^2}} \end{bmatrix} e^{-i\lambda_2 t}$$

$$C_1(t) e^{-i\frac{\Delta}{2}t} = \frac{a'}{a'-a} e^{-i\lambda_1 t} - \frac{a}{a'-a} e^{-i\lambda_2 t}$$

$$C_2(t) e^{i\frac{\Delta}{2}t} = \frac{aa'}{a'-a} e^{-i\lambda_1 t} - \frac{aa'}{a'-a} e^{-i\lambda_2 t}$$

$$\frac{a'}{a'-a} = \frac{\sqrt{\Omega_R^2 + \Delta^2} - \Delta}{2\sqrt{\Omega_R^2 + \Delta^2}} = \gamma_1 \quad \frac{aa'}{a'-a} = \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Delta^2}} = \gamma_3$$

$$\frac{a}{a'-a} = -\frac{\sqrt{\Omega_R^2 + \Delta^2} + \Delta}{2\sqrt{\Omega_R^2 + \Delta^2}} = -\gamma_2$$

$$C_1(t) e^{-i\frac{\Delta}{2}t} = \gamma_1 e^{-i\lambda_1 t} + \gamma_2 e^{-i\lambda_2 t} \Rightarrow C_1(t) = (\gamma_1 e^{-i\lambda_1 t} + \gamma_2 e^{-i\lambda_2 t}) e^{i\frac{\Delta}{2}t}$$

$$C_2(t) e^{i\frac{\Delta}{2}t} = \gamma_3 (e^{-i\lambda_1 t} - e^{-i\lambda_2 t}) \Rightarrow C_2(t) = \gamma_3 (e^{-i\lambda_1 t} - e^{-i\lambda_2 t}) e^{-i\frac{\Delta}{2}t}$$

$$|C_1(t)|^2 = \gamma_1^2 + \gamma_2^2 + \gamma_1 \gamma_2 e^{i(\lambda_1 - \lambda_2)t} + \gamma_1 \gamma_2 e^{i(\lambda_2 - \lambda_1)t}$$

$$|C_2(t)|^2 = \gamma_3^2 \left[1 + 1 - e^{i(\lambda_1 - \lambda_2)t} - e^{i(\lambda_2 - \lambda_1)t} \right]$$

$$\lambda_1 - \lambda_2 = -\lambda - \lambda = -2\lambda \quad \lambda_2 - \lambda_1 = 2\lambda$$

$$|C_1(t)|^2 = \gamma_1^2 + \gamma_2^2 + \gamma_1 \gamma_2 e^{-i2\lambda t} + \gamma_1 \gamma_2 e^{i2\lambda t} = \gamma_1^2 + \gamma_2^2 + 2\gamma_1 \gamma_2 \cos(2\lambda t)$$

$$|C_2(t)|^2 = \gamma_3^2 \left[2 - e^{-i2\lambda t} - e^{i2\lambda t} \right] = \gamma_3^2 \left[2 - 2\cos(2\lambda t) \right]$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{4(\Omega_R^2 + \Delta^2)} \cdot 2(1 - \cos(2\lambda t)) = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot 2\sin^2(\lambda t) = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2(\lambda t)$$

$$\lambda = \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}$$

$$\gamma_1^2 + \gamma_2^2 = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2 + \Delta^2)}, \quad 2\gamma_1 \gamma_2 = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \Rightarrow \gamma_1^2 + \gamma_2^2 + 2\gamma_1 \gamma_2 = 1 \quad 2\gamma_3^2 = 2\gamma_1 \gamma_2$$

$$\gamma_3^2 = \gamma_1 \gamma_2$$

$$|C_1(t)|^2 = 1 - 2\gamma_1 \gamma_2 + 2\gamma_1 \gamma_2 \cos(2\lambda t) = 1 + 2\gamma_1 \gamma_2 [\cos(2\lambda t) - 1]$$

$$|C_1(t)|^2 = 1 + \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot (-2) \sin^2(\lambda t)$$

$$|C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \sin^2(\lambda t) \quad \text{ήτοι αναμετρώται}$$

$$\text{όπου } |C_1(t)|^2 + |C_2(t)|^2 = 1$$

Συνοπτικώς:

$$|C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \sin^2(\lambda t)$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \sin^2(\lambda t)$$

$$\lambda = \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}$$

$$|C_1(t)|^2 = 1 - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t)$$

$$|C_1(t)|^2 = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2 + \Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t) = P_1(t)$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t) = P_2(t)$$

περίοδος
ταλαντώσεων

$$T_R = \frac{2\pi}{2\lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Delta^2}} = \frac{1}{\nu_R}$$

$$\alpha_R = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}$$

μεγιστο ποσοστό
μεταβιβάσεως
maximum transfer
percentage

$$\Delta \uparrow \Rightarrow \alpha_R \downarrow \text{ και } \nu_R \uparrow (T_R \downarrow)$$

$$\Delta = 0 \Rightarrow \alpha_R = 1 \text{ και } T_R = \frac{2\pi}{\Omega_R}$$

$$\langle P_1(t) \rangle = \langle |C_1(t)|^2 \rangle = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2 + \Delta^2)}$$

μέση πιθανότητα παρουσία στη στάση 1

$$\langle P_2(t) \rangle = \langle |C_2(t)|^2 \rangle = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)}$$

μέση πιθανότητα παρουσία στη στάση 2

μέγιστος ρυθμός μεταβίβασης
(maximum transfer rate)

$$\frac{A_R}{T_R} = \frac{\Omega_R^2 \sqrt{\Omega_R^2 + \Delta^2}}{(\Omega_R^2 + \Delta^2) 2\pi} = \frac{\Omega_R^2}{2\pi \sqrt{\Omega_R^2 + \Delta^2}}$$

$t_{2\text{mean}}$:= ο χρόνος, ο οποίος απαιτείται ώστε η $P_2(t)$ να μείσει τη φορά των $\langle P_2(t) \rangle$

$$\Rightarrow \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t_{2\text{mean}}) = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)}$$

$$\Rightarrow \cos(2\lambda t_{2\text{mean}}) = 0 \Rightarrow 2\lambda t_{2\text{mean}} = \frac{\pi}{2} \Rightarrow$$

$$t_{2\text{mean}} = \frac{\pi}{4\lambda}$$

μέσος ρυθμός μεταβίβασης
(mean transfer rate)

$$k := \frac{\langle |C_2(t)|^2 \rangle}{t_{2\text{mean}}} = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \frac{4}{\pi} \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} = \frac{\Omega_R^2}{\pi \sqrt{\Omega_R^2 + \Delta^2}}$$

$$\Rightarrow k = 2 \frac{A_R}{T_R}$$

* όταν $|\Delta| \uparrow$ (δηλ απομακρυνόμαστε από το συντονισμό) $\Rightarrow A_R \downarrow$
 $T_R \downarrow$

δηλ το φαινόμενο γίνεται πιο μικρό και πιο γρήγορο

* όταν $\Omega_R \ll |\Delta|$ (μικρή διαταραχή σε σχέση με την απόλυτη τιμή του αποσυντονισμού)

$$P_2(t) = |c_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2\left(\frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} t\right)$$

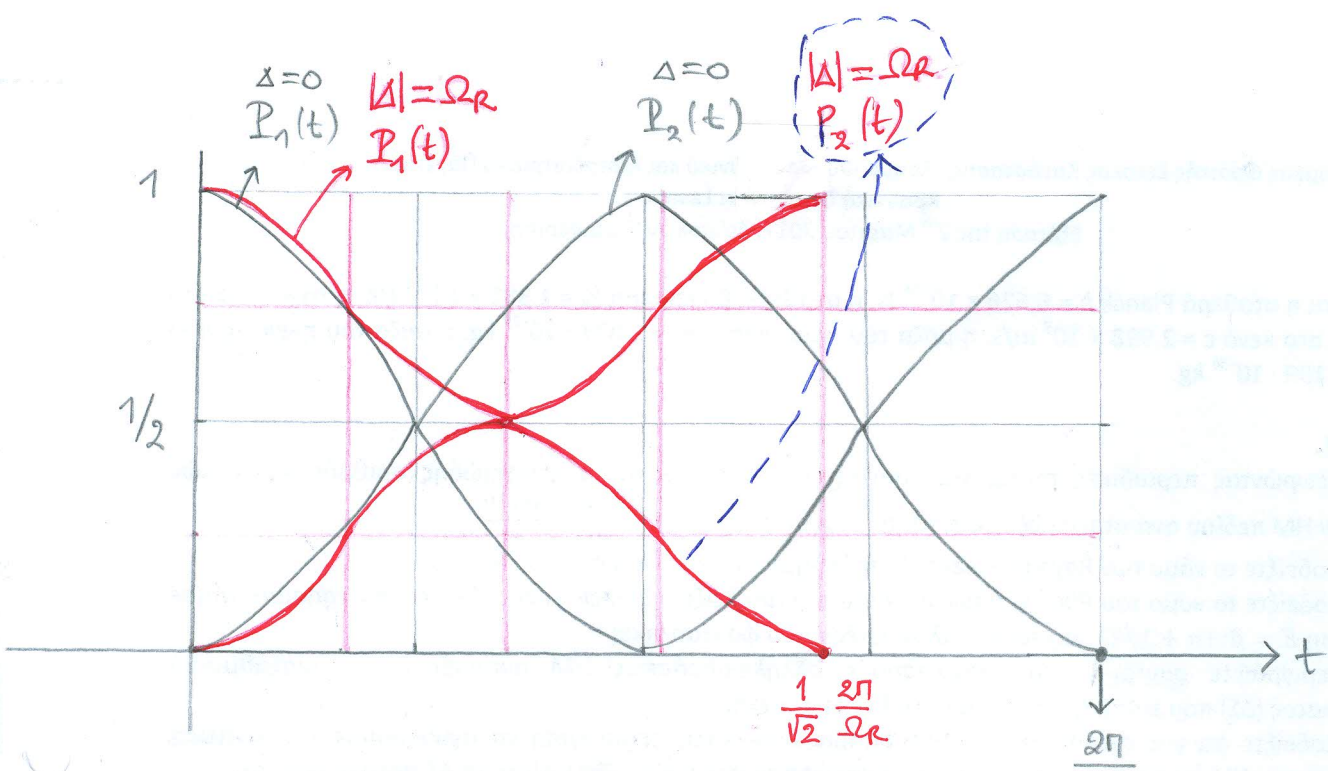
$$\approx \frac{\Omega_R^2}{\Delta^2} \sin^2\left(\frac{|\Delta|}{2} t\right)$$

$$\text{η } P_2(t) = |c_2(t)|^2 = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cos\left(2 \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} t\right)$$

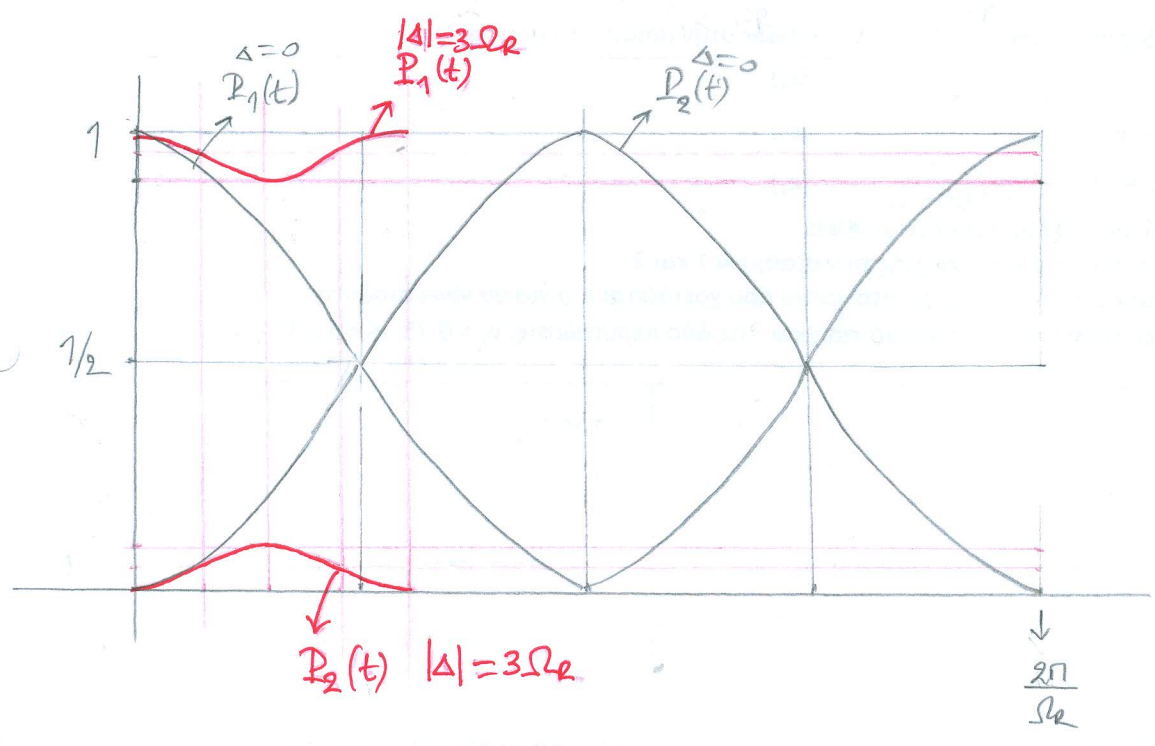
$$\approx \frac{\Omega_R^2}{2\Delta^2} - \frac{\Omega_R^2}{2\Delta^2} \cos(|\Delta| \cdot t)$$

$$\Rightarrow T_R \approx \frac{2\pi}{|\Delta|} \quad A_R \approx \frac{\Omega_R^2}{\Delta^2}$$

$$\lim_{\Omega_R \rightarrow 0} T_R = \frac{2\pi}{|\Delta|} \quad \lim_{\Omega_R \rightarrow 0} A_R = 0$$



av n.x. $|\Delta| = \Omega_R \Rightarrow \mathcal{A}_R = \frac{1}{2}$ $\& T_R = \frac{1}{\sqrt{2}} \frac{2\pi}{\Omega_R} \approx 0.707 \frac{2\pi}{\Omega_R}$



av n.x. $|\Delta| = 3\Omega_R \Rightarrow \mathcal{A}_R = \frac{1}{10}$ $\& T_R = \frac{2\pi}{\sqrt{10} \Omega_R} \approx 0.316 \frac{2\pi}{\Omega_R}$