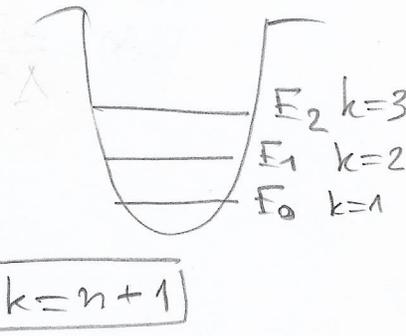


ΤΡΙΣΤΑΘΜΙΚΟ ΜΕ ΑΑΤ

$$\dot{C}_k = -\frac{i}{\hbar} \sum_k C_k(t) e^{i(\Omega_k - \Omega_k)t} U_{\varepsilon k'k}(t) \quad (1)$$

AAT  $E_n = \hbar \Omega (n + \frac{1}{2})$   $E_{n+1} - E_n = \hbar \Omega$   
 $E_0 = \frac{\hbar \Omega}{2}, E_1 = \frac{3\hbar \Omega}{2}, E_2 = \frac{5\hbar \Omega}{2}$



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2} \hat{x}^2$$

As θεωρούμε ότι γράφουμε να περιγράψουμε ότι 3 κινήσεις σταθμής με ΑΑΤ

$$U_{\varepsilon k'k}(t) = e E_0 \cos \omega t z_{k'k}$$

$$a = \left(\frac{\hbar}{m\Omega}\right)^{1/2} \quad \rightarrow 3\Delta$$

$$z_{kk} = \int d^3r \Phi_k^*(\vec{r}) z \Phi_k(\vec{r})$$

$$\Phi_n(x) = u_n(x) \exp\left(-\frac{m\Omega x^2}{2\hbar}\right)$$

$$z_{kk} = \int d^3r |\Phi_k(\vec{r})|^2 z = 0$$

k	n	$u_n(x)$
1 (A)	0	$(1/a\sqrt{\pi})^{1/2}$
2 (B)	1	$(1/2a\sqrt{\pi})^{1/2} 2 \frac{x}{a}$
3 (A)	2	$(1/8a\sqrt{\pi})^{1/2} [2 - 4(\frac{x}{a})^2]$
4 (B)	3	$(1/48a\sqrt{\pi})^{1/2} [12(\frac{x}{a}) - 8(\frac{x}{a})^3]$

$$z_{12} = \int d^3r \Phi_1^*(\vec{r}) z \Phi_2(\vec{r}) \neq 0$$

$$u_n = (1/n! 2^n a \sqrt{\pi})^{1/2} H_n\left(\frac{x}{a}\right)$$

↑ πολ. Hermite

$$z_{21} = \int d^3r \Phi_2^*(\vec{r}) z \Phi_1(\vec{r}) \neq 0$$

$$z_{13} = \int d^3r \Phi_1^*(\vec{r}) z \Phi_3(\vec{r}) = 0$$

$$z_{31} = \dots = 0$$

$$z_{23} = \int d^3r \Phi_2^*(\vec{r}) z \Phi_3(\vec{r}) \neq 0$$

γὰρ / / οὐ

$$z_{12} = z_{21} \neq z_{23} = z_{32}$$

$$z_{32} = z_{23}$$

$$U_{\varepsilon 12}(t) = e E_0 \cos \omega t z_{12} = -\mathcal{F}_{212} \cdot E_0 \cos \omega t$$

$$U_{\varepsilon 21}(t) = e E_0 \cos \omega t z_{21} = -\mathcal{F}_{221} E_0 \cos \omega t$$

$$-e z_{12} = -e z_{21} \Rightarrow \mathcal{F}_{212} = \mathcal{F}_{221} := \mathcal{F}$$

**Θέμα 3.**

- (α) Έστω ότι ένα αντικείμενο έχει τις συμμετρίες  $z$ ,  $yz$ . Ποια είναι η μικρότερη ομάδα συμμετρίας στην οποία αυτό μπορεί να ανήκει;
- (β) Έστω  $x, y$  και  $y, z$  οι άξονες του επιπέδου  $xy$  που τέμνονται στην αρχή των αξόνων  $O$ . Εκφράστε την ανακλάση σε επίπεδο που είναι κάθετο στο επίπεδο  $xy$  και διέρχεται από τη διχοτόμο της γωνίας (προσοχή...!)  $xOy$  σε μορφή πίνακα  $3 \times 3$ .

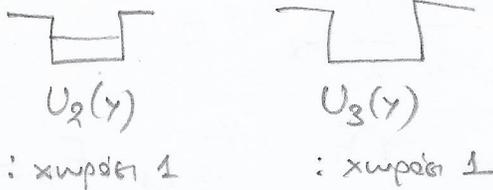
- (γ) Εξηγήστε τη μεταθετική και την αντιμεταθετική ιδιότητα. Πότε μια ομάδα ονομάζεται αβελιανή;
- (δ) Έχουν όλα τα περιοδικά κρυστάλλικα πλέγματα συμμετρία αντιστροφής; Έχει κάθε περιοδικός κρυστάλλος συμμετρία αντιστροφής; Δικαιολογήστε τις απαντήσεις σας.

**Θέμα 4.**

- Για το (ισοπλέυρο) τριγωνικό διδιστάτο κρυστάλλικο πλέγμα,
- (α) εκφράστε τα θεμελιώδη ανυστήματα με τα τοπίστας συναρτήσει των μοναδιαίων ανυσμάτων  $\hat{i}, \hat{j}$  καταλλήλου καρτεσιανού συστήματος αναφοράς,
- (β) υπολογίστε το εμβαδόν της θεμελιώδους κυψελίδας,  $S_{0K}$ ,
- (γ) ορίστε τη θεμελιώδη κυψελίδα Wigner-Seitz και αποδείξτε ότι το εμβαδόν της  $S_{WS}$ , είναι ίσο με  $S_{0K}$ .

**Θέμα 5.**

- (α) Θεωρήστε την καταστατική εξίσωση van der Waals (vdW)  $p = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$ , όπου  $a, b > 0$ . Περιγράψτε δύο μειονεκτήματα της vdW σε σχέση με την περιβαματική ιδιότητα.
- (β) Αποποιήστε την vdW θεωρώντας ότι δεν υπάρχουν απωστικές δυνάμεις. Εκφράστε την αλγοποιημένη vdW σε μορφή Virial.
- (γ) Περιγράψτε ποιοτικά, με ένα κατάλληλο, μια περιβαματική ιδιότητα  $p(V_m)$  που να περιλαμβάνει μεταρροπή φάσας αερίου-υγρού. Αποδείξτε ότι η θεμελιώδης εξίσωση είναι θετική. Τι είναι η λανθάνουσα θεμελιώδης;



n.x.

$$U_1(x) = \frac{m\omega^2}{2} x^2$$

$$\boxed{\begin{aligned} \mathcal{F}_{223} = \mathcal{F}_{232} &:= \mathcal{F}' \\ &= -e z_{23} = -e z_{32} \end{aligned}}$$

→ system

(2)

$$\dot{C}_1(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_1 - \Omega_1)t} U_{\mathcal{F}11}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_1 - \Omega_2)t} U_{\mathcal{F}12}(t) - \frac{i}{\hbar} C_3(t) e^{i(\Omega_1 - \Omega_3)t} U_{\mathcal{F}13}(t)$$

$$\boxed{\dot{C}_1(t) = +\frac{i}{\hbar} C_2(t) e^{-i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t} \quad (1)$$

$$\dot{C}_2(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_2 - \Omega_1)t} U_{\mathcal{F}21}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_2 - \Omega_2)t} U_{\mathcal{F}22}(t) - \frac{i}{\hbar} C_3(t) e^{i(\Omega_2 - \Omega_3)t} U_{\mathcal{F}23}(t)$$

$$\boxed{\dot{C}_2(t) = +\frac{i}{\hbar} C_1(t) e^{i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t + \frac{i}{\hbar} C_3(t) e^{-i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t} \quad (2)$$

$$\dot{C}_3(t) = -\frac{i}{\hbar} C_1(t) e^{i(\Omega_3 - \Omega_1)t} U_{\mathcal{F}31}(t) - \frac{i}{\hbar} C_2(t) e^{i(\Omega_3 - \Omega_2)t} U_{\mathcal{F}32}(t) - \frac{i}{\hbar} C_3(t) e^{i(\Omega_3 - \Omega_3)t} U_{\mathcal{F}33}(t)$$

$$\boxed{\dot{C}_3(t) = +\frac{i}{\hbar} C_2(t) e^{i\Omega t} \mathcal{F}' \epsilon_0 \cos \omega t} \quad (3)$$

μετασχηματισμός (M)

$$\begin{cases} C_1(t) = \tilde{C}_1(t) e^{\frac{i\Delta t}{2}} \\ C_2(t) = \tilde{C}_2(t) e^{-\frac{i\Delta t}{2}} \\ C_3(t) = \tilde{C}_3(t) e^{\frac{3i\Delta t}{2}} \end{cases}$$

$$\cos \omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

ώστε να κέρσουμε RWA

$$\textcircled{1} \Rightarrow C_1(t) = \frac{i}{\hbar} \mathcal{J} \epsilon_0 C_2(t) e^{-i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

RWA

$$\Delta := \omega - \Omega$$

$$\Omega_R = \frac{\mathcal{J} \epsilon_0}{\hbar} > 0 \text{ για } \mathcal{J} > 0 \dots$$

$$\dot{C}_1(t) = \left( \frac{i}{2\hbar} \mathcal{J} \epsilon_0 C_2(t) \right) e^{-i(\omega - \Omega)t} \Rightarrow \dot{C}_1(t) = \frac{i}{2} \Omega_R C_2(t) e^{i\Delta t} \textcircled{1'}$$

$$\textcircled{2} \Rightarrow C_2(t) = \frac{i}{\hbar} \mathcal{J} \epsilon_0 C_1(t) e^{i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

$$+ \frac{i}{\hbar} \mathcal{J}' \epsilon_0 C_3(t) e^{-i\Omega t} \frac{e^{i\omega t} - e^{-i\omega t}}{2}$$

RWA

$$\Omega_R = \frac{-\mathcal{J} \epsilon_0}{\hbar}$$

για  $\mathcal{J} < 0$

$$\Omega_R' = \frac{\mathcal{J}' \epsilon_0}{\hbar}$$

κλπ...

$$\dot{C}_2(t) = \frac{i \mathcal{J} \epsilon_0}{2\hbar} C_1(t) e^{-i(\omega - \Omega)t} + \frac{i \mathcal{J}' \epsilon_0}{2\hbar} C_3(t) e^{i(\omega - \Omega)t}$$

$$\Omega_R' = \frac{\mathcal{J}' \epsilon_0}{\hbar} > 0$$

για  $\mathcal{J}' > 0$   
...

$$\dot{C}_2(t) = \frac{i}{2} \Omega_R C_1(t) e^{-i\Delta t} + \frac{i}{2} \Omega_R' C_3(t) e^{i\Delta t} \textcircled{2'}$$

$$\textcircled{3} \Rightarrow C_3(t) = \frac{i \mathcal{J}' \epsilon_0}{\hbar} C_2(t) e^{i\Omega t} \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

RWA

ατ όριζουμε  
τις  
 $\Omega_R, \Omega_R'$   
αυτές

$$\dot{C}_3(t) = \frac{i \mathcal{J}' \epsilon_0}{2\hbar} C_2(t) e^{-i\Delta t} \textcircled{3'}$$

$$\left. \begin{aligned} \dot{C}_1(t) &= C_1(t) e^{\frac{j\Delta t}{2}} + C_1(t) \frac{j\Delta}{2} e^{\frac{j\Delta t}{2}} \\ \dot{C}_2(t) &= C_2(t) e^{-\frac{j\Delta t}{2}} + C_2(t) \left(-\frac{j\Delta}{2}\right) e^{-\frac{j\Delta t}{2}} \\ \dot{C}_3(t) &= C_3(t) e^{\frac{3j\Delta t}{2}} + C_3(t) \left(\frac{-3j\Delta}{2}\right) e^{\frac{3j\Delta t}{2}} \end{aligned} \right\} \leftarrow \textcircled{M} \quad \textcircled{4}$$

$$\textcircled{1} \textcircled{M} \quad \cancel{C_1(t) e^{\frac{j\Delta t}{2}} + C_1(t) \frac{j\Delta}{2} e^{\frac{j\Delta t}{2}}} = \frac{j}{2} \Omega_R C_2(t) e^{-\frac{j\Delta t}{2}} \quad \cancel{e^{j\Delta t}}$$

$$\boxed{\dot{C}_1(t) = -i \frac{\Delta}{2} C_1(t) + i \frac{\Omega_R}{2} C_2(t)} \quad \textcircled{1''}$$

$$\textcircled{2} \textcircled{M} \quad \cancel{C_2(t) e^{-\frac{j\Delta t}{2}} + C_2(t) \left(-\frac{j\Delta}{2}\right) e^{-\frac{j\Delta t}{2}}} = i \frac{\Omega_R}{2} C_1(t) e^{\frac{j\Delta t}{2}} \quad \cancel{e^{-j\Delta t}} \\ + i \frac{\Omega_R}{2} C_3(t) e^{\frac{3j\Delta t}{2}} \quad \cancel{e^{j\Delta t}}$$

$$\boxed{\dot{C}_2(t) = -i \frac{\Omega_R}{2} C_1(t) + i \frac{\Delta}{2} C_2(t) + i \frac{\Omega_R}{2} C_3(t)} \quad \textcircled{2''}$$

$$\textcircled{3} \textcircled{M} \quad \cancel{C_3(t) e^{\frac{3j\Delta t}{2}} + C_3(t) \left(-\frac{3j\Delta}{2}\right) e^{\frac{3j\Delta t}{2}}} = i \frac{\Omega_R}{2} C_2(t) e^{-\frac{j\Delta t}{2}} \quad \cancel{e^{-j\Delta t}}$$

$$\boxed{\dot{C}_3(t) = i \frac{\Omega_R}{2} C_2(t) + i \frac{3\Delta}{2} C_3(t)} \quad \textcircled{3''}$$

$$\begin{bmatrix} \dot{C}_1(t) \\ \dot{C}_2(t) \\ \dot{C}_3(t) \end{bmatrix} = \begin{bmatrix} -i\frac{\Delta}{2} & -i\frac{\Omega_R}{2} & 0 \\ +i\frac{\Omega_R}{2} & i\frac{\Delta}{2} & -i\frac{\Omega_R'}{2} \\ 0 & -i\frac{\Omega_R'}{2} & i\frac{3\Delta}{2} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

$\vec{x}'(t) \quad \quad \quad \tilde{A} \quad \quad \quad \vec{x}(t)$

Assume

$$\vec{x}(t) = \vec{u} e^{\tilde{\alpha} t}$$

$$\tilde{A} = -iA$$

$$\vec{x}'(t) = \tilde{A} \vec{x}(t)$$

$$\vec{u} \tilde{\alpha} e^{\tilde{\alpha} t} = \tilde{A} \vec{u} e^{\tilde{\alpha} t}$$

$$\tilde{A} \vec{u} = \tilde{\alpha} \vec{u}$$

$$\tilde{\alpha} = -i\lambda$$

$$A \vec{u} = \lambda \vec{u}$$

$$A = \begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} \end{bmatrix}$$

$$(A - \lambda I) \vec{u} = \vec{0}$$

$$\det \begin{bmatrix} \frac{\Delta}{2} - \lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} - \lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} - \lambda \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{3\Delta}{8} - \frac{2\Delta\Omega_R'^2}{8} + \frac{3\Delta - \lambda}{8} = 0 \Rightarrow 3\Delta^3 - 2\Delta\Omega_R'^2 + 3\Delta - \lambda^2 = 0$$

1724 για  $\Delta = 0$

(6)

$$\begin{bmatrix} -\lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\lambda \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow -\lambda \begin{vmatrix} -\lambda & +\frac{\Omega_R'}{2} \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} + \frac{\Omega_R}{2} \begin{vmatrix} +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} = 0$$

$$-\lambda \left[ \lambda^2 - \frac{\Omega_R'^2}{4} \right] + \frac{\Omega_R}{2} \frac{\Omega_R}{2} \lambda = 0$$

2'ris 280 y nepa 180...

$$-\lambda^3 + \lambda \frac{\Omega_R'^2}{4} + \lambda \frac{\Omega_R^2}{4} = 0 \Rightarrow \lambda \left[ -\lambda^2 + \frac{\Omega_R'^2}{4} + \frac{\Omega_R^2}{4} \right] = 0$$

$$\lambda = 0$$

(11)  
3)

$$\lambda^2 = \frac{\Omega_R^2 + \Omega_R'^2}{4}$$

$$\lambda = \pm \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} \quad \lambda_2 = 0 \quad \lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\Lambda < 0$$

$$\lambda_2 = 0$$

$$\lambda_3 = \Lambda > 0$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_e'^2}}{2} = -\Lambda < 0$$

(7)

$$\begin{bmatrix} \Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & \Lambda & -\frac{\Omega_e'}{2} \\ 0 & -\frac{\Omega_e'}{2} & \Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow \Lambda U_1 = \frac{\Omega_R}{2} U_2 \Rightarrow U_1 = \frac{\Omega_R}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} U_1 + \Lambda U_2 - \frac{\Omega_e'}{2} U_3 = 0$$

$$-\frac{\Omega_e'}{2} U_2 + \Lambda U_3 = 0 \Rightarrow \Lambda U_3 = \frac{\Omega_e'}{2} U_2 \Rightarrow U_3 = \frac{\Omega_e'}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} \frac{\Omega_R}{2\Lambda} U_2 + \Lambda U_2 - \frac{\Omega_e'}{2} \frac{\Omega_e'}{2\Lambda} U_2 = 0$$

$$\Rightarrow U_2 \left[ -\frac{\Omega_R^2}{4\Lambda} + \Lambda - \frac{\Omega_e'^2}{4\Lambda} \right] = 0$$

$$\Lambda = \frac{\sqrt{\Omega_R^2 + \Omega_e'^2}}{2}$$

$$\Rightarrow U_2 \left[ \frac{-\Omega_R^2 + 4\Lambda^2 - \Omega_e'^2}{4\Lambda} \right] = 0$$

$$\Lambda^2 = \frac{\Omega_R^2 + \Omega_e'^2}{4}$$

αν  $U_2 = 0$   
 $\Rightarrow U_1 = U_2 = U_3 = 0$   $\Rightarrow U_2$   $\neq 0$   $\Rightarrow$   $\vec{v}_1$   $\neq 0$   
 $\Rightarrow \vec{v}_1 \cdot \vec{v}_1 = 1$

$$\vec{v}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_e'}{2\Lambda} \end{bmatrix}$$

$$\vec{v}_1 = \beta \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_e'}{2\Lambda} \end{bmatrix}$$

$$\vec{v}_1 \cdot \vec{v}_1 = 1 \Rightarrow \beta^2 \left( \frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_e'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + 4\Lambda^2 + \Omega_e'^2}{4\Lambda^2} = 1$$

$$|\beta|^2 = \frac{2}{1} = 1 \Rightarrow \beta = \frac{1}{\sqrt{2}}$$

•  $\lambda_2 = 0$

(8)

$$\begin{bmatrix} 0 & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & 0 & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$-\frac{\Omega_R}{2} U_2 = 0 \quad (U_2 = 0)$$

$$-\frac{\Omega_R}{2} U_1 - \frac{\Omega_R'}{2} U_3 = 0$$

$$-\frac{\Omega_R}{2} U_1 = \frac{\Omega_R'}{2} U_3 \Rightarrow U_3 = -\frac{\Omega_R}{\Omega_R'} U_1$$

$$-\frac{\Omega_R'}{2} U_2 = 0 \quad (U_2 = 0)$$

$$\vec{V}_2 = \beta \begin{bmatrix} 1 \\ 0 \\ -\frac{\Omega_R}{\Omega_R'} \end{bmatrix} \quad \vec{V}_2 \cdot \vec{V}_2 = 1 \Rightarrow |\beta|^2 \left( 1 + \frac{\Omega_R^2}{\Omega_R'^2} \right) = 1 \Rightarrow |\beta|^2 \frac{\Omega_R'^2 + \Omega_R^2}{\Omega_R'^2} = 1$$

$$|\beta|^2 \frac{4\Lambda^2}{\Omega_R'^2} = 1 \Rightarrow \text{nx. } \beta = \frac{\Omega_R'}{2\sqrt{\Omega_R'^2 + \Omega_R^2}} = \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

•  $\lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} = \Lambda > 0$

$$\begin{bmatrix} -\Lambda & -\frac{\Omega_R}{2} & 0 \\ \frac{\Omega_R}{2} & -\Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & -\Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\vec{V}_2 = \begin{bmatrix} \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \\ 0 \\ -\frac{\Omega_R}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \end{bmatrix} = \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix}$$

$$-\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow -\frac{\Omega_R}{2} U_2 = \Lambda U_1 \Rightarrow U_1 = -\frac{\Omega_R}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} U_1 - \Lambda U_2 - \frac{\Omega_R'}{2} U_3 = 0$$

$$-\frac{\Omega_R'}{2} U_2 - \Lambda U_3 = 0 \Rightarrow -\frac{\Omega_R'}{2} U_2 = \Lambda U_3 \Rightarrow U_3 = -\frac{\Omega_R'}{2\Lambda} U_2$$

$$+\frac{\Omega_R}{2} \frac{\Omega_R}{2\Lambda} U_2 - \Lambda U_2 + \frac{\Omega_R'}{2} \frac{\Omega_R'}{2\Lambda} U_2 = 0 \Rightarrow U_2 \left[ \frac{\Omega_R^2}{4\Lambda} - \frac{4\Lambda^2}{4\Lambda} + \frac{\Omega_R'^2}{4\Lambda} \right] = 0$$

$$U_2 \left[ \frac{\Omega_R^2 + \Omega_R'^2 - 4\Lambda^2}{4\Lambda} \right] = 0 \Rightarrow U_2 \text{ sly sly sly } \text{nx. } U_2 = 1$$



$$\vec{v}_3 = \beta \begin{bmatrix} -\frac{\Omega_R}{2\Lambda} \\ 1 \\ -\frac{\Omega_I}{2\Lambda} \end{bmatrix}$$

$$\vec{v}_3 \cdot \vec{v}_3 = 1 \Rightarrow |\beta|^2 \left( \frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_I^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + \Omega_I^2 + 4\Lambda^2}{4\Lambda^2} = 1 \Rightarrow |\beta|^2 \cdot 2 = 1 \Rightarrow \alpha \times \beta = -\frac{1}{\sqrt{2}}$$

$$\vec{v}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_I}{2\Lambda} \end{bmatrix}$$

γενική λύση

$$\vec{x}(t) = \sum_{k=1}^3 \sigma_k \vec{v}_k e^{-i\lambda_k t}$$

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_I^2}$$

$$4\Lambda^2 = (\Omega_R^2 + \Omega_I^2)$$

$$16\Lambda^4 = (\Omega_R^2 + \Omega_I^2)^2$$

$$C_1(0) = 1$$

$$C_2(0) = 0$$

$$C_3(0) = 0$$

ΣΕΤΩΘΕΝ  
αρχ. συνθήκες  $\begin{matrix} \text{---} \\ \bullet \\ \text{---} \end{matrix}$  (10)

$$\vec{x}(t) = \begin{bmatrix} C_1(t) e^{-\frac{i\Delta t}{2}} \\ C_2(t) e^{i\frac{\Delta t}{2}} \\ C_3(t) e^{3i\frac{\Delta t}{2}} \end{bmatrix} = \frac{\sigma_1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_1 t} + \sigma_2 \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ \frac{\Omega_R}{2\Lambda} \end{bmatrix} e^{-i\lambda_2 t} + \frac{\sigma_3}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_3 t}$$

$$1 = \frac{\sigma_1}{\sqrt{2}} \frac{\Omega_R}{2\Lambda} + \sigma_2 \frac{\Omega_R'}{2\Lambda} + \frac{\sigma_3}{\sqrt{2}} \frac{\Omega_R}{2\Lambda}$$

$$0 = \frac{\sigma_1}{\sqrt{2}} - \frac{\sigma_3}{\sqrt{2}} \Rightarrow \boxed{\sigma_1 = \sigma_3 = \sigma}$$

$$0 = \frac{\sigma}{\sqrt{2}} \frac{\Omega_R'}{2\Lambda} - \sigma_2 \frac{\Omega_R}{2\Lambda} + \frac{\sigma}{\sqrt{2}} \frac{\Omega_R}{2\Lambda}$$

$$0 = \left(\frac{\sigma}{\sqrt{2}}\right) \left(\frac{\Omega_R'}{2\Lambda}\right) - \frac{\sigma_2 \Omega_R}{2\Lambda} + \left(\frac{\sigma}{\sqrt{2}}\right) \left(\frac{\Omega_R}{2\Lambda}\right) \Rightarrow 0 = \frac{\sigma \Omega_R'}{\sqrt{2} \Lambda} - \frac{\sigma_2 \Omega_R}{\sqrt{2} \Lambda} \Rightarrow$$

$$\frac{\sigma_2 \Omega_R}{2\Lambda} = \frac{\sigma \Omega_R'}{\sqrt{2} \Lambda} \Rightarrow \boxed{\sigma_2 = \sigma \sqrt{2} \frac{\Omega_R'}{\Omega_R}}$$

$$1 = \left(\frac{\sigma}{\sqrt{2}}\right) \left(\frac{\Omega_R}{2\Lambda}\right) + \left(\sigma \sqrt{2} \frac{\Omega_R'}{\Omega_R}\right) \left(\frac{\Omega_R'}{2\Lambda}\right) + \left(\frac{\sigma}{\sqrt{2}}\right) \left(\frac{\Omega_R}{2\Lambda}\right)$$

$$2\Lambda = \sigma \left( \frac{\Omega_R}{\sqrt{2}} + \sqrt{2} \frac{\Omega_R'^2}{\Omega_R} + \frac{\Omega_R}{\sqrt{2}} \right) = \sigma \frac{\Omega_R^2 + 2\Omega_R'^2 + \Omega_R^2}{\sqrt{2} \Omega_R} = \sigma \frac{2}{\sqrt{2}} \frac{\Omega_R^2 + \Omega_R'^2}{\Omega_R}$$

$$\sigma = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2} \sqrt{2} \Omega_R}{2(\Omega_R^2 + \Omega_R'^2)} \Rightarrow \boxed{\sigma = \frac{\Omega_R}{\sqrt{2} \sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{\Omega_R}{\sqrt{2} 2\Lambda}}$$

$$\sigma = \frac{\Omega_R}{\sqrt{2} 2\Lambda}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_I^2}} \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_I^2}} e^{+i\Delta t} + \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_I^2}} \frac{\Omega_I}{\Omega_R \sqrt{\Omega_R^2 + \Omega_I^2}} e^{-i0} + \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_I^2}} \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_I^2}} e^{-i\Delta t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_I^2)} e^{i\Delta t} + \frac{\Omega_R^2}{\Omega_R^2 + \Omega_I^2} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_I^2)} e^{-i\Delta t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{2\Omega_R^2}{2(\Omega_R^2 + \Omega_I^2)} \cos \Delta t + \frac{\Omega_R^2}{\Omega_R^2 + \Omega_I^2}$$

$$C_2(t) e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{2}\sqrt{2}} \frac{1 \cdot e^{-i\Delta t + i\Delta t}}{\sqrt{2}\sqrt{2}} \frac{\Omega_R}{\sqrt{2}\sqrt{2}} e^{-i\Delta t}$$

$$= \frac{\Omega_R}{2\sqrt{\Omega_R^2 + \Omega_I^2}} (e^{+i\Delta t} - e^{-i\Delta t})$$

$\frac{\cos + i\sin}{-\cos + i\sin}$

$$C_2(t) e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{\Omega_R^2 + \Omega_I^2}} \sin \Delta t$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_I^2} \sin^2(\Delta t) = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_I^2} \left( \frac{1}{2} - \frac{\cos(2\Delta t)}{2} \right)$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2} \cdot \left( \frac{1}{2} - \frac{\cos(2\Omega t)}{2} \right)$$
  
$$= \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_R'^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Omega_R'^2)} \cos(2\Omega t)$$

$$A_2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2}$$

maximum transfer percentage  
μέγιστο ποσοστό μεταβίβασης

$$T_2 = \frac{2\pi}{2\Omega} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

για  $\Omega_R = \Omega_R' \Rightarrow A_2 = \frac{1}{2}$

$$T_2 = \frac{2\pi}{\sqrt{2} \Omega_R} = \frac{1}{\sqrt{2}} \left( \frac{2\pi}{\Omega_R} \right)$$

↑  
περίοδος ετήσιου  
σταθμισμού

$$|C_2(t)|^2 = \frac{1}{2} \left( \frac{1}{2} - \frac{\cos(\sqrt{2} \Omega_R t)}{2} \right)$$

$$2\Omega = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2} \Omega_R$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\underbrace{\sqrt{2} \Omega_R t}_{2\omega_1 = \omega_2})$$

$$|C_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\omega_2 t)$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2 \cdot 4\Lambda^2} e^{+i\Lambda t} + \frac{\Omega_R'^2}{4\Lambda^2} + \frac{\Omega_R^2}{2 \cdot 4\Lambda^2} e^{-i\Lambda t}$$

$$C_1(t) e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{4\Lambda^2} \cos(\Lambda t) + \frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + 2 \frac{\Omega_R^2}{4\Lambda^2} \cdot \cos(\Lambda t) \cdot \frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot \cos(\Lambda t)$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos(2\Lambda t) + 1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot \cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \quad T_B = \frac{2\pi}{\Lambda} \quad \frac{T_B}{T_A} = 2 \Rightarrow \text{περιοδική κίνηση}$$

με περίοδο  $T_1 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \cdot 2 = 2T_2$

$$\begin{aligned} \left| C_1\left(\frac{2\pi}{\Lambda}\right) \right|^2 &= \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos\left(2\Lambda \frac{2\pi}{\Lambda}\right) + 1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot \cos\left(\Lambda \frac{2\pi}{\Lambda}\right) = \\ &= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 + \Omega_R'^2)^2}{16\Lambda^4} = 1 \end{aligned}$$

$$\begin{aligned} \left| C_1\left(\frac{2\pi}{2\Lambda}\right) \right|^2 &= \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\cos\left(2\Lambda \frac{2\pi}{2\Lambda}\right) + 1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot \cos\left(\Lambda \frac{2\pi}{2\Lambda}\right) \\ &= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2} \end{aligned}$$

ή τιμή της  $|C_1(t)|^2$  στο ήμισιο της περιόδου

$$\frac{d}{dt} |C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot (2 \cos(\Lambda t) \cdot (-\Lambda) \sin(\Lambda t)) + \frac{2 \Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-\Lambda) \sin(\Lambda t)$$

$$\frac{d}{dt} |C_1(t)|^2 = \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \sin(\Lambda t) \cdot [\Omega_R^2 \cos(\Lambda t) + \Omega_R'^2]$$

$$\begin{aligned} \frac{d^2}{dt^2} |C_1(t)|^2 &= \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \Lambda \cdot \cos(\Lambda t) \cdot [\Omega_R^2 \cos(\Lambda t) + \Omega_R'^2] + \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \sin(\Lambda t) \cdot \Omega_R^2 (-\Lambda) \cdot \sin(\Lambda t) \\ &= \frac{(-2\Lambda) \Omega_R^2}{16\Lambda^4} \cdot \Lambda \cdot \left\{ \Omega_R^2 \cdot \cos^2(\Lambda t) + \Omega_R'^2 \cos(\Lambda t) - \Omega_R^2 \sin^2(\Lambda t) \right\} \end{aligned}$$

$$\frac{d|G_1(t)|^2}{dt} = \frac{(-2\Lambda) \cdot \Omega_R^2}{16\Lambda^4} \cdot \sin(\Lambda t) \cdot [\Omega_R^2 \cdot \cos(\Lambda t) + \Omega_R'^2]$$

$$\frac{d^2|G_1(t)|^2}{dt^2} = \frac{(-2\Lambda^2) \cdot \Omega_R^2}{16\Lambda^4} [\Omega_R^2 \sin^2(\Lambda t) - \Omega_R^2 \cos^2(\Lambda t) - \Omega_R'^2 \cos(\Lambda t)]$$

$$\frac{d|G_1(t)|^2}{dt} = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$$

$\Downarrow$   
 $\Lambda t = n\pi, n \in \mathbb{Z}$   
Π1

προβοχή, πρέπει  
 $\Omega_R'^2 \leq \Omega_R^2$   
Π2

Π2

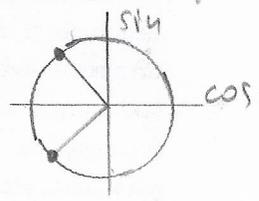
$$\frac{d^2|G_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} \left[ \Omega_R^2 \cdot \sin^2(\Lambda t) - \Omega_R^2 \frac{\Omega_R'^4}{\Omega_R^4} + \Omega_R^2 \frac{\Omega_R'^2}{\Omega_R^2} \right]$$

$$= \frac{2\Lambda^2}{16\Lambda^4} \left[ \Omega_R^4 \cdot \sin^2(\Lambda t) - \Omega_R'^4 + \Omega_R^4 \right] = \frac{2\Lambda^2}{16\Lambda^4} \cdot \Omega_R^4 \cdot \sin^2(\Lambda t) > 0$$

Οπλ. συν Π2 έχουμε έλαχιστα, με τιμή μηδενική όταν τότε:

$$|G_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot \frac{\Omega_R'^4}{\Omega_R^4} + \frac{\Omega_R^4}{16\Lambda^4} + \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-1) \frac{\Omega_R'^2}{\Omega_R^2} = \frac{2\Omega_R^4}{16\Lambda^4} - \frac{2\Omega_R'^4}{16\Lambda^4} = 0$$

Μάλιστα σε μία περίοδο  $T_1 = \frac{2\pi}{\Lambda}$ , η όσια είναι και η περίοδος του όρου  $\cos(\Lambda t)$ ,  $\exists$  2 φορές όπου έχουμε  $\cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$   
 Δηλαδή θα έχουμε 2 μηδενισμούς σε μία περίοδο  $T_1$ .



Όποτε τότε,  $G_1 = 1$  μέγιστο ποσοστό μεταβιβάσεως  
 maximum transfer percentage

Π1

$$\frac{d^2|G_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} \left[ -\Omega_R^2 \cos^2(\Lambda t) - \Omega_R'^2 \cos(\Lambda t) \right]$$

$\sin(\Lambda t) = 0 \Rightarrow \cos(\Lambda t) = \pm 1$

Π1α  $\Lambda t = 0, 2\pi, 4\pi, \dots \cos(\Lambda t) = 1$   
Π1β  $\Lambda t = \pi, 3\pi, 5\pi, \dots \cos(\Lambda t) = -1$

$\rightarrow t = 0, T_1, 2T_1, \dots$   
 $\rightarrow t = \frac{T_1}{2}, \frac{3T_1}{2}, \frac{5T_1}{2}, \dots$

Π1α  $\frac{d^2 |C_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} (-\Omega_R^2 - \Omega_R'^2) < 0 \Rightarrow$  τοπικό μέγιστο

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με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} \cdot 1 + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 + \Omega_R'^2)^2}{16\Lambda^4} = 1$$

δηλ. είναι δίκως μέγιστο

Π1β  $\frac{d^2 |C_1(t)|^2}{dt^2} = \frac{2\Lambda^2 \Omega_R^2}{16\Lambda^4} (-\Omega_R^2 + \Omega_R'^2) > 0 \Rightarrow$  τοπικό ελάχιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

ή δικά είναι η τιμή της  $|C_1(t)|^2$  στο ήμισυ της περιόδου

$$d_1 = 1 - \frac{(\Omega_R^2 - \Omega_R'^2)^2}{(\Omega_R^2 + \Omega_R'^2)^2} = \frac{4 \cdot \Omega_R^2 \cdot \Omega_R'^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

για  $\Omega_R = \Omega_R'$

$$T_1 = \frac{2\pi}{\sqrt{2} |\Omega_R|} \cdot 2 = \sqrt{2} \left( \frac{2\pi}{\Omega_R} \right)$$

↓  
περίοδος αντίστοιχου διασπασμού

Π2  $\Rightarrow d_1 = 1$   $\cos(\Lambda t) = -1 \Rightarrow \Lambda t = +\pi, 3\pi, \dots$   $t = \frac{\pi}{\Lambda} = \frac{2\pi}{2\Lambda} =$  το ήμισυ της περιόδου

Π1  $\Rightarrow d_1 = 1$  στο ήμισυ της περιόδου 1 φορά μηδενισμός...

δηλ. για  $\Omega_R = \Omega_R'$  οι Π1, Π2 ταυτίζονται

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2} \Omega_R$$

$$4\Lambda^2 = 2\Omega_R^2$$

$$16\Lambda^4 = 4\Omega_R^4$$

π.χ. όπως στο GG, GGG

$$T_{GG} \approx 20.6783 \text{ fs}$$

$$T_{GGG} = 29.2436 \text{ fs} = \sqrt{2} T_{GG}$$

$$T_{31} = 14.6218 \text{ fs} = \frac{T_{GG}}{\sqrt{2}}$$

$$T_{32} = 29.2436 \text{ fs} = T_{GGG} = \sqrt{2} T_{GG}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4} \cdot \frac{\cos(\sqrt{2}\Omega_R t) + 1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} + \frac{2\Omega_R^4}{4\Omega_R^4} \cdot \cos\left(\frac{\sqrt{2}}{2}\Omega_R t\right)$$

$$|C_1(t)|^2 = \frac{1}{4} \left( \frac{1}{2} \cdot \cos(\sqrt{2} \omega_1 t) + \frac{1}{2} \right) + \frac{1}{4} + \frac{1}{2} \cos\left(\frac{\sqrt{2}}{2} \omega_1 t\right)$$

$\underbrace{\qquad\qquad\qquad}_{2\omega_1 = \omega_2}$ 
 $\underbrace{\qquad\qquad\qquad}_{\omega_1}$

$$\frac{1}{8} \cos(2\omega_1 t) + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} \cos(\omega_1 t) =$$

$$\frac{1}{8} \left( \underbrace{\cos(2\omega_1 t) + 1 + 2 + 4 \cos(\omega_1 t)}_{2 \cos^2(\omega_1 t)} \right) = \frac{1}{8} \left( 2 \cos^2(\omega_1 t) + 2 + 4 \cos(\omega_1 t) \right)$$

$$= \frac{1}{4} \left( \cos^2(\omega_1 t) + 1 + 2 \cos(\omega_1 t) \right) = \frac{1}{4} \left( \underbrace{\cos(\omega_1 t) + 1}_{2 \cos^2\left(\frac{\omega_1 t}{2}\right)} \right)^2 = \cos^4\left(\frac{\omega_1 t}{2}\right)$$

$$C_3(t) e^{3i\frac{\Delta t}{2}} = \frac{\Omega_R}{4\Lambda} \cdot \frac{\Omega_R'}{2\Lambda} e^{+i\Lambda t} + \frac{\Omega_R}{2\Lambda} \frac{\Omega_R'}{\cancel{\Omega_R}} (-1) \frac{\Omega_R}{2\Lambda} + \frac{\Omega_R}{4\Lambda} \cdot \frac{\Omega_R'}{2\Lambda} e^{-i\Lambda t} \quad (14)$$

$$C_3(t) e^{3i\frac{\Delta t}{2}} = \frac{\Omega_R \Omega_R'}{4\Lambda^2} \cos(\Lambda t) - \frac{\Omega_R \Omega_R'}{4\Lambda^2}$$

$$|C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos^2(\Lambda t) + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - 2 \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$|C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \frac{\cos(2\Lambda t) + 1}{2} + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \quad T_B = \frac{2\pi}{\Lambda} \quad \frac{T_B}{T_A} = 2 \Rightarrow \text{η κίνηση είναι περιόδου } T$$

με περίοδο

$$T_3 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \cdot 2 = 2T_2 = T_1$$

$$\frac{d}{dt} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2 \cos(\Lambda t) (-\Lambda) \sin(\Lambda t) - 2 \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (-\Lambda) \sin(\Lambda t)$$

$$\frac{d}{dt} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2(-\Lambda) [\cos(\Lambda t) \cdot \sin(\Lambda t) - \sin(\Lambda t)]$$

$$\frac{d}{dt} |C_3(t)|^2 = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = 1$$

$$\Rightarrow t = T_3, 2T_3, \dots$$

$$(π1) \quad \Lambda t = 0, 2\pi, 4\pi, \dots \Rightarrow \sin(\Lambda t) = 0 \text{ και } \cos(\Lambda t) = 1$$

$$(π2) \quad \Lambda t = \pi, 3\pi, 5\pi, \dots \Rightarrow \sin(\Lambda t) = 0 \text{ και } \cos(\Lambda t) = -1$$

$$\Rightarrow t = \frac{T_3}{2}, \frac{3T_3}{2}, \dots$$

$$(π1) \quad |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \cdot 1 + \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} (\pm 1) = \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \mp \frac{2\Omega_R^2 \Omega_R'^2}{16\Lambda^4}$$

$$(π1) \Rightarrow |C_3(t)|^2 = 0 \text{ ελάχιστο}$$

$$(π2) \Rightarrow |C_3(t)|^2 = \frac{4\Omega_R^2 \Omega_R'^2}{16\Lambda^4} \text{ μέγιστο δίνω...}$$

$$\frac{d^2}{dt^2} |C_3(t)|^2 = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2(-\Lambda) \left[ \frac{1}{2} 2\Lambda \cos(2\Lambda t) - \Lambda \cos(\Lambda t) \right] = \frac{\Omega_R^2 \Omega_R'^2}{16\Lambda^4} 2\Lambda^2 (\cos(\Lambda t) - \cos(2\Lambda t))$$

$$(π1) \Rightarrow (\dots) = \cos(2\pi) - \cos(4\pi) = 0$$

$$(π2) \Rightarrow (\dots) = \cos(\pi) - \cos(2\pi) = -1 - 1 = -2 \Rightarrow \frac{d^2}{dt^2} |C_3(t)|^2 < 0 \text{ μέγιστο}$$

Αν τα παράγωγα είναι 0, τότε

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$$\frac{d^3 |G_3(t)|^2}{dt^3} = \vartheta \cdot (\Lambda(-1) \cdot \sin(\Lambda t) + 2\Lambda \sin(2\Lambda t))$$

$$= \vartheta \Lambda (2 \sin(2\Lambda t) - \sin(\Lambda t))$$

και για  $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} = \vartheta \Lambda (2 \sin(4\pi) - \sin(2\pi)) = 0$

$$\frac{d^4 |G_3(t)|^2}{dt^4} = \vartheta \Lambda (2 \cdot 2\Lambda \cos(2\Lambda t) - \Lambda \cdot \cos(\Lambda t))$$

$$= \vartheta \Lambda^2 (4 \cos(2\Lambda t) - \cos(\Lambda t))$$

και για  $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} = \vartheta \Lambda^2 (4 \cdot \underbrace{\cos(4\pi)}_1 - \underbrace{\cos(2\pi)}_1) > 0$

ΞΑΝΑ

$$\frac{d^2 |G_3(t)|^2}{dt^2} = \vartheta (\underbrace{\cos(\Lambda t)}_1 - \underbrace{\cos(2\Lambda t)}_1)$$

για  $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} \quad 1 \quad 1 \quad = 0$

$$\frac{d^3 |G_3(t)|^2}{dt^3} = \vartheta (-\Lambda \sin(\Lambda t) + 2\Lambda \sin(2\Lambda t))$$

για  $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} \quad \underbrace{\phantom{0}}_0 \quad \underbrace{\phantom{0}}_0 \quad = 0$

$$\frac{d^4 |G_3(t)|^2}{dt^4} = \vartheta (-\Lambda^2 \cos(\Lambda t) + 2\Lambda \cdot 2\Lambda \cos(2\Lambda t)) = \vartheta \Lambda^2 (4 \underbrace{\cos(2\Lambda t)}_1 - \underbrace{\cos(\Lambda t)}_1)$$

για  $\Lambda t = 2\pi \begin{matrix} \text{π} \\ \downarrow \end{matrix} \quad = \vartheta \Lambda^2 \cdot 3 > 0$

δηλαδή στα ελάχιστα μηδενίζονται η  $|G_3(t)|^2$ , καθώς και η 1η, 2η και 3η παράγωγός της!

ένω, η 4η παράγωγός της είναι θετική...

μοιάζει κάπως με έντονη ανάρτηση (flat function)

[όπου μηδενίζονται σε κάποιο σημείο αλλά οι παράγωγοι]

21 Δρα

$$A_3 = \frac{4\Omega_R^2 \Omega_R'^2}{(\Omega_R^2 + \Omega_R'^2)^2}$$

μέγιστο ποσοστό μεταβίβασης  
maximum transfer percentage

14"

για  $\Omega_R' = \Omega_R$

$$T_3 = \frac{2\pi}{\sqrt{2}\Omega_R} \cdot 2 = \sqrt{2} \left( \frac{2\pi}{\Omega_R} \right)$$

$$2\omega_1 = \sqrt{2}\Omega_R^2 = \sqrt{2}\Omega_R$$

↓ περίοδος αντίστοιχου διαταθμικού

$$|G_3(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4} \frac{\cos(\sqrt{2}\Omega_R t) + 1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} - \frac{2\Omega_R^4}{4\Omega_R^4} \cos\left(\frac{\sqrt{2}\Omega_R t}{2}\right)$$

$$|G_3(t)|^2 = \frac{1}{4} \left( \frac{1}{2} \cos(\sqrt{2}\Omega_R t) + \frac{1}{2} + \frac{1}{4} - \frac{1}{2} \cos\left(\frac{\sqrt{2}\Omega_R t}{2}\right) \right)$$

$\underbrace{\hspace{10em}}_{2\omega_3 = 2\omega_1 = \omega_2}$ 
 $\underbrace{\hspace{10em}}_{\omega_1 = \omega_3}$

$$= \frac{1}{8} \cos(2\omega_3 t) + \frac{1}{8} + \frac{1}{4} - \frac{1}{2} \cos(\omega_3 t)$$

$$= \frac{1}{8} \left( \cos(2\omega_3 t) + 1 + 2 - 4 \cos(\omega_3 t) \right)$$

$$= \frac{1}{8} \left( 2 \cos^2(\omega_3 t) + 2 - 4 \cos(\omega_3 t) \right)$$

$$= \frac{1}{4} \left( \cos^2(\omega_3 t) + 1 - 2 \cos(\omega_3 t) \right)$$

$$= \frac{1}{4} \left( 1 - \cos(\omega_3 t) \right)^2$$

$$= \frac{1}{4} \left( 2 \sin^2\left(\frac{\omega_3 t}{2}\right) \right)^2 = \sin^4\left(\frac{\omega_3 t}{2}\right)$$



$$2\psi^2 - 4\psi - 1 = 0$$

$$\psi = \cos x$$

$$\Delta = 16 + 4 \cdot 2 = 24$$

$$\frac{4 \pm \sqrt{24}}{2 \cdot 2} = 1 \pm \frac{\sqrt{24}}{4} = 1 \pm \sqrt{\frac{24}{16}} = 1 \pm \sqrt{\frac{3 \cdot 8}{2 \cdot 8}}$$

$$= 1 \pm \frac{\sqrt{3}}{\sqrt{2}}$$

$$1 - \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2}}$$

$$1 + \frac{\sqrt{3}}{\sqrt{2}} > 1$$

ἀπορριπόμενα

$$\cos(\Lambda t_{3mean}) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\cos\left(\frac{\sqrt{\Omega_a^2 + \Omega_k^2}}{2} t_{3mean}\right) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\frac{\sqrt{\Omega_a^2 + \Omega_k^2}}{2} t_{3mean} = 1.797477 \dots$$

$$t_{3mean} = \frac{2 \cdot 1.797477}{\sqrt{\Omega_a^2 + \Omega_k^2}}$$

mean transfer rate

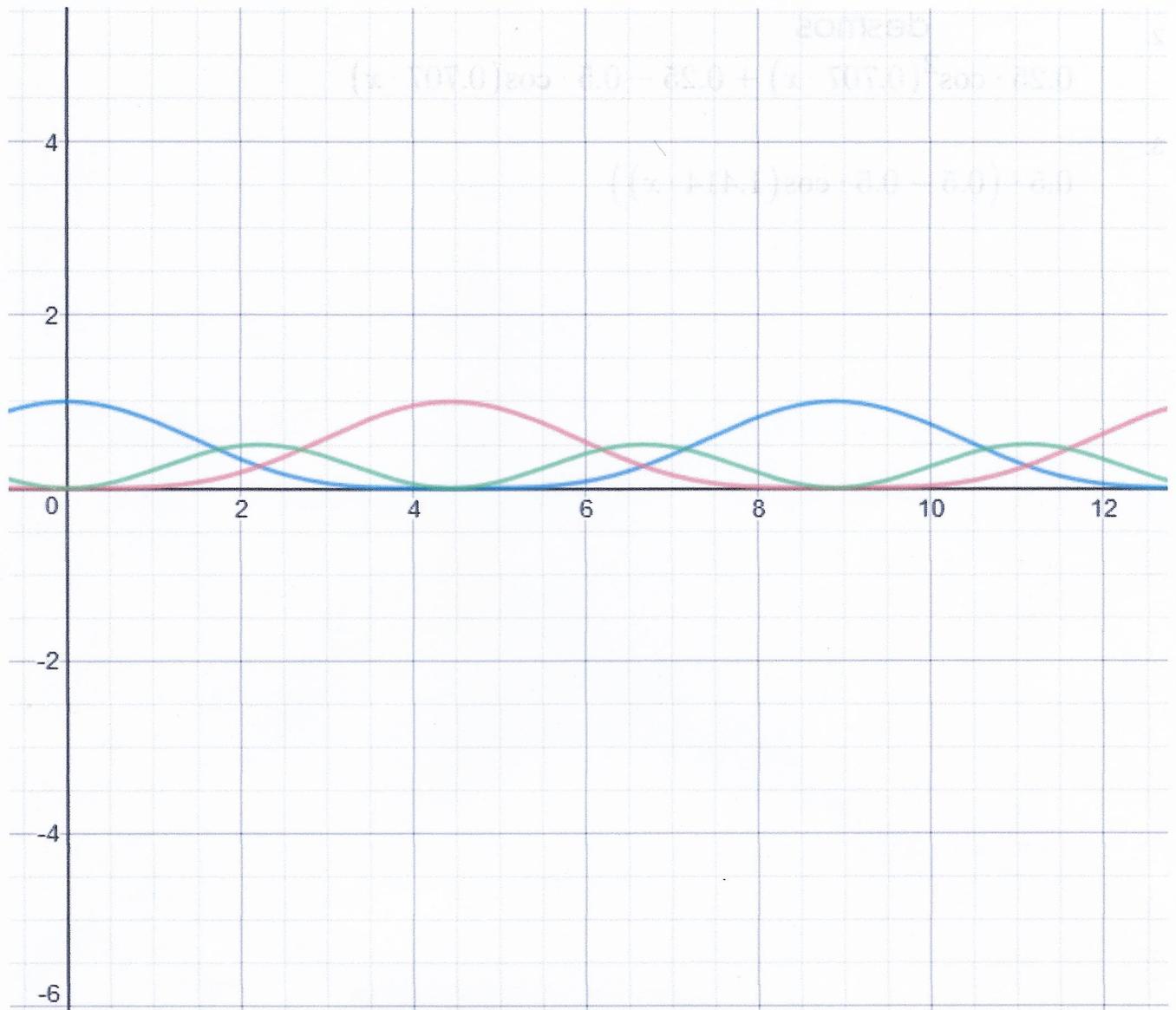
μέσος ρυθμός μεταφοράς

$$k := \frac{\langle |G_3(t)|^2 \rangle}{t_{3mean}} = \frac{3 \Omega_a^2 \Omega_k^2}{2 \cdot 1.614} \frac{\sqrt{\Omega_a^2 + \Omega_k^2}}{2 \cdot 1.797477}$$

$$\frac{k}{\frac{d_3}{T_3}} = \frac{\cancel{3 \Omega_a^2 \Omega_k^2}}{\cancel{2 (\Omega_a^2 + \Omega_k^2)^2}} \frac{\sqrt{\Omega_a^2 + \Omega_k^2}}{2 \cdot 1.797477} \cdot \frac{(\Omega_a^2 + \Omega_k^2)^2}{\cancel{4 \Omega_a^2 \Omega_k^2} \sqrt{\Omega_a^2 + \Omega_k^2}} \quad (2) \quad (2)$$

$$= \frac{3 \cdot \pi}{4 \cdot 1.797477} \approx 1.21083 \dots$$

(12)



—  $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 + 0.5 \cdot \cos(0.707 \cdot x)$

—  $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 - 0.5 \cdot \cos(0.707 \cdot x)$

—  $0.5 \cdot (0.5 - 0.5 \cdot \cos(1.414 \cdot x))$   
 η πρόστιμο

για

$$\Omega_R = \Omega'_R = 1$$

$$2\Lambda = \sqrt{1^2 + 1^2} = \sqrt{2} \Rightarrow \Lambda = \frac{\sqrt{2}}{2}$$

ΑΣΚΗΣΗ (matlab)

Χρησιμοποιώ για το πρόγραμμα Oscillator.m να γίνει η γραφική παράσταση των μεταβολών ΔΣ

με  $|\Delta| = \sqrt{3} \Omega_R \Rightarrow \alpha = \frac{1}{4}, T = \frac{1}{2} \frac{2\pi}{\Omega_R}$

ΑΣΚΗΣΗ (matlab)

Να φτιαχθεί αντίστοιχο πρόγραμμα για Τρισταθμικό Σύστημα

και να γίνει η γραφική παράσταση για  $\Omega_R = \Omega_d = 1$

$\Delta = 0$  ή  $\Delta = \Omega_R$