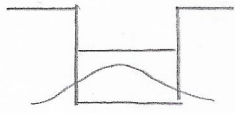




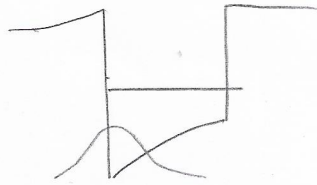
Ας υποθέσουμε 3D κρατικό ξέτασιμό (κρατική ζέλετα)
 π.χ με τα χαρακτηριστικά

①



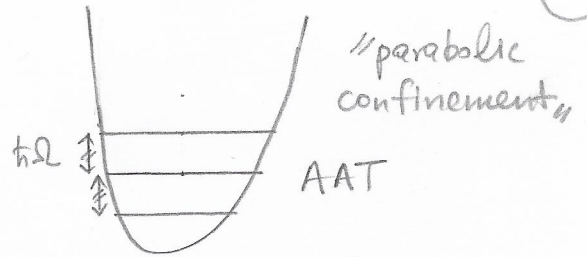
$$U_1(x)$$

π.χ. χωράει για στάθμη



$$U_2(y)$$

π.χ. χωράει για στάθμη



$$U_3(z) = \frac{m\Omega^2}{2} z^2$$

π.χ. στρωματική έπιταξία

Αλκ βασι-x As

μεταβλητός x

$$E_n = \hbar\Omega \left(n + \frac{1}{2}\right)$$

άν λοιπόν,
 η δυναμική ενέργεια
 μπορεί να γραφεί
 ως άθροισμα

$$U(\vec{r}) = U_1(x) + U_2(y) + U_3(z)$$

χωρε
 1 μόν
 στάθμη

χωρε
 1 μόν
 στάθμη

AAT
 ...

τότε μπορούμε
 να χωρίσουμε
 μεταβλητές

$$\Phi_k(\vec{r}) = X_1(x) Y_1(y) Z_n(z)$$

$$E_k = E_{1x} + E_{1y} + E_n$$

ΤΡΙΣΤΑΘΜΙΚΟ ΜΕ ΑΑΤ

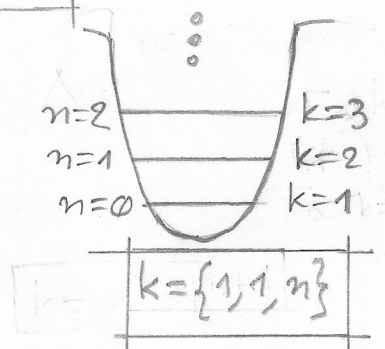
$$\dot{C}'_k = -\frac{i}{\hbar} \sum_k C_k(t) e^{i(\Omega_k' - \Omega_k)t} U_{\Sigma k'k}(t)$$

(1)

Ιδιοenergeies 1Δ ΑΑΤ

$$E_n = \hbar \Omega \left(n + \frac{1}{2} \right) \quad E_{n+1} - E_n = \hbar \Omega$$

$$E_0 = \frac{\hbar \Omega}{2}, \quad E_1 = \frac{3\hbar \Omega}{2}, \quad E_2 = \frac{5\hbar \Omega}{2}$$



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2} \hat{z}^2$$

* ΔΕΙΤΕ (1)
 Υποτίθεται ότι
 $\Phi_n(\vec{r}) = X_1(x) Y_1(y) Z_n(z)$

Αν θεωρήσουμε ότι γράφουμε να περιγράψουμε τις 3 κοινές στάθμες ως ΑΑΤ

$$U_{\Sigma k'k}(t) = e E_0 \cos \omega t z_{k'k}$$

$$z_{kk} = \int d^3r \Phi_k^*(\vec{r}) z \Phi_k(\vec{r})$$

$$z_{kk} = \int d^3r |\Phi_k(\vec{r})|^2 z = 0$$

(A) (Π)

$$z_{12} = \int d^3r \Phi_1^*(\vec{r}) z \Phi_2(\vec{r}) \neq 0$$

A Π Π

$$z_{21} = \int d^3r \Phi_2^*(\vec{r}) z \Phi_1(\vec{r}) \neq 0$$

$$z_{13} = \int d^3r \Phi_1^*(\vec{r}) z \Phi_3(\vec{r}) = 0$$

A Π A

$$z_{31} = \dots = 0$$

$$z_{23} = \int d^3r \Phi_2^*(\vec{r}) z \Phi_3(\vec{r}) \neq 0$$

Π Π A

$$z_{32} = z_{23}$$

$$-e z_{12} = -e z_{21}$$

$$f_{z_{12}} = f_{z_{21}} := f$$

Ιδιοenergeies 1Δ ΑΑΤ

$$a = \left(\frac{\hbar}{m\Omega} \right)^{1/2}$$

$$Z_n(z) = u_n(z) \exp\left(-\frac{m\Omega z^2}{2\hbar}\right)$$

k	n	$u_n(z)$
1 (A)	0	$(1/\sqrt{\pi})^{1/2}$
2 (Π)	1	$(1/2\sqrt{\pi})^{1/2} z/\alpha$
3 (A)	2	$(1/8\sqrt{\pi})^{1/2} [4(\frac{z}{\alpha})^2 - 2]$
4 (Π)	3	$(1/48\sqrt{\pi})^{1/2} [8(\frac{z}{\alpha})^3 - 12(\frac{z}{\alpha})]$
k	n	$(1/n! 2^n \sqrt{\pi})^{1/2} H_n(\frac{z}{\alpha})$

↑
 πολώνυμα Hermite

μάλλον...

$$z_{12} = z_{21} \neq z_{23} = z_{32}$$

$$U_{\Sigma 12}(t) = e E_0 \cos \omega t z_{12}$$

$$= -f_{z_{12}} E_0 \cos \omega t$$

$$U_{\Sigma 21}(t) = e E_0 \cos \omega t z_{21}$$

$$= -f_{z_{21}} E_0 \cos \omega t$$

$$\mathcal{J}_{223} = \mathcal{J}_{232} := \mathcal{J}'$$

$$= -eZ_{23} = -eZ_{32}$$

→

σ ομοίως...

(2)

$$\dot{C}_1(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_1 - \Omega_4)t} U_{\mathcal{J}11}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_1 - \Omega_2)t} U_{\mathcal{J}12}(t) - \frac{i}{\hbar} C_3(t) e^{i(\Omega_1 - \Omega_3)t} U_{\mathcal{J}13}(t)$$

$$\dot{C}_1(t) = +\frac{i}{\hbar} C_2(t) e^{-i\Omega t} \mathcal{J} \epsilon_0 \cos \omega t \quad (1)$$

$$\dot{C}_2(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_2 - \Omega_1)t} U_{\mathcal{J}21}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_2 - \Omega_2)t} U_{\mathcal{J}22}(t) - \frac{i}{\hbar} C_3(t) e^{i(\Omega_2 - \Omega_3)t} U_{\mathcal{J}23}(t)$$

$$\dot{C}_2(t) = +\frac{i}{\hbar} C_1(t) e^{i\Omega t} \mathcal{J} \epsilon_0 \cos \omega t + \frac{i}{\hbar} C_3(t) e^{-i\Omega t} \mathcal{J}' \epsilon_0 \cos \omega t \quad (2)$$

$$\dot{C}_3(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_3 - \Omega_1)t} U_{\mathcal{J}31}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_3 - \Omega_2)t} U_{\mathcal{J}32}(t) - \frac{i}{\hbar} C_3(t) e^{i(\Omega_3 - \Omega_3)t} U_{\mathcal{J}33}(t)$$

$$\dot{C}_3(t) = +\frac{i}{\hbar} C_2(t) e^{i\Omega t} \mathcal{J}' \epsilon_0 \cos \omega t \quad (3)$$

μετασχηματισμός

$$(M) \begin{cases} C_1(t) = \tilde{C}_1(t) e^{\frac{i\Delta t}{2}} \\ C_2(t) = \tilde{C}_2(t) e^{-\frac{i\Delta t}{2}} \\ C_3(t) = \tilde{C}_3(t) e^{\frac{3i\Delta t}{2}} \end{cases}$$

$$\cos \omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

ώστε να κάνουμε RWA

ΔΥΣΤΗ

$$Z_{12} = \int d^3r \Phi_1^*(\vec{r}) z \Phi_2(\vec{r})$$

$$\mathcal{F}_{Z_{12}} = -e Z_{12}$$

$$U_{Z_{12}} = e \epsilon_0 \cos \omega t \cdot Z_{12} = -\mathcal{F}_{Z_{12}} \epsilon_0 \cos \omega t \quad (2')$$

$$Z_{23} = \int d^3r \Phi_2^*(\vec{r}) z \Phi_3(\vec{r})$$

$$\mathcal{F}_{Z_{23}} = -e Z_{23}$$

$$U_{Z_{23}} = e \epsilon_0 \cos \omega t \cdot Z_{23} = -\mathcal{F}_{Z_{23}} \epsilon_0 \cos \omega t$$

$$\Phi_1(\vec{r}) = X_1(x) \cdot Y_1(y) \cdot Z_1(z)$$

$$\Phi_2(\vec{r}) = X_1(z) \cdot Y_1(y) \cdot Z_2(z)$$

$$\Phi_3(\vec{r}) = X_1(z) \cdot Y_1(z) \cdot Z_3(z)$$

$$\mathcal{F} := \mathcal{F}_{Z_{12}} \quad \Omega_{\mathcal{F}} = \frac{\mathcal{F} \epsilon_0}{\hbar}$$

$$\mathcal{F}' := \mathcal{F}_{Z_{23}} \quad \Omega_{\mathcal{F}'} = \frac{\mathcal{F}' \epsilon_0}{\hbar}$$

$$Z_{12} = \int dx |X_1(x)|^2 \int dy |Y_1(y)|^2 \int dz Z_1(z) z Z_2(z)$$

$$Z_{23} = \int dx |X_1(x)|^2 \int dy |Y_1(y)|^2 \int dz Z_2(z) z Z_3(z)$$

ΑΣΚΗΣΗ

Να υπολογιστεί ο λόγος

$$\frac{\mathcal{F}}{\mathcal{F}'} = \frac{\Omega_{\mathcal{F}}}{\Omega_{\mathcal{F}'}}$$

το σύστημα

κι αν οι $X_1(x), Y_1(y)$ είναι κανονικοποιημένες $\int |X_1|^2 dx = 1$

$$\begin{aligned} Z_{12} &= \int dz \left(\frac{1}{\alpha \sqrt{\pi}} \right)^{1/2} \exp\left(-\frac{m\Omega z^2}{2\hbar}\right) \cdot z \cdot \left(\frac{1}{\alpha \sqrt{\pi} 2} \right)^{1/2} \left(\frac{z}{a} \right) \exp\left(-\frac{m\Omega z^2}{2\hbar}\right) \\ &= \frac{2\alpha}{\sqrt{\pi} \sqrt{2}} \int \frac{dz}{\alpha} \exp\left(-\frac{z^2}{2a^2}\right) \left(\frac{z}{a} \right) \cdot \left(\frac{z}{a} \right) \exp\left(-\frac{z^2}{2a^2}\right) \quad \frac{z}{a} = \theta \\ &= \frac{2\alpha}{\sqrt{\pi} \sqrt{2}} \int d\theta \exp(-\theta^2) \cdot \theta^2 = \frac{2\alpha}{\sqrt{\pi} \sqrt{2}} \frac{\sqrt{\pi}}{2} = \frac{\alpha}{\sqrt{2}} \end{aligned}$$

$$Z_{23} = \int dz \left(\frac{1}{2\alpha \sqrt{\pi}} \right)^{1/2} \left(\frac{z}{a} \right) \exp\left(-\frac{z^2}{2a^2}\right) \cdot z \cdot \left(\frac{1}{8\alpha \sqrt{\pi}} \right)^{1/2} \left[4 \left(\frac{z}{a} \right)^2 - 2 \right] \exp\left(-\frac{z^2}{2a^2}\right)$$

$$\frac{2\alpha^2}{\sqrt{\pi} \sqrt{2} \sqrt{8} \alpha} \int \frac{dz}{a} \left(\frac{z}{a} \right)^2 \exp\left(-\frac{z^2}{a^2}\right) \left[4 \left(\frac{z}{a} \right)^2 - 2 \right]$$

$$\frac{\alpha}{2\sqrt{\pi}} \int d\theta \cdot \exp(-\theta^2) \cdot \theta^2 [4\theta^2 - 2] = \frac{\alpha}{2\sqrt{\pi}} \cdot (+2)\sqrt{\pi} = +\alpha$$

Eisberg $\psi_n(\xi) = \left(\frac{1}{\sqrt{\pi} 2^n n!} \right)^{1/2} e^{-\xi^2/2} \cdot H_n(\xi), n \in \mathbb{N}$ $\xi = \frac{z}{a}$ $d\xi = \frac{dz}{a}$

$$\textcircled{1} \Rightarrow \dot{C}_1(t) = \frac{i}{\hbar} \mathcal{F} \epsilon_0 C_2(t) e^{-i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

RWA

$$\Delta := \omega - \Omega$$

$$\Omega_R = \frac{\mathcal{F} \epsilon_0}{\hbar} > 0 \text{ για } \mathcal{F} > 0$$

$$\dot{C}_1(t) = \left(\frac{i}{2\hbar}\right) \mathcal{F} \epsilon_0 C_2(t) e^{-i(\omega-\Omega)t} \Rightarrow \dot{C}_1(t) = \frac{i}{2} \Omega_R C_2(t) e^{i\Delta t} \textcircled{1'}$$

$$\textcircled{2} \Rightarrow \dot{C}_2(t) = \frac{i}{\hbar} \mathcal{F} \epsilon_0 C_1(t) e^{i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2} + \frac{i}{\hbar} \mathcal{F}' \epsilon_0 C_3(t) e^{-i\Omega t} \frac{e^{i\omega t} - e^{-i\omega t}}{2}$$

RWA

για $\mathcal{F} < 0$

$$\Omega_R = \frac{-\mathcal{F} \epsilon_0}{\hbar}$$

μηδεν...

$$\Omega_R' = \frac{-\mathcal{F}' \epsilon_0}{\hbar}$$

κλπ...

$$\dot{C}_2(t) = \frac{i \mathcal{F} \epsilon_0}{2\hbar} C_1(t) e^{-i(\omega-\Omega)t} + \frac{i \mathcal{F}' \epsilon_0}{2\hbar} C_3(t) e^{i(\omega-\Omega)t}$$

$$\Omega_R' = \frac{\mathcal{F}' \epsilon_0}{\hbar} > 0$$

για $\mathcal{F}' > 0$

...

$$\dot{C}_2(t) = \frac{i}{2} \Omega_R C_1(t) e^{-i\Delta t} + \frac{i}{2} \Omega_R' C_3(t) e^{i\Delta t} \textcircled{2'}$$

Η 2 είναι ο διαμεσοσφαιρικός.

από ορίσουμε τις Ω_R, Ω_R' όπως

$$\textcircled{3} \Rightarrow \dot{C}_3(t) = \frac{i \mathcal{F}' \epsilon_0}{\hbar} C_2(t) e^{i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

RWA

$$\dot{C}_3(t) = \frac{i \mathcal{F}' \epsilon_0}{2\hbar} C_2(t) e^{-i\Delta t} \textcircled{3'}$$

$\textcircled{1'} \textcircled{2'} \textcircled{3'}$

οι παράγωγοι των συναρτήσεων σχετίζονται με τις συναρτήσεις με χρονικά εφάρμομενους συντελεστές

$$\left. \begin{aligned} \dot{C}_1(t) &= C_1(t) e^{\frac{i\Delta t}{2}} + C_1(t) \frac{i\Delta}{2} e^{\frac{i\Delta t}{2}} \\ \dot{C}_2(t) &= C_2(t) e^{-\frac{i\Delta t}{2}} + C_2(t) \left(-\frac{i\Delta}{2}\right) e^{-\frac{i\Delta t}{2}} \\ \dot{C}_3(t) &= C_3(t) e^{\frac{3i\Delta t}{2}} + C_3(t) \left(\frac{-3i\Delta}{2}\right) e^{\frac{3i\Delta t}{2}} \end{aligned} \right\} \leftarrow \textcircled{M}$$

$$\textcircled{1'} \textcircled{M} \quad \cancel{C_1(t) e^{\frac{i\Delta t}{2}} + C_1(t) \frac{i\Delta}{2} e^{\frac{i\Delta t}{2}}} = \cancel{\frac{i}{2} \Omega_R C_2(t) e^{-\frac{i\Delta t}{2}}} \quad \cancel{e^{i\Delta t}}$$

$$\dot{C}_1(t) = -i \frac{\Delta}{2} C_1(t) + i \frac{\Omega_R}{2} C_2(t) \quad \textcircled{1''}$$

$$\textcircled{2'} \textcircled{M} \quad \cancel{C_2(t) e^{-\frac{i\Delta t}{2}} + C_2(t) \left(-\frac{i\Delta}{2}\right) e^{-\frac{i\Delta t}{2}}} = \cancel{i \frac{\Omega_R}{2} C_1(t) e^{\frac{i\Delta t}{2}}} \quad \cancel{e^{-i\Delta t}} \\ + \cancel{i \frac{\Omega_R}{2} C_3(t) e^{\frac{3i\Delta t}{2}}} \quad \cancel{e^{i\Delta t}}$$

$$\dot{C}_2(t) = -i \frac{\Omega_R}{2} C_1(t) + i \frac{\Delta}{2} C_2(t) + i \frac{\Omega_R}{2} C_3(t) \quad \textcircled{2''}$$

$$\textcircled{3'} \textcircled{M} \quad \cancel{C_3(t) e^{\frac{3i\Delta t}{2}} + C_3(t) \left(-\frac{3i\Delta}{2}\right) e^{\frac{3i\Delta t}{2}}} = \cancel{\frac{i}{2} \Omega_R C_2(t) e^{-\frac{i\Delta t}{2}}} \quad \cancel{e^{-i\Delta t}}$$

$$\dot{C}_3(t) = -i \frac{\Omega_R}{2} C_2(t) + i \frac{3\Delta}{2} C_3(t) \quad \textcircled{3''}$$

$\textcircled{1''} \textcircled{2''} \textcircled{3''}$

οι παράγωγοι των συναρτήσεων
 οξειδώνονται με τις συναρτήσεις
 με χρονικά αντίστοιχους συντελεστές

$$\begin{bmatrix} \dot{C}_1(t) \\ \dot{C}_2(t) \\ \dot{C}_3(t) \end{bmatrix} = \begin{bmatrix} -i\frac{\Delta}{2} & -\frac{i\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & i\frac{\Delta}{2} & -i\frac{\Omega_R'}{2} \\ 0 & -i\frac{\Omega_R'}{2} & i\frac{3\Delta}{2} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

$\vec{x}'(t) \quad \quad \quad \tilde{A} \quad \quad \quad \vec{x}(t)$

Ans

$$\vec{x}(t) = \vec{u} e^{\tilde{\lambda} t}$$

$$\tilde{A} = -iA$$

$$\vec{x}'(t) = \tilde{A} \vec{x}(t)$$

$$\vec{u} \tilde{\lambda} e^{\tilde{\lambda} t} = \tilde{A} \vec{u} e^{\tilde{\lambda} t}$$

$$\tilde{A} \vec{u} = \tilde{\lambda} \vec{u} \quad \tilde{\lambda} = -i\lambda$$

$$A \vec{u} = \lambda \vec{u}$$

$$A = \begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} \end{bmatrix}$$

$$(A - \lambda I) \vec{u} = \vec{0}$$

$$\det \begin{bmatrix} \frac{\Delta}{2} - \lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} - \lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} - \lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{3\Delta}{2} - \frac{\Omega_R \Omega_R'}{2} + \frac{\Delta - \lambda}{2} = 0 \implies 3\Delta^2 - 2\Delta \Omega_R'^2 + 3\Delta \Omega_R^2 = 0$$

ΛΥΣΗ
για

$$\Delta = 0$$

6

$$\begin{bmatrix} -\lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\lambda \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow -\lambda \begin{vmatrix} -\lambda & +\frac{\Omega_R'}{2} \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} + \frac{\Omega_R}{2} \begin{vmatrix} +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} = 0$$

$$-\lambda \left[\lambda^2 - \frac{\Omega_R'^2}{4} \right] + \frac{\Omega_R}{2} \frac{\Omega_R}{2} \lambda = 0$$

δύο εδω γ παρα 1 εδω...

$$-\lambda^3 + \lambda \frac{\Omega_R'^2}{4} + \lambda \frac{\Omega_R^2}{4} = 0 \Rightarrow \lambda \left[-\lambda^2 + \frac{\Omega_R'^2}{4} + \frac{\Omega_R^2}{4} \right] = 0$$

$$\lambda = 0$$

(2)
3)

$$\lambda^2 = \frac{\Omega_R^2 + \Omega_R'^2}{4}$$

$$\lambda = \pm \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_2 = 0$$

$$\lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\Lambda < 0$$

$$\lambda_2 = 0$$

$$\lambda_3 = \Lambda > 0$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_a'^2}}{2} = -\Lambda < 0$$

(7)

$$\begin{bmatrix} \Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & \Lambda & -\frac{\Omega_a'}{2} \\ 0 & -\frac{\Omega_a'}{2} & \Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow \Lambda U_1 = \frac{\Omega_R}{2} U_2 \Rightarrow U_1 = \frac{\Omega_R}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} U_1 + \Lambda U_2 - \frac{\Omega_a'}{2} U_3 = 0$$

$$-\frac{\Omega_a'}{2} U_2 + \Lambda U_3 = 0 \Rightarrow \Lambda U_3 = \frac{\Omega_a'}{2} U_2 \Rightarrow U_3 = \frac{\Omega_a'}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} \frac{\Omega_R}{2\Lambda} U_2 + \Lambda U_2 - \frac{\Omega_a'}{2} \frac{\Omega_a'}{2\Lambda} U_2 = 0$$

$$\Rightarrow U_2 \left[-\frac{\Omega_R^2}{4\Lambda} + \Lambda - \frac{\Omega_a'^2}{4\Lambda} \right] = 0$$

$$\Lambda = \frac{\sqrt{\Omega_R^2 + \Omega_a'^2}}{2}$$

$$\Rightarrow U_2 \left[\frac{-\Omega_R^2 + 4\Lambda^2 - \Omega_a'^2}{4\Lambda} \right] = 0$$

$$\Lambda^2 = \frac{\Omega_R^2 + \Omega_a'^2}{4}$$

αν $U_2 = 0$
 $\Rightarrow U_1 = U_2 = U_3 = 0$ μηδέν
 $\Rightarrow U_2$ ο,τι θέλουμε (μη μηδενικό)
 π.χ. $U_2 = 1$

$$\vec{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_a'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_1 = \beta \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_a'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_1 \cdot \vec{V}_1 = 1 \Rightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_a'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + 4\Lambda^2 + \Omega_a'^2}{4\Lambda^2} = 1$$

$$|\beta|^2 = \frac{2}{1} = 1 \Rightarrow \text{π.χ. } \beta = \frac{1}{\sqrt{2}}$$

• $\lambda_2 = 0$

$$\begin{bmatrix} 0 & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & 0 & -\frac{\Omega'_R}{2} \\ 0 & -\frac{\Omega'_R}{2} & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$-\frac{\Omega_R}{2} U_2 = 0 \quad (U_2 = 0)$$

$$-\frac{\Omega_R}{2} U_1 - \frac{\Omega'_R}{2} U_3 = 0$$

$$-\frac{\Omega_R}{2} U_1 = \frac{\Omega'_R}{2} U_3 \Rightarrow U_3 = -\frac{\Omega_R}{\Omega'_R} U_1$$

$$-\frac{\Omega'_R}{2} U_2 = 0 \quad (U_2 = 0)$$

$$\vec{V}_2 = \beta \begin{bmatrix} 1 \\ 0 \\ -\frac{\Omega_R}{\Omega'_R} \end{bmatrix}$$

$$\vec{V}_2 \cdot \vec{V}_2 = 1 \Rightarrow |\beta|^2 \left(1 + \frac{\Omega_R^2}{\Omega'^2_R} \right) = 1 \Rightarrow |\beta|^2 \frac{\Omega'^2_R + \Omega_R^2}{\Omega'^2_R} = 1$$

$$|\beta|^2 \frac{4\Lambda^2}{\Omega'^2_R} = 1 \Rightarrow \beta = \frac{\Omega'_R}{2\sqrt{\Omega'^2_R + \Omega_R^2}} = \frac{\Omega'_R}{\sqrt{\Omega'^2_R + \Omega_R^2}}$$

• $\lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega'^2_R}}{2} = \Lambda > 0$

$$\begin{bmatrix} -\Lambda & -\frac{\Omega_R}{2} & 0 \\ \frac{\Omega_R}{2} & -\Lambda & -\frac{\Omega'_R}{2} \\ 0 & -\frac{\Omega'_R}{2} & -\Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\vec{V}_3 = \begin{bmatrix} \frac{\Omega'_R}{\sqrt{\Omega'^2_R + \Omega_R^2}} \\ 0 \\ -\frac{\Omega_R}{\sqrt{\Omega'^2_R + \Omega_R^2}} \end{bmatrix} = \begin{bmatrix} \frac{\Omega'_R}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix}$$

$$-\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow -\frac{\Omega_R}{2} U_2 = \Lambda U_1 \Rightarrow U_1 = -\frac{\Omega_R}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} U_1 - \Lambda U_2 - \frac{\Omega'_R}{2} U_3 = 0$$

$$-\frac{\Omega'_R}{2} U_2 - \Lambda U_3 = 0 \Rightarrow -\frac{\Omega'_R}{2} U_2 = \Lambda U_3 \Rightarrow U_3 = -\frac{\Omega'_R}{2\Lambda} U_2$$

$$+\frac{\Omega_R}{2} \frac{\Omega_R}{2\Lambda} U_2 - \Lambda U_2 + \frac{\Omega'_R}{2} \frac{\Omega'_R}{2\Lambda} U_2 = 0 \Rightarrow U_2 \left[\frac{\Omega_R^2}{4\Lambda} - \frac{4\Lambda^2}{4\Lambda} + \frac{\Omega'^2_R}{4\Lambda} \right] = 0$$

$$U_2 \left[\frac{\Omega_R^2 + \Omega'^2_R - 4\Lambda^2}{4\Lambda} \right] = 0 \Rightarrow U_2 \text{ μηδενικό (μη μηδενικό)} \Rightarrow \text{πχ. } U_2 = 1$$

$$\vec{V}_3 = \beta \begin{bmatrix} -\frac{\Omega_R}{2\Lambda} \\ 1 \\ -\frac{\Omega_I}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_3 \cdot \vec{V}_3 = 1 \Rightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_I^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + \Omega_I^2 + 4\Lambda^2}{4\Lambda^2} = 1 \Rightarrow |\beta|^2 \cdot 2 = 1 \Rightarrow \text{π.χ. } \beta = -\frac{1}{\sqrt{2}}$$

$$\vec{V}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_I}{2\Lambda} \end{bmatrix}$$

γενική λύση

$$\vec{x}(t) = \sum_{k=1}^3 \sigma_k \vec{V}_k e^{-i\lambda_k t}$$

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_I^2}$$

$$4\Lambda^2 = (\Omega_R^2 + \Omega_I^2)$$

$$16\Lambda^4 = (\Omega_R^2 + \Omega_I^2)^2$$

$$C_1(0) = 1$$

$$C_2(0) = 0$$

$$C_3(0) = 0$$

ΣΤΩΘΑΝ
αρχ. συνθήκες $\begin{matrix} \text{---} \\ \text{---} \\ \bullet \end{matrix}$ (10)

$$\vec{x}(t) = \begin{bmatrix} C_1(t) e^{-\frac{i\Delta t}{2}} \\ C_2(t) e^{i\frac{\Delta t}{2}} \\ C_3(t) e^{3i\frac{\Delta t}{2}} \end{bmatrix} = \frac{\sigma_1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_1 t} + \sigma_2 \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix} e^{-i\lambda_2 t} + \frac{\sigma_3}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_3 t}$$

$$1 = \frac{\sigma_1}{\sqrt{2}} \frac{\Omega_R}{2\Lambda} + \sigma_2 \frac{\Omega_R'}{2\Lambda} + \frac{\sigma_3}{\sqrt{2}} \frac{\Omega_R}{2\Lambda}$$

$$0 = \frac{\sigma_1}{\sqrt{2}} - \frac{\sigma_3}{\sqrt{2}} \Rightarrow \boxed{\sigma_1 = \sigma_3 = \sigma}$$

$$0 = \frac{\sigma_1}{\sqrt{2}} \frac{\Omega_R'}{2\Lambda} - \sigma_2 \frac{\Omega_R}{2\Lambda} + \frac{\sigma_3}{\sqrt{2}} \frac{\Omega_R}{2\Lambda}$$

$$\downarrow 0 = \frac{\sigma}{\sqrt{2}} \frac{\Omega_R'}{2\Lambda} - \frac{\sigma_2 \Omega_R}{2\Lambda} + \frac{\sigma}{\sqrt{2}} \frac{\Omega_R}{2\Lambda} \Rightarrow 0 = \frac{\sigma \Omega_R'}{\sqrt{2} \Lambda} - \frac{\sigma_2 \Omega_R}{\sqrt{2} \Lambda} \Rightarrow$$

$$\frac{\sigma_2 \Omega_R}{2\Lambda} = \frac{\sigma \Omega_R'}{\sqrt{2} \Lambda} \Rightarrow \boxed{\sigma_2 = \sigma \sqrt{2} \frac{\Omega_R'}{\Omega_R}}$$

$$1 = \frac{\sigma}{\sqrt{2}} \frac{\Omega_R}{2\Lambda} + \frac{\sigma \sqrt{2} \Omega_R'}{\Omega_R} \frac{\Omega_R'}{2\Lambda} + \frac{\sigma}{\sqrt{2}} \frac{\Omega_R}{2\Lambda}$$

$$2\Lambda = \sigma \left(\frac{\Omega_R}{\sqrt{2}} + \sqrt{2} \frac{\Omega_R'^2}{\Omega_R} + \frac{\Omega_R}{\sqrt{2}} \right) = \sigma \frac{\Omega_R^2 + 2\Omega_R'^2 + \Omega_R^2}{\sqrt{2} \Omega_R} = \sigma \frac{2}{\sqrt{2}} \frac{\Omega_R^2 + \Omega_R'^2}{\Omega_R}$$

$$\sigma = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2} \sqrt{2} \Omega_R}{2(\Omega_R^2 + \Omega_R'^2)} \Rightarrow \boxed{\sigma = \frac{\Omega_R}{\sqrt{2} \sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{\Omega_R}{\sqrt{2} 2\Lambda}}$$

$$\sigma = \frac{\Omega_R}{\sqrt{2} 2\Lambda}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k^2}} \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k^2}} e^{+i\Delta t} + \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k^2}} \frac{\Omega_k}{\Omega_R} \frac{\Omega_k}{\sqrt{\Omega_R^2 + \Omega_k^2}} e^{-i0} + \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k^2}} \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k^2}} e^{-i\Delta t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_k^2)} e^{i\Delta t} + \frac{\Omega_R^2}{\Omega_R^2 + \Omega_k^2} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_k^2)} e^{-i\Delta t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{2\Omega_R^2}{2(\Omega_R^2 + \Omega_k^2)} \cos \Delta t + \frac{\Omega_R^2}{\Omega_R^2 + \Omega_k^2}$$

$$C_2(t) e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{2}\sqrt{2}} 1 \cdot e^{i\Delta t + i\Delta t} - \frac{\Omega_R}{\sqrt{2}\sqrt{2}} e^{-i\Delta t}$$

$$= \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_k^2}} (e^{+i\Delta t} - e^{-i\Delta t})$$

$\frac{\cos + i\sin}{- \cos + i\sin}$

$$C_2(t) e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{\Omega_R^2 + \Omega_k^2}} i \sin \Delta t$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_k^2} \sin^2(\Delta t) = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_k^2} \left(\frac{1}{2} - \frac{\cos(2\Delta t)}{2} \right)$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \cdot \left(\frac{1}{2} - \frac{\cos(2\Lambda t)}{2} \right)$$

$$= \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_R'^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_R'^2)} \cos(2\Lambda t)$$

$$d_2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2}$$

maximum transfer percentage
 μέγιστο ποσοστό μεταβίβασης

$$T_2 = \frac{2\pi}{2\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

για $\Omega_R = \Omega_R' \Rightarrow d_2 = \frac{1}{2}$

$$T_2 = \frac{2\pi}{\sqrt{2} \Omega_R} = \frac{1}{\sqrt{2}} \left(\frac{2\pi}{\Omega_R} \right)$$

↑
 περίοδος άπλοσίχου
 διαστήματος

$$|C_2(t)|^2 = \frac{1}{2} \left(\frac{1}{2} - \frac{\cos(\sqrt{2} \Omega_R t)}{2} \right)$$

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2} \Omega_R$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\underbrace{\sqrt{2} \Omega_R t}_{2\omega_1 = \omega_2})$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\omega_2 t)$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2 \cdot 4\Lambda^2} e^{+i\Lambda t} + \frac{\Omega_k'^2}{4\Lambda^2} + \frac{\Omega_k^2}{2 \cdot 4\Lambda^2} e^{-i\Lambda t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{4\Lambda^2} \cos(\Lambda t) + \frac{\Omega_k'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_k'^4}{16\Lambda^4} + 2 \frac{\Omega_R^2}{4\Lambda^2} \cdot \cos(\Lambda t) \cdot \frac{\Omega_k'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_k'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_k'^2}{16\Lambda^4} \cdot \cos(\Lambda t)$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos(2\Lambda t) + 1}{2} + \frac{\Omega_k'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_k'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \quad T_B = \frac{2\pi}{\Lambda} \quad \frac{T_B}{T_A} = 2 \Rightarrow \text{περιοδική κίνηση}$$

με περίοδο $T_1 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_k'^2}} \cdot 2 = 2 T_2$

$$\left| C_1\left(\frac{2\pi}{\Lambda}\right) \right|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos\left(2\Lambda \frac{2\pi}{\Lambda}\right) + 1}{2} + \frac{\Omega_k'^2}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_k'^2}{16\Lambda^4} \cdot \cos\left(\Lambda \frac{2\pi}{\Lambda}\right) =$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_k'^2}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_k'^2}{16\Lambda^4} = \frac{(\Omega_R^2 + \Omega_k'^2)^2}{16\Lambda^4} = 1$$

$$\left| C_1\left(\frac{2\pi}{2\Lambda}\right) \right|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos\left(2\Lambda \frac{2\pi}{2\Lambda}\right) + 1}{2} + \frac{\Omega_k'^2}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_k'^2}{16\Lambda^4} \cdot \cos\left(\Lambda \frac{2\pi}{2\Lambda}\right)$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_k'^2}{16\Lambda^4} - \frac{2 \Omega_R^2 \Omega_k'^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_k'^2)^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_k'^2)^2}{(\Omega_R^2 + \Omega_k'^2)^2}$$

ή τιμή της $|C_1(t)|^2$ στο ήμισιο της περιόδου

$$\frac{d}{dt} |C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot (2 \cos(\Lambda t) \cdot (-\Lambda) \sin(\Lambda t)) + \frac{2 \Omega_R^2 \Omega_k'^2}{16\Lambda^4} (-\Lambda) \sin(\Lambda t)$$

$$\frac{d}{dt} |C_1(t)|^2 = \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \sin(\Lambda t) \cdot [\Omega_R^2 \cos(\Lambda t) + \Omega_k'^2]$$

$$\frac{d^2}{dt^2} |C_1(t)|^2 = \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \Lambda \cdot \cos(\Lambda t) \cdot [\Omega_R^2 \cos(\Lambda t) + \Omega_k'^2] + \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \sin(\Lambda t) \cdot \Omega_R^2 (-\Lambda) \cdot \sin(\Lambda t)$$

$$= \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \Lambda \cdot \left\{ \Omega_R^2 \cdot \cos^2(\Lambda t) + \Omega_k'^2 \cos(\Lambda t) - \Omega_R^2 \sin^2(\Lambda t) \right\}$$

$$\frac{d|G_1(t)|^2}{dt} = \frac{(-2\Lambda) \cdot \Omega_R^2}{16\Lambda^4} \cdot \sin(\Lambda t) \cdot [\Omega_R^2 \cdot \cos(\Lambda t) + \Omega_R'^2]$$

$$\frac{d^2|G_1(t)|^2}{dt^2} = \frac{(-2\Lambda^2) \cdot \Omega_R^2}{16\Lambda^4} [\Omega_R^2 \sin^2(\Lambda t) - \Omega_R^2 \cos^2(\Lambda t) - \Omega_R'^2 \cos(\Lambda t)]$$

$$\frac{d|G_1(t)|^2}{dt} = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$$

\Downarrow
 $\Lambda t = n\pi, n \in \mathbb{Z}$

πρόβλημα, πρέπει
 $\Omega_R'^2 \leq \Omega_R^2$

Π1

Π2

Π2

$$\begin{aligned} \frac{d^2}{dt^2} |G_1(t)|^2 &= \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} \cdot \left[\Omega_R^2 \cdot \sin^2(\Lambda t) - \Omega_R^2 \cdot \frac{\Omega_R'^4}{\Omega_R^4} + \Omega_R^2 \cdot \frac{\Omega_R'^2}{\Omega_R^2} \right] \\ &= \frac{2\Lambda^2}{16\Lambda^4} \left[\Omega_R^4 \cdot \sin^2(\Lambda t) - \Omega_R'^4 + \Omega_R^4 \right] = \frac{2\Lambda^2}{16\Lambda^4} \cdot \Omega_R^4 \cdot \sin^2(\Lambda t) > 0 \end{aligned}$$

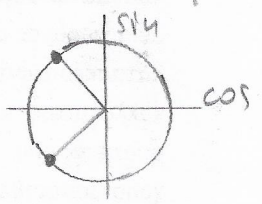
Οπλ. συν Π2 έχουμε έλαχιστα, με τιμή μηδενική όταν τότε:

$$|G_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\Omega_R'^4}{\Omega_R^4} + \frac{\Omega_R^4}{16\Lambda^4} + \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-1) \frac{\Omega_R^2}{\Omega_R^2} = \frac{2\Omega_R^4}{16\Lambda^4} - \frac{2\Omega_R'^4}{16\Lambda^4} = 0$$

Μάλιστα σε μία περίοδο $T_1 = \frac{2\pi}{\Lambda}$, ή όποια είναι και η περίοδος του όρου

$\cos(\Lambda t)$, \exists 2 φορές όπου έχουμε $\cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$

δηλαδή θα έχουμε 2 μηδενισμούς σε μία περίοδο T_1 .



Όποτε τότε, $G_1 = 1$ μέγιστο ποσοστό μεταβιβάσεων
 maximum transfer percentage

Π1

$$\frac{d^2}{dt^2} |G_1(t)|^2 = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} [-\Omega_R^2 \cos^2(\Lambda t) - \Omega_R'^2 \cos(\Lambda t)]$$

$$\sin(\Lambda t) = 0 \Rightarrow \cos(\Lambda t) = \pm 1$$

Π1α

$\rightarrow t = 0, T_1, 2T_1, \dots$
 $\Lambda t = 0, 2\pi, 4\pi, \dots \cos(\Lambda t) = 1$

Π1β

$\Lambda t = \pi, 3\pi, 5\pi, \dots \cos(\Lambda t) = -1$

$\rightarrow t = \frac{T_1}{2}, \frac{3T_1}{2}, \frac{5T_1}{2}, \dots$

π1α) $\frac{d^2 |C_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} (-\Omega_R^2 - \Omega_R'^2) < 0 \Rightarrow$ τοπικό μέγιστο 13''

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot 1 + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 + \Omega_R'^2)^2}{16\Lambda^4} = 1$$

δηλ. είναι δίκως μέγιστο

π1β) $\frac{d^2 |C_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} (-\Omega_R^2 + \Omega_R'^2) > 0 \Rightarrow$ τοπικό ελάχιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

ή δοσκά είναι η τιμή της $|C_1(t)|^2$ στο ήμισυ της περιόδου

$$d_1 = 1 - \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2} = \frac{4 \cdot \Omega_R^2 \cdot \Omega_R'^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

για $\Omega_R = \Omega_R'$

$$T_1 = \frac{2\pi}{\sqrt{2} |\Omega_R|} \cdot 2 = \sqrt{2} \left(\frac{2\pi}{\Omega_R} \right)$$

↓
περίοδος αντίστοιχου διαταγμένου

π2 \Rightarrow $d_1 = 1$ $\cos(\Lambda t) = -1 \Rightarrow \Lambda t = +\pi, 3\pi, \dots$ $t = \frac{\pi}{\Lambda} = \frac{2\pi}{2\Lambda} =$ το ήμισυ της περιόδου

π1 \Rightarrow $d_1 = 1$ στο ήμισυ της περιόδου 1 φορά μηδενισμός...

δηλ. για $\Omega_R = \Omega_R'$ οι π1, π2 ταυτίζονται

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2} \Omega_R$$

$$4\Lambda^2 = 2\Omega_R^2$$

$$16\Lambda^4 = 4\Omega_R^4$$

π.χ. όπες στο GG, GGG

$$T_{GG} \approx 20.6783 \text{ fs}$$

$$T_{GGG} = 29.2436 \text{ fs} = \sqrt{2} T_{GG}$$

$$T_{31} = 14.6218 \text{ fs} = \frac{T_{GG}}{\sqrt{2}}$$

$$T_{32} = 29.2436 \text{ fs} = T_{GGG} = \sqrt{2} T_{GG}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4} \cdot \frac{\cos(\sqrt{2}\Omega_R t) + 1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} + \frac{2\Omega_R^4}{4\Omega_R^4} \cdot \cos\left(\frac{\sqrt{2}}{2}\Omega_R t\right)$$

$$|C_1(t)|^2 = \frac{1}{4} \left(\frac{1}{2} \cdot \cos(\sqrt{2} \omega_1 t) + \frac{1}{2} \right) + \frac{1}{4} + \frac{1}{2} \cos\left(\frac{\sqrt{2}}{2} \omega_1 t\right)$$

13'''

$$\frac{1}{8} \cos(2\omega_1 t) + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} \cos(\omega_1 t) =$$

$$\frac{1}{8} \left(\underbrace{\cos(2\omega_1 t) + 1}_{2 \cos^2(\omega_1 t)} + 2 + 4 \cos(\omega_1 t) \right) = \frac{1}{8} \left(2 \cos^2(\omega_1 t) + 2 + 4 \cos(\omega_1 t) \right)$$

$$= \frac{1}{4} \left(\cos^2(\omega_1 t) + 1 + 2 \cos(\omega_1 t) \right) = \frac{1}{4} \left(\underbrace{\cos(\omega_1 t) + 1}_{2 \cos^2\left(\frac{\omega_1 t}{2}\right)} \right)^2 = \cos^4\left(\frac{\omega_1 t}{2}\right)$$

$$C_3(t) e^{3i\frac{\Delta t}{2}} = \frac{\Omega_R}{4\Lambda} \cdot \frac{\Omega_R'}{2\Lambda} e^{+i\Lambda t} + \frac{\Omega_R}{2\Lambda} \frac{\Omega_R'}{\Omega_R} (-1) \frac{\Omega_R}{2\Lambda} + \frac{\Omega_R}{4\Lambda} \cdot \frac{\Omega_R'}{2\Lambda} e^{-i\Lambda t} \quad (14)$$

$$C_3(t) e^{3i\frac{\Delta t}{2}} = \frac{\Omega_R \Omega_R'}{4\Lambda^2} \cos(\Lambda t) - \frac{\Omega_R \Omega_R'}{4\Lambda^2}$$

$$|C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - 2 \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$|C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \frac{\cos(2\Lambda t) + 1}{2} + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \quad T_B = \frac{2\pi}{\Lambda} \quad \frac{T_B}{T_A} = 2 \Rightarrow \text{η κίνηση είναι περιοδική με}$$

με περίοδο

$$T_3 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \cdot 2 = 2T_2 = T_1$$

$$\frac{d}{dt} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2 \cos(\Lambda t) (-\Lambda) \sin(\Lambda t) - 2 \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-\Lambda) \sin(\Lambda t)$$

$$\frac{d}{dt} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2(-\Lambda) [\cos(\Lambda t) \sin(\Lambda t) - \sin(\Lambda t)]$$

$$\frac{d}{dt} |C_3(t)|^2 = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = 1$$

$$\Rightarrow t = T_3, 2T_3, \dots$$

$$\textcircled{\Pi 1} \quad \Lambda t = 0, 2\pi, 4\pi, \dots \Rightarrow \sin(\Lambda t) = 0 \text{ και } \cos(\Lambda t) = 1$$

$$\textcircled{\Pi 2} \quad \Lambda t = \pi, 3\pi, 5\pi, \dots \Rightarrow \sin(\Lambda t) = 0 \text{ και } \cos(\Lambda t) = -1$$

$$\Rightarrow t = \frac{T_3}{2}, \frac{3T_3}{2}, \dots$$

$$\textcircled{\Pi 1} \quad |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot 1 + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (\pm 1) = \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \mp \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4}$$

$$\textcircled{\Pi 1} \Rightarrow |C_3(t)|^2 = 0 \quad \text{ελάχιστο}$$

$$\textcircled{\Pi 2} \Rightarrow |C_3(t)|^2 = \frac{4\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \quad \text{μέγιστο δίνω...}$$

$$\frac{d^2}{dt^2} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2(-\Lambda) \left[\frac{1}{2} 2\Lambda \cos(2\Lambda t) - \Lambda \cos(\Lambda t) \right] = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2\Lambda^2 (\cos(\Lambda t) - \cos(2\Lambda t))$$

$$\textcircled{\Pi 1} \Rightarrow (\dots) = \cos(2\pi) - \cos(4\pi) = 0$$

$$\textcircled{\Pi 2} \Rightarrow (\dots) = \cos(\pi) - \cos(2\pi) = -1 - 1 = -2 \Rightarrow \frac{d^2}{dt^2} |C_3(t)|^2 < 0 \quad \text{μέγιστο}$$

Αν τα παράγωγα είναι 0, τότε

$$\frac{d^3 |G_3(t)|^2}{dt^3} = \vartheta \cdot (\Lambda(-1) \cdot \sin(\Lambda t) + 2\Lambda \sin(2\Lambda t))$$

$$= \vartheta \Lambda (2 \sin(2\Lambda t) - \sin(\Lambda t))$$

και για $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} = \vartheta \Lambda (2 \sin(4\pi) - \sin(2\pi)) = 0$

$$\frac{d^4 |G_3(t)|^2}{dt^4} = \vartheta \Lambda (2 \cdot 2\Lambda \cos(2\Lambda t) - \Lambda \cdot \cos(\Lambda t))$$

$$= \vartheta \Lambda^2 (4 \cos(2\Lambda t) - \cos(\Lambda t))$$

και για $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} = \vartheta \Lambda^2 (4 \cdot \underbrace{\cos(4\pi)}_1 - \underbrace{\cos(2\pi)}_1) > 0$

ΕΑΝΑ $\frac{d^2 |G_3(t)|^2}{dt^2} = \vartheta (\underbrace{\cos(\Lambda t)}_1 - \underbrace{\cos(2\Lambda t)}_1)$

για $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} \quad 1 \quad 1 = 0$

$$\frac{d^3 |G_3(t)|^2}{dt^3} = \vartheta (-\Lambda \underbrace{\sin(\Lambda t)}_0 + 2\Lambda \underbrace{\sin(2\Lambda t)}_0)$$

για $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} \quad 0 \quad 0 = 0$

$$\frac{d^4 |G_3(t)|^2}{dt^4} = \vartheta (-\Lambda^2 \underbrace{\cos(\Lambda t)}_1 + 2\Lambda \cdot 2\Lambda \underbrace{\cos(2\Lambda t)}_1) = \vartheta \Lambda^2 (4 \cos(2\Lambda t) - \cos(\Lambda t))$$

για $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} = \vartheta \Lambda^2 \cdot 3 > 0$

δηλαδή στα ελάχιστα μηδενίζονται η $|G_3(t)|^2$, καθώς και η 1η, 2η και 3η παράγωγός της!

Ενώ, η 4η παράγωγός της είναι θετική...

μοιάζει κάπως με έντονη ανύψωση (flat function)

[όπου μηδενίζονται σε κάποια στιγμή αλλά οι παράγωγοι]

2) αρα

$$I_3 = \frac{4\Omega_R^2 \Omega_R'^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

μέγιστο ποσοστό μεταβίβασης
maximum transfer percentage

14"

για $\Omega_R' = \Omega_R$

$$T_3 = \frac{2\pi}{\sqrt{2}\Omega_R} \cdot 2 = \sqrt{2} \left(\frac{2\pi}{\Omega_R} \right)$$

$$\Omega_1 = \sqrt{2\Omega_R^2} = \sqrt{2}\Omega_R$$

↓ περίοδος αλληλοίχου σιγαδύκως

$$|G_3(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4} \frac{\cos(\sqrt{2}\Omega_R t) + 1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} - \frac{2\Omega_R^4}{4\Omega_R^4} \cos\left(\frac{\sqrt{2}\Omega_R t}{2}\right)$$

$$|G_3(t)|^2 = \frac{1}{4} \left(\frac{1}{2} \cos(\underbrace{\sqrt{2}\Omega_R t}_{2\omega_3 = 2\omega_1 = \omega_2}) + \frac{1}{2} + \frac{1}{4} - \frac{1}{2} \cos\left(\underbrace{\frac{\sqrt{2}\Omega_R t}{2}}_{\omega_1 = \omega_3}\right) \right)$$

$$= \frac{1}{8} \cos(2\omega_3 t) + \frac{1}{8} + \frac{1}{4} - \frac{1}{2} \cos(\omega_3 t)$$

$$= \frac{1}{8} \left(\underbrace{\cos(2\omega_3 t) + 1}_{2\cos^2(\omega_3 t)} + 2 - 4\cos(\omega_3 t) \right)$$

$$= \frac{1}{8} \left(2\cos^2(\omega_3 t) + 2 - 4\cos(\omega_3 t) \right)$$

$$= \frac{1}{4} \left(\cos^2(\omega_3 t) + 1 - 2\cos(\omega_3 t) \right)$$

$$= \frac{1}{4} \left(1 - \cos(\omega_3 t) \right)^2$$

$$= \frac{1}{4} \left(2\sin^2\left(\frac{\omega_3 t}{2}\right) \right)^2 = \sin^4\left(\frac{\omega_3 t}{2}\right)$$

$$A_3 = \frac{4 \Omega_r^2 \Omega_{r'}^2}{(\Omega_r^2 + \Omega_{r'}^2)^2}$$

μέγιστος ρυθμός μεταβιβάσεων
 maximum transfer rate 1 → 3

$$\frac{A_3}{T_3} = \frac{A_3}{T_3} = \frac{4 \Omega_r^2 \Omega_{r'}^2}{(\Omega_r^2 + \Omega_{r'}^2)^2} \frac{\sqrt{\Omega_r^2 + \Omega_{r'}^2}}{2\pi \cdot 2}$$

Μέσες πιθανότητες παρουσίατος σε κάθε σταθμό

$$\langle |C_3(t)|^2 \rangle = \frac{\Omega_r^2 \Omega_{r'}^2}{16 \Lambda^4} \frac{1}{2} + \frac{\Omega_r^2 \Omega_{r'}^2}{16 \Lambda^4} = \frac{3 \Omega_r^2 \Omega_{r'}^2}{2 \cdot 16 \Lambda^4}$$

$$\langle |C_1(t)|^2 \rangle = \frac{\Omega_r^4}{16 \Lambda^4} \frac{1}{2} + \frac{\Omega_{r'}^4}{16 \Lambda^4} = \frac{\Omega_r^4 + 2 \Omega_{r'}^4}{2 \cdot 16 \Lambda^4}$$

$$\langle |C_2(t)|^2 \rangle = \frac{\Omega_r^2}{\Omega_r^2 + \Omega_{r'}^2} \frac{1}{2} = \frac{\Omega_r^2}{4 \Lambda^2} \frac{1}{2} = \frac{4 \Omega_r^2 \Lambda^2}{2 \cdot 16 \Lambda^4}$$

$$\langle |C_1(t)|^2 \rangle + \langle |C_2(t)|^2 \rangle + \langle |C_3(t)|^2 \rangle = \frac{3 \Omega_r^2 \Omega_{r'}^2 + \Omega_r^4 + 2 \Omega_{r'}^4 + 4 \Omega_r^2 \Lambda^2}{2 \cdot 16 \Lambda^4}$$

$$\frac{3 \Omega_r^2 \Omega_{r'}^2 + \Omega_r^4 + 2 \Omega_{r'}^4 + \Omega_r^2 (\Omega_r^2 + \Omega_{r'}^2)}{2 \cdot 16 \Lambda^4} = \frac{4 \Omega_r^2 \Omega_{r'}^2 + 2 \Omega_r^4 + 2 \Omega_{r'}^4}{2 \cdot 16 \Lambda^4}$$

$$\frac{\Omega_r^4 + 2 \Omega_r^2 \Omega_{r'}^2 + \Omega_{r'}^4}{(\Omega_r^2 + \Omega_{r'}^2)^2} = 1$$

όριση
 t_{3mean}

$$\frac{3 \Omega_r^2 \Omega_{r'}^2}{2 \cdot 16 \Lambda^4} = \frac{\Omega_r^2 \Omega_{r'}^2}{16 \Lambda^4} \cos^2(\Lambda t_{3mean}) + \frac{\Omega_r^2 \Omega_{r'}^2}{16 \Lambda^4} - \frac{2 \Omega_r^2 \Omega_{r'}^2}{16 \Lambda^4} \cos(\Lambda t_{3mean})$$

ο άπλογμένος
 χρόνος για να
 γιάσει τη
 μέγιστη τιμή

$$\frac{3}{2} = \cos^2(\Lambda t_{3mean}) + 1 - 2 \cos(\Lambda t_{3mean})$$

$$\frac{1}{2} = \cos^2(\Lambda t_{3mean}) - 2 \cos(\Lambda t_{3mean})$$

$$1 = 2 \cos^2 x - 4 \cos x$$

$$2\psi^2 - 4\psi - 1 = 0 \quad \psi = \cos x$$

$$\Delta = 16 + 4 \cdot 2 = 24$$

$$\frac{4 \pm \sqrt{24}}{2 \cdot 2} = 1 \pm \frac{\sqrt{24}}{4} = 1 \pm \sqrt{\frac{24}{16}} = 1 \pm \sqrt{\frac{3 \cdot 8}{2 \cdot 8}}$$

$$= 1 \pm \frac{\sqrt{3}}{\sqrt{2}}$$

$$\left(1 - \frac{\sqrt{3}}{\sqrt{2}}\right) = \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2}} \quad 1 + \frac{\sqrt{3}}{\sqrt{2}} > 1$$

approximate

$$\cos(\Lambda_{\text{mean}}) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\cos\left(\frac{\sqrt{\Omega_a^2 + \Omega_i^2}}{2} t_{\text{mean}}\right) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\frac{\sqrt{\Omega_a^2 + \Omega_i^2}}{2} t_{\text{mean}} = 1.797477 \dots$$

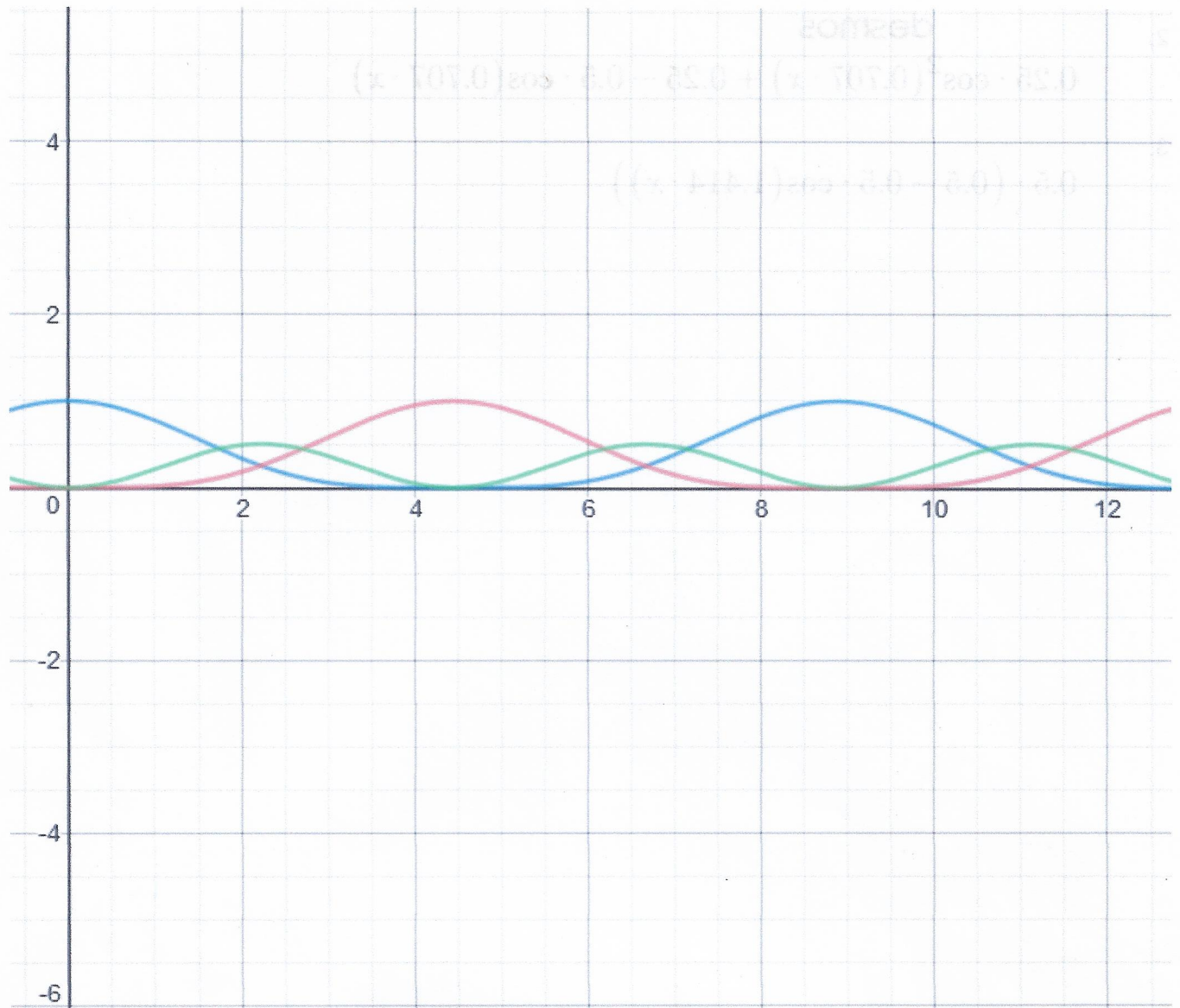
$$t_{\text{mean}} = \frac{2 \cdot 1.797477}{\sqrt{\Omega_a^2 + \Omega_i^2}}$$

mean transfer rate
μεγος πυδης μεταφορας

$$k := \frac{\langle |z_3(t)|^2 \rangle}{t_{\text{mean}}} = \frac{3 \Omega_a^2 \Omega_i^2}{2 \cdot 1614} \frac{\sqrt{\Omega_a^2 + \Omega_i^2}}{2 \cdot 1.797477}$$

$$\frac{k}{\frac{d_3}{13}} = \frac{\cancel{3 \Omega_a^2 \Omega_i^2}^2}{\cancel{2 (\Omega_a^2 + \Omega_i^2)^2}} \cdot \frac{\sqrt{\Omega_a^2 + \Omega_i^2}}{2 \cdot 1.797477} \cdot \frac{(\Omega_a^2 + \Omega_i^2)^2}{\cancel{4 \Omega_a^2 \Omega_i^2} \sqrt{\Omega_a^2 + \Omega_i^2}} \cdot \cancel{27} \cdot \cancel{2}$$
$$= \frac{3 \cdot \pi}{4 \cdot 1.797477} \approx 1.21083 \dots$$

(12)



— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 + 0.5 \cdot \cos(0.707 \cdot x)$

— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 - 0.5 \cdot \cos(0.707 \cdot x)$

— $0.5 \cdot (0.5 - 0.5 \cdot \cos(1.414 \cdot x))$
 ηρέσινα

για

$$\Omega_R = \Omega'_R = 1$$

$$2\Lambda = \sqrt{1^2 + 1^2} = \sqrt{2} \Rightarrow \Lambda = \frac{\sqrt{2}}{2}$$

ΑΣΚΗΣΗ (matlab)

Χρησιμοποιώντας το πρόγραμμα Oscillation.m να γίνει η γραφική παράσταση των μεταβολών ΔΣ

με $|\Delta| = \sqrt{3} \Omega_e \Rightarrow \alpha = \frac{1}{4}, T = \frac{1}{2} \frac{2\pi}{\Omega_e}$

ΑΣΚΗΣΗ (matlab)

Να φτιαχθεί αντίστοιχο πρόγραμμα για Τρισταθμικό Σύστημα

και να γίνει η γραφική παράσταση για $\Omega_e = \Omega_d = 1 \quad \Delta = \infty$

Θέμα: Άσκηση με matlab 2

Χρησιμοποιώντας τα προγράμματάκια, τα οποία υπάρχουν στην η-τάξη (έγγραφα) callnoRWA.m, noRWA.m, setGlobalOmegaROmegaomega.m

δοκιμάστε να συγκρίνετε και να σχολιάσετε

τη λύση με RWA (rotating wave approximation, προσέγγιση περιστρεφόμενου κύματος) με την αριθμητική λύση χωρίς προσέγγιση περιστρεφόμενου κύματος (noRWA).

Περιπτώσεις (μονάδες, ας πούμε fs):

α) $\Omega R = 1, \omega = 1, \Omega = 0.9$

$\Delta = 0.1$ ή 1 ή 10

β) $\Delta = 1, \omega = 10, \Omega = 9$

$\Omega R = 0.1$ ή 1 ή 10

γ) $\Omega R = 1, \Delta = 1,$

$\omega = 10, \Omega = 9$

$\omega = 100, \Omega = 99$

$\omega = 1000, \Omega = 999$

Θέμα: Άσκηση με matlab 1

Χρησιμοποιώντας το προγράμματάκι, το οποίο υπάρχει στην η-τάξη (έγγραφα)

Oscillations.m

δοκιμάστε να συγκρίνετε και να σχολιάσετε

τη λύση με RWA (rotating wave approximation, προσέγγιση περιστρεφόμενου κύματος)

σε συντονισμό και χωρίς συντονισμό για τέσσερις περιπτώσεις, δικής σας επιλογής, π.χ.

$\Delta = \Omega R, \Delta = 3 \Omega R, \dots$