Είχαμε βρει, για
$\Delta = \emptyset$

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} \frac{c_1}{\sqrt{2}} e^{i\frac{\Omega_R t}{2}} + \frac{c_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \\ \frac{c_1}{\sqrt{2}} e^{i\frac{\Omega_R t}{2}} - \frac{c_2}{\sqrt{2}} e^{-i\frac{\Omega_R t}{2}} \end{bmatrix}$$

ας βάλουμε αρχικές συνθήκες $C_1(0) = \frac{1}{\sqrt{2}} e^{i\theta}$ κ $C_2(0) = \frac{1}{\sqrt{2}} e^{i\varphi} \Rightarrow$

$$|C_1(0)|^2 = \frac{1}{2} = |C_2(0)|^2$$

- $\frac{1}{\sqrt{2}} e^{i\theta} = \frac{c_1}{\sqrt{2}} + \frac{c_2}{\sqrt{2}} \Rightarrow c_1 + c_2 = e^{i\theta}$ $\qquad c_1 = \frac{e^{i\theta} + e^{i\varphi}}{2}$

- $\frac{1}{\sqrt{2}} e^{i\varphi} = \frac{c_1}{\sqrt{2}} - \frac{c_2}{\sqrt{2}} \Rightarrow \underline{c_1 - c_2 = e^{i\varphi}}$ $\qquad c_2 = \frac{e^{i\theta} - e^{i\varphi}}{2}$

$\oplus \; 2c_1 = e^{i\theta} + e^{i\varphi}$

$\ominus \; 2c_2 = e^{i\theta} - e^{i\varphi}$

- $C_1(t) = \frac{e^{i\theta} + e^{i\varphi}}{2\sqrt{2}} e^{i\frac{\Omega_R t}{2}} + \frac{e^{i\theta} - e^{i\varphi}}{2\sqrt{2}} e^{-i\frac{\Omega_R t}{2}}$ $\left.\begin{array}{c}\\\\\end{array}\right\} \Rightarrow$

- $C_2(t) = \frac{e^{i\theta} + e^{i\varphi}}{2\sqrt{2}} e^{i\frac{\Omega_R t}{2}} - \frac{e^{i\theta} - e^{i\varphi}}{2\sqrt{2}} e^{-i\frac{\Omega_R t}{2}}$

- $2\sqrt{2}\, C_1(t) = e^{i\theta} e^{i\frac{\Omega_R t}{2}} + e^{i\varphi} e^{i\frac{\Omega_R t}{2}} + e^{i\theta} e^{-i\frac{\Omega_R t}{2}} - e^{i\varphi} e^{-i\frac{\Omega_R t}{2}}$ $\left.\begin{array}{c}\\\\\end{array}\right\} \Rightarrow$

- $2\sqrt{2}\, C_2(t) = e^{i\theta} e^{i\frac{\Omega_R t}{2}} + e^{i\varphi} e^{i\frac{\Omega_R t}{2}} - e^{i\theta} e^{-i\frac{\Omega_R t}{2}} + e^{i\varphi} e^{-i\frac{\Omega_R t}{2}}$

- $2\sqrt{2}\, C_1(t) = e^{i\theta} 2\cos\left(\frac{\Omega_R t}{2}\right) + e^{i\varphi} 2i \sin\left(\frac{\Omega_R t}{2}\right)$ $\left.\begin{array}{c}\\\\\end{array}\right\} \Rightarrow$

- $2\sqrt{2}\, C_2(t) = e^{i\theta} 2i\sin\left(\frac{\Omega_R t}{2}\right) + e^{i\varphi} 2\cos\left(\frac{\Omega_R t}{2}\right)$

- $8|C_1(t)|^2 = 4\cos^2\left(\frac{\Omega_R t}{2}\right) + 4\sin^2\left(\frac{\Omega_R t}{2}\right) + e^{i\theta} 2\cos\left(\frac{\Omega_R t}{2}\right) e^{-i\varphi} 2(-i)\sin\left(\frac{\Omega_R t}{2}\right)$

$\qquad\qquad + e^{i\varphi} 2i\sin\left(\frac{\Omega_R t}{2}\right) 2 e^{-i\theta}\cos\left(\frac{\Omega_R t}{2}\right) \Rightarrow$

$2|C_1(t)|^2 = \cos^2\left(\frac{\Omega_R t}{2}\right) + \sin^2\left(\frac{\Omega_R t}{2}\right) - i e^{i\theta} e^{-i\varphi} \cos\left(\frac{\Omega_R t}{2}\right)\cdot\sin\left(\frac{\Omega_R t}{2}\right)$

$\left\{\begin{array}{c}\\\\\end{array}\right. \quad + i e^{i\varphi} e^{-i\theta} \cos\left(\frac{\Omega_R t}{2}\right)\cdot\sin\left(\frac{\Omega_R t}{2}\right)$

$\frac{1}{2}\sin(\Omega_R t)\, i\left\{ e^{i(\varphi - \theta)} - e^{-i(\varphi-\theta)} \right\} = \frac{i}{2}\sin(\Omega_R t)\, 2i\sin\psi = -\sin(\Omega_R t)\sin\psi$

$\qquad\qquad \psi := \varphi - \theta \qquad\qquad = \sin(\Omega_R t)\cdot\sin(\theta - \varphi)$

$\qquad\qquad\qquad\qquad\qquad \cos\psi \; i\sin\psi$

$\qquad\qquad\qquad\qquad\qquad -\cos\psi + i\sin\psi$

$$\boxed{|C_1(t)|^2 = \frac{1}{2} + \frac{1}{2}\sin(\Omega_R t)\sin(\theta - \varphi)} \qquad \gamma\epsilon\nu\iota\kappa\dot\omega\varsigma,\ \exists\ \tau\alpha\lambda\dot\alpha\nu\tau\omega\sigma\eta$$

$\overset{\alpha\nu}{\text{όμως}}\ \theta = \varphi \Rightarrow |C_1(t)|^2 = \frac{1}{2}$ $\ \zeta\ \not\exists\ \tau\alpha\lambda\dot\alpha\nu\tau\omega\sigma\eta$ $\qquad$ <span style="color:magenta">$2\sin A \sin B = \cos(A-B) - \cos(A+B)$</span>

Αν θέλω $\quad \frac{1}{2} + \frac{1}{2}\sin(\Omega_R t)\sin(\theta - \varphi) = \frac{1}{2} + \frac{1}{2}\cos(\Omega_R t + \frac{\pi}{2}) \Rightarrow$

$$\sin(\Omega_R t)\sin(\theta - \varphi) = -\sin(\Omega_R t) \Rightarrow$$

$$\sin(\theta - \varphi) = -1 \Rightarrow \theta - \varphi = -\frac{\pi}{2} \Rightarrow \theta = \varphi - \frac{\pi}{2}$$

○ $8|C_2(t)|^2 = 4\sin^2\left(\frac{\Omega_R t}{2}\right) + 4\cos^2\left(\frac{\Omega_R t}{2}\right) + e^{i\theta}2i\sin\left(\frac{\Omega_R t}{2}\right)\cdot e^{-i\varphi}2\cos\left(\frac{\Omega_R t}{2}\right)$

$$e^{i\varphi}2\cos\left(\frac{\Omega_R t}{2}\right)\cdot e^{-i\theta}2(-i)\sin\left(\frac{\Omega_R t}{2}\right) \Rightarrow$$

$$2|C_2(t)|^2 = 1 + \frac{1}{2}\sin(\Omega_R t)\,i\left\{e^{i\theta}e^{-i\varphi} - e^{i\varphi}e^{-i\theta}\right\}$$

$$\underset{e^{i(\theta-\varphi)}}{\quad} \quad \underset{e^{-i(\theta-\varphi)}}{\quad}$$

$$\psi := \theta - \varphi$$

$$\underset{\cos\varphi \quad i\sin\varphi}{e^{i\psi}} - \underset{-\cos\psi + i\sin\psi}{e^{-i\psi}}$$

$$2|C_2(t)|^2 = 1 + \frac{1}{2}\sin(\Omega_R t)\,i\,2i\sin\psi$$

$$\boxed{|C_2(t)|^2 = \frac{1}{2} - \frac{1}{2}\sin(\Omega_R t)\sin(\theta - \varphi)} \qquad \gamma\epsilon\nu\iota\kappa\dot\omega\varsigma,\ \exists\ \tau\alpha\lambda\dot\alpha\nu\tau\omega\sigma\eta$$

$\overset{\alpha\nu}{\text{όμως}}\ \theta = \varphi \Rightarrow |C_2(t)|^2 = \frac{1}{2}$ $\ \zeta\ \not\exists\ \tau\alpha\lambda\dot\alpha\nu\tau\omega\sigma\eta$

Αν θέλω $\quad \frac{1}{2} - \frac{1}{2}\sin(\Omega_R t)\sin(\theta - \varphi) = \frac{1}{2} - \frac{1}{2}\cos(\Omega_R t + \frac{\pi}{2}) \Rightarrow$

$$\sin(\Omega_R t)\sin(\theta - \varphi) = -\sin(\Omega_R t) \Rightarrow$$

$$\sin(\theta - \varphi) = -1 \Rightarrow \theta - \varphi = -\frac{\pi}{2} \Rightarrow \theta = \varphi - \frac{\pi}{2}$$

ΔΣ  $\Delta = 0$  αρχικές συνθήκες  $C_1(0) = 1$, $C_2(0) = 0$  εδώ $=$

απο συντονισμός

$$P_1(t) = |C_1(t)|^2 = \cos^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} + \frac{1}{2}\cos(\Omega_R t)$$

$\Delta := \omega - \Omega$

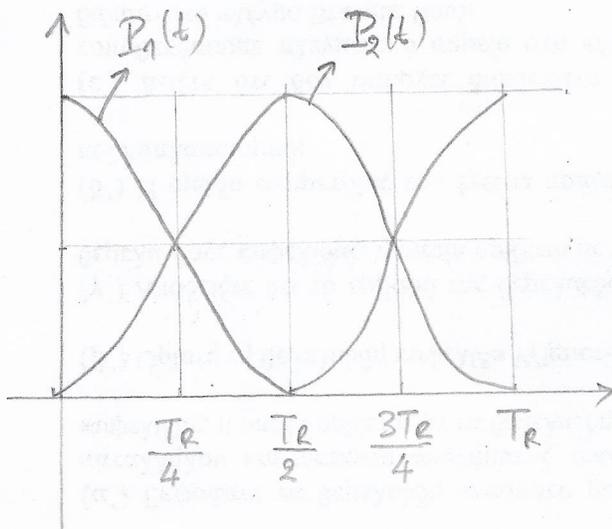$$P_2(t) = |C_2(t)|^2 = \sin^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos(\Omega_R t)$$

$\Omega_R := \dfrac{\not{p} \mathcal{E}_0}{\hbar}$  $(\not{p} > 0)$

περίοδος (period)

συχνότητα Rabi
ορίζεται θετική

$$T_R = \frac{2\pi}{\Omega_R}$$

$\Omega_R := \dfrac{-\not{p}\mathcal{E}_0}{\hbar}$  $(\not{p} < 0)$



$A_R = 1$

μέγιστο ποσοστό
μεταβιβάσεως
(maximum
transfer
percentage)

$\Omega_R$: εκφράζει την ταχύτητα
διαταραχής

$\Delta$: εκφράζει την απόσταση
των $\omega$ (ΗΜ πεδίο) ή
των $\Omega$ (ΔΣ) .

$$\langle P_1(t) \rangle = \langle |C_1(t)|^2 \rangle = \frac{1}{2}$$   μέση πιθανότητα παρουσίας στη στάθμη 1

$$\langle P_2(t) \rangle = \langle |C_2(t)|^2 \rangle = \frac{1}{2}$$   μέση πιθανότητα παρουσίας στη στάθμη 2

μέγιστος ρυθμός μεταβιβάσεως
(maximum transfer rate)

$$\boxed{\frac{A_R}{T_R} = \frac{1}{\frac{2\pi}{\Omega_R}} = \frac{\Omega_R}{2\pi}}$$

$t_{2mean} :=$ ο χρόνος, ο οποίος απαιτείται ώστε ή $P_2(t)$ να πιάσει 1η φορά

την $\langle P_2(t) \rangle$

$$\Rightarrow \frac{1}{2} - \frac{1}{2}\cos(\Omega_R t_{2mean}) = \frac{1}{2} \Rightarrow \cos(\Omega_R t_{2mean}) = 0 \Rightarrow$$

$$\Omega_R t_{2mean} = \frac{\pi}{2} \Rightarrow \boxed{t_{2mean} = \frac{\pi}{2\Omega_R}}$$

μέσος ρυθμός μεταβιβάσεως
(mean transfer rate)

$$k := \frac{\langle |C_2(t)|^2 \rangle}{t_{2mean}} = \frac{\frac{1}{2}}{\frac{\pi}{2\Omega_R}} = \frac{\Omega_R}{\pi} \Rightarrow \boxed{k = 2\frac{A_R}{T_R}}$$

- ΛΥΣΗ για $\Delta \neq 0$

για μηλε $\Im \varepsilon < 0$

$$\begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \lambda \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\lambda_{2,1} = \pm \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} = \pm \lambda$$

$$\lambda > 0$$

$$\vec{U_1} = \begin{bmatrix} \dfrac{1}{\sqrt{1+a^2}} \\ \dfrac{a}{\sqrt{1+a^2}} \end{bmatrix} \qquad a = \dfrac{\dfrac{\Delta}{2} + \dfrac{\sqrt{\Omega_R^2+\Delta^2}}{2}}{\dfrac{\Omega_R}{2}}$$

οί πράξεις υπάρχουν στο βιβλίο

$$\vec{U_2} = \begin{bmatrix} \dfrac{1}{\sqrt{1+a'^2}} \\ \dfrac{a'}{\sqrt{1+a'^2}} \end{bmatrix} \qquad a' = \dfrac{\dfrac{\Delta}{2} - \dfrac{\sqrt{\Omega_R^2+\Delta^2}}{2}}{\dfrac{\Omega_R}{2}}$$

Γενική λύση

$$\vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} C_1(t) e^{-i\frac{\Delta}{2}t} \\ C_2(t) e^{i\frac{\Delta}{2}t} \end{bmatrix} = \sum_{k=1}^{2} C_k \vec{U_k} e^{-i\lambda_k t} = C_1 \vec{U_1} e^{-i\lambda_1 t} + C_2 \vec{U_2} e^{-i\lambda_2 t}$$

$$= C_1 \begin{bmatrix} \dfrac{1}{\sqrt{1+a^2}} \\ \dfrac{a}{\sqrt{1+a^2}} \end{bmatrix} e^{-i\lambda_1 t} + C_2 \begin{bmatrix} \dfrac{1}{\sqrt{1+a'^2}} \\ \dfrac{a'}{\sqrt{1+a'^2}} \end{bmatrix} e^{-i\lambda_2 t}$$

"Εστωσαν

Αρχικές Συνθήκες $C_1(0) = 1 \quad C_2(0) = 0$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{c_1}{\sqrt{1+a^2}} + \dfrac{c_2}{\sqrt{1+a'^2}} \\ \dfrac{c_1 a}{\sqrt{1+a^2}} + \dfrac{c_2 a'}{\sqrt{1+a'^2}} \end{bmatrix} \Rightarrow \dots$$

$$c_1 = \dfrac{a'\sqrt{1+a^2}}{a'-a}$$

$$c_2 = -\dfrac{a\sqrt{1+a'^2}}{a'-a}$$

Άρα...

$$\begin{bmatrix} C_1(t) e^{-i\frac{\Delta}{2}t} \\ C_2(t) e^{i\frac{\Delta}{2}t} \end{bmatrix} = \dfrac{a'\sqrt{1+a^2}}{a'-a} \begin{bmatrix} \dfrac{1}{\sqrt{1+a^2}} \\ \dfrac{a}{\sqrt{1+a^2}} \end{bmatrix} e^{-i\lambda_1 t} - \dfrac{a\sqrt{1+a'^2}}{a'-a} \begin{bmatrix} \dfrac{1}{\sqrt{1+a'^2}} \\ \dfrac{a'}{\sqrt{1+a'^2}} \end{bmatrix} e^{-i\lambda_2 t}$$

$$C_1(t)\, e^{-i\frac{\Delta}{2}t} = \frac{a'}{a'-a}\, e^{-i\lambda_1 t} - \frac{a}{a'-a}\, e^{-i\lambda_2 t}$$

$$C_2(t)\, e^{i\frac{\Delta}{2}t} = \frac{aa'}{a'-a}\, e^{-i\lambda_1 t} - \frac{aa'}{a'-a}\, e^{-i\lambda_2 t}$$

$$\frac{a'}{a'-a} = \frac{\sqrt{\Omega_R^2+\Delta^2} - \Delta}{2\sqrt{\Omega_R^2+\Delta^2}} = \gamma_1 \qquad \frac{aa'}{a'-a} = \frac{\Omega_R}{2\sqrt{\Omega_R^2+\Delta^2}} = \gamma_3$$

$$\frac{a}{a'-a} = -\frac{\sqrt{\Omega_R^2+\Delta^2} + \Delta}{2\sqrt{\Omega_R^2+\Delta^2}} = -\gamma_2$$

$$C_1(t)\, e^{-i\frac{\Delta}{2}t} = \gamma_1 e^{-i\lambda_1 t} + \gamma_2 e^{-i\lambda_2 t} \Rightarrow C_1(t) = \left(\gamma_1 e^{-i\lambda_1 t} + \gamma_2 e^{-i\lambda_2 t}\right) e^{i\frac{\Delta}{2}t}$$

$$C_2(t)\, e^{i\frac{\Delta}{2}t} = \gamma_3\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right) \Rightarrow C_2(t) = \gamma_3\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right) e^{-i\frac{\Delta}{2}t}$$

$$|C_1(t)|^2 = \gamma_1^2 + \gamma_2^2 + \gamma_1\gamma_2\, e^{i(\lambda_1-\lambda_2)t} + \gamma_1\gamma_2\, e^{i(\lambda_2-\lambda_1)t}$$

$$|C_2(t)|^2 = \gamma_3^2\left[1 + 1 - e^{i(\lambda_1-\lambda_2)t} - e^{i(\lambda_2-\lambda_1)t}\right]$$

$$\lambda_1 - \lambda_2 = -\lambda - \lambda = -2\lambda \qquad \lambda_2 - \lambda_1 = 2\lambda$$

$$|C_1(t)|^2 = \gamma_1^2 + \gamma_2^2 + \gamma_1\gamma_2\, e^{-i2\lambda t} + \gamma_1\gamma_2\, e^{i2\lambda t} = \gamma_1^2 + \gamma_2^2 + \gamma_1\gamma_2\, 2\cos(2\lambda t)$$

$$|C_2(t)|^2 = \gamma_3^2\left[2 - e^{-i2\lambda t} - e^{i2\lambda t}\right] = \gamma_3^2\left[2 - 2\cos(2\lambda t)\right]$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{4(\Omega_R^2+\Delta^2)} \cdot 2\left(1 - \cos(2\lambda t)\right) = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)}\, 2\sin^2(\lambda t) = \frac{\Omega_R^2}{\Omega_R^2+\Delta^2} \cdot \sin^2(\lambda t)$$

$$\lambda = \frac{\sqrt{\Omega_R^2+\Delta^2}}{2}$$

$$\gamma_1^2 + \gamma_2^2 = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2+\Delta^2)}, \quad 2\gamma_1\gamma_2 = \frac{\Omega_R^2}{2(\Omega_R^2+\Delta^2)} \Rightarrow \gamma_1^2 + \gamma_2^2 + 2\gamma_1\gamma_2 = 1 \qquad 2\gamma_3^2 = 2\gamma_1\gamma_2$$

$$\gamma_3^2 = \gamma_1\gamma_2$$

$$|C_1(t)|^2 = 1 - 2\gamma_1\gamma_2 + 2\gamma_1\gamma_2\cos(2\lambda t) = 1 + 2\gamma_1\gamma_2\left[\cos(2\lambda t) - 1\right]$$

$$|C_1(t)|^2 = 1 + \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot (-2) \sin^2(\lambda t)$$

$$|C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \sin^2(\lambda t) \qquad \text{όπως αναμενόταν}$$

$$\text{διότι} \quad |C_1(t)|^2 + |C_2(t)|^2 = 1$$

## Συνοπτικώς:

$$\boxed{\begin{aligned} |C_1(t)|^2 &= 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \sin^2(\lambda t) \\[2mm] |C_2(t)|^2 &= \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \sin^2(\lambda t) \end{aligned}}$$

$$\boxed{\lambda = \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}}$$

$$|C_1(t)|^2 = 1 - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t)$$

$$\boxed{\begin{aligned} |C_1(t)|^2 &= \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2 + \Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t) = P_1(t) \\[2mm] |C_2(t)|^2 &= \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t) = P_2(t) \end{aligned}}$$

περίοδος
Ταλαντώσεων

$$\boxed{T_R = \frac{2\pi}{2\lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Delta^2}} = \frac{1}{\nu_R}}$$

NA ΔΙΟΡΘΩΘΕΙ
& ΣΤΟ ΒΙΒΛΙΟ
~~α/2αος~~
SOS ~~Ταλαντώσεων~~

$$\boxed{d_R = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}}$$

μέγιστο ποσοστό μεταβιβάσεων
(maximum transfer percentage)

$$\Delta \uparrow \Rightarrow d_R \downarrow \quad \text{και} \quad \nu_R \uparrow \ (T_R \downarrow)$$

$$\Delta = 0 \Rightarrow d_R = 1 \quad \text{και} \quad T_R = \frac{2\pi}{\Omega_R}$$

$$\langle P_1(t) \rangle = \langle |C_1(t)|^2 \rangle = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2 + \Delta^2)}$$ μέση πιθανότητα παρουσίας στη στάθμη 1

$$\langle P_2(t) \rangle = \langle |C_2(t)|^2 \rangle = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)}$$ μέση πιθανότητα παρουσίας στη στάθμη 2

μέγιστος ρυθμός μεταβιβάσεως
(maximum transfer rate)

$$\boxed{\frac{A_R}{T_R} = \frac{\Omega_R^2 \sqrt{\Omega_R^2 + \Delta^2}}{(\Omega_R^2 + \Delta^2) \, 2\pi} = \frac{\Omega_R^2}{2\pi \sqrt{\Omega_R^2 + \Delta^2}}}$$

$t_{2mean} := $ ὁ χρόνος, ὁ ὁποῖος ἀπαιτεῖται ὥστε ἡ $P_2(t)$ να πιάσει 1η φορά την $\langle P_2(t) \rangle$

$$\Rightarrow \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t_{2mean}) = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)}$$

$$\Rightarrow \cos(2\lambda t_{2mean}) = 0 \Rightarrow 2\lambda t_{2mean} = \frac{\pi}{2} \Rightarrow \boxed{t_{2mean} = \frac{\pi}{4\lambda}}$$

μέσος ρυθμός μεταβιβάσεως
(mean transfer rate)

$$\boxed{k := \frac{\langle |C_2(t)|^2 \rangle}{t_{2mean}} = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \frac{4}{\pi} \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} = \frac{\Omega_R^2}{\pi \sqrt{\Omega_R^2 + \Delta^2}}}$$

$$\Rightarrow \boxed{k = 2 \frac{A_R}{T_R}}$$

* όταν $|\Delta| \uparrow$ (δηλ απομακρυνόμαστε από το συντονισμό) $\Rightarrow$ $A_R \downarrow$
  $T_R \downarrow$

δηλ το φαινόμενο γίνεται πιο μικρό και πιο γρήγορο

* όταν $\Omega_R \ll |\Delta|$ (μικρή διαταραχή σε σχέση με την απόλυτη τιμή του απορυντονισμού)

$$P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2\left(\frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} t\right)$$

$$\approx \frac{\Omega_R^2}{\Delta^2} \sin^2\left(\frac{|\Delta|}{2} t\right)$$

$$\overset{\eta}{\underset{}{}} P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos\left(\not{2}\frac{\sqrt{\Omega_R^2 + \Delta^2}}{\not{2}} t\right)$$

$$\approx \frac{\Omega_R^2}{2\Delta^2} - \frac{\Omega_R^2}{2\Delta^2} \cos\left(|\Delta| \cdot t\right)$$

$$\Rightarrow T_R \approx \frac{2\pi}{|\Delta|} \qquad A_R \approx \frac{\Omega_R^2}{\Delta^2}$$

---

$$\lim_{\Omega_R \to 0} T_R = \frac{2\pi}{|\Delta|} \qquad \lim_{\Omega_R \to 0} A_R = 0$$

---

$\Delta=0$  $P_1(t)$    $|\Delta|=\Omega_R$  $P_1(t)$      $\Delta=0$  $P_2(t)$    $|\Delta|=\Omega_R$  $P_2(t)$

$1$

$\dfrac{1}{2}$

$t$

$\dfrac{1}{\sqrt{2}}\dfrac{2\pi}{\Omega_R}$      $\dfrac{2\pi}{\Omega_R}$

$$\text{αν η.χ. } |\Delta|=\Omega_R \Rightarrow \mathcal{A}_R=\frac{1}{2} \quad , \quad T_R=\frac{1}{\sqrt{2}}\frac{2\pi}{\Omega_R}\simeq 0.707\,\frac{2\pi}{\Omega_R}$$

$\Delta=0$  $P_1(t)$    $|\Delta|=3\Omega_R$  $P_1(t)$      $P_2(t)$  $\Delta=0$

$1$

$\dfrac{1}{2}$

$P_2(t)$  $|\Delta|=3\Omega_R$

$\dfrac{2\pi}{\Omega_R}$
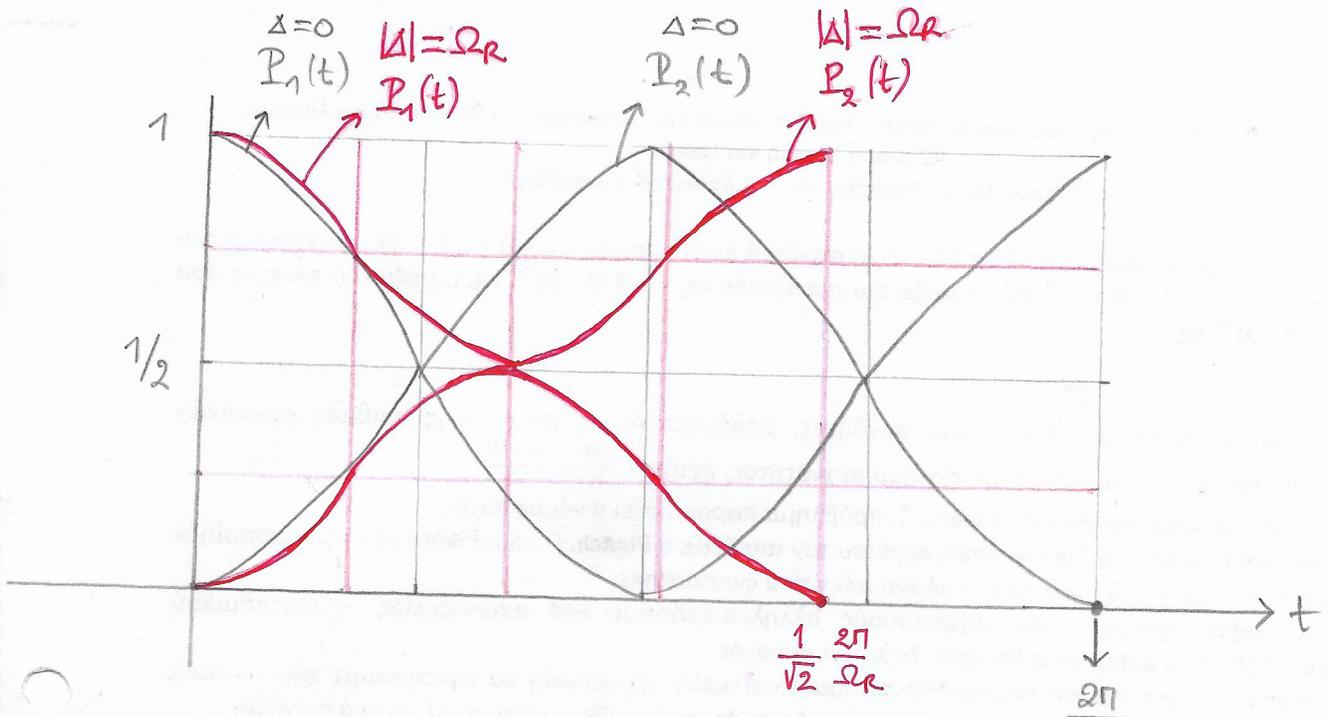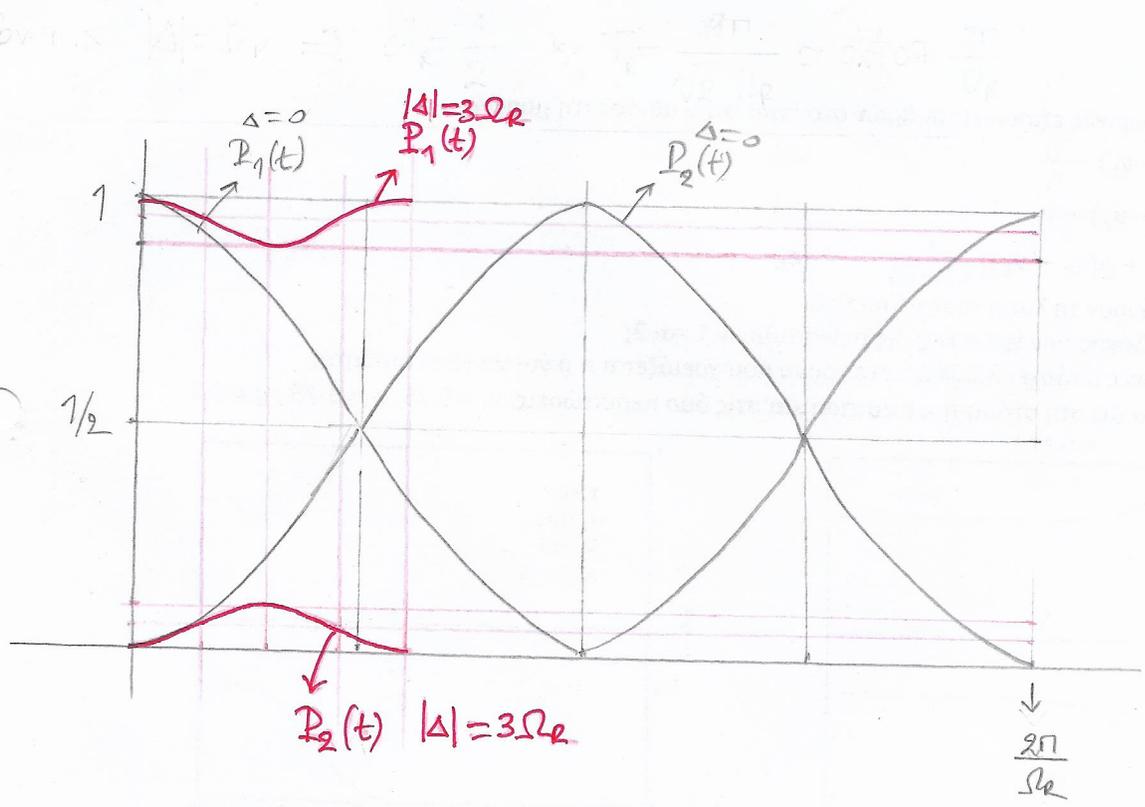
$$\text{αν η.χ. } |\Delta|=3\Omega_R \Rightarrow \mathcal{A}_R=\frac{1}{10} \quad , \quad T_R=\frac{2\pi}{\sqrt{10}\,\Omega_R}\simeq 0.316\,\frac{2\pi}{\Omega_R}$$

ΑΣΚΗΣΗ

Να λυθεί το πρόβλημα $\Delta = 0$ και αρχική συνθήκη

$$C_1(0) = \frac{1}{\sqrt{2}} = C_2(0)$$

$$\Rightarrow |C_1(0)|^2 = \frac{1}{2} = |C_2(0)|^2$$

Δηλ. το $\frac{1}{2}$ ηλεκτρόνιο βρίσκεται εξ ίσου στις δύο στάθμες τη χρονική στιγμή ⓪.

ΛΥΣΗ

Είχαμε βρεί για $\Delta = 0$

$$\begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \begin{bmatrix} \dfrac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} + \dfrac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \\ \dfrac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} - \dfrac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t} \end{bmatrix}$$

με αρχική συνθήκη $C_1(0) = \frac{1}{\sqrt{2}} = C_2(0) \Rightarrow$

$$\frac{1}{\sqrt{2}} = \frac{C_1}{\sqrt{2}} + \frac{C_2}{\sqrt{2}} \Rightarrow 1 = C_1 + C_2$$

$$\frac{1}{\sqrt{2}} = \frac{C_1}{\sqrt{2}} - \frac{C_2}{\sqrt{2}} \Rightarrow \underline{1 = C_1 - C_2}$$

$$2 = 2C_1 \Rightarrow C_1 = 1$$

$$C_2 = 0$$

$$C_1(t) = \frac{1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} \Rightarrow |C_1(t)|^2 = \frac{1}{2} = \text{σταθερό}$$

$$C_2(t) = \frac{1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} \Rightarrow |C_2(t)|^2 = \frac{1}{2} = \text{σταθερό}$$

Δηλαδή δεν υπάρχει ταλάντωση φορτίου...

ΑΣΚΗΣΗ

Να λυθεί το πρόβλημα $\Delta=0$ με αρχική συνθήκη $C_1(0)=0$, $C_2(0)=1$

δηλ. το ηλεκτρόνιο βρίσκεται αρχικά στην ΑΝΩ ΣΤΑΘΜΗ

ΛΥΣΗ

Είχαμε βρεί
για $\Delta=0$

$$C_1(t) = \frac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} + \frac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t}$$

$$C_2(t) = \frac{C_1}{\sqrt{2}} e^{i\frac{\Omega_R}{2}t} - \frac{C_2}{\sqrt{2}} e^{-i\frac{\Omega_R}{2}t}$$

με αρχική συνθήκη $C_1(0)=0$, $C_2(0)=1$ $\Rightarrow$

$$0 = \frac{C_1+C_2}{\sqrt{2}} \Rightarrow C_2 = -C_1 := -c$$

$$1 = \frac{C_1-C_2}{\sqrt{2}} \Rightarrow \sqrt{2} = c+c \Rightarrow 2c=\sqrt{2} \Rightarrow c=\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Άρα $C_1(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} - \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} = i\sin\left(\frac{\Omega_R}{2}t\right)$

$C_2(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} + \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} = \frac{1}{2}2\cos\left(\frac{\Omega_R}{2}t\right) = \cos\left(\frac{\Omega_R}{2}t\right)$

$$\hat{H} = \hat{H}_0 + U_{\varepsilon}(\vec{r}, t)$$

$$i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = \hat{H} \Psi(\vec{r}, t)$$

αρχ. συν. $\Psi(\vec{r}, 0) = \Phi(\vec{r}) =$ γνωστή

$$\Phi(\vec{r}) = \sum_k f_k \Phi_k(\vec{r})$$

$$\Psi(\vec{r}, t) = \sum_k C_k e^{-i\Omega_k t} \Phi_k(\vec{r})$$

$$\hat{H}_0 \Phi_k(\vec{r}) = E_k \Phi_k(\vec{r})$$

$$E_k = \hbar \Omega_k$$

$$\Downarrow$$

$$\dot{C}_k(t) = -\frac{i}{\hbar} \sum_k C_k(t) e^{i(\Omega_{k'} - \Omega_k)t} U_{\varepsilon k' k}(t)$$

όπου $U_{\varepsilon k' k}(t) = \int d^3 r \, \Phi_{k'}^*(\vec{r}) U_{\varepsilon}(\vec{r}, t) \Phi_k(\vec{r}) = \langle \Phi_{k'} | U_{\varepsilon}(\vec{r}, t) | \Phi_k \rangle$

$$U_{\varepsilon k' k}(t) = \begin{cases} \wp \varepsilon_0 \cos\omega t, & k \neq k' \\ 0, & k = k' \end{cases}$$

$$\wp = \wp_{z12} = -e z_{12} = -e z_{21} = \wp_{z21} \qquad \text{για } \Phi_k(\vec{r}) \text{ πραγματικές}$$

$$\vec{\varepsilon}(t) = \varepsilon_0 \hat{z} \cos\omega t \qquad \text{μετά από προσέγγιση διπόλου} \quad |\vec{r}| \sim a_0 \ll \lambda \text{ για οπτικά μήκη κύματος}$$

συχνοτική απόκλιση ΗΜ πεδίου – ΔΣ

$$\Delta := \omega - \Omega$$ detuning αποσυντονισμός

$$\odot \Delta\Sigma$$

$$\Omega := \Omega_2 - \Omega_1 = \frac{E_2 - E_1}{\hbar}$$

RWA

γρήγοροι   διώχνομε όρους   $e^{\pm i(\omega + \Omega)t}$

αργοί   κρατάμε όρους   $e^{\pm i(\omega - \Omega)t}$

$$\Omega_R := \frac{\wp \varepsilon_0}{\hbar} \qquad \text{συχνότητα Rabi}$$

$\downarrow$ ισχύς διαταραχής

$$\Downarrow$$

$$\dot{C}_1(t) = \frac{i}{2} \Omega_R e^{i\Delta t} C_2(t)$$

$$\dot{C}_2(t) = \frac{i}{2} \Omega_R e^{-i\Delta t} C_1(t)$$

χρονικώς εξαρτώμενοι συντελεστές

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Ⓜ $\begin{cases} C_1(t) = \mathcal{C}_1(t) e^{i\frac{\Delta}{2} t} \\ C_2(t) = \mathcal{C}_2(t) e^{-i\frac{\Delta}{2} t} \end{cases}$ $\longrightarrow$ $\begin{bmatrix} \dot{\mathcal{C}}_1(t) \\ \dot{\mathcal{C}}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{i\Delta}{2} & \frac{i\Omega_R}{2} \\ \frac{i\Omega_R}{2} & \frac{i\Delta}{2} \end{bmatrix} \begin{bmatrix} \mathcal{C}_1(t) \\ \mathcal{C}_2(t) \end{bmatrix}$

χρονικώς ανεξάρτητοι συντελεστές

$$\dot{\vec{x}}(t) = \tilde{A} \vec{x}(t)$$

ΔΛΜ $\quad \vec{x}(t) = \vec{v} e^{\tilde{\lambda} t}$

$$\tilde{A} \vec{v} = \tilde{\lambda} \vec{v}$$

$$A = \begin{bmatrix} \frac{\Delta}{2} & -\frac{\Omega_R}{2} \\ -\frac{\Omega_R}{2} & -\frac{\Delta}{2} \end{bmatrix}$$

$$\overset{\gamma}{\tilde{\lambda}} \quad A\vec{v} = \lambda \vec{v} \qquad \tilde{A} = -iA$$
$$\tilde{\lambda} = -i\lambda$$

γενική λύση $\qquad \vec{x}(t) = \sum_k C_k \vec{v}_k e^{-i\lambda_k t}$

$$A\vec{U} = \lambda \vec{U}$$

$$\begin{bmatrix} \dfrac{\Delta}{2} & -\dfrac{\Omega_R}{2} \\[2mm] -\dfrac{\Omega_R}{2} & -\dfrac{\Delta}{2} \end{bmatrix} \begin{bmatrix} U_1 \\[2mm] U_2 \end{bmatrix} = \lambda \begin{bmatrix} U_1 \\[2mm] U_2 \end{bmatrix} \iff \begin{bmatrix} \dfrac{\Delta}{2}-\lambda & -\dfrac{\Omega_R}{2} \\[2mm] -\dfrac{\Omega_R}{2} & -\dfrac{\Delta}{2}-\lambda \end{bmatrix} \begin{bmatrix} U_1 \\[2mm] U_2 \end{bmatrix} = \begin{bmatrix} 0 \\[2mm] 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow \lambda_{2,1} = \pm \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}$$

$$\lambda_{2,1} = \pm \frac{\Omega_R}{2} \quad \text{στην περίπτωση συντονισμού } (\Delta=0)$$

Θα χρησιμοποιήσουμε άρχικές συνθήκες $C_1(0)=1$, $C_2(0)=0$ $\left(\dfrac{t=0}{\cdot}\right)$

$$\Downarrow$$

$$C_1(0) = 1, \quad C_2(0) = 0$$

ΛΥΣΗ για $\Delta = 0$

$$\vec{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \lambda_1 = -\frac{\Omega_R}{2}$$

$$\vec{V}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \lambda_2 = \frac{\Omega_R}{2}$$

$$\vec{x}(t) = \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \sum_{k=1}^{2} C_k \vec{V}_k e^{i\lambda_k t} = \frac{C_1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{+i\frac{\Omega_R}{2}t} + \frac{C_2}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-i\frac{\Omega_R}{2}t}$$

αρχ. συν.

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{C_1 + C_2}{\sqrt{2}} \\[3mm] \dfrac{C_1 - C_2}{\sqrt{2}} \end{bmatrix} \Rightarrow C_1 = C_2 = \frac{\sqrt{2}}{2}$$

$$\Rightarrow C_1(t) = \cos\left(\frac{\Omega_R t}{2}\right) \qquad \Rightarrow P_1(t) = \cos^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} + \frac{1}{2}\cos(\Omega_R t)$$

$$C_2(t) = i\sin\left(\frac{\Omega_R t}{2}\right) \qquad P_2(t) = \sin^2\left(\frac{\Omega_R t}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos(\Omega_R t)$$
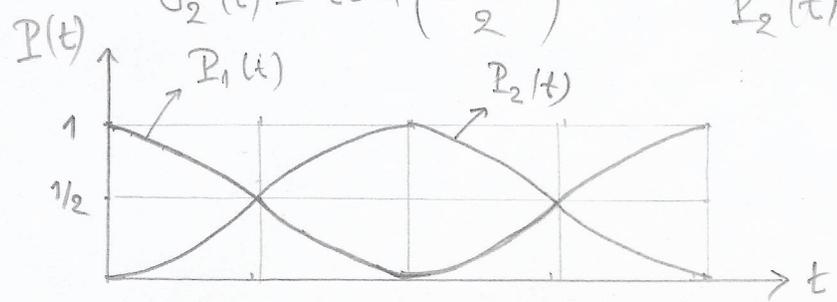


$$T = \frac{2\Pi}{\Omega_R} \quad \text{περίοδος}$$

$$\mathcal{A} = 1 \quad \text{πλάτος}$$

♪ ΛΥΣΗ για $\Delta \neq 0$

$$\vec{U_1} = \begin{bmatrix} \dfrac{1}{\sqrt{1+a^2}} \\[2mm] \dfrac{a}{\sqrt{1+a'^2}} \end{bmatrix}$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} := -\lambda < 0$$

$$a = \frac{\dfrac{\Delta}{2} + \dfrac{\sqrt{\Omega_R^2 + \Delta^2}}{2}}{\dfrac{\Omega_R}{2}}$$

$$\vec{V_2} = \begin{bmatrix} \dfrac{1}{\sqrt{1+a'^2}} \\[2mm] \dfrac{a'}{\sqrt{1+a'^2}} \end{bmatrix}$$

$$\lambda_2 = +\frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} := \lambda > 0$$

(C) ΒΙΒΛΙΟ

$$a' = \frac{\dfrac{\Delta}{2} - \dfrac{\sqrt{\Omega_R^2 + \Delta^2}}{2}}{\dfrac{\Omega_R}{2}}$$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos 2x = \cos^2 x - \sin^2 x$

$\left.\begin{array}{l}\sin 2x = 2\sin x \cos x \\ \cos^2 x + \sin^2 x = 1\end{array}\right\rangle \cos 2x = 2\cos^2 x - 1$

$$P_1(t) = |C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}\sin^2(\lambda t)$$

$\cos^2 x = \dfrac{\cos 2x + 1}{2}$

$\sin^2 x = 1 - \cos^2 x$

$$P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}\sin^2(\lambda t)$$

$\sin^2 x = 1 - \dfrac{\cos 2x + 1}{2}$

$\sin^2 x = \dfrac{1}{2} - \dfrac{\cos 2x}{2}$

$$P_1(t) = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}\left(\frac{1}{2} - \frac{\cos(2\lambda t)}{2}\right) = \frac{\Omega_R^2 + 2\Delta^2}{2(\Omega_R^2 + \Delta^2)} + \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)}\cos(2\lambda t)$$

$$P_2(t) = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}\left(\frac{1}{2} - \frac{\cos(2\lambda t)}{2}\right) = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)}\cdot\cos(2\lambda t)$$

ΝΑ ΔΙΟΡΕΘΕΙ
~~Αλλαγ~~ 4
ΣΤΟ ΒΙΒΛΙΟ
SOS

$$\mathcal{U} = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}$$

μέγιστο ποσοστό μεταβιβάστων
maximum transfer percentage

για $\Delta = 0 \Rightarrow \mathcal{U} = 1$

$T = \dfrac{2\pi}{\Omega_R}$

περίοδος

$$T = \frac{2\pi}{2\lambda} = \frac{2\pi}{2\frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Delta^2}}$$

όπως αναμενόταν

$$:= \frac{A}{T} = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \frac{\sqrt{\Omega_R^2 + \lambda^2}}{2\pi} = \frac{\Omega_R^2}{2\pi \sqrt{\Omega_R^2 + \Delta^2}}$$

ὁρισμός

$t_{2\,mean}$

ὁ ἀπαιτούμενος χρόνος

γιὰ νὰ πιάσει τη μέση τιμή

$\downarrow$

$$\frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t_{2\,mean})$$

$$\Rightarrow \cos(2\lambda t_{2\,mean}) = 0 \Rightarrow 2\lambda t_{2\,mean} = \frac{\pi}{2}$$

$$\Rightarrow t_{2\,mean} = \frac{\pi}{4\lambda} = \frac{2\pi}{4 \cdot \sqrt{\Omega_R^2 + \Delta^2}}$$

mean transfer rate

$$k := \frac{\langle |C_2(t)|^2 \rangle}{t_{2\,mean}} = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \frac{4\sqrt{\Omega_R^2 + \Delta^2}}{2\pi} = \frac{\Omega_R^2}{\sqrt{\Omega_R^2 + \Delta^2} \cdot \pi}$$

$$\frac{k}{\frac{A}{T}} = 2 \Rightarrow k = 2\frac{A}{T}$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos(2x)}{2}$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2(\lambda t) =$$

$$= \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} - \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)} \cdot \cos(2\lambda t) \Rightarrow$$

$$\langle |C_2(t)|^2 \rangle = \frac{\Omega_R^2}{2(\Omega_R^2 + \Delta^2)}$$

$$P_1(t) = |C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2(\lambda t) \qquad \lambda = \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2}$$

$$P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2(\lambda t)$$

$$\sin^2(\lambda t) = \frac{1}{2} - \frac{\cos(2\lambda t)}{2} \qquad A = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2}$$

$$\langle |C_1(t)|^2 \rangle = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \frac{1}{2} \qquad T = \frac{2\pi}{2\lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Delta^2}}$$

$$\langle |C_2(t)|^2 \rangle = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \frac{1}{2}$$

$$\langle |C_1(t)|^2 \rangle + \langle |C_2(t)|^2 \rangle = 1$$

αν $\Delta = 0 \Rightarrow \langle |C_1(t)|^2 \rangle = \frac{1}{2} = \langle |C_2(t)|^2 \rangle$

αλλά για $\Delta \neq 0$ οι πιθανότητες δεν είναι ίσες

π.χ. με $\Delta = \sqrt{3}\,\Omega_R$

$$\langle |C_1(t)|^2 \rangle = 1 - \frac{\Omega_R^2}{4\Omega_R^2} \cdot \frac{1}{2} = \frac{7}{8}$$

$$\langle |C_2(t)|^2 \rangle = \frac{\Omega_R^2}{4\Omega_R^2} \cdot \frac{1}{2} = \frac{1}{8}$$

pure maximum transfer rate $:= \frac{A}{T} = \frac{\Omega_R^2}{(\Omega_R^2 + \Delta^2)} \cdot \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2\pi} = \frac{\Omega_R^2}{2\pi} \cdot \frac{1}{\sqrt{\Omega_R^2 + \Delta^2}}$

$t_{2mean} \xrightarrow{\text{ορισμός}} \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \frac{1}{2} = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \sin^2(\lambda t)_{2mean} \Rightarrow \sin^2(\lambda t)_{2mean} = \frac{1}{2}$

$$\sin(\lambda t_{2mean}) = \pm \frac{\sqrt{2}}{2} = \pm \sin\left(\frac{\pi}{4}\right)$$

$\lambda t_{2mean} = \frac{\pi}{4} \Rightarrow \frac{\sqrt{\Omega_R^2 + \Delta^2}}{2} \cdot t_{2mean} = \frac{\pi}{4} \Rightarrow t_{2mean} = \frac{\pi}{2} \cdot \frac{1}{\sqrt{\Omega_R^2 + \Delta^2}}$

$$k := \frac{\langle |C_2(t)|^2 \rangle}{t_{2mean}} = \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \cdot \frac{1}{2} \cdot \frac{2\sqrt{\Omega_R^2 + \Delta^2}}{\pi} = \frac{\Omega_R^2}{\pi} \cdot \frac{1}{\sqrt{\Omega_R^2 + \Delta^2}}$$

$$k = 2 \frac{A}{T}$$