$$\text{(A)}\begin{cases}\dot{C}_1(t)=\dfrac{i\wp\mathcal{E}_0}{2\hbar}e^{i\Delta t}C_2(t)\\[2mm]\dot{C}_2(t)=\dfrac{i\wp\mathcal{E}_0}{2\hbar}e^{-i\Delta t}C_1(t)\end{cases}$$

$\boxed{\Delta\Sigma \text{ μετά την RWA}}$

$\Omega_R=\dfrac{\wp\mathcal{E}_0}{\hbar}$

$\Delta:\omega-\Omega$

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εναλλακτική λύση των διαφορικών εξισ. με Χ.Ε.

$$\text{(A')}\begin{cases}\dot{C}_1(t)=\dfrac{i\Omega_R}{2}e^{i\Delta t}C_2(t)\\[2mm]\dot{C}_2(t)=\dfrac{i\Omega_R}{2}e^{-i\Delta t}C_1(t)\end{cases}$$

$$\ddot{C}_1(t)=\frac{i\Omega_R}{2}i\Delta e^{i\Delta t}C_2(t)+\frac{i\Omega_R}{2}e^{i\Delta t}\dot{C}_2(t)$$

$$=\frac{i\Omega_R}{2}i\Delta e^{i\Delta t}\frac{\dot{C}_1(t)}{\frac{i\Omega_R}{2}e^{i\Delta t}}+\frac{i\Omega_R}{2}\cancel{e^{i\Delta t}}\frac{i\Omega_R}{2}\cancel{e^{-i\Delta t}}C_1(t)\Rightarrow$$

$$\ddot{C}_1(t)=i\Delta\dot{C}_1(t)-\frac{\Omega_R^2}{4}C_1(t)\Rightarrow\boxed{\ddot{C}_1(t)-i\Delta\dot{C}_1(t)+\frac{\Omega_R^2}{4}C_1(t)=0}\quad(1)$$

$$\ddot{C}_2(t)=\frac{i\Omega_R}{2}(-i\Delta)e^{-i\Delta t}C_1(t)+\frac{i\Omega_R}{2}e^{-i\Delta t}\dot{C}_1(t)$$

$$=\frac{i\Omega_R}{2}(-i\Delta)e^{-i\Delta t}\frac{\dot{C}_2(t)}{\frac{i\Omega_R}{2}e^{-i\Delta t}}+\frac{i\Omega_R}{2}\cancel{e^{-i\Delta t}}\frac{i\Omega_R}{2}\cancel{e^{i\Delta t}}C_2(t)\Rightarrow$$

$$\ddot{C}_2(t)=(-i\Delta)\dot{C}_2(t)-\frac{\Omega_R^2}{4}C_2(t)\Rightarrow\boxed{\ddot{C}_2(t)+i\Delta\dot{C}_2(t)+\frac{\Omega_R^2}{4}C_2(t)=0}\quad(2)$$

ΔΛΜ $\quad C_k(t)=U_k e^{i\mu_k t}$ $\qquad$ ΔΛΜ = Δοκιμάζω Λύσεις της Μορφής

$$\dot{C}_k(t)=U_k i\mu_k e^{i\mu_k t}$$

$$\ddot{C}_k(t)=U_k(i\mu_k)^2e^{i\mu_k t}$$

$$(1)\Rightarrow\cancel{U_1}(i\mu_1)^2\cancel{e^{i\mu_1 t}}-i\Delta\cancel{U_1}i\mu_1\cancel{e^{i\mu_1 t}}+\frac{\Omega_R^2}{4}\cancel{U_1}\cancel{e^{i\mu_1 t}}=0\Rightarrow$$

$$\boxed{-\mu_1^2+\Delta\mu_1+\frac{\Omega_R^2}{4}=0}\quad \text{ΧΕ1}$$

∄ συνθήκη για το U_1

$$(2)\Rightarrow\cancel{U_2}(i\mu_2)^2\cancel{e^{i\mu_2 t}}+i\Delta\cancel{U_2}i\mu_2\cancel{e^{i\mu_2 t}}+\frac{\Omega_R^2}{4}\cancel{U_2}\cancel{e^{i\mu_2 t}}=0\Rightarrow$$

$$\boxed{-\mu_2^2-\Delta\mu_2+\frac{\Omega_R^2}{4}=0}\quad \text{ΧΕ2}$$

∄ συνθήκη για το U_2

$$\Delta_1 = \Delta^2 - 4(-1)\frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2 \qquad \lambda_{2,1} := \pm\frac{\sqrt{\Delta^2+\Omega_R^2}}{2} = \pm\lambda$$

$$\mu_1 = \frac{-\Delta \pm \sqrt{\Delta^2+\Omega_R^2}}{-2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2+\Omega_R^2}}{2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \lambda$$

$$\Delta_2 = (-\Delta)^2 - 4(-1)\frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2$$

$$\mu_2 = \frac{+\Delta \pm \sqrt{\Delta^2+\Omega_R^2}}{-2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2+\Omega_R^2}}{2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \lambda$$

Άρα, οι λύσεις θα είναι

$$\begin{cases} C_1(t) = \alpha e^{i\frac{\Delta}{2}t} e^{i\lambda t} + \beta e^{i\frac{\Delta}{2}t} e^{-i\lambda t} \\ C_2(t) = \gamma e^{-i\frac{\Delta}{2}t} e^{i\lambda t} + \delta e^{-i\frac{\Delta}{2}t} e^{i\lambda t} \end{cases}$$

$$\begin{cases} C_1(t) = e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \\ C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] \end{cases} \xRightarrow{\Delta=0} \begin{cases} C_1(t) = \alpha e^{i\lambda t} + \beta e^{-i\lambda t} \\ C_2(t) = \gamma e^{i\lambda t} + \delta e^{-i\lambda t} \end{cases}$$

Τις δοκιμάζουμε τώρα στις (Α') για $\Delta = 0$, οπότε $\lambda = \frac{\Omega_R}{2}$

$$\alpha i\lambda e^{i\lambda t} + \beta(-i\lambda)e^{-i\lambda t} = \frac{i\Omega_R}{2}\left(\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right) \Rightarrow \gamma = \alpha \quad \beta = -\delta$$

$$\gamma i\lambda e^{i\lambda t} + \delta(-i\lambda)e^{-i\lambda t} = i\frac{\Omega_R}{2}\left(\alpha e^{i\lambda t} + \delta e^{-i\lambda t}\right) \Rightarrow \gamma = \alpha \quad \beta = -\delta$$

$$\begin{cases} C_1(t) = \alpha e^{i\frac{\Omega_R}{2}t} + \beta e^{-i\frac{\Omega_R}{2}t} \\ C_2(t) = \alpha e^{i\frac{\Omega_R}{2}t} - \beta e^{-i\frac{\Omega_R}{2}t} \end{cases}$$

Έστωσαν οι αρχικές συνθήκες (Α.Σ.) $C_1(0) = 1$, $C_2(0) = 0$

$$\left.\begin{aligned} 1 &= \alpha + \beta \\ 0 &= \alpha - \beta \Rightarrow \beta = \alpha \end{aligned}\right\} \quad 2\alpha = 1 \Rightarrow \alpha = \frac{1}{2}$$

$$C_1(t) = \frac{1}{2}e^{i\frac{\Omega_R}{2}t} + \frac{1}{2}e^{-i\frac{\Omega_R}{2}t} \Rightarrow \boxed{C_1(t) = \cos\left(\frac{\Omega_R}{2}t\right)}$$

$$C_2(t) = \frac{1}{2}e^{i\frac{\Omega_R}{2}t} - \frac{1}{2}e^{-i\frac{\Omega_R}{2}t} \Rightarrow \boxed{C_2(t) = i\sin\left(\frac{\Omega_R}{2}t\right)}$$

Τις δοκιμάζουμε τώρα στις (A') για $\Delta \neq 0$ $\quad \lambda = \frac{\sqrt{\Delta^2+\Omega_R^2}}{2} \Rightarrow 4\lambda^2 = \Delta^2 + \Omega_R^2$ (3)

$$i\frac{\Delta}{2} e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] + e^{i\frac{\Delta}{2}t}\left[\alpha i\lambda e^{i\lambda t} + \beta(-i\lambda) e^{-i\lambda t}\right] =$$

$$i\frac{\Omega_R}{2} e^{i\Delta t} e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] \Rightarrow$$

$$\frac{\Delta}{2}\alpha + \alpha\lambda = \frac{\Omega_R}{2}\gamma \quad \text{και} \quad \frac{\Delta}{2}\beta - \beta\lambda = \frac{\Omega_R}{2}\delta$$

$$\frac{\Delta\alpha + 2\alpha\lambda}{\cancel{2}} = \frac{\Omega_R}{\cancel{2}}\gamma \quad \text{και} \quad \frac{\Delta\beta - 2\beta\lambda}{2} = \frac{\Omega_R}{2}\delta$$

$$\boxed{\gamma = \frac{\Delta + 2\lambda}{\Omega_R}\alpha} \quad \text{και} \quad \boxed{\delta = \frac{\Delta - 2\lambda}{\Omega_R}\beta}$$

$$-i\frac{\Delta}{2} e^{-i\frac{\Delta}{2}t}\left[\gamma e^{i\lambda t} + \delta e^{-i\lambda t}\right] + e^{-i\frac{\Delta}{2}t}\left[\gamma(i\lambda) e^{i\lambda t} + \delta(-i\lambda) e^{-i\lambda t}\right] =$$

$$i\frac{\Omega_R}{2} e^{-i\Delta t} e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \Rightarrow$$

$$-\frac{\Delta}{2}\gamma + \gamma\lambda = \frac{\Omega_R}{2}\alpha \quad \text{και} \quad -\frac{\Delta}{2}\delta - \delta\lambda = \frac{\Omega_R}{2}\beta$$

$$\frac{-\Delta\gamma + 2\gamma\lambda}{\cancel{2}} = \frac{\Omega_R}{\cancel{2}}\alpha \quad \text{και} \quad \frac{-\Delta\delta - 2\delta\lambda}{\cancel{2}} = \frac{\Omega_R}{\cancel{2}}\beta$$

$$\boxed{\alpha = \frac{-\Delta + 2\lambda}{\Omega_R}\gamma} \quad \text{και} \quad \boxed{\beta = \frac{-\Delta - 2\lambda}{\Omega_R}\delta}$$

$$\gamma = \frac{(2\lambda + \Delta)}{\Omega_R} \cdot \frac{(2\lambda - \Delta)}{\Omega_R}\gamma \Rightarrow \frac{4\lambda^2 - \Delta^2}{\Omega_R^2} = 1 \quad \text{τό οποίον ισχύει}$$

$$\delta = \frac{(\Delta - 2\lambda)}{\Omega_R} \cdot \frac{-(\Delta + 2\lambda)}{\Omega_R}\delta \Rightarrow -\frac{\Delta^2 - 4\lambda^2}{\Omega_R^2} = 1 \quad \text{τό οποίον ισχύει}$$

$$\begin{cases} C_1(t) = e^{i\frac{\Delta}{2}t}\left[\alpha e^{i\lambda t} + \beta e^{-i\lambda t}\right] \\ C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\alpha\frac{\Delta + 2\lambda}{\Omega_R} e^{i\lambda t} + \frac{\Delta - 2\lambda}{\Omega_R}\beta e^{-i\lambda t}\right] \end{cases}$$

Έστωσαν οι αρχικές συνθήκες (Α.Σ) $C_1(0)=1$, $C_2(0)=0$

$$\left.\begin{array}{l} 1 = \alpha + \beta \\ 0 = \alpha \dfrac{\Delta+2\lambda}{\Omega_R} + \beta \dfrac{\Delta-2\lambda}{\Omega_R} \Rightarrow \alpha(\Delta+2\lambda) = \beta(2\lambda-\Delta) \Rightarrow \\ \beta = \dfrac{2\lambda+\Delta}{2\lambda-\Delta}\alpha \end{array}\right\} \Rightarrow$$

$$1 = \alpha + \frac{2\lambda+\Delta}{2\lambda-\Delta}\alpha = \alpha\frac{2\lambda-\Delta+2\lambda+\Delta}{2\lambda-\Delta} \Rightarrow (2\lambda-\Delta) = \alpha 4\lambda$$

$$\Rightarrow \boxed{\alpha = \frac{2\lambda-\Delta}{4\lambda}} \qquad \beta = \frac{2\lambda+\Delta}{2\lambda-\Delta}\,\frac{2\lambda-\Delta}{4\lambda} \Rightarrow \boxed{\beta = \frac{2\lambda+\Delta}{4\lambda}}$$

$$C_1(t) = e^{i\frac{\Delta}{2}t}\left[\frac{2\lambda-\Delta}{4\lambda}e^{i\lambda t} + \frac{2\lambda+\Delta}{4\lambda}e^{-i\lambda t}\right]$$

$$C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\underbrace{\frac{\Delta+2\lambda}{\Omega_R}\,\frac{2\lambda-\Delta}{4\lambda}}\,e^{i\lambda t} + \underline{\frac{\Delta-2\lambda}{\Omega_R}\,\frac{2\lambda+\Delta}{4\lambda}}\,e^{-i\lambda t}\right]$$

$$\frac{4\lambda^2-\Delta^2}{4\Omega_R\lambda} = \frac{\Delta^2+\Omega_R^2-\Delta^2}{4\Omega_R\lambda} = \frac{\Omega_R}{4\lambda}$$

$$\frac{\Delta^2-4\lambda^2}{4\Omega_R\lambda} = \frac{\Delta^2-\Delta^2-\Omega_R^2}{4\Omega_R\lambda} = \frac{-\Omega_R}{4\lambda}$$

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$$C_2(t) = e^{-i\frac{\Delta}{2}t}\left[\frac{\Omega_R}{4\lambda}e^{i\lambda t} - \frac{\Omega_R}{4\lambda}e^{-i\lambda t}\right] \Rightarrow C_2(t) = e^{-i\frac{\Delta}{2}t}\,\frac{\Omega_R}{4\lambda}\,2i\sin(\lambda t)$$

Άρα $P_2(t) = |C_2(t)|^2 = \dfrac{\Omega_R^2}{\Delta^2+\Omega_R^2}\sin^2(\lambda t)$

$$P_1(t) = |C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2+\Delta^2}\sin^2(\lambda t)$$