DISCOURSE ON THE CAUSE OF GRAVITY

(DISCOURS DE LA CAUSE DE LA PESANTEUR)

Christiaan Huygens

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PREFACE

(p.125) Nature acts in a manner so secret and so imperceptible in drawing what we call heavy bodies towards the Earth that, regardless of the attention or industry invested, the senses are unable to detect anything of it. This has compelled philosophers of centuries past to look for the cause of this admirable effect in the bodies themselves and to attribute it to some internal and inherent quality that makes them tend downward and toward the center of the Earth, or to a tendency of the parts to join together. All of this did not reveal causes but rather assumed vague and ill-understood principles. In many cases, those who were bound by such solutions can be forgiven, but we cannot so easily excuse Democritus and those of his sect, who, in their attempt to bring reason to everything through atoms, excluded only gravity, which they linked **(p. 126)** to terrestrial bodies and to the atoms themselves without looking into how these things could acquire gravity.[1] Among the modern authors and restorers of philosophy, many have correctly considered that it was mistaken to introduce something outside bodies as the cause of the attractions and connections we observe in them. However, these progress scarcely farther than the first. Some have resorted to a thin, heavy air that causes bodies to descend by pressing them but this is hardly progress because it already assumes one gravity. It is also strongly contrary to the laws of mechanics to claim that a heavy, liquid matter presses down the bodies it surrounds, since it would, on the contrary, make them rise, assuming they are without any weight in themselves, in entirely the same way that water causes an empty vial plunged into it to rise. Still others have resorted to certain spirits and immaterial emanations, which elucidates nothing since we have no understanding of how something immaterial gives motion to a corporeal substance.

Mr. Descartes was more aware than those who preceded him that we will never understand anything more in physics than what we can give an account of with certain principles that do not exceed the reach of our mind, such as those that depend on bodies,

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[1] (First occurrence of 'pesanteur'.)

considered without qualifications, and their motions. The greatest difficulty, however, **(p.127)** consists in showing how so many diverse things are affected by these principles alone, and it is in doing this that Mr. Descartes has not been very successful in several particular subjects that he has proposed to examine – among them, in my opinion, gravity. This will be evaluated in the remarks that I make in a few places concerning what he has written on it, and I have been able here to join some others to these remarks. Nevertheless, I acknowledge that his efforts and his views, however false, have opened to me the path to my discoveries on the very same subject.

I do not claim to be exempt from all doubt, nor from objections. It is very difficult to reach such a point in research of this nature. All the same, I believe that if the principal hypothesis on which I myself relied is not the true one, there is little hope that one will be found within the limits of true and sound Philosophy.

Moreover, insofar as it concerns only the cause of gravity, what I bring about here will not appear new to those who have read Mr. Rohault's *Treatise on Physics* because my theory is reported almost entirely therein. Having seen my experiment with turning water and having understood my application of it (which he **(p.128)** cleverly recognized), this philosopher found my view plausible enough to desire adopting it. Because, however, he mixes among my thoughts not only those of Mr. Descartes but also his own, and because he omits several things pertaining to this matter, some of which he could not know, I am very pleased that people may see it as I have treated it myself.

The greatest part of this Discourse, up to the place where the it speaks of the alteration of the pendulums by the motion of the Earth, was written during the time I resided in Paris and is recorded in the Registers of the Royal Academy of Science. The remainder was added several years later, followed by the Addition, which was added on the occasion that will be found indicated in the beginning of that section.

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DISCOURSE ON THE CAUSE OF GRAVITY

(p.129) In order to find an intelligible cause of gravity we must see how gravity can come about while presupposing in nature only bodies that are made from a like matter, and considering in these neither a quality nor a tendency to draw near one another, but only their different magnitudes, figures, and motions. How, I say, might it still come about that several of these bodies tend directly toward a common center, and are held together around it? This is the most ordinary and most important phenomenon of what we call gravity.

The simplicity of the principles that I assume does not leave much choice in this research. We judge rightly from the start that there is nothing in the appearance attributed to the figure, or in the smallness of its corpuscles, with an effect that resembles that of gravity, which, being an endeavor or a tendency to motion, must in all likelihood be produced by a motion. So, there remains only to find in what manner it can act, and in which bodies it can be encountered.

(p.130) While examining merely the bodies, without this quality that we call gravity, we find that their motion is naturally either straight or circular.[2] The former concerns when they move unhindered; the latter, when they are held around a particular center, or when they turn on their own center. We know (aucunement) the nature of straight motion and the laws that govern bodies in the communication of their motions when they meet.[3] But so long as we consider only this sort of motion and the reflections that occur between particles of matter, we find nothing that forces them to tend toward a center. We must necessarily

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[2] In the 1669 version of the text this sentence read, "We see two sorts of motion in the world, straight and circular."

[3] Even at the time of the original Huygens had long since completed, though not published, his masterful paper *De Motu Corporum ex Percussione*, giving the laws of motion of perfectly elastic spheres under head-on impact (see *OCCH*, Vol. 16, pp. 29-91). The first hypothesis of that paper is, "Any body once moved continues to move, if nothing prevents it, at the same constant speed and along a straight line." (Unclear use of 'aucunement' in the French; several queries to historians of the language pending.)

then turn to the properties of circular motion and see if there is anything there that can help us.

I know that Mr. Descartes has also tried, in his *Physics*, to explain gravity by the motion of certain matter that turns around the Earth, and there is much to having had this idea first.[4] But we will see, by the remarks that I will make in the rest of this discourse, how his way differs from what I am going to propose and also how it appears to me to be faulty.

He has considered, as I have, the endeavor that makes bodies in circular motion move away from the center, which experience[5] does not permit us to doubt. When we turn a stone in a slingshot, we feel that it pulls our hand and that it is correspondingly stronger as we turn it faster, until it is so fast that the cord reaches its breaking point. I have displayed this particular property of circular motion before, by attaching some heavy bodies to a round table, pierced in the center, that turns on a pivot. I have found the strength of the force and several theorems that concern it, as one can see at the end of the book that I wrote on the Motion of Pendulums.[6] For example, I say that, if a body turning in a circle at the end of a **(p.131)** string stretched horizontally travels with the velocity it would be able to acquire by its fall, dropping from a height equal to half of that same string, that is to say a quarter of the diameter of the circumference that it describes, the string would be pulled with exactly as much force as if it had supported the same body suspended in air.[7]

The endeavor to move away from the center is then a constant effect of circular motion. And even though this effect appears directly opposed to that of gravity[8] and, as some had objected to Copernicus, houses and people ought to be thrown in the air by the

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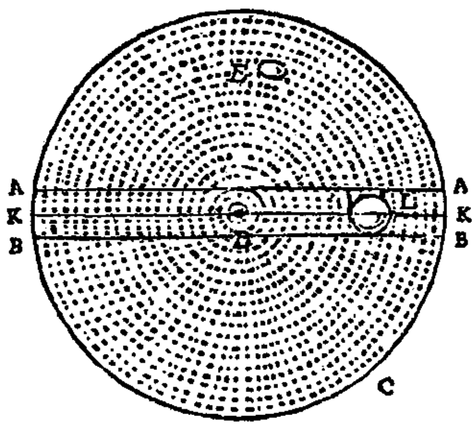
[4] The second clause of this sentence was not in the 1669 version.
[5] French '*experience*' = 'experiment' or 'experience' in English.
[6] "Horologium Oscillatorium," *OCCH*, Vol. 18, pp. 366-368; in English translation: *Christiaan Huygens' The Pendulum Clock or Geometrical Demonstration Concerning the Motion of Pendula as Applied to Clocks*, tr. Richard J. Blackwell (Ames: The Iowa State University Press, 1986), pp. 176-178. A posthumous paper giving demonstrations of these theorems can be found in *OCCH*, Vol. 16, pp. 253-301.
[7] Ibid., Theorem V.
[8] '*Gravité*' in the French rather than '*pesanteur*'.

rotation of the Earth in 24 hours, I will nevertheless point out that the same endeavor that causes bodies turning in a circle to be drawn away from the center is the cause of other bodies converging toward this center.

7.[9] Let us imagine that, around center D, there turns a fluid matter contained in space ABC, from which it cannot escape because of the other bodies that surround it. It is certain that all the parts of this fluid endeavor[10] to move away from the center D, but without any effect, since those which must follow in their place have the same tendency to move away from the center. But if among the parts of this matter there is something, like E[11], which does not follow the circular motion of the others, or which moves less quickly than the others surrounding it, I say that it will be pushed toward the center. Because, not



(p.132) endeavoring to move away or doing so less than the nearby parts, it will yield to the endeavor of those that would be less drawn from the center D, and its position relative to them will be approaching toward the center, since it could not do otherwise.

We can see this in effect in an experiment that I have done expressly for the purpose, which surely merits being remarked on because it reveals to the eye a picture of gravity. I took a cylindrical vessel, around eight or ten inches in diameter, the base of which was clean and solid, with a height only a half or a third of its width. Having filled it with water, I threw in some crushed Spanish beeswax[12] which, being just a bit heavier

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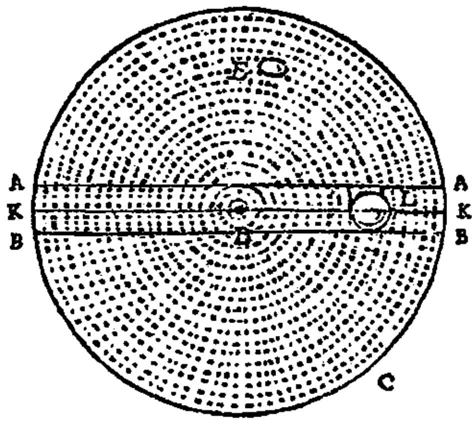
[9] The 1669 version had numbered sections; the section beginning here was numbered 5.
[10](First use of 'endeavor' for the French 'effort'.)
[11] In the *OCCH* version this letter was incorrectly 'F' in the text, though 'E' in the figure. The original has 'E' in both places.
[12] Check Boyle's collected works for this experiment and clue to why Spanish.



than the water, goes to the bottom. I then covered it with a piece of glass laid directly on the water, which I attached all the way around with some cement so that nothing could escape. Having arranged it thus, I placed this vessel in the middle of the round table of which I spoke a little earlier and, causing it to turn, I saw immediately that the bits of Spanish wax, which were touching the bottom and following more the motion of the vessel than that of the water, gathered all around the sides, the reason being that they have more force than the water to move away from the center. Having continued to turn the vessel with the table for a little while, causing the water to acquire more and more circular motion, I suddenly stopped the table. At that instant all the Spanish wax fell to the center in a pile, which represented to me the effect of **(p.133)** gravity. The reason for this was that the water, notwithstanding the vessel's rest, still maintained its circular motion and consequently its endeavor to move away from the center, whereas the Spanish wax had lost its motion, or very nearly so, on account of touching the bottom of the stopped vessel. I noticed also that this powder returned to the center in spiral lines because the water still carried it a bit. But if we arrange some body in this vessel so that it cannot fully follow the motion of the water but can only go toward the center, it will then be pushed entirely straight. For example, if L is a small ball that can roll freely on the bottom between threads AA, BB, and a third one a little higher, KK, held horizontally through the middle of the vessel, we see immediately that if the motion of the vessel is stopped, the ball will go to the center D. And we must notice that we can make body L of the same gravity as the water in this last experiment and it will succeed even better; so that, without any difference in the gravity of the bodies that are in the vessel, the motion alone produces the effect here.

The experiment that Mr. Descartes proposes in one of his published letters[13] differs quite a bit from this one because he fills vessel ABC with small lead balls mixed with some pieces of wood, or some other material lighter than the lead. Turning everything together, he says that the pieces of wood will be driven towards the middle of the vessel, which I can well believe, provided that one continuously taps lightly on the sides of the vessel in order to facilitate the separation of these two materials. But what occurs here is not at all appropriate to represent the effect of gravity, since we would have to conclude from this experiment that bodies that contain less matter are those that weigh more, which is contrary to what is observed of true gravity. In another **(p.134)** letter, he suggests throwing small bits of wood into some turning water, and he says that they will go towards the middle of the water. If he means at this point that some wood floats on the water, as it appears he does, there will be no concentration of wood in the middle. But if he means that it settles on the bottom, this will be exactly the same experiment that I proposed a little earlier, and the wood will amass in the center; but this will be because its circular motion will be slowed by touching the bottom of the vessel, a judgment about which Mr. Descartes has not spoken.

So, having found in nature an effect similar to that of gravity with a known cause, it remains to be seen if we can suppose that something similar happens with regard to the Earth, that is to say whether there would be some motion of matter that forces bodies to tend to the center and that at the same time accommodates all the other phenomena of gravity.

Supposing that the daily motion of the Earth and the air and ether that surround it have this same motion, gravity still might not be produced. Because, according to the experiment reported a little earlier, the terrestrial bodies might not follow the circular motion

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[13] Letter to Mersenne of 16 October 1639. Letter no. 174 in A&T (excerpted in Cottingham et al., The Philosophical Writings of Descartes, Vol III, (Cambridge: Cambridge University Press, 1991), pp. 138ff.

of the celestial matter, but might be as at rest in relation to it, if they were pushed by it to the center.

But, if we say that celestial matter turns in the same direction as the Earth, but much faster, it would follow that this rapid motion of a matter that moves continuously and all in the same direction would be detectable and would carry with it the bodies that are on the Earth, the same as the water bearing the Spanish wax in our experiment – which it does not in the least. Beyond that, the circular motion around the axis of the Earth could in any case push the bodies that were not following **(p.135)** this motion towards the axis in such a way that we would not see weighted bodies fall perpendicularly to the horizon, but rather in lines perpendicular to the axis of the world – which is also contrary to experience.

Thus, in order to explain gravity of the sort that I envisage here,[14] I will suppose that in a spherical space, which includes the Earth and the bodies that surround it up to a great distance, there is a fluid matter that consists of very small parts and that is diversely agitated in all directions with great speed. Since the matter cannot leave this space, which is surrounded by other bodies, I say that its motion must become in part circular around the center; yet not so much so that it begins to turn all in the same direction, but in such a way that most of these different motions would be made on spherical surfaces around the center of said space, which in this case would also be the center of the Earth.

The reason for this circular motion is that matter contained in such a space is more easily moved in this manner than by straight motions opposed to one another, which, being likewise reflected (because this matter is not able to leave the space that encloses it), are forced to change to circular.

We see this effect of motion when we test silver in a small cup. The small lead ball mixed with the silver, having its parts strongly agitated by heat, turns unceasingly around its center, first in one direction and then in the other, constantly changing so quickly that the eye struggles to perceive it. The same thing also happens when we draw a drop of

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[14] In the 1669 version, this reads, "In order then to provide a possible cause for gravity, I will suppose that ...."

tallow from a candle near to the flame, holding it suspended at the tip of a pair of snuffers, for it is set to turn with a very great speed.

It is true that ordinarily this drop turns entirely in one **(p.136)** direction or the other, depending on how the candle flame touches it. But it need not happen this way in the celestial matter, as I have supposed, because once it is moving in all directions it must always remain thus. Although it might become spherical, because there is no reason why the motion of one part of the matter would impose itself on that of the others in order to make all the mass turn in the same direction. On the contrary, the law of nature, which I discuss elsewhere, is such that it always maintains the same quantity of motion in the same direction in the collision of diversely agitated bodies.[15]

And though these circular motions in so many diverse directions in the same space must often appear to oppose or to prevent each other, the great mobility of the matter, aided by the smallness of its parts, which far surpasses the imagination, nevertheless causes it to endure all these different agitations more easily. We see when we boil some water in a glass vial how many different motions its parts are capable of, and we must imagine the liquidity of the celestial matter incomparably greater than what we note in water, which, being composed of heavy parts stacked up one upon the other, is rendered sluggish in motion. Instead the celestial matter, moving freely on all sides, very easily takes different impressions from the diverse meetings of its parts, or from the weaker impetus of other bodies. Were the air thus, it would not yield as easily as it does to the motion of our hands. So, we must take into account that the circular motions of this fluid matter around the Earth are very often interrupted and changed by other motions, but that the circular

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[15] This sentence was not in the 1669 version, but it did appear in the 1693 version. None of the numbered propositions in Huygens's unpublished paper on motion under impact expressly states the indicated result, which is a version of the principle of conservation of linear momentum. Huygens did, however, present this result in his *Journal des Sçavans* article of 18 March 1669 listing his results on impact; this article was published in *Phil. Trans.* 46 (12 April 1669), pp. 925-928; see especially p. 928, paragraph 5.

motion[16] persists there always more than those motions that follow different paths. This suffices for the present illustration.

(p.137) It is not difficult now to explain how gravity is produced by this motion. If within the fluid matter that turns in the space that we have supposed, the circular motion[17] encounters some parts much greater than those that compose the fluid matter, or some bodies formed by an amassing of small parts hooked together, these bodies will necessarily be pushed towards the center of motion, since they do not follow the rapid motion of the aforementioned matter; and if there is a sufficient amount of them, they will form there the terrestrial globe, supposing that the Earth was not yet formed. The reason is the same as the one reported in the experiment above, which showed that the Spanish wax amassed in the center of the vessel. This then is in all likelihood what the gravity of bodies truly consists of: we can say that this is the endeavor that causes the fluid matter, which turns circularly around the center of the Earth in all directions, to move away from the center and to push in its place bodies that do not follow this motion.

Now, the reason why heavy bodies that we see descend in the air do not follow the spherical motion of the fluid matter is obvious enough. Since this motion is directed toward all its sides, the impulses that a body receives follow so suddenly one upon the other that less time passes than would be necessary for it to acquire a detectable motion.[18] But as this reason alone does not suffice to prevent bodies smaller than the eye can perceive, such as the bits of dust which fly about in the air, from being chased here and there by the rapidity of this motion, it is necessary to be aware that these small bodies do not float only in the liquid matter that causes gravity, but that outside of that one there are other matters, composed of much larger particles, which fill the greater part of the space

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[16] The singular masculine pronoun *il* has no apparent anaphora. We have taken it to be referring to circular motion, meaning the circular component of the motions.

[17] Here again we have taken the masculine pronoun *il*, which has no apparent anaphora other than the '*il*' referred to in the prior footnote, to be referring to circular motion.

[18] The remainder of this paragraph does not appear in the 1669 version, but does appear in the 1693 version.

around them, and likewise the spaces of the **(p.138)** heavens. These particles, although diversely agitated and reflected among themselves, do not follow the sudden motion of the fluid matter because, as they are contiguous or scarcely distant from one another, an excessively large quantity would have to move all at once. We know that there are primarily particles of air around the Earth, which we will soon show to be larger than those of the fluid matter that we have supposed. I say moreover that there is a matter, the particles of which are smaller than those of air but larger than those of this fluid matter. This is proven by our experiment conducted with a machine that voids air.[19] There we notice the effect of an invisible matter that has weight where there is no air, seeing that it supports water suspended in a glass tube, the open end of which is plunged in some other water, and it causes water from a curved siphon to run down, as it does in the air, provided that the water in these experiments has been purged of air, which is done by leaving it for some hours in the void.[20] It appears from the beginning that particles of this heavy[21] and invisible body are smaller than those of the air since they pass through the glass that excludes air and in there they offer a glimpse of their gravity. It appears moreover that they must be larger than the particles of fluid matter that cause gravity, so that the body that they form does not follow the motion of this matter, because were it following the motion, it would not be heavy. We might have around us still other sorts of matter with different degrees of tenuity, although all larger than the matter that causes gravity. These then will have a part in preventing the small bits of dust from being carried by the rapid motion of this matter, because they[22] do not follow this motion themselves.

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19 The air pump; Huygens developed his own version of this device after seeing Boyle's while he was in London.

20 The anomalous results Huygens describes here were a source of major controversy during the 1660s and 1670s. See Steven Shapin and Simon Schaffer, *Leviathan and the Air-Pump* (Princeton: Princeton University Press, 1985), Chapter 6, pp. 223-282.

21 The French word '*pesant*' is here translated 'heavy', meaning having weight. This translation will be used throughout.

22 The referent of 'they' is ambiguous in the French as well. From the content its referent is surely 'sorts of matter all larger than the matter that causes gravity.'

Moreover, it is not necessary to find these different degrees **(p.139)** of small corpuscles or their extreme smallness strange. For, although we have reason to believe that some bodies, scarcely visible, are already as small as they can be, reason tells us that the same proportion as there is from a mountain to a grain of sand, this grain can have to another small body, and that again to another, and thus as many times as we wish.

The extreme smallness of the parts of our fluid matter is still absolutely necessary to provide a reason for one notable effect of gravity, namely that heavy bodies, enclosed on all sides in a vessel of glass, metal, or any other existing material, are always found to weigh equally. So it must be that the matter that we have said is the cause of gravity passes very easily through all bodies that we regard to be the most solid, and with the same ease as passing through the air.

This is again confirmed because if there were not this freedom of passage, a glass bottle would weigh as much as a solid glass body of the same size, and all solid bodies of equal volume would weigh equally, since, according to our theory, the gravity of each body is proportional to the quantity of fluid matter that must rise up in its place.

This matter then passes easily through the interstices of the particles that compose the bodies, but not through the particles themselves; and this causes the various gravities, for example, of rocks, metals, etc. This is because the heavier of these bodies contain more of such particles, not in number but in volume: for, only in their place is the fluid matter able to rise. But because one could doubt if these particles, being impenetrable to the fluid matter, are as such entirely solid (because not being solid, or equally **(p.140)** being empty, they would still have to cause the same effect, for the reason I just stated), I will show that they have this perfect solidity and that consequently the gravity of bodies corresponds precisely to the proportion of matter that composes them.[23]

To this end, I will point out what occurs during the impact of two bodies when they meet in horizontal motion. It is certain that the resistance that causes bodies to be moved

23 [Is this in the 1693 version as well, or is it post-*Principia*? If it is post-*Principia*, we should include a note or a comment in the introduction contrasting the claim here with Descartes's view.]

horizontally, as a ball of marble or lead placed on a very level table would be, is not caused by their weight toward the Earth, since the lateral motion does not tend to draw them away from the Earth, and so is not at all contrary to the action of the gravity that pushes them down.

There is nothing then in the quantity of matter attached together contained in each body that produces this resistance. So, if two bodies each contain as much matter as the other, they will reflect equally, or both will remain completely motionless, depending on whether they are hard or soft. But experience shows that every time two bodies reflect equally in this way or stop one another, having come to meet with equal velocities, these bodies are of equal gravity. It follows then from this that those bodies that are composed of equal quantities of matter are also of equal gravity. This was to be shown.

Mr. Descartes was of another sentiment on this, especially in regard to the free passage of the matter that causes gravity through bodies on which it acts. Concerning this last point, he claims that this matter would be prevented from continuing its motion in a straight line by its meeting with the Earth and, on account of this, it moves away from the Earth as much as it can. In this he appears to have overlooked that property of gravity that I had commented on a little earlier. **(p.141)** For, if the motion of this matter is prevented by the Earth, it will penetrate neither metal bodies nor those of glass any more freely. From this it would follow that lead enclosed in a vial would lose its weight in relation to the vial itself, or that at least this weight would be diminished. Moreover, if a heavy body were carried to the bottom of a pit, or into some deep shaft or mine, it ought to lose much of its gravity. But we have not found by experience, as far as I know, that it loses anything at all.

As for the other point, Mr. Descartes claims that although a mass of gold would be 20 times heavier than a portion of water of the same size, the gold nevertheless can only contain 4 or 5 times as much matter as the water: first, because it is necessary to subtract, we should rather say add, one pound equally to both on account of the air in which we weigh them; and second, because water and other liquids have some lightness with regard to solid bodies, particularly as the parts of the former are in continual motion.

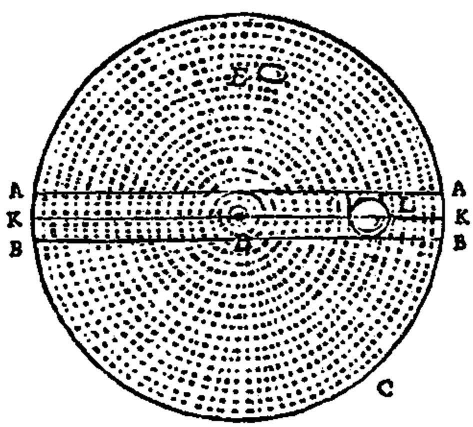
But to the first of these two ideas we can respond that, since the gravity of the air around us is to that of the water as 1 to 800,[24] the weight that will have to be added equally to that of the gold and the water, found by the balance, will not be noticeable. And for the other idea, were it valid, it would have to be the case that the same portion of water would weigh much more when frozen than it would when a liquid; and likewise metals in bulk would weigh more than when they are melted, which is contrary to experience. In addition, I do not see how he conceived that the motion of the parts of liquid bodies will give them some lightness, that is to say some endeavor to move away from the center, since this would require their motion to be circular around the center of the Earth, or to be stronger toward the top than the bottom, which he has never claimed. Entirely **(p.142)** to the contrary, he has said that the parts of liquids move in all directions indiscriminately.

He seems also to have overlooked how great the velocity of the fluid matter must be in order to yield as much gravity as we find for the most part in bodies. Otherwise, he would have certainly reasoned that the motion that the particles of water and some similar liquids can have is not at all comparable to the motion of this matter that causes gravity.

As for myself I have carefully researched the degree of this velocity and I believe it possible to determine more or less how high it has to be. And since several other natural effects can be obtained from my calculation, it would not be unhelpful to show here what gives rise to it and on what it is founded. Going back then to the figure that I used above, seeing that the gravity of body E is exactly equal to the endeavor with which an equal portion of the fluid matter tends to move away from the center D – or rather that this is the same thing – it is necessary that a pound of **(p.143)** lead, for example, weigh as much towards[25] the Earth as a mass of fluid matter of the same size (I mean of the size of its

[24] In the 1669 version Huygens said that the ratio of the density of air to that of water was around 1 to 900 or 1 to 1000. Newton said this ratio was 1 to 800 or 900 in the first edition of the *Princpia*. The modern value is 1 to 830.
[25] '*vers*' = 'near', 'around', 'towards.'

solid parts) weighs in an upward direction,[26] in order to move away from the center by virtue of its circular motion. But the lead matter and the fluid matter do not differ at all according to our hypothesis. We can then say that a pound of lead weighs as much towards the bottom as it will weigh towards the top if, remaining at the same distance from the center of the Earth, it turned around with as much velocity as the fluid matter does. But I find with my theory of circular motion, which agrees perfectly with experience, that if we claim that the endeavor of a body turning in a circle to move away from the center exactly equals the endeavor of its gravity alone, it must make each turn in as much time as a pendulum of the length of the semidiameter of this circle takes to make two arcs.[27] We



must then determine in how much time a pendulum the length of a semidiameter of the Earth would make these two arcs. This is easy, by the known property of pendulums, and by the length of those that beat seconds, which is 3 feet 8 1/2 lines, measured in Paris. I find it would be necessary for these two vibrations to be 1 hour 24 1/2 minutes, supposing, according to the exact measurement of Mr. Picard, that the semidiameter of the Earth is some 19,615,800 [Paris] feet by the same measure. The velocity then of the fluid matter near the surface of the Earth must be equal to that of a body that would make the circuit of the Earth in this time of 1 hour 24 1/2 minutes. This velocity is very nearly 17 times greater than that of a point below the equator, which makes the same revolution in relation to the fixed stars as we must take here, in 23 hours 56 minutes.

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26 In the French, *du costé d'enhaut*, literally "in the direction from above."

27 The claim is Proposition X of the Appendix to *Horologium Oscillatorium*.

This comes out in a proportion between this time and that of 1 hour 24 1/2 minutes, which is very nearly 17 to 1.[28]

I know this speed will seem strange to someone who wishes to compare it with the motions that are seen here in our midst. But this should not cause any difficulty; nor likewise, will it appear extraordinary in proportion to its sphere[29] or the size of the Earth. If, for example, looking at a terrestrial globe such as we make for use in geography, **(p.144)** we imagine a point that advances only one degree in 14 seconds or beats of the pulse, which is the velocity of the matter as I have just stated, we will find this motion very moderate, and it could even appear to be slow.

There are several other natural effects that appear to require an extremely agitated matter that easily penetrates the pores of bodies. One is the force of powder from a cannon, which does not get its violent motion from the lighting of it alone, nor from what approaches the fuse. Consequently it must be that it comes from some other matter that has this motion and that is found everywhere, causing its effect every time it finds a suitable setting. Another, I understand, is the force of elasticity, for steel and other solid bodies as well as for the air. To what can we attribute the force of animal muscles? We explain this very well by the fermentation that the juice of the nerves causes in the blood but where will the force of the fermentation come from if not from some external motion? The forceful action of frost no longer appears inconceivable if we have recourse to a violent impulse of some matter that causes either the expansion of the ice, by introducing some other particles there, or the expansion of the bubbles that form there, by augmenting the air they contain. This is done with so much violence that I have seen some musket barrels in which water has been trapped burst.

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[28] Huygens has refined some of the numbers here versus the 1669 version: 1 hour 24 1/2 minutes was 1 hour 25 minutes, 23 hours 56 minutes was 24 hours, and 19,615,800 Paris feet was 19,595,154 Rhenish feet (18,935,926 Paris feet). In the second edition of the *Principia* Newton used 19,695,539 Paris feet for the radius of the Earth, but changed to 19,615,800 in the third; in the first edition Newton gave no number for the radius, but did assert that the squared ratio of the times was 290 4/5 to 1, in contrast to the 289 to 1 he used in the second and third editions, corresponding to Huygens's value.

[29] That is, the sphere the motion describes.

But to return to gravity, the extreme velocity of the matter that causes it serves also to explain how weighted bodies, when falling, will always accelerate their motion even though they have already reached a very great degree of velocity. The motion of the fluid matter far surpasses, for example, the motion of the cannon ball that falls from the air after having been pulled up in it perpendicularly. The cannon ball, up to the end of its fall, sustains an almost constant **(p.145)** pressure from the matter, and consequently its speed is constantly increased by it. If instead the matter had only a moderate motion, the ball having acquired as much, would no longer accelerate its fall because, otherwise, the ball would have to push this very matter to succeed in its place with more velocity than it would have for doing this by its own motion.

We can finally find here proof of the principle that Galileo had used to demonstrate the proportion of the acceleration of falling bodies, namely that their velocity is increased equally in equal times. Bodies are pushed successively by the particles of matter that try to rise in their place and that, as we are coming to see, constantly act on them with the same force, at least in the descents that lie within our experience. It is then a necessary consequence that the increase in velocities will be proportional to the times.

So then I have explained, with one hypothesis that contains nothing impossible, why terrestrial bodies tend to the center; why the action of gravity[30] cannot be prevented by any known body; why the parts within each body all contribute to its gravity; and finally why falling bodies constantly increase their velocity in proportion to the times. Such are the properties of gravity as we have distinguished them so far.[31]

There still remains one property, that until now we believed less certain, namely that bodies weigh as much in one place on the Earth as they do in another. Since this has been shown otherwise by some recently made observations, it is worth the trouble to examine its origin and its consequences.

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30 '*Gravité*' in the French rather than '*pesanteur*'.

31 This is the end of the tract on the cause of gravity of 1669 as well as of its 1687 rewrite.

It is claimed that a seconds pendulum is found to be one and a quarter lines shorter in Cayenne, a country **(p.146)** in America only four or five degrees distant from the equator, than it is in Paris. From this it follows that, if we use pendulums of equal length, the one in Cayenne makes its arc more slowly than the one in Paris. Supposing this is true, we can conjecture only that this would be a sure sign that weighted bodies descend more slowly in Cayenne than in France. And because this difference would cause a completely opposite effect, were it known to be attributable to the tenuity of the air, which is greater in the torrid zone, I do not see that there could be any other reason except that like bodies weigh less at the equator[32] than in regions separated from it. I realized, as quickly as this new phenomenon had been communicated to us, that the cause could be attributed to the daily motion of the Earth. This, being greater in each country the nearer to the equator[33], must produce a proportional endeavor to push bodies from the center and thus to rid them of a certain part of their gravity. Given what was explained above, it is easy to know which part this must be in bodies that are found at the equator. Having found, as we have seen, that if the Earth turned 17 times faster than it does, the centrifugal force at the equator would be equal the total gravity of a body, it must be the case that the motion of the Earth, such as it is now, remove one part of the gravity, which would be to the entire gravity as one to the square of 17, which is to say 1/289; because the forces of the bodies to move away from the center around which they turn are as the squares of their velocities, according to my Theorem Three in *Vi Centrifuga*.[34] Since each body at the equator then weighs less by 1/289 of what it would be if the Earth did not turn on its axis, it follows from the laws of mechanics that the length **(p.147)** of a pendulum in this place must also be diminished by 1/289 in order to make its arcs in the same time that it would make them on an immobile Earth.

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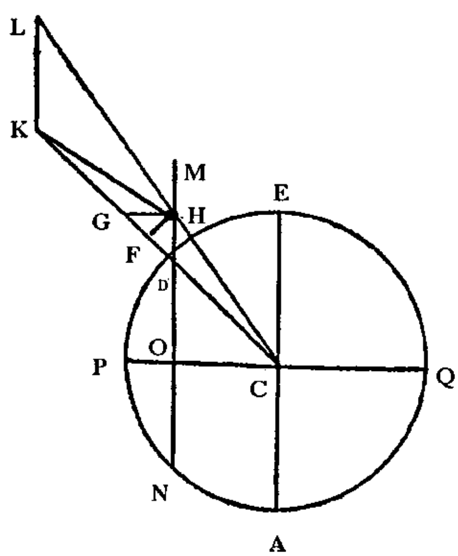
32 '*Sous la ligne*' in French.
33 '*La ligne équinoxiale*' in French.
34 That is, Theorem 3 of the Appendix to *Horologium Oscillatorium* cited above.

But to know how much shorter a pendulum, transported from Paris to the equator must be, it is necessary to consider that its length at Paris is already less than if the Earth were still, because the daily motion at this parallel also generates an endeavor to draw bodies away from the center of the Earth. This endeavor is not, however, as great as it is at the equator; this is as much because the circle of motion is less, as it is because the endeavor does not drive the bodies straight up, but along the perpendicular to the axis of the Earth, as we will see in the figure. The circle PAQE here represents the Earth, cut by a plane which passes through its two poles, P and Q. The center is C; the equator is ECA; the parallel of Paris DON, supposing that Paris is at D. KH represents a string that supports a lead shot H, turned aside from the perpendicular KDC because it is pushed back by the circular motion along line ODM, which I suppose to pass through the weight H.



In order to know now what the position of thread KH must be and how much less the lead shot H weighs in this way than if it were to hang perpendicularly **(p.148)** along KD, it is necessary to consider point H being pulled by three threads, HC, HM and HK. HC pulls it toward the center of the Earth with all the weight that the lead shot would have if the Earth was immobile. But HM pulls it from this direction with the force that the motion of the Earth gives in circle DN. And the third thread HK pulls or is pulled by a force that is the one that we seek. Having then lengthened CH and drawn KL parallel to DM, we know that the three sides of triangle HLK are proportional to the forces[35] that pull point H: side LH corresponding to the pull from HC; side KL to the pull from HM; and side HK to the force that pulls or holds up the lead shot by thread KH. But the triangle KDH is supposed to have all its sides equal to

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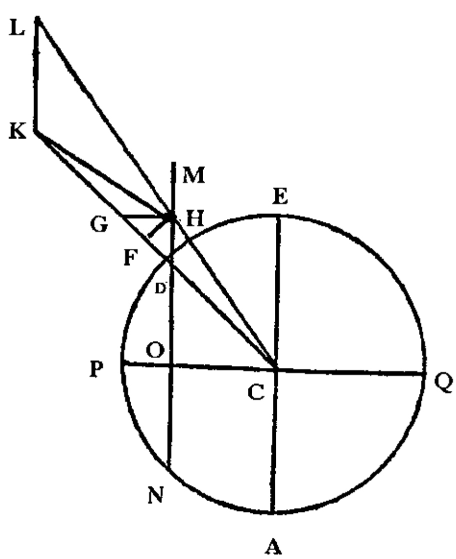
[35] This occurrence of the English 'force' and all subsequent ones in the paragraph are translations of '*puissance*'; all other occurrences of the English 'force' throughout are translations of the French '*force*'.

those of triangle HLK because CHL is almost parallel to CDK. The sides of KDH then correspond to those same forces: namely side KD to the absolute gravity of the weight of H, as it would be if the Earth did not turn; DH to the force imparted to it by daily motion; and KH to the gravity that we seek. So this triangle KHD is given, seeing that we know that the circular endeavor at the equator at E is 1/289 of the absolute weight; and since this endeavor is to that at D, or at H, as EC to DO, which are in a given proportion, we will **(p.149)** then also know what part of the absolute weight the centrifugal endeavor is at D or H. This is to say that the proportion of DK to DH will be known, being composed of 289 to 1 and of EC to DO.[36] But the angle HDK is also known, being equal to that of the latitude of Paris, namely as some 48 degrees 51 minutes. Then we will know the proportion of DK to KH, which is that of the absolute gravity of bodies to that which they have in Paris, and which is likewise the length of the pendulum on the immobile Earth to the length that it must have at this parallel, according to what has already been said. And seeing that the length of a seconds pendulum is given in Paris, we will also know what the seconds pendulum on an immobile Earth would be, and what the difference is, and how much less this difference is than this 1/289 that we have found at the equator.



In order to make this computation with ease and without the calculation of the triangles, it is necessary to know, and we will now prove it, that as the square of the radius EC is to the square of DO, the sine of the complement of the latitude of Paris, so is 1/289, the difference or shortening of the pendulum at the equator, to the difference or shortening

[36] That is, the product of 289/1 and EC/DO; in other words, the ratio of the centrifugal endeavor at Paris to gravity at Paris in the absence of rotation is sin(90-latitude)/289.

at Paris.[37] This is found by that to be 1/668[38] of the length of a pendulum on an immobile Earth, or at the pole. Seeing that the seconds pendulum in Paris is some 3 feet 8 1/2 lines, it follows that the length of a pendulum on an immobile Earth, or at the pole, would be some 3 feet 9 1/6 lines. Removing from that 1/289, which makes 1 1/2 lines, we would have the length of the seconds pendulum at the equator as some 3 feet 7 2/3 lines.[39] Thus, this pendulum would be shorter than that in Paris by 5/6 of a line, which is a little less than what had been found at Cayenne by Mr. Richer, namely one and a quarter lines.

But we cannot entirely trust these first observations, the occurance of which we do not see as conspicuous in any way[40] – and we can trust still less, given what I believe, in those that are said to have **(p.150)** been made in Guadeloupe, where the shortening of the Paris pendulum had been found to be two lines. We must hope that in time we will be informed exactly of these different lengths, at the equator as well as in other regions; and certainly it is something that well deserves being researched with care, even if it would only be to correct, according to this theory, the motions of pendulum clocks, in order to make them serve as a measure of longitudes at sea. So a clock, for example, which was accurately adjusted in Paris, being transported to some place at the equator, will be slowed down around one minute and 5 seconds in 24 hours, as is easy to calculate following the preceding reasoning. And likewise in proportion for each different degree of latitude. We will find then that these retardations follow almost precisely the same proportion[41] as the reduction in the length of the pendulum, and that the greatest retardation, namely that of a clock at the equator that had been set at the pole, would be very near to 2 1/2 minutes per

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37 Let Δl_{Paris} be the amount the seconds pendulum is shorter at Paris than at the pole, and Δl_{Equat} be the amount it is shorter at the equator than at the pole, namely 1/289 of its length at the pole. Huygens is saying that $\Delta l_{Paris}/\Delta l_{Equat}$ = the square of sin(90-latitude of Paris); this generalizes to give $\Delta l/\Delta l_{Equat}$ at other latitudes as well.

38 More exactly, 1/667.4.

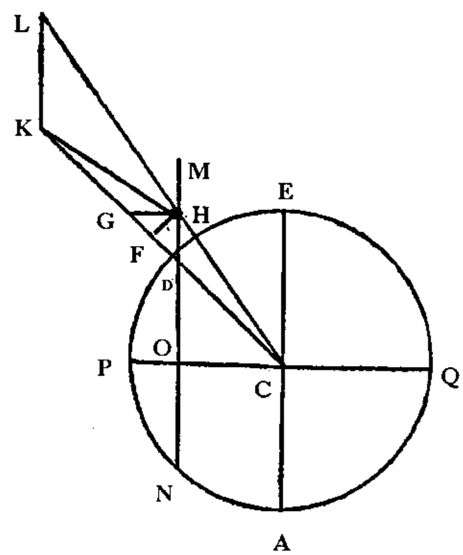
39 In the *OCCH* version the length of the seconds pendulum at the equator is incorrectly given as 3 ft 7 1/2 lines; the original has 3 ft 7 2/3 lines. If we carry out his calculation using decimals and 1/667.4, the numbers are 3 ft 9.16 lines at the equator and 3 ft 7.63 lines.

40 [There must be a better translation of '*desquelles on ne voit marqué aucune circonstance*'.]

41 The time lost per day at any latitude φ to the time lost per day at the equator is very nearly as the square of the sin (90–φ).

day. Having then calculated some tables, we could correct, by their average, the motion of the clocks, and use the result with the same certainty as if this motion were equal everywhere.[42]

In order to demonstrate what was put forward a little before, **(p.151)** while seeking the shortening of the pendulum at Paris (and likewise in any other place) when we know how much shorter it is at the equator: in the same figure, set KF equal to KH, and let HG be parallel to the axis PQ. It has been shown that HD is to DK as the endeavor to move away from the center, at D or H, is to the absolute weight on an immobile Earth. But as EC or CD is to DO, that is to say as GD is to HD, so the centrifugal endeavor at E, at the equator, is to that at D. Then as GD is to DK, so will the centrifugal endeavor at E be to the absolute weight on an immobile Earth. And line GD will be the shortening of the pendulum that is required at the equator, according to what was said previously. But FD is the shortening at Paris, and GD is to DF as the square of GD to the square of DH, because the smallness of the angle DKH makes it possible to consider HF as perpendicular to GD. The shortening then at the equator, to that agreed upon in Paris, is as the square of GD to the square of DH, which is to say as the square of CD, or of EC, is to the square of DO. This had to be shown.



It remains to consider angle HKD in this figure, which indicates how much the lead shot KH, while at rest, declines from the perpendicular KD. Here I find that at the parallel of Paris this angle is some 5 minutes 54 seconds, and that it must be still a little greater at 45 degrees of latitude.

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[42] Huygens's table for this purpose is given in his Report to the Directors of the Dutch East India Company; see page ?? below.

This declination is certainly contrary to what we had always supposed as a very certain truth - namely that a chord holding a suspended lead shot tends directly toward the center of the Earth. This angle, some 1/10 of a degree, is considerable enough to cause us to think that we would have noticed it, either in astronomical observations or in those that we make with the level.[43] So to speak only of the latter, **(p.152)** would it not be necessary that, looking from the direction of the north, the line of the level would drop visibly below the horizon? This has certainly never been noticed, nor surely will it ever occur. And the reason for it, which is another paradox, is that the earth is not entirely spherical, but in the figure of a sphere pulled down toward the two poles, so that it would be close to an ellipse, turning on its small axis. This comes from the daily motion of the Earth, and it is a necessary consequence of the aforementioned declination of the lead shot. Since the descent of weighted bodies is parallel to the line of this suspension, it is necessary that the surface of all liquid be inclined in a way that this line would be perpendicular to it, because otherwise the lead shot could descend further. Therefore the surface of the ocean is such that a suspended thread is perpendicular to it in all places.[44] From this it follows that the line of the level, which is to say the one that cuts the thread of the suspended lead weight at right angles, must mark the horizon, as it does, save only for the height of the place where the level is put, which would cause it to point a little higher. But the edges of the ground being generally raised about the same amount everywhere with regard to the ocean, it follows that the total combination of land and sea is reduced to the same spherical figure that the surface of the ocean necessarily gives. And it is believed that the earth took this figure when it was assembled by the effect of gravity, its matter having at the time a circular motion of 24 hours.

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[43] For details on the Level, see the Introduction, p. ??? above.
[44] This claim has come to be known as Huygens's principle.

ADDITION

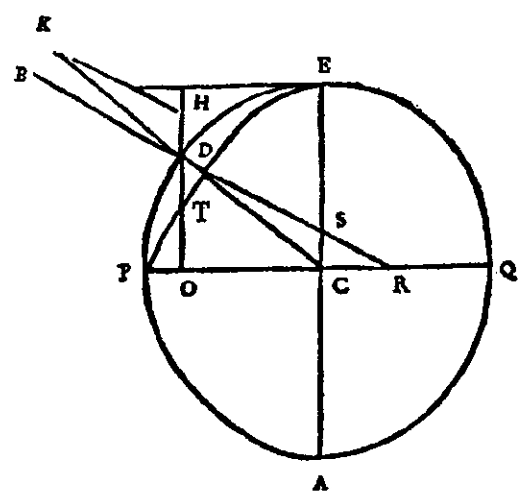
Some time after I had finished writing the preceding, I received and examined the journal of the voyage, which, by order of the Directors of the Dutch East India Company, had been made with our pendulum clocks **(p.153)** as far as the Cape of Good Hope. Since then I have also read the very scholarly work of Mr. Newton, entitled *Philosophiae Naturalis Principia Mathematica.* Both provided me with material to extend this Discourse further. First, concerning the different lengths of the pendulums in different regions, which he has also addressed, I believe I have from the average of these clocks a clear confirmation not only of this effect of the motion of the Earth but also of the measure of these lengths, which agrees very well with the calculation that I have just given. For, having corrected and adjusted, according to this calculation, the longitudes that were measured with the clocks on the return from the Cape of Good Hope to Texel in Holland (because going they were not of service), I have found that the route of the vessel was much better marked on the map than it was without this correction; so much so that arriving at this port there was not 5 or 6 leagues of error in the longitude thus adjusted. This presupposes that the aforementioned Cape had been well surveyed by the Jesuit fathers when they passed by there on the way to Siam in the year 1685, and that it is located some 18 degrees more to the East than Paris, which I know moreover to be scarcely far from the truth. The detail of this whole matter is deduced in full in the Report that I have made to said Honorable Directors concerning this voyage of the pendulums.[45] After this report had been examined by knowledgeable persons, it pleased them to direct us to conduct a second trial in order to be assured by several experiments of the soundness of this discovery. We will see what the success of this other voyage will be, and particularly what the variation of

[45] This Report, which is translated below, existed only in Dutch at the time. In spite of what Huygens says here, he did not publish it, and hence it appears that the only qualified people who had an opportunity to read the handwritten manuscript were Johan Hudde and Burchard de Volder, the "knowledgeable persons" whom the Directors asked to review it.

the pendulums is, making certain that, in order to know the variation well, these clocks, by their acceleration and their deceleration, give an average more reliable than actually measuring the length of the seconds-pendulum in different countries. Meanwhile, **(p.154)** because experience in the trial of which I have been speaking is so well in accord with what I have found by reasoning, I trust enough in this to want to continue this speculation, considering first what the figure of the Earth is, since, as has been said, it is not spherical.

For this, it is useful to consider the Earth as completely covered with water, or as if all its mass[46] were another matter. And then, by what was explained above, it appears that the surface must be such that a thread supporting a lead weight, in any place whatever, would meet it at right angles, taking into account the gravity and the centrifugal force together that divert the thread from its course toward the center. For, if the thread does not meet the surface at right angles, the surface could not remain at the foundation where it is.

Suppose then the same things as in the last figure of the preceding discourse, and



also what was explained by it, but, making the shape of the Earth a little diminished and flattened toward the poles such that the axis PQ would be shorter than the diameter EA. Let BDSR be drawn parallel to KH, cutting EA and PQ at S and R. The thread KH, which supports the lead weight, or rather its parallel BD, must meet the surface of the ocean at right angles. This thread hangs such that KD is to DH, or DC to **(p.155)** CS, as the absolute gravity is to the centrifugal force at D. This ratio is composed of the ratio of the absolute gravity to the centrifugal force at E, which is as 289 to 1, and of the ratio of that force to the centrifugal force at D, which is as EC to

46 Even though Huygens had read the *Principia* at the time he wrote this, there is no reason to think that he is here using the term '*masse*' as it has come to be used in modern physics.

DO.[47] It thus appears that the nature of the curved line EDP is determined by the property of its perpendicular, DR. That is to say, in drawing such a perpendicular the ratio of DC to CS must always be composed of this given ratio, and that of EC to DO. Or equally, as we can easily infer, that the ratio of DO to CS, or of OR to RC, must be composed of this given ratio and that of EC to CD.

Now, it is difficult to find curved lines in this way from the given property of their perpendiculars, or, equivalently, from the property of their tangents.[48] But there is an easy enough method for this particular curve, based on the equilibrium of certain canals of which Mr. Newton has given the first sketch.[49]

The canal that he supposes is represented in our figure by ECP, forming a right angle at the center of the Earth. It is necessary to think of it as being a somewhat small cavity filled with water. This being the case, it is certain that the two legs, EC and CP, must stay in equilibrium if we suppose that the Earth, being completely composed of water, takes a shape with the diameters EA and PQ. For this water in the canal, thinking of it without the canal, will otherwise no longer remain in its place, contrary to what we supposed. From this it is easy to find the ratio of EA to PQ. For, taking EC to be a and CP to be b, and representing the absolute gravity by a line p, and the centrifugal force at E by the line n, the weight of the canal PC is pb, obtained by multiplying all the parts of this canal equally by the line p. But the weight of canal EC, which would be **(p.156)** pa, is diminished by the centrifugal force of all its parts, of which the highest, at E, has force n; all the other parts are proportional to that one, according to their distance from the center D, which makes $1/2\ na$ for all the centrifugal force of the water of canal EC. This being removed from its weight pa, leaves $pa - 1/2\ na$, which must be equal to the weight pb of

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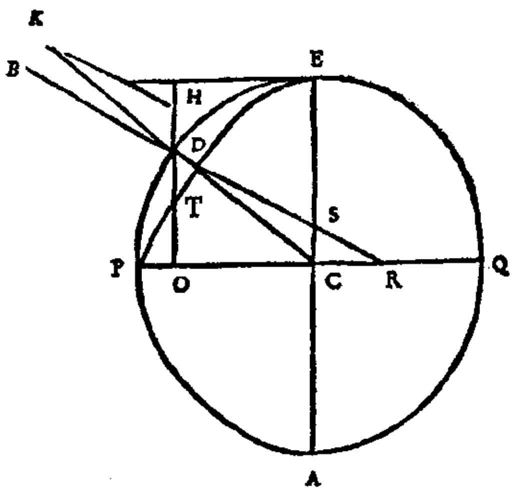
47 In other words, DC/CS = 289*EC/DO.

48 The required mathematics consists of an integral of the tangents, starting from the pole. If Newton, with his knowledge of the calculus, found this integral too imposing to attempt in the *Principia*, Huygens, who had not yet acquired any detailed knowledge of the calculus from Leibniz, must have regarded it as intractable.

49 Newton introduced this ingenious method of canals to solve the problem of the shape of a fluid Earth under universal gravity in Proposition 19 of Book 3 of the *Principia*. Huygens saw that the method could be used equally with other rules of gravity.

canal PC. From this it appears that a is to b as p to p-1/2 n. That is to say, that the diameter EA of the Earth is to its axis PQ as 289 to 288 1/2, or as 578 to 577, because the ratio of p to n would be as 289 to 1.[50]



In order to find next what the curved line EDP is, I imagine the canal ECD full of water, and drawing DO perpendicular to the axis PC, I take CO to be x, and OD to be y, the other lines being named as before. It is certain that the water from EC and that from DC must again be counterbalanced. And likewise, this must happen regardless of how we understand the canal to be made, provided that it ends on both sides at the surface – as, for example, if it came through DOCE, or DOP, or DCP. Now, the centrifugal force of all the water at CD is equal to that of the water that would refill canal OD, supposing it to be the same size, which is easily seen from the mechanics of inclined planes. But as EC, taken to be a, is to DO, taken to be y, so is the centrifugal force at E, which would be n, to the centrifugal force at D, **(p.157)** which will then be ny/a. Now multiplying the contents of canal DO, taken to be y, by half of this, makes the centrifugal force of this canal equal to $1/2(ny^2/a)$, which is also the centrifugal force of canal CD. But the gravity of this canal CD towards the center C is $p\sqrt{x^2/y^2}$. Then its

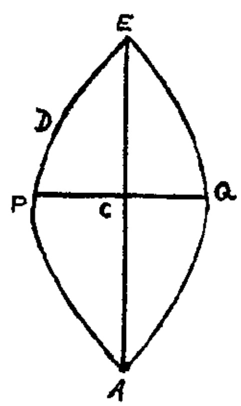
[50] Huygens's analysis presupposes that the force of gravity is the same throughout the interior of the Earth as it is at the surface. Under Newton's rule of universal gravity, the force of gravity varies linearly with the radius of a sphere from the center to the surface, and then as $1/r^2$ beyond the surface -- this under the assumption that the density of the sphere is uniform. In contrast to Huygens's value, the value Newton gives in Proposition 19 of Book 3 of the first edition of the *Principia* for the ratio of the radii of the equator and the pole is 692 to 689 -- again assuming uniform density. In effect, then, Huygens is claiming that the radius at the equator is around 6.8 miles larger than the radius to the poles, in contrast to the value of 17 miles Newton gave in the first edition.

pressure, which remains toward C, will be $p\sqrt{(x^2/y^2) - 1/2\,(ny^2/a)}$, which must be equal to *pa* – 1/2 *an*, the pressure of canal EC, found previously.

This equation, supposing *ap/n* to be *f*, amounts to the following:[51]

$$y^4 = 4f^2y^2 - 4a^2f^2 + 4f^2x^2$$
$$- 4afy^2 + 4a^3f$$
$$+ 2a^2y^2 - a^4$$

This shows that the curved line EDP is not a conic section, unless p and n are equal – that is to say when the centrifugal force of a body placed at E is supposed equal to its gravity toward center C. For, then it comes out that f is equal to a; and the equation becomes $y^4 = 2a^2y^2 - a^4 + 4f^2x^2$; or better $y^4 - 2a^2y^2 + a^4 = 4f^2x^2$ and finally $y^2 - a^2 = 2ax$. This indicates that in this case EDP is a parabola, as in the figure, having vertex P, axis PC equal to half of CE, and parameter double the same CE.



So, if the Earth, with diameter EA of its true length, turned 17 times faster than it does on its axis PQ , (for then the centrifugal force at E would be equal to the gravity toward the center, by the demonstration that is in **(p.158)** this Discourse), it would have the figure of a body like the two half parabolas opposed, PEC and QEC, turning around its axis PQ. And we see that there is here the greatest centrifugal force that we can suppose; for, if we had made it greater than the gravity, bodies placed at E would be carried away into the air.

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51 In modern form, Huygens's equation is

$$p\sqrt{x^2 + y^2} - \omega^2y^2/2 = pa - \omega^2a^2/2$$

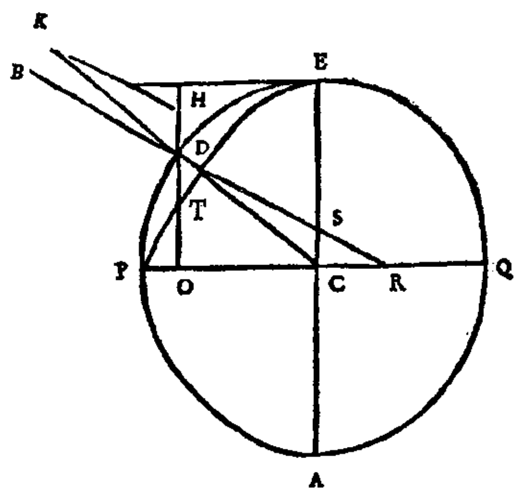
where ω is the angular velocity of the Earth. Huygens's parameter f is then $ap/(a\omega^2) = p/\omega^2$.

Outside of this case, if in the derived equation we make $y^2 = az$, z being an undetermined line, we would have

$$z = a - 2f + 2f^2/a - \sqrt{4f^2 - 8f^3 + 4f^4/a^2 + 4f^2x^2/a^2}\,.$$

And putting d for $f^2/a - f$, we will get $z = a + 2d - \sqrt{4d^2 + 4f^2x^2/a^2}\,.$

From this I know that if, CO being x, the perpendicular OT is named z, then point T will be in a hyperbola, the axis of which added to CE would be $4d$. And as $4f^2$ is to a^2, thus will be the axis to the parameter, which will then be a^2d/f^2, that is to say $a - na/p$, replacing the values of d and f. And because y^2 was equal to az, it follows that DO = y will be the mean proportional between OT and EC.[52] From this we can find the points through which the curved line EDP must pass.



But this line also satisfies what I claimed is necessary; namely that drawing DR at right angles to it, the ratio **(p.159)** of OR to RC will be composed of the ratio of p to n and EC to CD, as can be shown by algebraic calculation.[53]

I have supposed throughout this reasoning that gravity is the same inside the Earth as it is at the surface, which seems to me to be very likely, notwithstanding the reason one can have to doubt it, of which I will speak later. But were it otherwise, this will change almost nothing of what has been discovered about the figure of the Earth,[54] though certainly when the centrifugal force makes up a considerable part of the gravity, or when it

[52] That is, OT/DO=DO/EC.

[53] That is OR/RC=(p/n)(EC/CD).

[54] In other words, if gravity in the interior of the Earth were to vary with radius, the non-spherical figure of the Earth would change little. As Todhunter has shown, Huygens is correct about this. Todhunter has also shown that, if gravity in the interior of the Earth is taken to vary as $1/r^2$, the eccentricity would be 1/579 instead of 1/578. (See Isaac Todhunter, *A History of the Mathematical Theories of Attraction and The Figure of the Earth* (New York: Macmillan, 1878, reissued by Dover, 1962), p. 31ff.) As we shall see, this is the alternative of principal concern to Huygens, alluded to in the preceding sentence.

is equal to it, as in the case of the parabolic figure, it would be completely different. Moreover, when the centrifugal force at E is very small in proportion to gravity, as it is here on the Earth, the hyperbola ETP, on account of its great distance from the center, closely approaches a parabola, and consequently EDP differs scarcely from an ellipse, and also very little from a circle, because EC then surpasses CP by very little. As was found a little before, this excess is only 1/578 of EC, the semi-diameter of the Earth.

Mr. Newton came up with 1/231 of EC, and hence that the figure of the Earth differs much more from the spherical, using a completely different calculation that I will not examine here because I am not especially in agreement with a Principle that he supposes in this calculation and others, namely, that all the small parts that we can imagine in two or more different bodies attract one another or tend to approach each other mutually. This I could not concede, because I believe I see clearly that the cause of such an attraction is not explicable either by any principle of mechanics or by the laws of motion. Nor am I at all persuaded of the necessity of the mutual attraction of whole bodies, having shown that, were there no Earth, bodies would not cease to tend toward a center because of what we call their gravity.

(p.160) I have nothing against *Vis Centripeta*, as Mr. Newton calls it, which causes the planets to weigh (or gravitate)[55] toward the Sun, and the Moon toward the Earth, but here I remain in agreement without difficulty because not only do we know through experience that there is such a manner of attraction or impulse in nature, but also that it is explained by the laws of motion, as we have seen in what I wrote above on gravity. Because nothing hinders the action of this *Vis Centripeta* toward the Sun, it would be similar to what pushes bodies that we call heavy to descend toward the Earth. I thought for a long time that the spherical figure of the Sun could be produced by the same thing that, according to me, produces the sphericity of the Earth, but I had not extended the action of gravity to such great distances as from the Sun to the planets, or from the Earth to the

55 '*Peser*' in the French.

Moon, because the vortices of Mr. Descartes, which formerly appeared very likely to me, and which I still had in mind, cut across it.[56] I had not thought at all about the regular diminution of gravity, namely that it is in inverse proportion to the squares of the distances from the center. This is a new and quite remarkable property of gravity, the basis of which is well worth the trouble of investigating. But seeing now from the demonstrations of Mr. Newton that, if one supposes such a gravity towards the Sun that diminishes according to said proportion, it counterbalances the centrifugal force of the planets so well and produces exactly the effect of elliptical motion that Kepler had predicted and verified by observations, I can scarcely doubt that these hypotheses[57] concerning gravity would be true, or that the System of Mr. Newton, insofar as it is founded thereupon, would likewise be true. This should appear so much the more probable as we find in it the solutions of several difficulties that are a problem for the vortices **(p.161)** supposed by Descartes. We see now how the eccentricities of the planets are able to remain constantly the same; why the planes of their orbits do not join together, but retain their different inclinations with respect to the plane of the ecliptic; and why the planes of all these orbits necessarily pass through the Sun. We see how the motion of the planets can accelerate and decelerate to the extents that we observe, which could occur in this way with difficulty if they floated in a vortex around the Sun. Finally, we see how comets can pass through our system. For, while we know that they often enter in the region of the planets, we had some difficulty imagining how they could sometimes go in a motion contrary to that of the vortex that had enough force to carry the planets. But this doubt is also removed with the doctrine of Mr. Newton, since nothing prevents the comets from traveling in elliptical paths around the Sun, like the planets, but in more extended paths, and in a figure more different from circular so that

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56 "venoient a la traverse"? The 'it' meaning 'the action of gravity'.

57 Newton's famous remark, "*hypotheses non fingo*", followed in the next sentence by the comment that hypotheses have no place in experimental philosophy, was introduced in the second edition of the *Principia*; nothing in the first edition displays the aversion to hypotheses that he expressed in the second and third.

these bodies have their own periodic revolutions, as certain ancient and modern planets, but and astronomers had imagined.[58]

There is only this difficulty: in rejecting the vortices of Mr. Descartes, Mr. Newton claims, in order that the planets and the comets encounter the fewest obstacles in their paths, that the celestial spaces contain only a very rarefied matter. If we suppose that rarity, it does not appear possible to explain either the action of gravity or that of light,[59] at least through the mediums that I have made use of. In order to examine this point then, I propose that the ethereal matter can be supposed rarefied in two ways: either its particles may be distant from each other, with a great deal of empty space between them, or they may touch each other, with the tissue of each being very thin, and **(p.162)** with very many small empty spaces mixed with them. I admit without difficulty that there is some void. Moreover, I believe it necessary for the motion of small corpuscles amongst themselves, not being of the sentiment of Mr. Descartes, who claims that extended matter alone is the essence of bodies; but I add perfect hardness to them as well, rendering them impenetrable and incapable of being either broken or impaired. Considering the rarity of the first type, however, I do not see how we could provide a reason for gravity; and, concerning light, it seems to me entirely impossible with such voids to explain its prodigious speed, which must be 600,000 times greater than that of sound according to the demonstration of Mr. Römer, which I reported in the *Treatise on Light.*[60] This is why I hold that this sort of rarity cannot be thought to suit the celestial spaces.

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58 Confirmation of Newton's conclusion that (at least some) comets revolve periodically around the Sun did not come until decades after Huygens's death; Newton put much more emphasis on this conclusion in the second edition of the *Principia* than in the first.

59 Because Newton thought light most likely to consist of particles, he had no problem with light being transmitted across empty celestial spaces. Huygens, however, thought light consists of longitudinal waves much like sound waves, and this led him to require a medium for transmitting these waves.

60 Descartes's theory seems to entail that the transmission of light is instantaneous. In 1677 Römer noticed an anomaly in the orbits of the satellites of Jupiter, which he found he could account for by supposing that light is transmitted with a finite velocity. The magnitude of this velocity he inferred, to within 10 percent of the modern value, from the anomaly. Newton, like Huygens, accepted Römer's conclusion. Many, however, including Cassini, had not accepted it as of 1690.

There is a greater likelihood of understanding this rarity if it is of the other type, since the particles are able to touch each other, as I had supposed in the aforementioned *Treatise* and, nevertheless, resist the motion of the planets very little because of the lightness of their material. We know to what extent nature can go to compose tough bodies with little matter, especially since some very slender and delicate, or even hollow, particles can be infinitely strong. But I believe that, without considering the rarity, the great agitation of the ethereal matter can contribute a great deal to its penetrability. For, if the slight motion of particles of water makes it liquid, and of much less resistance with respect to the bodies that swim within it, like the sand or other very fine powder, is it not necessary that a matter more subtle and infinitely more agitated would also be that much easier to penetrate?

Whatever the case may be, we see that nature does not lack ingenuity in making spaces in which bodies move with very little resistance, as is clear **(p.163)** from what our hands sense in the air, and more so through the experiments that we conducted in the glass vessels, from which we drew out all the air, where the lightest feather falls with the same velocity as a ball of lead. If one wanted to maintain that this is a result of the great rarity of the matter that remains in this airless void, I will urge to the contrary that we perceive there the effect of a matter which weigh[61] considerably, as we saw in the experiment reported above.[62]

With regard to the reasoning of Mr. Newton in Prop. 6 of Book 3[63] to prove the extreme rarity of the ether – namely that the gravities of bodies are as the quantity of matter that they contain, and that, this being so, if the spaces of air or of ether were also full of

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61 '*Pèse*' in the French.

62 Huygens is referring to his (anomalous) result of the column of mercury still receiving some support after he had evacuated all of the air in the vessel. See note 18 above.

63 The parts of Proposition 6 and its corollaries to which Huygens alludes here remained almost entirely the same from the first edition of the *Principia* to the second and third. The most important change in the latter editions, which Huygens of course never saw, was the appeal to the third rule of reasoning in justifying the conclusion that gravity is universal among all bodies. This rule and the appeal to it in Proposition 6 were added in the second edition apparently in an effort to fortify the reasoning to universality.

matter like gold or silver, these metals would not fall, because a solid body which does not have a greater specific gravity than a liquid would not be able to sink – I say that I agree that the gravity of bodies corresponds to the quantity of their matter, and I have even demonstrated this in the present Discourse.[64] But I have also shown that the gravity can well be imparted to these bodies that we call heavy, by the centrifugal force of a matter that does not itself weigh (or gravitate)[65] toward the center of the Earth, because of its very rapid and circular motion, but that tends to move away from it. This matter then can very well fill all the space around the Earth that other corpuscles do not occupy without hindering the descent of bodies that we call heavy, being on the contrary the only cause that holds them there. This would be otherwise if we supposed that the gravity were an inherent quality of corporeal matter. But it is with this that I do not believe Mr. Newton agrees, because such a hypothesis would distance us a great deal from mathematical or mechanical principles.

He will perhaps say to me that even if it was granted me that, in order to transmit light, **(p.164)** ethereal matter consists of particles that are touching, it would not yet be clear that light would not observe this law of extending itself only in a straight line, as it does; for this is contrary to his Propos. 42 of Book 2,[66] which says that motion that spreads itself out in a fluid matter does not extend only in a straight line from its origin after having passed through some opening, but it scatters also to the sides. To this I respond in advance, that what I have claimed in order to prove that light (excepting in reflection or refraction) extends only directly nonetheless remains, notwithstanding the cited proposition. I do not deny that when the Sun shines through a window it is spread out by motion to the sides of the lighted space; but I say that these indirect waves are too weak to

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64 Descartes denies that the weight of bodies is proportional to their quantity of matter. Newton's Proposition 6 and especially its corollaries are meant to counter this Cartesian view.

65 The French word, '*pèse*', is the same here as the word translated 'weigh' in the last line of the preceding paragraph.

66 Newton's Proposition 42 of Book 2, concerning wave motion in fluids generally and sound in particular, reads, "All motion propagated through a fluid diverges from a straight path into the motionless spaces." [change to IBC&AW translation] Huygens's phrasing describes the figure accompanying the proposition.

produce any light. And, although he would claim the emanation of sound proves that these outpourings to the side are sensible, I am quite sure that it rather proves the opposite. Because if sound, having passed through an opening, likewise extends to the sides, as Mr. Newton claims, in an echo it would not retain the equality of angles of incidence and of reflection so exactly; so, when one is situated in a place where the sound cannot fall perpendicularly on the reflecting plane of a wall a little removed, one does not distinguish the responding echo from the noise made in this place, as I have experienced very frequently. I likewise do not doubt that the experience that he offers on sound, in which we would hear notwithstanding a house[67] interposed, would be quite different provided that this house were placed in the middle of some large body of water or were placed so that there was nothing around that could return any bit of the sound by reflection.

To this he says that, anywhere we might be in a room with an open window we hear the sound from outside, not by reflection from the walls but coming **(p.165)** directly from the window; we see how easy it is to be mistaken because of the multitude of reiterated reflections that occur in an instant, so that the sound, which is heard as if coming immediately from the open window, can come from there or from any place very near after a double reflection. I acknowledge that for some waves or rings that are made on the surface of the water, it occurs very nearly as Mr. Newton asserts. That is to say that a wave, having crossed an opening, expands in sequence on both sides, and always more feebly there than in the middle. But as for sound, I say that these emanations on the sides are nearly insensible to the ear, and that in the case of light, they have no effect on the eyes at all.

I thought it necessary to go to the root of these objections that Mr. Newton's book could evoke, knowing the great esteem that there is for this work, and with reason, since

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[67] In the second and third editions of the *Principia* Newton referred to a mountain interposed; in the first edition it was a house interposed. The full passage Huygens is responding to is as follows: "We find this by experience in the case of sounds, which are heard when there is a mountain in the way or which expand into all parts of a room when let in through a window and are heard in all corners, being not so much reflected from the opposite walls as propagated directly from the window, as far as the senses can tell."

one cannot imagine anyone more wise in these matters or who evidences a greater sharpness of mind. There still remain two things in his System for me to comment on, things which seem very elegant to me, and that will give me occasion to offer some reflection. Following that, I will add what I have discovered among my papers concerning the motion of bodies passing through the air, or other resisting medium, which he discusses at length in Book 2.

We have seen how in Mr. Newton's System the gravities, as much of the Planets toward the Sun as of the satellites towards their planets, are supposed to be in inverse square proportion to their distances from the center of their orbits.[68] This is confirmed admirably by what he demonstrates concerning the Moon, namely that its centrifugal force, caused by its motion, precisely equals its gravity toward the Earth, and so these two contrary forces hold it suspended where it is. Because the **(p.166)** distance from here to the Moon is 60 semi-diameters of the Earth, and therefore the gravity in its region is 1/3600 of what we feel, it is necessary that the centrifugal force of a body, which is moved like the Moon, would likewise equal 1/3600 of the weight that it would have at the surface of the Earth. This is in effect found in this manner, and the calculation can easily be done, since we already know that the centrifugal force at the equator is 1/289 of our gravity here below.[69]

But since this example of the Moon proves so well the decrease in weight according to the inverse proportion of the square of the distances from the center of the Earth, one might wonder whether there would not be another inequality in the pendulums beside that which was caused by daily motion. For, if the Earth is not spherical but closer to

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68 Huygens's reference to the "center of their orbits," rather than the focus of their elliptical orbits, need not be construed as an error or a failure to read the *Principia* thoroughly. In many places, including the "Moon test," which Huygens goes on to discuss immediately below, Newton adopts the idealization of a circular orbit.

69 In the first edition of the *Principia* Newton concludes that the Moon would fall 15 1/12 Paris feet in 1 second were it at the surface of the Earth, in agreement with the value of 15 1/12 feet in 1 second that Huygens had inferred for surface gravity from the seconds-pendulum. In subsequent editions, the numbers Newton gives are more precise: 15 feet 1 inch 1 4/9 lines for the Moon, and 15 feet 1 inch 1 7/9 lines for Huygens's value.

spheroidal, and if a point at the equator is further away from the center than a point at the pole in the ratio of 578 to 577,[70] as had been said above, the gravities in these places being in reverse proportion to the squares of the distances, the pendulum at the equator must also be shorter than the one at the pole in this same reverse proportion. That is to say that these pendulums would be as 288 to 289, or that the pendulum at the equator would be shorter by 1/289 of what it would be at the pole. This is nearly the same difference that is produced above for daily motion or centrifugal force. Thus a clock with the same length pendulum would run slower at the equator than at the pole by twice what it would be slowed by the motion of the Earth; and so this daily difference at the equator would be about 5 minutes. At the other parallels, it would everywhere be more than twice what it was previously. But I strongly doubt that experience confirms this large of a variation, since I have observed, in the voyage that I mentioned, that the first equation alone suffices, and it would give more than twice too great a difference around **(p.167)** the middle of the course between the route of the vessel calculated with the pendulum and the route estimated by the mariners.[71] In order to explain why the second variation would not occur, I say that it would not be strange ~~be by coincidence~~ if the gravity near the surface of the Earth does not behave in precisely the same way as in higher regions, the decrease corresponding to different distances from the center; because it may be that the motion of the matter that causes gravity is not altered in the least near the surface of the Earth, such as it apparently is in the interior. If this were not the case, would have to be said that gravity would increase to infinity as it approached the center, which is not likely. On the contrary, according to Mr. Newton, the

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[70] As indicated in note 54 above, if gravity varies as the inverse-square of the distance from the center of the Earth in the interior of the Earth, as Huygens is here considering, then the ratio of radii at the equator and the pole is 579/578, not 578/577. The latter number has the virtue of simplifying his exposition.

[71] Even though Huygens was misled by the voyage when he concluded that the loss of time by a pendulum clock from the pole to the equator would be 2 1/2 minutes per day, he was correct that a 5 minute loss is far too much.

gravity in the interior of the Earth diminishes as bodies approach the center; but he uses his principle to prove it, with which I have said I do not agree.[72]

There still remains something for me to remark on concerning his System, something which has pleased me greatly. It is that he finds means, supposing the distance from here to the Sun to be known, to define what gravity the inhabitants of Jupiter and Saturn would feel, compared to what we feel here on the Earth, and also what its measure is at the surface of the Sun. These are things which previously seemed quite removed from our knowledge, and which nevertheless are some of the consequences of the principles that I reported a little earlier.

This determination takes place in the planets that have one or several satellites since the periodic time of those and their distances from the planets that they accompany must enter into the calculation. Through this Mr. Newton finds the gravities at the surfaces of the Sun, Jupiter, Saturn, and the Earth in proportion to these numbers: 10000, 804 1/2, 536, 805 1/2. It is true that there is some uncertainty because of the distance of the Sun, which is not sufficiently well known, and which has been taken in these calculations to be around 5,000 Earth diameters, **(p.168)** instead of following Mr. Cassini's dimension of around 10,000, which tolerably approaches what I have found previously through similar reasoning in my *System of Saturn*, namely 12,000.[73] I also disagree somewhat with the diameters of the planets. So, by my calculation the gravity in Jupiter, in relation to what we have here on Earth, is found to be 13 to 10, as opposed to Mr. Newton's having made

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72 Because Newton's law of universal gravity entails that gravity varies linearly with the distance from the center in the interior of a solid sphere (Book 1, Proposition 83), his inverse-square gravity does not go to infinity at the center of the Earth; but a strictly inverse-square rule of the sort Huygens is here considering would face this problem.

73 Newton changed his distance from the Earth to the Sun in subsequent editions, crediting Huygens for the correction; the relative values for surface gravity for the Sun, Jupiter, Saturn, and the Earth in the third edition of the *Principia* are 10000, 743, 529, and 435. Only the last of these numbers depends on the Earth-Sun distance; the other changes are from improved astronomical measurements of the orbital radii of the satellites of Jupiter and Saturn. For more details, see van Helden, *The Measure of the Universe* (Chicago: University of Chicago Press, 1985), Chapters 11-13.

them equal, or insensibly different.[74] But the gravity in the Sun, which by the numbers that we have seen, would be around twelve times greater than ours on Earth, I find 26 times greater. From this it follows, in explaining gravity in the manner that I have, that the fluid matter close to the Sun must have a velocity 49 times greater than what we have found near the Earth, which was already 17 times greater than the velocity of a point at the equator. This then is an awesome speed, which has made me wonder whether it would not be sufficient to be the cause of the brilliant light of the Sun, supposing that the light were produced as I have explained in what I have written here; namely that the solar particles, swimming in a more subtle and extremely agitated matter, knock against the particles of the ether that surround it. For, if the agitation of any such matter, with the motion that it has here on the Earth, could cause the light of the flame of a candle or of ignited camphor, how much greater would this light be with a motion 49 times more swift and more violent?

I have seen with pleasure what Mr. Newton writes concerning the descent and projection of heavy bodies in the air, or in any other medium that resists motion, having previously applied myself to the same research. Seeing that this matter concerns gravity in part, I think I can report here what I discovered. I will, however, give it only in summary **(p.169)** and without adding demonstrations, having neglected to finish them because this speculation had not appeared to me to be useful enough, or of consequence, in proportion to the difficulty that accompanies it.

I first examined these motions supposing that the forces of resistance are as the velocities of the bodies, which then appeared to me very likely. But having found what I was looking for, I learned at about the same time, through experiments that we conducted in Paris at the Royal Academy of Sciences, that the resistance of air and of water was as the square of the velocities.[75] The reasoning is sufficiently easy to comprehend: a body

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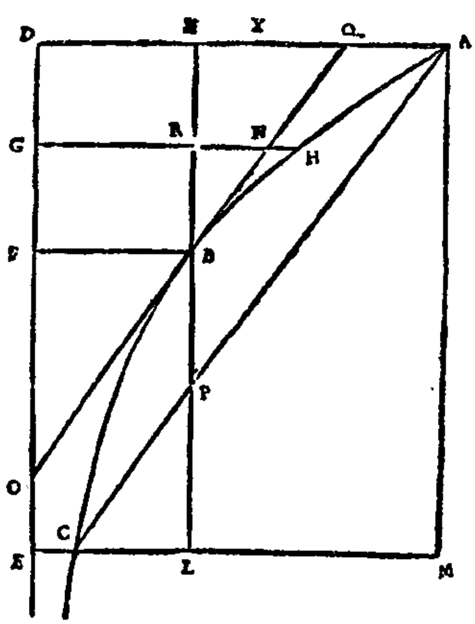
74 In this sentence and the next Huygens for some reason contrasts gravity "in" Jupiter and "in" the Sun -- *dans Jupiter* and *dans le Soleil* -- with gravity on the Earth -- *sur la Terre*. Clearly he is speaking of surface gravity throughout.

75 The experiments are described in "Experiences de 1669 sur la force de l'eau ou de l'air en mouvement et sur les résistances éprouvées par des corps traversant ces milieux," *OCCH*, Vol. 19, pp. 120-143.

moving, for example, with twice the velocity meets twice as many particles of air or of water, with twice the speed.[76] Thus I saw my new theory destroyed, or at least rendered useless. After this I wanted to find out what happens when we suppose this real basis of resistances, whence I saw that the thing was much more difficult, particularly concerning the curved line that bodies thrown obliquely pass through.[77]

In the first supposition, where the resistance is as the velocity, I noticed that, in order to find the space traversed in given times when bodies fall or rise perpendicularly, and to know the velocities at the end of these times, there was a curved line I had examined a long time before that was of great use in this research. We can call it the *logarithm* or the *logistic*, as I do not see that it has been given a name yet, although others have also previously taken into account. This infinite line ABC, has a vertical line for an asymptote,



such as DE; if in this we take any equal parts whatsoever that follow one another, like DG and GF, and we draw from points D, G, and F some perpendiculars up to the curve, **(p.170)** namely DA, GH, and FB, these lines will be continuously proportional.[78] From this we see it is easy to find as many points as we want on this curve. I will report in time some properties of it that are worth considering. To explain the falling of bodies I repeat here what I wrote first at the end of the *Treatise on the Center of Oscillation:*[79] namely that

[76] The tacit step in the reasoning is that the force of impact (between the body and the particles) is proportional to the velocity. Newton employed this same reasoning in Book 2 of the *Principia.*

[77] Even Newton was unable to find the solution to the trajectory problem when resistance varies as v^2. The best he could do was to devise approximations, given in Proposition 10 of Book 2.

[78] That is, DA/GH=GH/FB=.... In modern notation, where DGF is taken to be the x-axis and AKD the y-axis, the curve ABC drawn generically in the figure, which Huygens calls the logarithm or logistic, is $y=ca^{bx}$, where $0<a<1$ and $0<b,c$; this amounts to the same thing as $x=\log_a(y/c)/b$.

[79] In Part IV of *Horologium Oscillatorium*, p. 358f of *OCCH*, Vol. 18; p. 172 in the Blackwell translation.

a body falling through the air continuously increases its velocity, but in such a way that it can never exceed, nor even attain, a certain degree, which is the velocity of the air that is required if blown upwards from below to hold the body suspended without being able to descend, because then the force of the air against the body equals its gravity. I call this velocity, in each body, the *terminal* velocity.

If then a heavy body is thrown up perpendicularly, with a velocity in a given ratio to the terminal velocity, for example as AK to KD in the ordinate AD, perpendicular to the asymptote DE, let KB be drawn parallel to this asymptote, and let the straight line BO, which meets DE at O and DA at Q, be tangent to the curve at point B. This tangent is found by taking FO, from the ordinate **(p.171)** BF, equal to a determined length, which is the same for all the tangents, as I will show in the following.[80] Then let AC be parallel to this tangent, cutting KB extended at P, and from the point C, where it encounters the curve, let CLM be drawn parallel to AD, cutting KB extended and AM parallel to the asymptote, at points L and M. Now the time that the body takes to rise to the height that it is able to attain is to the time of its descent from this same height as KB to BL.

And the time that it takes to rise through the air, being thrown as described, is to the time that it would take without encountering any resistance, as KB to KP.

And the height to which it will rise in the air is to that where it would rise without resistance as the space ABK to the triangle APK, or as QA to AX, which I suppose to be half of a third proportional[81] to the lines DK and KA.

And its velocity, commencing to rise, is to that which it has returning to the earth as ML to LC.[82]

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80 With $y = ca^{bx}$, as above, x_1 the point B, and x_2 the point F where the tangent at B meets the x-axis, $(x_2 - x_1)$ is always $-1/\{b\ln(a)\}$; this is a fundamental property of this curve. In particular, when $\xi = e^{-\eta}$ the form we will take Huygens's curve to represent below, $(\eta_2 - \eta_1)$ is always equal to 1.

81 Specifically, 2AX/AK = AK/DK.

82 In modern terms, let the acceleration dv/dt be $-g - \lambda v$ in ascent and $g - \lambda v$ in descent. The solution here is for the problem of the complete motion consisting of the vertical ascent produced by an initial velocity V_0 followed by the vertical descent back to the initial point. In modern notation, the solution for ascent is given by:

We find moreover, by this same line, what curve a body thrown obliquely traverses. In the same figure, suppose LMR is the angle of trajectory on the horizontal line with a given velocity, of which the motion upwards would be to the terminal velocity as AK to KD; let the preceding construction be repeated so that the straight line AS, which is tangent to the curve ABC at A, meets KB at S. Then as SP is to PB let RL be to LT; and on the base MC let a figure be drawn in proportion to the segment ABCP so that the lines

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$$\lambda t = -\ln\{(V_T + v_y)/(V_T + V_0)\}$$
$$v_y = -V_T + (V_T + V_0)\exp(-\lambda t)$$
$$s_y = -tV_T + (1/\lambda)(V_T + V_0)\{1 - \exp(-\lambda t)\}$$

where $V_T = g/\lambda$ is the terminal velocity, s_y is the distance covered, and v_y is taken positive upward. The solution given here for descent is given by:

$$\lambda t = -\ln\{(V_T + v_y)/V_T\}$$
$$v_y = V_T - (V_T + V_0)\exp(-\lambda t)$$
$$s_y = -tV_T + (1/\lambda)V_T\{1 - \exp(-\lambda t)\}$$

where v_y is taken positive downward, and t=0 when descent commences.

To relate Huygens's solution to the modern one, think of AD as corresponding to the quantity $\xi = \{(V_T + V_y)/(V_T + V_0)\}$, with $\xi=0$ at D and $\xi=1$ at A, and DE as corresponding to $\eta=\lambda t$, with $\eta=0$ at D, where t is now the total elapsed time and the vertical velocity V_y changes from positive to negative at the apex of the motion (that is, at K). The curve ABC, which represents the motion with resistance, then corresponds to

$$\eta = -\ln\xi \qquad \text{or} \qquad \xi = \exp(-\eta)$$

and the straight line APC, which represents the motion in the absence of resistance, corresponds to $\xi = -\xi_{ap}\eta + 1$, where $\xi_{ap} = V_T/(V_T + V_0)$ is the value at the apex. The non-dimensionalization of distance associated with η and ξ is $\zeta=\lambda s/V_T$. In particular, the altitude of the body at any time with resistance is given by

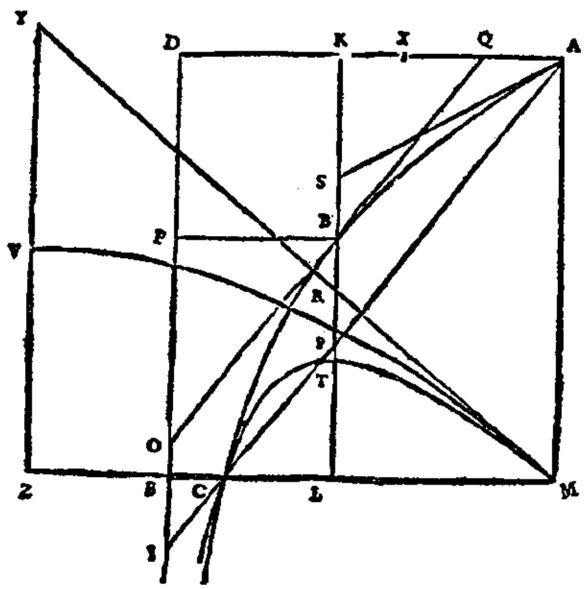
$$\zeta_y = \{\xi_{ap}\ln\xi + (1-\xi)\}/\xi_{ap}$$

and in the absence of resistance, by

$$\zeta_y = \{(1-\xi_{ap}) - (1-\xi)/2\}\{(1-\xi)/\xi_{ap}\}$$

The crucial fact underlying Huygens's solution is that the velocity at impact is the same as the velocity that would be acquired in the same elapsed time if the body were projected vertically in the same way without resistance, namely the velocity corresponding to EC, where C is the intersection of the logarithmic curve and the straight line. The full development of this solution can be found in "Théorie de 1668 du mouvement d'un point pesant dans un milieu dont la résistance est proportionnelle à la vitesse du mobile," *OCCH*, Vol. 19, pp. 102-119.

parallel and equally distant from the asymptote DE in both figures[83] would have throughout the same ratio of BP to TL. This will be the curve MTC that will indicate the requisite figure of the throw.[84]



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83 That is, the vertical distance between the straight line APC and the curve ABC, on the one hand, and the vertical distance between the base MLC and the curve MTC, on the other.

84 The modern solution for the horizontal component is given by

$$\lambda t = -\ln(u/U_0)$$

$$v_x = U_0 \exp(-\lambda t)$$

$$s_x = (1/\lambda) U_0 \{1 - \exp(-\lambda t)\}$$

$$= (1/\lambda) V_0 \{1 - \exp(-\lambda t)\} \cot\theta_0$$

where θ_0 is the angle of projection and $U_0 = V_0 \cot\theta_0$ is the horizontal component of the initial velocity. In terms of the non-dimensional quantities corresponding to the elements in Huygens's figure, this last equation can be rewritten as

$$\zeta_x = \{(1-\xi_{ap})/\xi_{ap}\}(1-\xi) \cot\theta_0$$

Thus, using the expression for ζ_y in note 78 above, the ratio of the altitude to the horizontal distance at any time is given by

$$\zeta_y/\zeta_x = \{\xi_{ap} \ln\xi + (1-\xi)\} \tan\theta_0 / \{(1-\xi_{ap})(1-\xi)\}$$

In the representation of the trajectory in the figure, MTC, the horizontal distance corresponds to $(1-\xi)$, so that the altitude corresponds to $\{\xi_{ap} \ln\xi + (1-\xi)\} \tan\theta_0/(1-\xi_{ap})$. In Huygens's solution SP is

And because the height of the elevation with resistance would be to the height of the throw free from resistance as QA is to AX, if we require TL to have the same ratio to another line VZ, this will be the height **(p.172)** of the parabola MV that this throw free from resistance makes, commencing at M with the same force and in the same direction MR as the other throw had. So, if in angle LMR we add YZ perpendicular to MC, and equal to twice VZ, we would have the summit of this parabola at V in the middle of YZ, and its half base or half amplitude MZ.[85]

It is of note that whatever the angle of elevation LMR might be, provided that the vertical velocity stays the same, we find here the same amplitude MC. But it is necessary to warn that these are only the figures of throws found in this manner, and not all the heights and amplitudes of various throws compared to one another. So, they must all be of the same height, when the **(p.173)** vertical speed is the same. This then is why then each figure of a throw, thus found, must be reduced to a proportional figure of equal height, if we want to know what the amplitudes and the heights of diverse throws are from one to another.

Here, I add also that the logarithmic line seems not only to determine the curves of the throws, but that it is this curve itself in one case, namely when we throw a body obliquely down so that the vertical descent is equal to the terminal velocity. Then this body would follow precisely the curvature of one such line, always approaching the asymptote without being able to reach it. And what determines the nature of the line is that its *Subtangent* (so I will call line FO, which is the same for all the tangents) will be twice the height to which the terminal velocity can make the body rise in the absence of the resistance of the medium.[86]

$(1-\xi_{ap})^2/\xi_{ap}$ and RL is $(1-\xi_{ap})\tan\theta_0$, so that $BP/TL=SP/RL=\{(1-\xi_{ap})/\xi_{ap}\}\cot\theta_0$, and the vertical distance between the straight line APC and the curve ABC, that is, $\eta_{APC}-\eta_{ABC}$, is $\{\xi_{ap}\ln\xi + (1-\xi)\}/\xi_{ap}$.

85 In other words, MZ corresponds to $(1/2)\{(1-\xi_{ap})/\xi_{ap}\}^2\cot\theta_0$.

86 If $V_0=V_T$ in descent, $s_y= tV_T$ and $s_x=(V_T/\lambda)\tan\alpha_0\{1-\exp(-\lambda t)\}$, where α_0 is the angle of projection downwards relative to the vertical. Thus $\zeta_x=\tan\alpha_0\{1-\exp(-\zeta_y)\}$, or letting $\delta_x=1-\zeta_x$, $\delta_x=\tan\alpha_0\exp(-\zeta_y)$. Since FO is 1 on the non-dimensionalization $\lambda s/V_T$, FO amounts to V_T/λ, which indeed is V_T^2/g.

These are the things that I found in supposing the resistance to be as the velocity, but all this theory being based on a principle that nature, as I have said, does not observe in the case of the resistances of air and water, I dismissed it entirely; and it is only on the occasion of the treatise of Mr. Newton that I have taken it up again, to see if what we had looked for in very different ways would agree, as it should. This is found to be so. For, the construction of the line of the throw that he gives in Prop. 4 of Book 2, however different from and more difficult than mine, nevertheless produces the same curve as the one we were able to show with the demonstration.

While examining what occurs in the true hypothesis of resistance, which is as the square of the velocity, I have only determined the particular case of a body thrown up with its terminal velocity; namely that the time of its full elevation in the air is to the time that it takes to rise to where **(p.174)** it can without resistance, as the area of the circle to the area of the square circumscribed in it. And the height of the first projectile is to the height of the other as the space between an hyperbola and its asymptote, bound by two parallels to the other asymptote that would be in a ratio of 2 to 1, is to the rectangle or parallelogram of the same hyperbola – that is to say, as in the following figure, the space AMDK to the square AC.[87] I have not researched the other cases that are included universally in Prop. 9 of Book 2 of Mr. Newton, which is very elegant. And what hindered me was that I did not find, by the path that I followed, the measure of the descent of the bodies, except by supposing the quadrature of a determined curved line, as I did not know that it depended on the quadrature of the hyperbola. I reduced the dimension of the space of this curve to an infinite series, $a + 1/3\, a^3 + 1/5\, a^5 + 1/7\, a^7$ etc., not knowing that the same series also gave the measure of the hyperbolic sector; this I have seen since, by comparing the demonstration of Mr. Newton with what I had found.[88]

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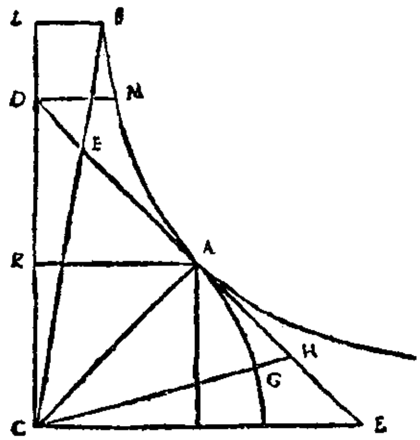
87 That is, the area of a square with sides AC.

88 Huygens's inability to get the solution to this problem in 1669 should not be surprising. Newton found himself having to use the calculus to obtain his solution, given in Proposition 9 of Book 2 for vertical motion with resistance as v^2. Huygens's efforts on motion under resitance varying as v^2 can be found in "Théorie de 1669 du mouvement ascendant ou descendant d'un point pesant dans un milieu dont la résistance est proportionnelle au carré de la vitesse du mobile," *OCCH*, Vol. 19, pp. 144-157.

But because this series, for the measure of the hyperbola, has not yet been remarked on that I know of, I wish to explain here how it is useful. Let AB be a



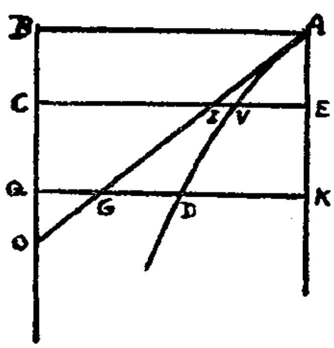
hyperbola, of which the asymptotes DC and CE form a right angle. Let the semi-axis be CA, perpendicular to DAE, which is tangent to the hyperbola; and let ACB be a sector, the line CB cutting AD at F. If we now take AC or AD for the unit, and let AF be called a, which is a fraction less than the unit, when AF and AD are commensurable; I say that, as the sum of the infinite series, $a + 1/3\ a^3 + 1/5\ a^5 + 1/7\ a^7$ etc. is to 1, so will be the sector ACB to the triangle ACD. Or if we bring the perpendiculars AK and BL to the asymptote, we can say the same thing of the space ABLK, which is equal to this sector, as we see easily by the equality of triangles CAK and CBL.[89] Thus, this series **(p.175)** for the hyperbola corresponds to what Mr. Leibniz has given for the circle, by which, if the sector of the circle is ACG, having AC for a ray, and CG cuts AE at H, with AH being called a and AE equal to 1, the sum of the series $a + 1/3\ a^3 + 1/5\ a^5 + 1/7\ a^7$ etc. is to 1 as the sector ACG is to the triangle ACE, or as the arc AG is to the vertical AE.

As for what the line of an oblique projectile is: if it sufficed in this sort of resistance to know the horizontal and vertical motion of a body in order to compose the oblique motion in the same way as in the first hypothesis, there would be a way of determining some points through which this line must pass; and the same logarithmic line would be useful here, being turned so that its asymptote is made parallel with the horizon, and it would itself again be the curve of the projectile in the case that I said that it served previously. But this composition of motion does not occur here. Because the diminution

[89] Let the right hyperbola, in modern notation, be xy=1, so that the vertex A is (1,1). Then the area of the triangle AKC is 1/2, and the area of the triangle CBL is xy/2=1/2.

of the retarded motion, in the diagonal of a rectangle, is not proportional to the diminution of the sides, it is extremely difficult, if not entirely impossible, to solve this problem.[90]

Horizontal motion being considered separately, as a ball rolled on a level plank, has a property worth noting here, namely that it must go at a distance to infinity notwithstanding the resistance **(p.176)** of the medium; for, when the resistance is as the velocity, it is confined and never reaches a determined end. This infinity is shown easily in Proposition 5 of Book 2 of Mr. Newton's treatise, because the space included between the hyperbola and its asymptotes is of infinite size.[91]



The properties of the logistic line that I promised to give an account of, certain ones of which have assisted in discovering what I have said concerning motion through the air, follow – other than the first, which I have already pointed out, concerning the proportionality of the ordinates to the asymptote when they are equally distant, through which we find certain points on this line.[92]

1. The spaces included between two ordinates and the asymptote are to one another as the differences of these ordinates. So in this figure, where AVD is the logistic, BO its asymptote, and the ordinates AB, VC, and DQ, the last two of which, being extended,

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90 The problem of a projectile with resistance varying as v^2 --the so-called ballistics problem -- still has no closed-form solution, for the very reason Huygens here gives. Johann Bernoulli obtained a solution "granting quadratures" in 1719, but the integral in his formula has no analytical solution. See A. R. Hall, *Ballistics in the Seventeenth Century* (Cambridge: Cambridge University Press, 1952), p. 154ff.

91 The solution for horizontal motion with resistance as v is given in note 80 above. The corresponding solution when the deceleration from resistance is κv^2 is

$$v_x = U_0/(U_0\kappa t+1)$$

$$s_x = (1/\kappa)\ln(U_0\kappa t+1)$$

So, as t grows indefinitely, s_x grows indefinitely in this case, while it is limited to U_0/λ in the earlier case.

92 Most of these properties are easy to demonstrate by applying the calculus to $\xi=\exp(-\eta)$, or even to $y=ca^{bx}$. Huygens established them without the benefit of the calculus in 1661 and 1662 (see *OCCH*, Vol. 14, pp. 451-482). See, also, *Horologium Oscillatorium*, pp. 214-221 in *OCCH*, Vol 18, and pp. 89-92 in the Blackwell translation.

meet AK parallel to the asymptote at E and K, the spaces ABCV and ABQD are to one another as the verticals EV and KD.

2. The same things being supposed, and AO being tangent to point A and cutting CE and QK at I and G, the spaces AVE and ADK are to one another as the verticals VI and DG.

3. The space included between two ordinates is to the infinite space which, from the least of these ordinates, extends itself between the logistic and its asymptote, as the difference **(p.177)** of these ordinates is to the least. When I say that the infinite space has a determined ratio to a finite space, this indicates that it approaches so near to the size of a given space that has this proportion to the finite space, that the difference can become less than any given space. In the preceding figure the space ABQD is to the infinite space that extends itself from DQ between the curve and the asymptote as KD is to DQ.

4. The subtangent, like BO in the same figure, is always the same length at any point on the logistic that the tangent pertains to.

5. This length is found by approximation, and it is to the part of the asymptote included between ordinates in double ratio,[93] as 434294481903251804 to 301029995663981195, or very nearly, as 13 to 9.[94]

(Spacing to accommodate graphic on next page.)

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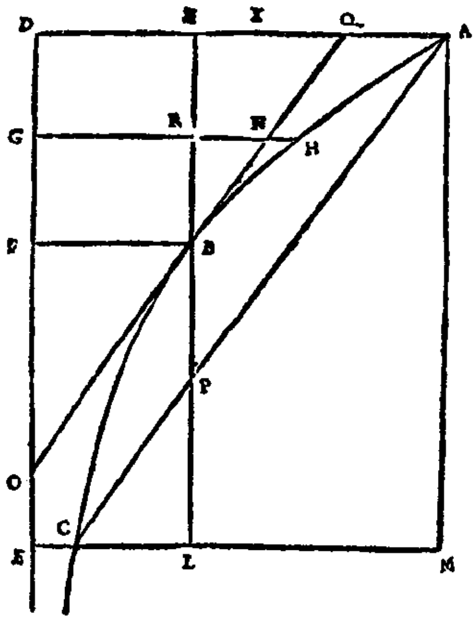
[93] In other words, one of the ordinates is twice the other. If, for example, $y=a^{-x}$, then taking $y_1/y_2=2$, $x_2-x_1=-\log_a(y_2/y_1)=\log_a 2$.

[94] Again taking $y=a^{-x}$, the ratio of the subtangent, $1/\ln(a)$, to $\log_a 2$ is $\log_a e/\log_a 2$ since $\ln(a)=1/\log_a e$. The first of the numbers Huygens gives, 434294481903251804, is $\log_{10} e$, and the second, 301029995663981195, is $\log_{10} 2$. (The original text of the *Discours* has 301039995663981195 instead of the correct number given above; Huygens had the correct value in the original work from the 1660s cited in note 88.) The ratio is simply 1/ln2, as can be seen by letting a=e. The number e was not singled out and given a special designation until the 18th century; Huygens is here encountering one of the many distinctive places where it emerges. For details, see Eli Maor. *e: The Story of a Number* (Princeton: Princeton University Press, 1994).

6. If there are three ordinates, as AD, HG, and BF are in the figure, and from the point on



the curve belonging to the least of these we draw a parallel to the asymptote that cuts the two other ordinates at R and K, and a tangent BQ that cuts them at N and Q, the trilinear spaces ABK and HBR are to one another as the parts of the ordinates between the curve and the tangent, namely AQ and HN.

(p.178) 7. The infinite space between an ordinate, the logistic, and its asymptote on the side of the ordinate where these last two approach one another is twice the triangle formed by the ordinate, the tangent drawn from the same point as the ordinate, and the subtangent. So, in the same figure, the infinite space from ordinate BF is twice the triangle BFO.

8. The space included between two ordinates is equal to the rectangle of the subtangent and the difference of these ordinates. So, in the same figure, space ADFB is equal to the rectangle of the subtangent FO and KA.

9. The solid formed by the infinite space from one ordinate, in turning around the asymptote, is 3/2 of the cone, in which the height is equal to the subtangent, and the semidiameter of the base is equal to this ordinate. So the solid formed by infinite space BFOC, revolving around FO, is 3/2 of the cone formed by triangle BFO, likewise revolving around FO.

10. The solid produced by the same infinite space, revolving around ordinate BF, from where it begins, is six times the cone formed by triangle BFO, through its conversion on BF. From the measure of solids it follows:

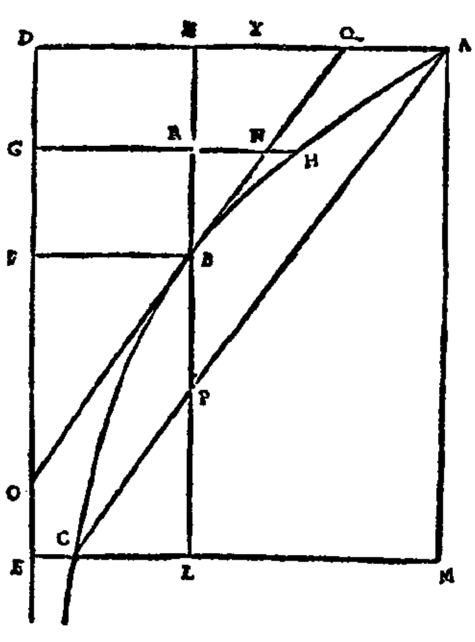
11. That the center of gravity[95] of the infinite space from one ordinate is the length of the subtangent away from this ordinate.

12. This same center of gravity[96] is a quarter of the ordinate away from the asymptote.

13. I have also found that the center of gravity[97] of the first of said infinite solids is distant from its base by half of its subtangent.

14. And the center of gravity[98] of the other solid is one eighth of its axis away from its infinite base.

15. We know enough, from the demonstrations of Father Gregoire de St. Vincent,[99] concerning the hyperbolic spaces included between two ordinates on one of the asymptotes, that this logistic line serves as the **(p.179)** quadrature of the hyperbola. And if there



are two such spaces, of which the ordinates of the one would be as AD to HG in the last figure, and the ordinates of the other as BF to CE, these spaces will be to one another as lines DG to FE. But we have not remarked, that I know, that these same hyperbolic spaces are to the parallelogram of the hyperbola (this is what I call the parallelogram the sides of which are the two ordinates on the asymptotes drawn from the same point of the section) as each of the lines DG and FE to the subtangent FO. So that, if the parallelogram of the

95 '*Gravité*' in the French rather than '*pesanteur*'.
96 Ibid.
97 Ibid.
98 Ibid.
99 Grégoire de Saint-Vincent (1584-1667) published his *Opus geometricum quadraturae circuli et sectionum coni* in 1647; for details see Maor, *op. cit.* Chapter 7, especially p. 66f.

hyperbola is assumed to be 0.4342944819 parts, each hyperbolic space included between two ordinates on one of the asymptotes will be to this parallelogram as the logarithm of the proportion of these same ordinates, that is to say as the difference of the logarithms of the numbers that express the proportion of the ordinates to the number 0.4342944819, taking the logarithms of the 10 characters beyond the characteristic.[100]

(p.180) And from here it is easy to verify the quadrature of the hyperbola that I gave in the "Traité de l'Evolution des Lignes Courbes", which is in my *Horologium Oscillatorium.*[101]

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[100] In particular, if the logarithmic curve is $y=a^{-*}$ so that the associated right hyperbola is $xy=-\log_a e$, then the "parallelogram" of the hyperbola is $-\log_a e$ and the area under the hyperbola between y_1 and y_2 is $-\log_a(y_2/y_1)$.

[101] See note 90.