Abstract

Mauro Zonta and Gad Freudenthal

Nicomachus of Gerasa in Spain, ca. 1100: Abraham bar Ḥiyya's Testimony

In the 1120s, the Jewish philosopher and scientist Abraham Bar Ḥiyya (d. ca. 1136) wrote a Hebrew encyclopedia, *Yesodei ha-tevunah u-migdal ha-ʾemunah;* a detailed chapter on mathematics is extant from this work. One source of its account of the nature of number has long been known to be Nicomachus of Gerasa's *Introduction to Arithmetic*. Three Arabic versions of Nicomachus are known to have existed: Ḥabib Ibn Bahrīz's translation from the Syriac, now lost; al-Kindī's revision of the latter, lost in Arabic but extant in a Hebrew translation by Qalonymos ben Qalonymos; and Thābit Ibn Qurra's translation from the Greek. A close comparison of Bar Ḥiyya's text with the others and with the Greek original leads to the conclusion that Bar Ḥiyya used Ḥabib Ibn Bahrīz's Arabic translation in its unrevised form and had no contact with Thābit Ibn Qurra's translation.

Mauro Zonta and Gad Freudenthal[1]

Nicomachus of Gerasa in Spain, Circa 1100: Abraham Bar Ḥiyya's Testimony

Abraham Bar Ḥiyya (d. ca. 1136) was the first scholar in Sefarad to engage in a systematic effort to transmit Greco-Arabic science and philosophy to the Jews north of the Pyrenees, whose cultural tongue was Hebrew. One of the works he composed with this in mind was his encyclopedia *Yesodei ha-tevunah u-migdal ha-ʾemunah* (The Foundations of Reason and Tower of Faith),[2] which, according to Bar Ḥiyya himself, was based

1 The authors are very grateful to a referee for *Aleph* whose thorough skepticism and meticulous critique of an early draft allowed them to improve the presentation of the argument.

2 *La obra enciclopédica "Yesodé ha-tebuná u-migdal ha-emuná"* de R. Abraham bar Ḥiyya ha-Bargeloni, Edición crítica con traducción, ed. by José Mª Millás Vallicrosa (Madrid-Barcelona, 1952). For a study of its mathematical section, see Martin Levey, "The Encyclopedia of Abraham Savasorda: A Departure in Mathematical Methodology," *Isis* 43 (1952), 257–64; for an overview see Mercedes Rubio, "The First Hebrew Encyclopedia of Science: Abraham bar Ḥiyya's *Yesodei ha-Tevunah u-Migdal ha-Emunah*," in *The Medieval Hebrew Encyclopedias of Science and Philosophy*, ed. Steven Harvey (Dordrecht, Boston, London: Kluwer Academic Publisher, 2000), 140–53.

on sources in Arabic. The extant part of *Yesodei ha-tevunah* contains a relatively detailed chapter on mathematics, including an account of the nature of number. The present article seeks to shed some new light on the literary sources of one section of this account.

José Maria Millás Vallicrosa, the editor of *Yesodei ha-tevunah u-migdal ha-ʾemunah*, noted that the section entitled "On the Science of Number" is based on the *Introduction to Arithmetic* by Nicomachus of Gerasa.[3] He did not comment, however, on the precise source through which Bar Ḥiyya became acquainted with Nicomachus.[4] Recent progress in our knowledge of the complex and multi-layered reception and transmission of Nicomachus' work in Arabic allows us to answer this question.

There are two distinct branches of the transmission of Nicomachus' work in Arabic. The first begins with Ḥabib Ibn Bahrīz's translation from Syriac into Arabic. The second derives from the slightly later translation by Thābit Ibn Qurra, made directly from the Greek. In what follows we schematically recapitulate what is known about these two traditions.[5]

The first line of transmission comprises (with some simplification) four phases, straddling some four centuries: (1) In the late eighth century (as it seems), Nicomachus' *Introduction to Arithmetic* was translated from Greek into Syriac. This anonymous translation, no longer extant, was apparently rather literal. (2) A few decades later, and certainly before 822, Ḥabib Ibn Bahrīz, a noted Nestorian metropolitan and translator, rendered the work from Syriac into Arabic. Ḥabib translated some sentences literally, but paraphrased others and included additional material—some of it his own work, some of it derived from other sources. Ḥabib's translation is not extant, but several quotations from it are imbedded in *Taʾriḫ Yaʿqūbī* (completed before 873).[6] (3) Ḥabib's Arabic version of the *Introduction to Arithmetic* soon reached al-Kindī (d. ca. 870). Realizing that Ḥabib had tinkered with the text and introduced numerous "errors," he set out to "revise" the

text. He apparently did not commit his revision to writing himself, but had this done by one of his students, who put it in circulation. In this new version, some paragraphs are explicitly ascribed to al-Kindī

3 Text in Richard Hoche, ed., *Nicomachi Geraseni Pythagorei Introductionis arithmeticae Libri II* (Leipzig: Teubner, 1866); English translation in Nicomachus of Gerasa, *Introduction to Arithmetic*. Translated into English by Martin Luther D'Ooge, with Studies in Greek Arithmetic by Frank Egleston Robbins and Louis Charles Karpinski (New York: MacMillan, 1926); see pp. 71–87 on Nicomachus' life and works.

4 *La obra enciclopédica*, p. 14. On Bar Ḥiyya's mathematical vocabulary, see Gad B.-A. Sarfatti, *Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages* (Jerusalem: Magnes Press, 1968), 61–129 (Heb.).

5 We draw on Gad Freudenthal and Tony Lévy, "De Gérase à Bagdad: Ibn Bahrīz, al-Kindi, et leur recension arabe de l'*Introduction arithmétique* de Nicomaque, d'après la version hébraïque de Qalonymos ben Qalonymos d'Arles," in Régis Morelon and Ahmed Hasnawi, eds., *De Zénon d'Élée à Poincaré. Recueil d'études en hommage à Roshdi Rashed* (Louvain and Paris: Éditions Peeters 2004), 479–544; Gad Freudenthal, "L'Introduction arithmétique de Nicomaque de Gérase dans les traditions syriaque, arabe et hébraïque," in R. Goulet, ed., *Dictionnaire des philosophes antiques*, vol. 4 (Paris: CNRS Édition, 2005), 690–94; Gad Freudenthal and Mauro Zonta, "Remnants of Ḥabib Ibn Bahrīz's Arabic Translation of Nicomachus of Gerasa's *Introduction to Arithmetic*," in Y. T. Langermann et al., eds., *Adaptations and Innovations: Studies on the Interaction between Jewish and Islamic Thought and Literature from the Early Middle Ages to the Late Twentieth Century*, dedicated to Prof. Joel L. Kraemer (Louvain: Éditions Peeters, 2007), 67-82. Some details not germane to the present subject have been omitted.

6 Ibn Wādih qui dicitur al-Jaʿqubī *Historiae*, ed. M. Th. Houtsma (Leiden, 1883; repr. 1969), 140–43. This section is translated in M. Klamroth, "Über die Auszüge aus griechischen Schriftstellern bei al-Jaʿqūbī," *Zeitschrift der deutschen morgenländischen Gesellschaft* 42 (1888): 1–44, on pp. 9–16. See also Fuat Sezgin, *Geschichte des arabischen Schrifttums*. Band V: *Mathematik* (Leiden: Brill, 1974), 164–66.

(they apparently derive from notes written by al-Kindī himself); but he certainly interfered thoroughly in other sections of the work as well. We therefore refer to the text produced by al-Kindī's student as the "Ḥabib-Kindī version." The Arabic text of this version is not known to be extant. (4) The Ḥabib-Kindī version is, however, preserved in a Hebrew translation, dated 1317 and extant in several manuscripts, by the well-known translator and scholar Qalonymos ben Qalonymos. Qalonymos is known to have translated rather literally, so the Hebrew version of the Ḥabib-Kindī version can be assumed to reflect it faithfully. Below, when we refer to the text of the Ḥabib-Kindī version, we have in mind this version as represented by Qalonymos' Hebrew.[7] The prologue of the Ḥabib-Kindī version, from which we derive much of what we know about the history of the work in the Arabic tradition, has recently been published (in Hebrew).[8]

In parallel, the well-known mathematician and translator Thābit Ibn Qurra (826–901) soon translated Nicomachus anew into Arabic, this time directly from the Greek. This translation is totally independent of the Ḥabib-Kindī version.[9]

Abraham Bar Ḥiyya composed his encyclopedia, *Yesodei ha-tevunah u-migdal ha-ʾemunah,* in the early decades of the twelfth century, perhaps in the 1120s, around the same time as he wrote the related geometrical treatise *Ḥibbur ha-mešiḥah we-ha-tišboret* and *Sefer ha-ʿibbur* (1123). His acquaintance with Nicomachus' teachings could thus have drawn on three Arabic texts: Ḥabib Ibn Bahrīz's original Arabic version, prior to its revision by al-Kindī; the Ḥabib-Kindī version; and Thābit's translation. Here we want to determine which source Bar Ḥiyya used. For this purpose we have identified and compared a number of sentences from Bar Ḥiyya's encyclopedia that have equivalents in the Ḥabib-Kindī version, in Thābit's translation, and in the Greek original.

One possibility can quickly be ruled out. It is certain that Bar Ḥiyya did not use Thābit Ibn Qurra's translation. As we shall see,

comparison of the sentences to be studied below with their parallels in Thābit's version shows them to be completely unrelated.

The main question, then, is whether Bar Ḥiyya depended on Ḥabib Ibn Bahrīz's version or on its revision by al-Kindī. A superficial comparison of the Ḥabib-Kindī version (as preserved in Hebrew) with Bar Ḥiyya's treatment suggests that Bar Ḥiyya did not use it: his encyclopedia never echoes any of the passages explicitly ascribed to al-Kindī in the Ḥabib-Kindī version. By process of elimination, Bar Ḥiyya used the Arabic translation by Ḥabib Ibn Bahrīz prior to its revision by Kindī. (This is also the text used by Yaʿqūbī.) This hypothesis guided the research whose results are presented here. We now examine whether this conjecture, which we reached by elimination, can be substantiated positively as well.

We proceeded as follows: We first identified all the sentences or phrases in Bar Ḥiyya's text that have exact or nearly exact counterparts in at least one of the two sources; viz. in the Greek text or in the Ḥabib-Kindī version.[10] We then compared each sentence or phrase with the

7 We refer to a single manuscript, which we have checked against several others: MS Halle, Universitäts- und Landesbibliothek Sachsen-Anhalt, Yb 4° 5. The other manuscripts are listed in Freudenthal-Lévy, "De Gérase à Bagdad," p. 513.

8 Included (Hebrew text with French translation) in Freudenthal and Lévy, "De Gérase à Bagdad."

9 Text in *Ṯābit b. Qurra's arabische Übersetzung der Arithmetike Eisagoge des Nikomachos von Gerasa*, ed. Wilhelm Kutsch (Beirut: Imprimerie catholique, 1958).

10 We thus leave out of consideration two possible categories that have no bearing on the present research: (1) Sentences or phrases that are in Bar Ḥiyya but are not found in the Greek and the Ḥabib-Kindī version. Bar Ḥiyya may have found them in Ḥabib's version (from which they were subsequently deleted by al-Kindī) or they may be his own additions. (2) Sentences or phrases that are in both the Greek text and the Ḥabib-Kindī version, but not in Bar Ḥiyya's encyclopedia: these must have been deleted by

corresponding sentence or phrase in the other texts. In principle the following "ideal types" of cases are possible:

(A) Sentences or phrases that are in Bar Ḥiyya, the Greek text, and the Ḥabib-Kindī version. These must have been in Ḥabib's version, from which they passed to the Ḥabib-Kindī version (surviving Kindī's revision). However, because Bar Ḥiyya may have found them in either of these two sources they are of little use for our investigation.

(B) Sentences or phrases that are in Bar Ḥiyya's text and in the Ḥabib-Kindī version but not in the Greek text. These are interpolations by either Ḥabib Ibn Bahrīz or Kindī. They obviously do not allow us to know what Bar Ḥiyya's source was and so they too are not germane to the present inquiry.

(C) Sentences or phrases that are in Bar Ḥiyya's text and in the Greek text, but not in the Ḥabib-Kindī version. Bar Ḥiyya must have borrowed these sentences or phrases, which go back to Nicomachus himself, from Ḥabib's version, the only channel through which they could reach him. (These would be sentences or phrases that fell victim to al-Kindī's purification of Ḥabib's version.) Sentences of this kind would obviously be most valuable, for they would positively confirm our hypothesis that Bar Ḥiyya depended on Ḥabib's version. Of course it is possible that in some cases the situation will not be so neat, but the essential point will remain of having identified material that Bar Ḥiyya could have derived only from Ḥabib's version.

By contrast, our hypothesis would be refuted if we find cases of the following type:

(D) Sentences or phrases that are in Bar Ḥiyya's text and that the Ḥabib-Kindī version explicitly ascribes to Kindī. (We

cannot know with certainty the origin of other material in the Ḥabib-Kindī version that is not in the Greek.)

To avoid possible misunderstandings, a few observations seem appropriate. First, our methodology rests on identifying material that Bar Ḥiyya could only have taken from a version of Nicomachus that is closer to the Greek than the Ḥabib-Kindī version is. Since the two texts are rather close (one being a reworking of the other), our best hope for arriving at conclusive results is via a word-to-word comparison of suitable passages. Unfortunately, most of the Nicomachean material in *Yesodei ha-tevunah* consists of paraphrases and is of little use for our purposes. We are left with seven texts that represent all the passages we have identified as meeting the requirement of a word-to-word comparability (it is of course possible that one or more have escaped us). Second, our method makes no assumption as to how literal any of the translations were. Rather, a word-to-word correspondence attests to the fact that the sentence or phrase in question was successively rendered literally by all translators. That they survived the paraphrastic zeal of Ḥabib and al-Kindī and were also used verbatim by Bar Ḥiyya is quite remarkable; this is why they are so few in number. Third and last, to establish our case, we do not need to discuss all the differences between a given text in Bar Ḥiyya and the other texts. What is relevant for our purpose is only to point out instances in which Bar Ḥiyya's encyclopedia contains material that could not have reached him through the Ḥabib-Kindī version. At the same time we must make sure that no counter examples (case D above) show up.

Let us now consider the passages that constitute our corpus. (For the original Greek and Hebrew texts, see the appendix. Where D'Ooge,

Bar Ḥiyya himself. (They obviously were in Ḥabib's original version and were thus certainly available to Bar Ḥiyya, no matter what source text he used.)

whose translation of the Greek we are using, re-arranged the order of the lines to facilitate comprehension, we have re-inserted them in the location corresponding to the Greek and Hebrew texts, within angle brackets and in smaller type, to facilitate the comparison.) We proceed in the order of the text; note that our strongest evidence comes from the later instances.

Exhibit 1

Our first example relates to one of the properties of numbers.

No.	Greek[11]	Ḥabib-Kindī	Abraham Bar Ḥiyya
1	Every number is at once half	Every number is half of its two "sides"	Every number is half of its two "sides,"
2	of the sum of the two [numbers] on either side of itself,	when they are added.	
3	and similarly half	I.e., when the [number] greater than that number by one and the [number] less than it by one	the "side" greater than it and the "side" less [than it].
4	of the sum of those next to but one in either direction,	are added.	
5	and of those beyond them too,	Similarly it [= every number] is half of the "sides of its sides,"	It is similarly the half of the two "sides of its sides,"
6		and also half of what comes next, when what is deficient on one of the two "sides" is equal to what is in excess on the other.	
7	and so on as far as it is possible to go.	And so it goes on until the deficiency in the smaller "side" reaches unity, which it cannot go past.	and so on until the end of all its "sides."

These three texts share a common core (lines 1, 3, 5 and 7)—phrases going back to the Greek original that were taken over into Ḥabib's original Arabic version and survived al-Kindī's revision. They are no help in determining the source on which Bar Ḥiyya drew (case A above).

Line 6 of the Ḥabib-Kindī version is a long sentence that is not in the Greek or in Bar Ḥiyya. It is thus an interpolation, which prima facie can be due either to Ḥabib or to al-Kindī. The latter possibility seems more likely: For one thing, the character of the interpolation is more in tune with Kindī's style. For another, were the interpolation Ḥabib's, Bar Ḥiyya had to have seen it, whatever the text he used, but chose to delete it. Yet it seems improbable to assume that Bar Ḥiyya omitted precisely those words we know to be an interpolation, but which he could not have identified as such. The simplest explanation, then, is that these words are an interpolation by al-Kindī that were not in the text used by Bar Ḥiyya. The conclusion, thus, is that Bar Ḥiyya depended on Ḥabib's version.

The words in lines 2 and 4 (the concep of "sum" or "addition"), which are in both the Greek and the Ḥabib-Kindī version, must also have been in Ḥabib's version and thus were certainly in the text used by Bar Ḥiyya. Their absence from his text is presumably the result of a deliberate deletion on his part. By contrast, the illustration added by Bar Ḥiyya immediately following the above quotation (10 is half of 11 and 9 and half of 12 and 8, which are numbers on its two "sides" in increasing distance), which is neither in the Greek nor in Ḥabib-Kindī,

11 Trans. D'Ooge, p. 192 (slightly modified).

was presumably added by Bar Ḥiyya, who knew he was addressing beginners.[12]

Thābit Ibn Qurra's version (ed. Kutsch, p. 20, ll. 10–13) is remote from that used by Bar Ḥiyya. Thābit refers to "numbers" (instead of "sides," as in the Greek and in the versions deriving from Ḥabib) and writes: "Each number is equal to half of the two *numbers* that are on its two sides." Clearly, Abraham Bar Ḥiyya did not see this text.

Exhibit 2

This passage bears on the special case of unity.

No.	Greek[13]	Ḥabib-Kindī	Abraham Bar Ḥiyya
1	Unity alone,	Unity	Unity,
2	because it does not have two numbers on either side of it,	adjoins only the smallest number, namely two.	
[5]	is half		
3	merely of the adjoining number.	And since it [unity] is not a number having two "sides,"	not being a number, does not have two "sides" [lit. you do not find it to have two sides].
4			And since it is simple [and] indivisible,
5	<is half>	it is the half of its single "side,"	it is half of its single "side,"
6		i.e. the half of two,	i.e. [of] two, of which one is the half.
7		for two results from its doubling.	
8			[One] is the half of one thing, of a single concept; it is not the half of two things nor does it correspond to two concepts.[14]
9	In fact, unity is the natural principle of all [numbers].	It has thus been explained that unity is the cause of the increase in the number [lit. the cause of the number when it grows].	

Here again the three texts share a common core, consisting of lines 1, 3 and 5 (case A). Bar Ḥiyya's text contains no words that, by corresponding to the Greek without having an equivalent in the Ḥabib-Kindī version, would testify to dependence on Ḥabib's version. The words "the unity adjoins only the smallest number, namely two" in the Ḥabib-Kindī version, which go back to the Greek (line 3), must have been in Ḥabib's text as well. We can assume that their absence in Bar Ḥiyya's text is due to a deliberate deletion. By contrast, the words "since it is simple [and] indivisible" in Bar Ḥiyya's text (line 4) are presumably his own addition.

Interestingly, both Ḥabib-Kindī and Bar Ḥiyya offer a remark about the number two that has no parallel in the Greek text. The remarks themselves are different. This parallelism suggests that Ḥabib's text already contained such a remark, possibly deriving from a very short gloss ("i.e. the half of two": line 6) in either the Greek or the Syriac

12 Ed. Millás-Vallicrosa, p. 12:16–21.

13 Trans. D'Ooge, p. 192 (modified; D'Ooge inverts the order of 3 and 5).

14 I.e. the number two is "one thing," falling under a single concept; one is not the half of two things.

text. It seems likely that Bar Ḥiyya preserved Ḥabib's text unaltered, for the repeated fold reference to "two" at the end of the passage (*šenei devarim*; *šenei ʿinyanim*; line 8) points to a literal translation from the dual form in Arabic. Al-Kindī, for his part, echoes a Greek statement (line 9) that he presumably found in Ḥabib's text and which Bar Ḥiyya did not reproduce.

Thābit Ibn Qurra's version (ed. Kutsch, p. 20, ll. 13–15) differs from both Ḥabib-Kindī's and Bar Ḥiyya's versions. Specifically, it does not state that one is not a number, nor does it allude to the fact that one does not have two "sides." Clearly, Bar Ḥiyya did not draw on it.

Exhibit 3

The next sentence to be considered introduces the distinction between even and odd numbers.

No.	Greek[15]	Ḥabib-Kindī	Abraham Bar Ḥiyya
1	The primary division of number is even and odd.	The number is primarily divided into two parts: one of them is the even [number], and the other is the odd [number].	The number is primarily divided into two parts, which are the even and the odd.
2	The even is that which can be divided into two equal parts without a unit intervening in the middle;	The even is divisible into two equal parts,	The even is the number which is divisible in the middle [lit. in between],
[4]	and the odd is that which cannot be divided into two equal parts		
3	because of the aforesaid intervention of a unit.	[so that] between them there is no unit in the middle, by which one would be greater than the other.	
4	<and the odd is that which cannot be divided into two equal parts>	The odd is that which cannot be divided into two equal parts.	and the odd is the number which is not divisible in the middle.

The common core of all three versions (case A) does not allow any definite conclusions to be drawn from this passage.

Thābit Ibn Qurra's version (ed. Kutsch, p. 19, ll. 10–13) is, again, very different from both Ḥabib-Kindī's and Bar Ḥiyya's versions. For instance, instead of writing that number is divided "into two parts" it simply states "the first division into which number is divided is: some of it is even and some odd."

Exhibit 4

The next passage introduces the characteristically Nicomachean notions of even-even and even-odd.

No.	Greek[16]	Ḥabib-Kindī	Abraham Bar Ḥiyya
1	By subdivision of the even, there are	An even number is divided	An even [number] is divisible
2		into three parts.	

15 Trans. D'Ooge, p. 190 (D'Ooge inverts the order of 3 and 4).

16 Trans. D'Ooge, p. 192 (D'Ooge inverts the order of 6 and 7).

3	the even-times even, the odd-times even, and the even-times odd.	One is the even-times-even; the second is the even-times-odd; and the third is the even-times-even-times-odd.	
4	The even-times even and the even-times odd are opposites to one another, like extremes,	Consequently, the first two parts, namely the even-times-even and the even-times-odd,	into two primary parts: the even-times-even and the even-times-odd.
5		are distinguished inasmuch as the even and the odd are distinguished by definition.	
[7]	and the odd-times even		
6	is common to them both like a mean term.		Together they give rise to
7	<and the odd-times even>	The third part, which is the even-times-even-times-odd [lit. and odd],	a third part, which is the even-times-even-times-odd.
8		is the mean of the two ends.	

Lines 1, 4, and 7 constitute the common core of all three versions: Ḥabib's literal translation was preserved by al-Kindī and also used by Bar Ḥiyya. Instructive for our purposes are the additions to this core. In line 5 the Ḥabib-Kindī version explains that even-times-even and even-times-odd numbers "are distinguished inasmuch as the even and the odd are distinguished by definition": this interpolation clearly smacks of al-Kindī's logical frame of mind. Its absence from Bar Ḥiyya's version is consistent with our hypothesis that the latter depended on Ḥabib's version and not on Kindī's revision thereof. The same holds of the words "into three parts" (line 2).

The words *we-yityalled beinehem* in Bar Ḥiyya's text (line 6, which we clumsily translated as "together they give rise to") seem to reflect the expression *koinòn dè amfotérōn hṓsper mesótēs* (= "common to them both like a mean term"), found in the Greek text. The words "[it] is the mean of the two ends" (line 8) in the Ḥabib-Kindī version also seem to go back to this expression. This phrase therefore allows no conclusions to be drawn.

Not surprisingly, Thābit Ibn Qurra's version (ed. Kutsch, p. 20, ll. 15–19) differs markedly from Bar Ḥiyya's: it does not refer to the existence of "two first parts" (line 4) and does not contain the definition of "even-times-even-times-odd" (line 7).

Exhibit 5

This passage identifies a particular species of odd numbers—those that are odd *relative to* another number; i.e., that do not have a common denominator with it.

No.	Greek[17]	Ḥabib-Kindī	Abraham Bar Ḥiyya
[10a]	Now while these two species of the odd are opposed to each other		
1		The definition	You find
2	a third one	of the third species of odd number:	that this species of odd [number] [viz. uneven-times-uneven] has a third part.

17 Trans. D'Ooge, pp. 203–204 (D'Ooge splits 10 in two and moves it to the start of the passage).

3	is conceived between them,		
[10b]	deriving, as it were, its specific form from them both		
4		This species supervenes upon the odd accidentally.	
5		For it is [a number] such that a comparison with one another of two composite numbers shows to have no common denominator counting them. However, each, when its nature is considered on its own, has a number counting it, [namely, one that] is a part [i.e. a divisor] of it.	
6	namely the number which is in itself secondary		This is the number that is secondary relatively to itself,
7	and composite,		
8	but relatively to another number is prime		but prime relatively to another[18] number.
9	and incomposite.		
10	<Now while these two species of the odd are opposed to each other (...) deriving, as it were, its specific form from them both>		You find this distinction only with respect to two odd numbers,

11	This exists when a number, in addition to the common measure, unity, is measured by some other number		[namely] when you compare one of them with the other.
12	and is therefore able to admit a fractional part, or parts, with denominator other than the number itself, as well as the one with itself as denominator. When this number is compared with another number of similar properties, it is found that it cannot be measured by a measure common to the other, nor does it have a fractional part with the same denominator as those in the other.		
13	As an illustration, let 9	E.g., 9,	E.g. the two numbers 9
14		which consists, as we have said, of the multiplication of 3 three times,	
15	be compared with 25.	when it is compared with 25,	and 25,
16		which consists of 5 multiplied five times,	

18 Ed. Millás-Vallicrosa, p. 14, l. 19, but correcting the misprint *eḥad* to *aḥer* (as in the translation, p. 43).

17		each one of these two numbers is primary [and] not composed with respect to the other,	
18	Each of them is secondary		each of which one is secondary relative to itself [but prime relative to one another],
19	and composite,		
20		since they have no common denominator counting them.	
21	but relatively to each other they have only the unity as a common measure,		for each of them has a number which counts it,
22	and no factors in them have the same denominator		and you do not find a number common to them which counts both of them.

Bar Ḥiyya's text closely corresponds to the Greek and evinces no traces of the loose paraphrase of the Ḥabib-Kindī version or its interpolations. Among the latter, the gloss "this species supervenes upon the odd accidentally" (l. 4) is particularly characteristic of al-Kindī's style. We can surmise that Ḥabib's translation offered an almost literal translation of the Greek, which Abraham Bar Ḥiyya quoted verbatim or almost so. (The few missing words may have dropped in any of the early stages of transmission.) The main pertinent point is that, whatever the (minor) differences between Bar Ḥiyya and the Greek, Bar Ḥiyya had at his disposal some material that is not in the Ḥabib-Kindī version and is closer to the Greek. The paraphrasis of the Ḥabib-Kindī version thus

resulted from al-Kindī's revision. This, we submit, is an unequivocal instance of Abraham Bar Ḥiyya's dependence on a pre-Kindī Arabic version of the *Introduction to Arithmetic* (case C). The present case suffices to establish that Bar Ḥiyya did not use the Ḥabib-Kindī version, but rather Ḥabib's version: the latter still included a literal translation of some passages that were later expunged by al-Kindī.

Thābit Ibn Qurra's text (ed. Kutsch, p. 31, ll. 4–13) is again totally different from the other versions. In particular, it translates (ll. 7–11) the Greek passage of ed. Hoche, p. 29, ll. 5–10, which does not appear in the other versions and must have been omitted either by the Syriac translator or by Ḥabib.

Exhibit 6

This passage presents the notion of a "perfect number."

No.	Greek[19]	Ḥabib-Kindī	Abraham Bar Ḥiyya
1	A different subdivision [lit. yet another]: of the simple even numbers,	The even numbers are divided into three classes:	From a different perspective [lit. in another way] number is divided into three classes:
2	<and some are intermediary between them and are called>		
3	<perfect. (...)>	balanced,	perfect,
4		i.e. that the sum of its parts is equal to it;	
5	some are superabundant,	abundant [lit. additional],	abundant [lit. exceeding],

19 Trans. D'Ooge, pp. 203, 209 (with some minor changes; D'Ooge relocated lines 2, 3, and 11).

6		i.e. [this sum is] greater than it;	
7	some deficient,	deficient,	and deficient.
8		i.e. that the sum of all its parts is smaller than it.	
9	like extremes set over against each other,		
[2]	and some are intermediary between them and are called		
[3]	perfect. (...)		
10	The so-called perfect number ...	We already said that the balanced number is that	A perfect number is
11	<but is always equal to its parts.>	which equals the sum of all its parts.	that whose parts counting it add up to it [lit. fill it],
12		This number is analogous to an animal whose limbs are evenly matched and whose form is equitable.	
13	neither makes its parts greater than itself, added together,		they neither exceed it,
14	nor shows itself greater than its parts,		nor are less than it.
[11]	but is always equal to its parts.		
15	Such numbers are 6 and 28;	E.g. the number 6 and the number 28,	E.g. the number 6,
16	for 6 has the factors half, third, and sixth,	for six has a half, and a third, and a sixth,	whose parts are a sixth, a third, and a half,
17			and you do not find any other part thereof.

18	3, 2, and 1, respectively,	viz. 3, 2 and 1,	
19	and these added together make 6	which together are 6.	And when you add its sixth and its third and its half you find that they amount to 6,
20	and are equal to the original number,	Thus, [taken together] these three are equal to the 6 of which they are the parts,	
21		they neither exceed it, nor are deficient.	neither less nor more.
22	28 has the factors half etc.	Much the same holds for 28.	The number 28 is analogous to it.

In *Introduction to Arithmetic* I:14–16 Nicomachus discusses abundant, defective, and perfect numbers. The parallel passages in the three texts allow a word-to-word comparison.[20] Here we consider only the discussion of the perfect numbers; the others lead to similar conclusions.

Comparison of the three texts confirms our previous analyses without shedding substantial new light on the transmission of the text. For our purposes, the salient point is that we again have evidence of Bar Ḥiyya's dependence on a text closer to the Greek than the Ḥabib-Kindī version is (case C), as reflected in a number of terms and expressions: e.g., line 1 (Greek *pálin dè ánōthen* 'yet another' is very close to Bar Ḥiyya's *mi-dereḵ aḥeret* 'in another way' but has no counterpart in Ḥabib-Kindī); lines 13–14; line 19 (the words *meqabbeṣ* and *timṣa*' seem to reflect the Greek *synkefalaiōthénta* and *genómena*, which have no equivalents in Ḥabib-Kindī). Where the Ḥabib-Kindī version

20 We thank the referee for *Aleph* for drawing our attention to this passage.

has interpolations that are not in Bar Ḥiyya, these are usually didactic explanations typical of al-Kindī's style (e.g. lines 4, 6, 8 and especially 12). Particularly noteworthy is a semantic shift introduced by Kindī. To denote the notion of perfect number, Nicomachus uses the term *téleios* 'perfect, complete, full', which Bar Ḥiyya faithfully renders as *maleʾ*, testifying to dependence on an Arabic version that in turn faithfully reflected the Greek. (Ḥabib may have used either *malīʾ* 'full' in the sense of "complete" or, less likely, *tāmm* 'complete, perfect', as Thābit did [see below].) The Ḥabib-Kindī version has *šaweh* 'balanced, equal,' instead. This would seem to be a deliberate, philosophically motivated choice by al-Kindī, reflecting his view that "this number is analogous to an animal whose limbs are evenly matched and whose form is equitable" (line 12).

A comparison of Thābit's translation (ed. Kutsch, p. 36, ll. 12–14; p. 38, ll. 11–13 and 18–20) with the Ḥabib and Ḥabib-Kindī versions shows that it is more detailed, literal, and extensive, differing from them in almost every word. The only noteworthy point is that Thābit translates the Greek *téleios* as *tāmma* 'perfect' (p. 36, l. 14). This confirms that the version used by Thābit had the reading *téleios*, so there is no reason to assume that the term "balanced" found in the Ḥabib-Kindī version goes back to a different Greek text. Whatever Arabic term may have rendered *téleios* in Ḥabib's translation, al-Kindī replaced it with a term denoting "balanced."

Exhibit 7

This passage discusses a property of odd numbers.

No.	Greek[21]	Ḥabib-Kindī	Abraham Bar Ḥiyya
1	The odd is a number which in any division whatsoever, which necessarily is a division into unequal parts,	An odd number is one that however you divide it, its parts will not be equal;	An odd number is one whose parts are never equal,
2			neither in their magnitude nor in their form.
3	shows both the two species of number together, never without intermixture of one with the other,	they must be jointly even and odd.	
4	but always in one another's company.	I.e. if one of the parts is odd, the other is even.	Rather, one will always be even, and the other odd.
5			You can divide an even number into two equal parts, but you cannot divide an odd number into two equal parts.
6		It is therefore manifest that the parts of an odd number are closest to being equal when the difference between them is a unit, by which one of them exceeds the other.	However, if you divide them [i.e. odd numbers] precisely, and bring them close to being equal, then you find that one part is greater or smaller than the other by one.

21 See D'Ooge, p. 191.

This passage is particularly interesting. A core common to all three versions (lines 1 and 4), is followed by a sentence (line 6) that is not in the Greek but is found in both the Ḥabib-Kindī version and Bar Ḥiyya (case B). This suggests that it is a late interpolation, which found its way into Ḥabib's translation and thence into the Ḥabib-Kindī version and into Bar Ḥiyya's encyclopedia. This hypothesis is confirmed by what comes after this sentence. In the Ḥabib-Kindī version,[22] it is followed by a short discussion, also absent from the Greek, of the method called *diallélos*: the method was originated by Iamblichos, and the sentence seems to have been interpolated either by the Syriac translator or, more likely, by Ḥabib himself, who had access to Greek sources.[23] Line 6 and the discussion of *diallélos* seem to be part of the same interpolation. Since we know that the discussion of *diallélos* must be a late (post-Iamblichos) addition, the remark that the closest one can come to dividing an odd number into two equal parts is two parts differing by one unit is also a late interpolation. We see again that Bar Ḥiyya depended on Ḥabib's text, from which he deleted the reference to *diallélos,* correctly judging it unsuitable for beginners. Finally, it should be observed that although most of the present passage naturally does not differentiate between Ḥabib's text and the Ḥabib-Kindī version, line 4 gives yet another indication that Bar Ḥiyya depended on Ḥabib: the Greek words *allá pàntote* ("but always") have no correspondence in Ḥabib-Kindī, but are rendered in Bar Ḥiyya's text (*aval ... le-ʿolam* = "rather... always"). By contrast, in the same line, both Ḥabib-Kindī and Bar Ḥiyya refer to even and odd, whereas the Greek does not: as in former similar cases, this variation presumably was already in Ḥabib's text, whence it passed to all subsequent versions.

As could be expected, here too Thābit's translation (ed. Kutsch, p. 20, ll. 3–8) differs from both Ḥabib-Kindī's text and Bar Ḥiyya. For example, the affirmation in the Greek that in odd numbers "the two species of number [are] together, never without intermixture of one with the other" (line 3), which is rather freely rendered by Ḥabib-Kindī, is

translated literally by Thābit: "and these two species (of number) are never unmixed in it one with the other" (p. 20, ll. 6–7). By the same token, the words "but … always" discussed above are translated by Thābit too (*lākin … abadan*; p. 20, ll. 7–8).

Conclusion

From a purely chronological point of view, Abraham Bar Ḥiyya could have had access to all three Arabic versions of Nicomachus of Gerasa's *Introduction to Arithmetic*: Ḥabib Ibn Bahrīz's original translation from the Syriac; the revision Ḥabib's version by al-Kindī; and the translation from the Greek by Thābit Ibn Qurra. All three had been executed about two centuries before his time.

We have shown that our section of *Yesodei ha-tevunah* depends on Ḥabib's version and shows no marks of the Ḥabib-Kindī version. First and significantly, there is not a single trace in *Yesodei ha-tevunah* of any of the extensive material explicitly attributed to al-Kindī in the Ḥabib-Kindī version. This *argumentum e silentio*, which seems to us to carry considerable weight in the present context, is confirmed by positive evidence. A close comparison of *Yesodei ha-tevunah* with corresponding *loci* in the Greek original on the one hand and in the Ḥabib-Kindī version on the other has demonstrated elements in Bar Ḥiyya's work that derive from the Greek text but are absent from the Ḥabib-Kindī version (they were presumably deleted by al-Kindī). Bar Ḥiyya could have found them only in Ḥabib's unmodified translation. Thus, although Ḥabib's text is lost, it has left marks that could be retrieved by a careful comparison of the extant texts. We have

22 MS Halle, f. 8b, ll. 19–21 (not reproduced here).

23 Freudenthal-Lévy, "De Gérase à Bagdad," p. 493–95.

also ruled out the possibility that Bar Ḥiyya had access to Thābit's translation.

As mentioned earlier, some direct quotations from Ḥabib's version are preserved in Yaʿqūbī's *Taʾrīḫ*, suggesting the possibility of comparing Bar Ḥiyya's and Yaʿqūbī's texts. Unfortunately, Yaʿqūbī's presentation is too hasty and superficial to allow any conclusions to be drawn from such a comparison.

Finally, we should note that Abraham Bar Ḥiyya may have depended directly on the full text of Nicomachus' work in Ḥabib's translation, or on some compendium or epitome based on it. We are not aware of any such work. In any event, such a work, if it existed, would have contained enough of Ḥabib's original text to allow Bar Ḥiyya to borrow some of Ḥabib's original phrasing, including those that have precise parallels in the Greek text.[24] Whether Bar Ḥiyya drew directly on Ḥabib's translation of Nicomachus or on a work that quoted from it, the salient point is that the Nicomachean material in *Yesodei ha-tevunah u-migdal ha-ʾemunah* derives from Ḥabib's translation and not from al-Kindī's revision.

En passant, the comparison of our three texts has shed some new light on Ḥabib's original, unrevised translation. By implication, it has also revealed more about the style of al-Kindī's revision. By making similar comparisons (including other passages), a future editor of Qalonymos' Hebrew translation of the Ḥabib-Kindī version will be able to identify at least some of the passages that go back to Ḥabib on the one hand and others on which al-Kindī left his mark.

We have shown that in the early twelfth century Nicomachus' important work was known in Spain in its first Arabic translation. We also know that the Ḥabib-Kindī version reached Andalusia before the beginning of the fourteenth century, when it was translated by Qalonymos.[25] Other Spanish authors depended on Thābit's translation.[26] The full story of the reception and influence of Nicomachus' arithmetic and philosophy of science remains to be written.[27]

24 The hypothesis that Bar Ḥiyya used a compendium is attractive, because it would account for the great amount of knowledge integrated into *Yesodei ha-tevunah*. Many of the sources drawn on in his discussions remain unidentified. One wonders whether he perhaps found them all in a limited number of works. See also Levey, "The Encyclopedia of Abraham Savasorda," p. 258.

25 The colophons of the extant version of the Ḥabib-Kindī version provide evidence of a minor revision made in Andalusia. See Freudenthal-Lévy, "De Gérase à Bagdad."

26 Julio Samsó, "The Exact Sciences in al-Andalus," in Salma Khadra Jayyusi, ed., *The Legacy of Muslim Spain* (= *Handbuch der Orientalistik*, tome 12) (Leiden: Brill, 1992), 952–73, on pp. 953–54; Sonja Brentjes, "Untersuchungen zum Nicomachus Arabus," *Centaurus* 30 (1987): 212–39.

27 On Nicomachus' great and enduring influence on Jewish circles writing in Hebrew, see Y. Tzvi Langermann, "Studies in Medieval Hebrew Pythagoreanism. Translations and Notes to Nicomachus' Arithmological Texts," in *Gli Ebrei e le Scienze. The Jews and the Sciences* (= *Micrologus* IX [2000]) (Florence: Sismel–Edizioni del Galluzzo, 2000), 219–36.

Exhibit 1

	Greek, book I, chapter 8, ed. Hoche, p. 14, ll. 13–16	Habib-Kindi version, MS Halle, f. 9r, ll. 4–10	Abraham bar Ḥiyya, *Yesodei ha-tevunah*, ed. Millás Vallicrosa, p. 12, ll. 15–16
1	πᾶς ἀριθμὸς… ἅμα ἥμισύς ἐστι	כל מספר הוא חצי שתי פאותיו	כל מספר הוא מחצית שני צדיו
2	τῶν παρ’ ἑκάτερα συντεθέντων	כאשר יקובצו	
3	καὶ… ὁμοίως ἥμισύς ἐστι	ר״ל מה שהוא מוסיף מאותו המספר באחד ופוחת ממנו באחד	הצד העודף עליו והצד החסר
4	τῶν ὑπὲρ ἕνα ἑκατέρωθεν κειμένων	כאשר יקובצו	
5	καὶ ἔτι τῶν ὑπὲρ ἐκείνους	וכמו כן הוא חצי שתי פאות פאותיו	וכן הוא מחצית שני צדי צדיו
6		וחצי גם כן מה שאחר זה כאשר היה מה שיחסר מאחת משתי הפיאות כשעור מה שיוסיף באחרת	
7	καὶ τοῦτο μέχρις οὗ δυνατόν	וכן יהיה תמיד עד שיגיע החסרון מהפאה הקטנה אל האחדות אשר לא יוכל לעברו	עד תכלית כל צדיו

Exhibit 2

	Greek, book I, chapter 8, ed. Hoche, p. 14, ll. 16–19	Habib-Kindi version, MS Halle, f. 9r, ll. 11–14	Abraham bar Ḥiyya, Yesodei ha-tevunah, ed. Millás Vallicrosa, p. 12, l. 21 – p. 13, l. 2
1	μονοτάτη δὲ ἡ μονάς	ואמנם האחדות	והאחד
2	διὰ τὸ μὴ ἔχειν ἑκατέρωθεν αὐτὴν δύο	הנה לא ילוה לו רק הפחות במספרים אשר הוא שנים	
3	ἀριθμοὺς ἑνὸς μόνου τοῦ παρακειμένου	ולפי שאינו מספר שיהיו לו שתי פיאות	מפני שאינו מספר, אין אתה מוצא לו ב׳ צדדים
4			ומפני שהוא פשוט בלי מתחלק
5	ἥμισύς ἐστιν·	היה חצי פאתו האחת	אתה מוצא אותה מחצית צדו האחד
6		ר״ל חצי השנים	והוא השנים אשר האחד מחציתו
7		אחר שתולדת השנים מכפלו	
8			והוא מחצית לדבר אחד מענין אחד ולא מחצה לשני דברים ולא על שני ענינים
9	ἀρχὴ ἄρα πάντων φυσικὴ ἡ μονάς	הנה כבר התבאר מזה שהאחדות עלת המספר בצמיחתו	

Exhibit 3

	Greek, book I, chapter 7, ed. Hoche, p. 13, ll. 9–13	Habib-Kindi version, MS Halle, f. 8r, l. 21 – f. 8v, l. 4	Abraham bar Ḥiyya, Yesodei ha-tevunah, ed. Millás Vallicrosa, p. 13, ll. 4–8
1	τοῦ δὲ ἀριθμοῦ πρώτη τομὴ τὸ μὲν ἄρτιον, τὸ δὲ περιττόν	והמספר יחלק חלוקה ראשונה אל שני חלקים, אחד מהם הזוג והאחר הנפרד	והמספר נחלק לב׳ חלקים ראשונים, והם הזוג והנפרד
2	ἔστι δὲ ἄρτιον μέν, ὃ οἷόν τε εἰς δύο ἴσα διαιρεθῆναι	והזוג יחלק לשני חלקים שוים	הזוג הוא המספר הנחלק בנתים
3	μονάδος μέσον μὴ παρεμπιπτούσης	אין ביניהם אחדות אמצעי יעדיף בו אחד מהם על האחר	
4	περιττὸν δὲ τὸ μὴ δυνάμενον εἰς δύο ἴσα μερισθῆναι	והנפרד הוא אשר אין דרך אל שיחלק בשני חלקים שוים	והנפרד הוא המספר שאינו יכול ליחלק בנתים

Exhibit 4

	Greek, book I, chapter 8, ed. Hoche, p. 14, l. 19 – p. 15, l. 3	Habib-Kindi version, MS Halle, f. 9r, ll. 16–19	Abraham bar Ḥiyya, *Yesodei ha-tevunah*, ed. Millás Vallicrosa, p. 13, ll. 13–14
1	καθ᾽ ὑποδιαίρεσιν δὲ τοῦ ἀρτίου	המספר הזוג יחלק	והזוג נחלק
2		לשלשה חלקים	
3	τὸ μὲν ἀρτιάκις ἄρτιον, τὸ δὲ περισσάρτιον, τὸ δὲ ἀρτιοπέριττον	אחד מהם זוג הזוג והשני זוג הנפרד והשלישי זוג הזוג והנפרד	
4	ἐναντία μὲν ἀλλήλοις ὥσπερ ἀκρότητες τὸ ἀρτιάκις ἄρτιον καὶ τὸ ἀρτιοπέρισσον	הנה אם כן שני החלקים הראשונים אשר הם זוג הזוג וזוג הנפרד	לשני חלקים ראשונים והם זוג הזוג וזוג הנפרד
5		נבדלים להבדל הזוג והנפרד בגדר	
6	κοινὸν δὲ ἀμφοτέρων ὥσπερ μεσότης		ויתילד ביניהם
7	τὸ περισσάρτιον	והחלק השלישי אשר הוא זוג הזוג והנפרד	חלק שלישי והוא זוג הזוג והנפרד
8		באמצעי משני הקצוות	

Exhibit 5

	Greek, book I, chapter 13, ed. Hoche, p. 28, l. 29 – p. 29, l. 14	Habib-Kindi version, MS Halle, f. 14v, ll. 13–19	Abraham bar Ḥiyya, *Yesodei ha-tevunah*, ed. Millás Vallicrosa, p. 14, ll. 18–22
1		תאר	ואתה מוצא
2	…τρίτον	המין השלישי מהמספר הנפרד	לנפרד הזה חלק שלישי
3	ἀνὰ μέσον τι θεωρεῖται… (p. 29, l. 1)		
4		שזה המין אמנם יתחדש לנפרד במקרה	

5		לפי שהוא אמנם יהיה עם ההקשה בין שני המספרים המורכבים אין להם מספר משותף ימנה אותם, ולכל אחד מהם כאשר הושב אל טבעו מספר ימנהו הוא חלק לו	
6	τὸ καθ᾽ αὑτὸ μὲν δεύτερον		והוא המספר שהוא שני לגבי עצמו
7	καὶ σύνθετον		
8	πρὸς ἄλλο δὲ πρῶτον		וראשון לגבי מספר אחר
9	καὶ ἀσύνθετον… (p. 29, ll. 2–4)		
10	ἀντικειμένον δὴ ἀλλήλοις τῶν δύο τούτων εἰδῶν τοῦ περισσοῦ… (p. 28, l. 20 – p. 29, l. 1) οἱονεὶ ἐξ ἀμφοτέρων εἰδοποιούμενον… (p. 29, l. 2)		ואי אתה מוצא החלוק הזה אלא בין שני מספרים נפרדים
11	ὅταν ἀριθμὸς πρὸς τῷ κοινῷ μέτρῳ τῇ μονάδι ἔτι καὶ ἑτέρῳ μετρεῖταί τινι μέτρῳ		כשאתה מקיש אחד מהם אל חברו

12	καὶ διὰ τούτο δυνάμενος καὶ ἑτερώνυμος μέρος ἢ μέρη ἐπιδέξασθαι πρὸς τῷ παρωνύμῳ, πρὸς ἄλλον τινὰ ὁμοίως ἔχοντα ἀντεξεταζόμενος εὑρίσκεται μήτη κοινῷ μέτρῳ μετρηθῆναι δυνάμενος πρὸς ἐκεῖνον, μήτε τὸ αὐτὸ ὁμώνυμον μέρος ἔχων τῶν ἁπλῶς ἐν ἐκείνῳ· (p. 29, ll. 5–10)		
13	οἷον ὁ θʹ	וזה כמו ט'	כגון שני מספרים ט'
14		אשר הם מורכבים כמו שאמרנו מהכפלת ג' שלשה פעמים	
15	πρὸς τὸν κεʹ	הנה כאשר הוקשו אל כ"ה	וכ"ה
16		אשר הם מורכבים מה' חמשה פעמים	
17		הנה כל אחד משני אלו המספרים אצל חבירו ראשון בלתי מורכב	
18	ἑκάτερος γὰρ καθ᾽ ἑαυτὸν δεύτερός ἐστι (p. 29, ll. 11–12)		אשר כל אחד מהם שני בפני עצמו
19	καὶ σύνθετος (p. 29, l. 12)		
20		לפי שאין להם מספר משותף ימנם	
21	πρὸς δὲ ἀλλήλους μονάδι μόνῃ κοινῷ μέτρῳ χρῶνται (p. 29, ll. 12–13)		מפני שיש לכל אחד מהם מספר שהוא מונה אותו

22	καὶ οὐδὲν μόριον ὁμωνυμεῖ ἐν ἀμφοτέροις (p. 29, ll. 13–14)		ואין אתה מוצא מנין שהוא מתחברים בו שהוא מונה לשניהם

Exhibit 6

	Greek, book I, chapters 14 and 16, ed. Hoche, p. 36, ll. 6–9; p. 39, ll. 6–10; p. 39, l. 19 – p. 40, l. 4	Habib-Kindi version, MS Halle, f. 16v, ll. 13–15; f. 17r, ll. 14–18	Abraham bar Ḥiyya, *Yesodei ha-tevunah*, ed. Millás Vallicrosa, p. 15, ll. 4–8
1	πάλιν δὲ ἄνωθεν· τῶν ἁπλῶς ἀρτίων ἀριθμῶν… (p. 36, ll. 6–7)	המספר הזוג יחלק לשלשה חלקים	והמספר נחלק מדרך אחרת לג' חלקים:
2	οἱ δὲ ἀνὰ μέσον ἀμφοτέρων, οἳ καὶ λέγονται		
3	τέλειοι.. (p. 36, ll. 8–9)	ואם שוה	למלא
4		ר"ל שהגעת כלל חלקיו שוה לכלו	
5	οἱ μέν εἰσιν ὑπερτελεῖς	אם נוסף	ועודף
6		ר"ל מוסיף על כלו	
7	οἱ δὲ ἐλλιπεῖς	ואם חסר	וחסר
8		ר"ל שהגעת כלל חלקיו יחסרו מכלו...	
9	καθάπερ ἀκρότητες ἀντικείμεναι ἀλλήλαις… (p. 36, ll. 7–8)		
10	ὁ λεγόμενος τέλειος… (p. 39, ll. 6–7)	כבר אמרנו שהמספר השוה הוא אשר	והמספר המלא הוא אשר
11	αἰεὶ ἴσος τοῖς ἑαυτοῦ μέρεσιν ὑπάρχων… (p. 39, ll. 9–10)	הגעת כלל חלקיו שוה לכלו	חלקיו המונין אותו ממלאים אותו
12		וזה המספר ידמה בעל חיים שוה האיברים ממוצע הצורה	
13	οὔτε τὰ μέρη ἑαυτοῦ πλείονα ἀποτελῶν συντεθέντα		ואין עודפים עליו
14	οὔτε ἑαυτὸν μείζονα τῶν μερῶν ἀποφαίνων… (p. 39, ll. 7–9)		ולא פוחתים ממנו
15	οἷον ὁ ϛʼ καὶ ὁ κη·	וזה כמו מספר ו' ומספר כ"ח	כגון מספר ו'
16	ὁ τε γὰρ ϛʼ ἔχει μέρη ἥμισυ, τρίτον, ἕκτον,	כי ששה יש לו חצי ושליש ושתות	אשר חלקיו שתות ושליש וחצי
17			ואי אתה מוצא לו חלק אחר
18	ἅπερ εἰσὶ γʼ, βʼ, αʼ,	והם גב"א	
19	ἅπερ συγκεφαλαιωθέντα ὁμοῦ καὶ γενόμενα ϛʼ	וכלם ששה	וכשאתה מקבץ שתיתו ושלישו וחציו תמצא בהן ששה
20	ἴσα τὸ ἐξ ἀρχῆς ὑπάρχει	הנה אלו השלשה[1]	
21	καὶ οὔτε πλείονα οὔτε ἐλάττονα·	לא יוסיפו ולא יחסרו	לא פחות ולא יותר
22	καὶ ὁ κηʼ… (pp. 39, l. 19 – 40, l. 4)	וכמו כן גם כן כ"ח	ודומה לו מספר כ"ח

Exhibit 7

	Greek, book I, chapter 7, ed. Hoche, p. 14, ll. 4–8	Habib-Kindi version, MS Halle, f. 8v, ll. 16–20	Abraham bar Ḥiyya, *Yesodei ha-tevunah*, ed. Millás Vallicrosa, p. 13, ll. 7–11
1	περισσὸς δέ ἐστιν ἀριθμὸς ὁ καθ᾽ ἡντιναοῦν τομὴν εἰς ἄνισα	והמספר הנפרד הוא אשר איך שתחלקהו לא ישתוו חלקיו	אבל הנפרד הוא שכל חלקיו לעולם אינם שוים
2			לא במנינם ולא בדמותם
3	πάντως γινομένην ἀμφότερα ἅμα ἐμφαίνων τὰ τοῦ ἀριθμοῦ δύο εἴδη οὐδέποτε ἄκρατα ἀλλήλον	ולא ימנעו מחלקיו הזוג והנפרד יחד	
4	ἀλλὰ πάντοτε σὺν ἀλλήλοις	ר"ל שאם היה אחד מחלקיו נפרד היה האחר זוג	אבל האחד לעולם זוג והשני נפרד
5			ואתה יכול לחלק את הזוג בנתים לשני חלקים שוים במספרם ואי אתה יכול לחלק את הנפרד לשני חלקים שוים במספרם
6		אם כן הוא מבואר שהיותר קרוב מה שיהיו חלקי הנפרד מן השווי כאשר היה בין שני חלקיו אחדות יעדיף בו אחד מהם לאחר	אבל אם אתה מדקדק בחלוקם ומקריב אותם מן השוה אתה מוצא חלק אחד מוסיף על השני אחד או גורע ממנו אחד

1 השלשה] הששה.