Newton on the relativity of motion and the method of mathematical physics

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I. Introduction

The work of George Smith has illuminated how Newton's scientific method, and its use in constructing the theory of universal gravitation, introduced an entirely new sense of what it means for a theory to be supported by evidence. This new sense goes far beyond Newton's well known dissatisfaction with hypothetico-deductive confirmation, and his preference for conclusions that are derived from empirical premises by means of mathematical laws of motion. It was a sense of empirical success that George was especially well placed to identify and to understand, through his experience as an engineer specializing in failure analysis. For Newton, to understand how well his theory was supported by evidence, he had to anticipate, as far as possible, all the ways in which it might be wrong.

Newton's dedication to this practice, as we've learned from George, was an essential part of what made the theory of universal gravitation so fruitful as an account of gravity, as a foundation for celestial mechanics, and as a model for the future development of theoretical physics. George's account of this particular practice, and its role in Newton's methodology as a whole, is well known (see, for example, Smith 2002a, 2002b). My purpose is only to point out some further aspects of it that deserve more attention. For the empiricist methodology that made universal gravitation an empirically successful theory is also deeply intertwined, more than has been generally acknowledged, with aspects of Newton's work that seem to be more purely philosophical. This particularly applies to Newton's theory of space, time, and motion.

For much of its history, of course, and especially during much of the 20th century, Newton's theory was viewed as quite separate from, and even as an embarrassment to, his stated empiricist method. According to Newton's laws, the physically meaningful quantities are force, mass, and acceleration, while velocity is relative; his conception of "absolute space," however, implies that there is a meaningful distinction in principle between uniform motion and "absolute rest" in space, even though they are physically indistinguishable. This seemingly naive metaphysical appendage to an empirically well-founded physics earned Newton a low reputation as a philosopher among 20th-century empiricists. Only Stein (1967) eventually convinced philosophers of science to take seriously, at least, the empirical motivations of Newton's theory of space, time, and motion, and its close connection with the physics of motion, in spite of the superfluous aspects of absolute space. I propose to take a further step in the same direction, one particularly inspired by George Smith's work: Newton's philosophical account of absolute space, time, and motion, with all its flaws, was an integral part of his empiricist methodology. It was integral to his effort to anticipate all the ways in which his causal account of celestial motion, through universal gravitation, might be wrong. To see this, we have to see the development of this theory in connection with Newton's developing understanding of the relativity of motion, through the preliminary drafts and successive editions of the *Principia*.

II. Absolute space and the principle of relativity

The internal flaw in Newton's theory of absolute space was first made clear, in a published work at least, by George Berkeley (1721). As Newton well knew, motion with respect to space itself is unobservable, and any treatment of motion must begin with a relative space identified by relatively fixed empirical markers: the surface of the earth, the inner walls of a moving ship, the fixed stars. For Newton's application of the laws of motion, all of the relevant phenomena were motions of small bodies with respect to the earth, and displacements of celestial bodies with respect to the fixed stars. The empirical frame of reference for his theory, therefore, along with the theory's evidentiary basis, was essentially the one that astronomers had relied upon for centuries, though the accuracy of the evidence had (beginning with Tycho Brahe) increased dramatically in the century preceding Newton's *Principia*. By interpreting these phenomena through the laws of motion, Newton could infer the forces at work from the apparent motions, and thus describe the solar system as a system of interacting masses moving about a common centre of gravity, calculated from their relative masses and positions. This analysis, in turn, led to an account of "true motion" in a certain restricted, yet decisive, sense. It led to a principled account of the true "frame of the system of the world": only the centre of gravity of such a system can be its true fixed centre (at rest or in uniform motion), and since the Sun has most of the mass of the system, the Sun, though "agitated" by its interactions with the planets, can never be far from the centre of gravity. In other words, the decision between the "two chief world systems" could be settled not by the most plausible hypothesis, but by reasoning from the phenomena with the help of the laws of motion. Newton himself noted, of course, that this solution was independent of whether the centre of gravity of the system is at rest, or moving uniformly.

It was Berkeley who, embracing Newton's reasoning thus far, drew what now seems to be the obvious inference: that "absolute space" in Newton's sense is superfluous to Newton's theory of motion. Newton had succeeded in finding an empirical frame of reference in which the forces at work within the system can be known, and the quasi-heliocentric structure of the system can be calculated. But it certainly does not follow from this success that there is a truly resting frame of reference— at rest in absolute space— with respect to which the centre of gravity (or any body at all) has its true velocity. Berkeley also pointed out, more clearly than Newton's other contemporaries, just how far Newton himself had gone to incorporate the relativity of motion into his theory, and to make it, as we would say, Galilei-invariant. The most obvious example is Corollary V to the laws of motion:

> Corollary V: The motions of bodies enclosed within in a given space are the same among themselves, whether that space is at rest, or moving uniformly in a straight line without circular motion.(1687b, p. 19.)[1]

This was Newton's version of the relativity principle that had been used by Galileo and made precise by Huygens, and that is generally known as the principle of "Galilean relativity."[2] Berkeley turned this relativity principle directly against the very idea of absolute motion:

.[1] Translations are my own, unless otherwise noted.

[2] In what follows, "relativity principle" and "relativity of motion" refer to the Galileo-Huygens principle, and "relativistic" is to be understood in in the sense of this principle.

> As it is clear that, according to the principles of those who introduce absolute motion, it cannot be known by any mark whether the entire frame of things is at rest or moved uniformly in a right line, it is evident that no absolute motion of any body can be known. (1721, section 65.)

Beyond this, Berkeley recognized a more subtle way in which Newton had integrated the relativity principle within his theory: the re-definition of inertia as an essentially relativistic concept. The "principle of inertia" as we know it was first introduced by Huygens, though unclear or incomplete forms of it had already been asserted by Galileo, Descartes, and Gassendi:

> Hypothesis I: Any body, once moved, if nothing opposes it, will continue to move perpetually with the same velocity in a straight line. (Huygens 1656, p. 31.)

Kepler had defined "inertia" as a body's natural tendency to stay at rest; the core of the newer idea was that a body's natural tendency is to maintain its velocity until acted upon by an external cause. In the thinking of Galileo and Huygens, this tendency was essential to the relativity principle, as it formed the dynamical basis for the indistinguishability of uniform motion from rest: since bodies naturally persist in uniform motion, any accelerations among bodies will be simply composed with any uniform motion that they share, and so the bodies will behave in a shared uniform motion exactly as they would in a shared state of rest. Galileo appealed to this principle in explaining the fall of a stone from a tower as the earth moves (1632, p. 149f.), and Huygens in explaining collisions of bodies in a moving canal boat (1656, p. 31).

Before the *Principia*, however (and for some time after), the persistence of motion was generally understood in a manner incompatible with the relativity principle. For it was generally assumed that a body's power to persist in motion, and its power to change the motion of other bodies in virtue of its own motion, were distinct from its property of resisting changes in its own motion. This implies a difference between the state of rest and the state of uniform motion that, strictly speaking, should have been rejected by all those who claimed to embrace the relativity principle. But it proved difficult to formulate the concept of inertia in a relativistic way. Leibniz, for example, always distinguished between the "active power" of a body to change the motion of another, and its "passive power" to maintain its own state (e.g. 1695, p. 146; 1699, p. 170). Perhaps the most useful example is Newton himself, who, in the manuscript *De Gravitatione et aequipondio fluidorum* (1684a), made the same distinction:

> Definition 7. Impetus is force in so far as it is impressed on another.
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> Definition 8: Inertia is the internal force of a body, so that its state may not be easily changed by an external force. (1684a.)

Sometime between writing *De Gravitatione* and beginning the first drafts of the *Principia*, Newton came to see that these Definitions divided into distinct concepts what was a single concept, the "vis insita" or "intrinsic force," seen from different points of view; he explicitly blamed his former division on a failure to take into account the relativity principle. His first statement of the relativity principle appeared in the manuscript "On the motion of spherical bodies in fluids" (1684b), as a "Law" rather than a Corollary (as it became in the *Principia*).

> Law 3: The motions of bodies included in a given space are the same among themselves, whether that space is at rest or moves uniformly in a straight line without circular motion. (1684b, p. 40r.)

Neither the explicit principle, nor any acknowledgement of it, appears in *De Gravitatione*. After introducing it, however, Newton gradually revised his conception of inertia toward the relativistic notion that eventually appeared in the *Principia*.[3] Every body has a degree of mass; whether this appears as the resistance to motion, or a power proportional to its motion, depends on whether it is regarded as moving uniformly or at rest. The "inertia of the mass" is the same regardless of the velocity attributed to it. A subsequent draft, "De motu corporum" (1685b) contains a relativistic definition of inertia nearly identical to Definition III of the *Principia*:

> Definition 3: The internal force of matter is the power of resistance by which any body persists in its state of rest or of moving uniformly in a straight line: it is proportional to the body and does not differ from the inertia of matter except in our mode of conceiving it. A body truly exerts this force only in a change of its state brought about by another force impressed upon it, and the exercise of this force is both resistance and impetus, which are distinct from one another only relatively: resistance in so far as the body, to maintain its state, opposes the impressed force; impetus insofar as the same body, yielding only with difficulty to the force of a resisting obstacle, endeavours to change the state

[3] Newton's development of his mature, relativistic conception of inertia was, evidently, an important part of his development of the concept of mass as presented in the *Principia*. The latter development is central to the history of the conceptual structure of the *Principia*. But it has received a thorough historical and philosophical treatment only recently, in work by Fox (2016)

> of that obstacle. Resistance is commonly attributed to resting bodies and impetus to moving bodies; but motion and rest, as commonly understood, are only relatively distinguished from each other; and bodies commonly seen as resting are not always truly at rest. (1685b, p. 315.)

In remarking that the internal force differs from the inertia of matter only " in our mode of conceiving it," and that impetus and resistance differ "only relatively," Newton was clearly correcting the views he had expressed in *De Gravitatione*.[4] The most critical innovation was the assertion that "a body exerts this force only in a change of state brought about by another force impressed upon it": this explicitly rejects the pre-relativistic idea of a force that is required to maintain a body in its motion, because motion and rest are in fact only relatively distinguished. Newton may have confused some readers of the *Principia* by referring to the intrinsic force as the "force of inertia" or "force of inactivity." In explicating it as he did, however— specifying that a body exerts it only when a force is impressed upon it by another body— Newton showed that he had cleared away the last remnant of the medieval impetus theory, that is, the notion of a special power by which bodies persist in their motions. This conceptual achievement was not lost on Berkeley:

> Leibniz confounds impetus with motion. According to Newton, impetus is in truth the same as the force of inertia….(1721, section 16.)

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[4] The date of *De Gravitatione* is unknown. However, the development of Newton's conception of inertia toward his mature view, in the drafts of *De Motu*, definitively places *De Gravitiatione* before those drafts. (See DiSalle 2020a, 2020b.)

> Inert body acts just as body moved acts, if the matter is truly examined. This is what Newton acknowledges, where he asserts that the force of inertia is the same as impetus… (ibid., section 26.)

In other words, Berkeley saw that Newton had defined a concept of inertia that expressed the profound connection between the principle of inertia and the relativity of motion, on which even a professed advocate of relativity such as Leibniz remained confused.

Generally, Berkeley, more clearly than other early critics of absolute space, saw that Newton had thoroughly integrated the principle of relativity into his theoretical physics. This was precisely the ground of his objection to absolute space: there was no need for it in a theory that had spectacularly solved the fundamental problems of terrestrial and celestial motion without any appeal to it.

> The laws of motions and all their effects, and the theorems containing the calculations of the same for different figures of the paths, as well as for accelerations and various directions, and for more or less resistant media, all these hold without the calculation of absolute motion.(1721, section 65.)

If we replace metaphysical hypotheses into the nature of things with Newtonian mathematical principles, avoid abstractions such as absolute space, and reject any but sensible measures of motion, according to Berkeley, we will

> leave untouched all those celebrated theorems of the mechanical philosophy, by which the recesses of nature are brought out and the system of the world is subjected to human calculation: And the study of motion will be freed from a thousand minutiae, subtleties, and abstract ideas. (1721, section 66.)

The "celebrated theorems" include, evidently, the derivation of universal gravity and of the structure of the solar system, which Newton had achieved by the use of attraction as a "mathematical hypothesis." That is, Newton had brought all observable motions under the rule of the laws of mechanics and the law of universal gravitation. The idea that there was another sense in which bodies could be said to be moving, with respect to a space that cannot be represented in any empirical problem addressed by Newton's mathematical methods, was indeed an "abstract idea" in Berkeley's sense. Berkeley's critique therefore raises, more cogently than anyone else had done, the question, why did Newton ever think that he required the concept of absolute space? We will return to this below. For now, it suffices to recall that in Berkeley's view, Newton began from a misguided belief that bodies, and the space in which they exist and move, exist independently of being perceived. So Berkeley was approaching this question with a particular aim: to show that not even the achievements of Newton's science could justify belief in a material world outside the mind. For anyone who maintained that belief, eliminating absolute space from Newton's physics was more challenging than it was for Berkeley.

It should be recalled here that to the problem of absolute space, within Newton's theory, there is no corresponding problem with the concept of absolute time, or corresponding motive to eliminate the concept from Newton's theory. Absolute time does not imply more than is strictly required by the laws of motion, and therefore does not invoke relations for which the laws do not provide, in principle or to some degree of approximation, empirical measures. Absolute time incorporates two principles: absolute simultaneity, and absolutely "equable flow," that is, absolute equality of time intervals. The first is evidently presupposed by the notion that there are objective spatial distances, and therefore objective relative motions; indeed, this seems to have been presupposed almost universally before the advent of special relativity. None of the historically distinguished advocates for the relativity of time, such as Leibniz or Mach, ever doubted that there is an objective measure of spatial relations at a given time, and successive spatial configurations of bodies— on this assumption rested the entire classical theory of the relativity of motion. Newton made this assumption explicit in the Scholium to the Definitions: all things are located "in time with regard to order of succession, and in space with regard to order of situation" (1687b, p. 7); and again in the General Scholium: "every particle of space is *always*, and every indivisible moment of time is *everywhere*" (1726, p. 528). The laws of motion provide, in principle, ways of determining when events are simultaneous, to some level of approximation. Given the finite velocity of light, which was well known in Newton's time, any such determination would necessarily be retrospective.

For the principle of equable flow, too, empirical content is provided by the laws of motion. The laws determine a physically distinguished state of uniform rectilinear motion, so that any freely moving body must move equal spatial distances in equal intervals of time. Such a body obviously represents an ideal case, not to be encountered where gravity is ubiquitous; Newton noted that, in general, there may be no truly equable movement of any actual body in the universe (1687b, p. 7). Yet the laws of motion provide, in principle, empirical measures of how well any motion approximates an inertial motion, and practical comparisons among motions to determine the best available approximation to an absolutely equable measure. For example, an ideal rigid sphere, left to rotate freely, will rotate through equal angles in equal times, but the rotation of the non-ideal earth generally varies; therefore, the day as defined by the actual rotation of the earth must be compared with sidereal motions, as in the traditional "equation of time" (ibid.). Ideally, the more closely any such motions approximate equable motion, the more they will tend toward agreement with each other, i.e. toward equal or mutually proportional intervals of time. The equable flow of time, then, is a theoretical concept for which the laws of motion determine a method of approximation. Newton could justly be confident that measurements of time for celestial motions, though imperfect, were ameliorable. For the concept of absolute velocity in absolute space, in sharp contrast, there is no method of approximation, that is, no sensible measure of velocity from which a true measure could be gathered. This is why absolute space could be eliminated on the basis of an internal analysis of Newton's mechanics (cf. below), while absolute time was not displaced until Einstein proposed an alternative theory.

III. The relativity of motion and Newton's method.

Newton's introduction of the relativity principle (1684b) marked the beginning of what I have called "Newton's theory of relativity".[5] For here Newton began to work on a broader task, namely, to explore the conceptual consequences of embracing the relativity principle: the review and replacement of existing theoretical concepts in accord with the principle, and the develop-

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[5] See DiSalle (2020b) for further discussion.

ment of a new dynamical account of systems of bodies in states of motion that may be indistinguishable from rest. We have already discussed an instance of the first point, in Newton's gradual development of a relativistic conception of inertia. A striking illustration of the second is Newton's novel application of the relativity principle to the traditional question between the heliocentric and the geocentric accounts of the solar system, which gave the question a new meaning. His treatment began with four laws:

> Law 1: By its intrinsic force alone, a body perseveres in uniform motion in a straight line if nothing hinders it.
>
> Law 2: The change in a body's state of motion or rest is proportional to the force impressed and acts along the straight line in which that force is impressed.
>
> Law 3: The motions of bodies included in a given space are the same among themselves, whether that space is at rest or moves uniformly in a straight line without circular motion.
>
> Law 4: By the mutual actions between bodies their common centre of gravity does not change its state of motion or rest.(1684b, p. 40r.)

Newton's essential step forward was to apply the third and fourth principles to the motion of the solar system as a whole. He now made it clear that the motion of the system could be considered, not as having a state of motion in space itself, but as contained in a space of its own, "the whole space of the planetary system," i.e., a space encompassing all of the planets whose motion can be traced with respect to the fixed stars. Then the configuration of the system can be determined from the actions of the bodies among themselves: by Law 3, these actions will be the same, whether the space is at rest or in uniform motion; by Law 4, these actions will not alter the state of motion or rest of the centre of gravity, which may be in uniform motion or rest along with the entire system. Newton concluded:

> Moreover the whole space of the planetary heavens either rests (as is commonly believed) or moves uniformly in a straight line, and hence the common centre of gravity of the planets (by Law 4) either rests or moves along with it. In both cases (by Law 3) the relative motions of the planets are the same, and their common centre of gravity rests in relation to the whole space, and so can certainly be taken for the still centre of the whole planetary system. (ibid, p. 47r.)

The solution to the problem of the system of the world thus becomes the problem of using the laws of motion to find the resting centre of the system, which can only be its centre of gravity. To solve this problem, it suffices to begin with what Newton would later call a "relative space."
It followed that the traditional question— "which body is at rest in the centre of the planetary system?" — rested on an unjustified supposition. In a system of interacting bodies, only their common centre of gravity will remain unaccelerated. So the nearest equivalent to the traditional question is, "which body is closest to the system's centre of gravity?" By "Law 3," the motions of the bodies in the system will be the same, whether its centre of gravity is at rest or in uniform rectilinear motion. In explicitly asserting the dynamical equivalence of "whole spaces" that may moving uniformly or at rest, Newton made it clear that the solution to the problem of "the system

of the world" is the same with respect to any such moving space as it is with respect to immobile space.

This discussion provides a further example of Newton's conceptual progress beyond *De gravitatione.* By the same token, it connects Newton's progress in thinking about relativity with his concern about ways in which his reasoning might be wrong. In *De Gravitatione*, Newton had criticized Descartes's conceptions of matter, space and motion; in particular, he noted the incoherence of the concept of matter as nothing but extension, and of motion as ("in the philosophical sense") nothing but the displacement of a body with respect to immediately contiguous bodies. Both concepts were contrary, not only to common sense ideas, but also to the Cartesian explanation of planetary motion, which depended, in spite of the philosophical account of motion, on the supposition that body follows a privileged trajectory in space (a straight line) unless acted upon by external causes. Newton incorporated some of the essential points of his criticism into the Scholium to the Definitions in the *Principia*, though without mentioning Descartes by name.

One particular argument against Descartes, however, was not included in the *Principia*, even though, in retrospect, it seems as if it might have proved a compelling one: Descartes' definitions introduced arbitrariness and uncertainty into the very idea of a trajectory, and therefore undermined the very possibility of a law of motion such as that on which Cartesian physics was founded.[6] If such a law is to be applicable, there must be a way to identify a path for any body, and to characterize its deviation from a rectilinear path. But this is impossible if space is a gyrat-

[6] On Newton's criticisms of Descartes, see Stein (1967, 2002). For further discussion and comparison of Newton's criticisms in *De Gravitatione* and the Scholium, see DiSalle (2006, chapter 2), and DiSalle (2020b).

ing fluid vortex, and positions of bodies are defined only by the particles to which they are contiguous.

> [N]o one can assign the place according to Descartes at which the body was in the beginning of the motion undergone, or rather he has not said from where it is possible for a body to be moved. And the reason is that, according to Descartes, it is not possible to define and assign the place except from the position of the surrounding bodies, and that after any motion having been undergone, the position of the surrounding bodies is no longer the same as it was before….It follows that Cartesian motion is not motion, for it has no velocity, no determination, and there is no space or distance that it traverses. Therefore it is necessary that the definition of places, and so of local motion, be referred to some immobile thing, such as extension alone, or space insofar as it is viewed as truly distinct from bodies.(1684a, pp. 9-11.)

Stein identified the essential point that Newton was trying to make: that dynamics as then understood, founded on the principle of inertia, required just that connection of space with time implied by the privileged status of uniform rectilinear motion; moreover, Newton's contemporaries were in no reasonable position to deny this, as they all adopted such a principle unquestioningly as the foundation for their program for explaining the planetary motions (cf. Stein, 1967).

Yet one can also see why Newton abandoned this argument, once having fully embraced the relativity principle. For the conclusion that he had drawn was too strong: there was no need to refer places and motions to "some immobile thing." Given the relativity principle and the centre of gravity principle, Newton could solve the problem of the system of the world without knowing how the system as a whole might be moving with respect to space itself. The decision between the heliocentric and geocentric models was no longer a matter of choosing the more plausible hypothesis, but a matter of calculation using the laws of motion and astronomical evidence.

> Hence truly the Copernican system is proved a priori. For if the common centre of gravity is calculated for any position of the planets it either falls in the body of the Sun or will always be very close to it (1684b, p. 47r).

By "a priori," Newton obviously did not mean anything like the usual philosophical meaning, that is, "from first principles," or independent of experience, since the "a priori" proof explicitly appeals to empirical facts. He seems to have meant, rather, a proof from what is previously established: empirical facts about the motions and magnitudes of the planets, combined with the established physical laws that permit such a calculation. This was in stark contrast to the a posteriori argument for the heliocentric view maintained by his mechanistic contemporaries: that a heliocentric theory is a more likely basis for a mechanistic explanation, such as a vortex theory in the Cartesian vein. Newton's conclusion dispensed with the idea of a true central body, but rigorously derived a quasi-heliocentric account from physical principles and phenomena. And he showed that our knowledge of it could not be affected by our ignorance of the motion of the whole system in immobile space.

IV. Newton's introduction of absolute space.

Given Newton's clear-sighted understanding of the principle of relativity, and his commitment to empirical methods for deciding theoretical questions (whenever possible), it is a subtle problem to understand the place of his conception of absolute space within his broader theoretical outlook. There are three points about this conception that should be more widely appreciated. First, the conception was, in fact, Newton's own: he coined the terms "absolute space" and "absolute time," and there is no established usage of these terms before Newton used them. There was, of course, a history of distinguishing "absolute motion" from "relative motion," and Barrow's use of this terminology (1685) was doubtless familiar to Newton. Moreover, Newton was evidently not the first to think of space as infinite, immobile, and homogeneous, or time as flowing equably. To refer to "absolute space" and "absolute time" as Newton's own theoretical terms, therefore, is not to deny that they denoted ideas that had much in common with those of previous philosophers. It is only to acknowledge something that ought to be obvious: that he introduced these terms in order to tell his readers exactly how he meant them to be understood. Newton never asserted that "space is absolute" or that "time is absolute," as if "absolute" were a predicate with an established philosophical significance in this context. Instead, he introduced the terms "absolute space" and "absolute time" along with clear explications of their meanings.

This first point is reinforced by the second, namely, that Newton first introduced "absolute space, time, place, and motion" explicitly as "Definitions," in an unpublished draft titled *De motu corporum in mediis regulariter cedentibus* ("On the motion of bodies in regularly yielding media," 1685a). The treatise begins with eighteen definitions, of which the first four distinguish absolute from relative time and space:

> Def. 1. Absolute time is that which by its own nature without relation to anything else flows uniformly. Such it is whose equation Astronomers investigate, and by another name is called Duration….
>
> Def. 2. Time looked at relatively is that which from something some other sensible passage or another flow or passage is measured in respect to the flow or passage of any sensible thing is considered as uniform….
>
> Def. 3. Absolute space so-called is that which by its own nature and unrelated to any other thing whatsoever always remains immobile. As the order of the parts of time is immutable, so also is that of the parts of space….
>
> Def. 4. Relative space is that which is considered immobile with respect to another any sensible thing: such as the space of our air with respect to the earth….(1685a)

Granting that Newton was influenced by contemporary ideas about space,[7] it is clear that he wished the reader of his treatise on motion to understand no more or less by these terms than he specified.

In the *Principia*, Newton used more or less the same words, speaking in the same definitional mode, to distinguish absolute from relative space, time, and motion. It might seem puzzling, therefore — not to mention adverse to my point— that he did not include them among the named Definitions (now only eight) with which the book begins. But the puzzle is solved by

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[7] Rynasiewicz (1995a, 1995b) offers a thorough account of Newton's immediate antecedents.

comparing Newton's statements of the role of definitions. In the earlier treatise, the definitions are to ensure

> that the reader, freed from certain common prejudices and imbued with distinct conceptions of mechanical principles, may agree to what follows….(1685a).

This statement differs little from one that follows the Definitions in the *Principia*:

> "Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which they are to be understood in what follows." (1687b, p. 5.)

But one important difference stands out: in the *Principia*, the logical function of definitions in the axiomatic structure— to explain how the terms "are to be understood in what follows"— excludes the definitions of absolute space, place, time, and motion. They were crucial to his account of the philosophical context of his theory of motion, and, especially, to his view of the problem of the true structure of the solar system— "the frame of the system of the world"— and how the physics of the *Principia* would solve it. They were not, however, "terms used in the following treatise" in a logical sense: no subsequent reasoning in the book attempts to establish the absolute motion of any actual body in absolute space.[8] Therefore these definitions no longer be-

[8] Interestingly, the nearest approach to such a claim occurs in Proposition LXIII, in his discussion of motions with respect to the centre of gravity of a uniformly moving relative space. He adds the hypothetical statement that "adding to this motion the uniform progressive motion of the entire system of the space and the bodies revolving in it, we will then have the absolute motion of the bodies in immovable space." (1687b, pp. 168-69.) He does not, of course, suggest any method by which the "uniform progressive motion" in immovable space might be known.

longed to the logical structure of the work, as they had in the first draft; instead, they are relegated to the Scholium, where Newton explicated his distinctions and their empirical content, and exposed the philosophical "prejudices" that must be removed.

The third point important point about Newton's definitions concerns "absolute space" specifically: the theory of absolute space is, in fact, a space-time theory. This was already noted by Stein (1967), in explicating Newton's theory in four-dimensional terms. But it should be emphasized that Newton's own definition is explicitly spatio-temporal. The term "space-time" is anachronistic, of course, but it is certainly not anachronistic to observe that Newton incorporated time into his definition of absolute space: it "*remains always* similar and immovable" (1687b, p. 5; my emphasis). Newton did not think of space-time, but he did think of absolute space as something essentially connected with time. Predicates such as "same position at different times," or "same moment of time at different places," and "same velocity at different times," evidently refer to spatio-temporal relations that were commonly understood before the 20th century. And when Newton spoke of the "absolute places" as constituting the proper reference-frame for absolute motion, he characterized them as those places that "from infinity to infinity maintain given positions with respect to one another" (1687b, p. 8-9). In short, rather than claiming that "space is absolute" — which would not have had an obvious meaning for his readers— Newton claimed that space has a peculiar connection with time, and "absolute space" was his term for this conception of space.

My purpose here is not to belabour a point that seems obvious as soon as one reads Newton's Scholium with care. It is, rather, to provide the right context for the earlier-mentioned question, why did Newton maintain the theory of absolute space in a physical theory that had no use for it? Briefly: Because he understood that physics, in his time, presupposed a certain connection between space and time. This was strictly implied by the accepted principle that the motion of any free body is uniform and rectilinear, and that any change in that motion requires a causal explanation. Neither he nor any of his contemporaries, nor anyone else for the next two centuries, proposed a more appropriate spatio-temporal account of that connection than Newton's own theory. Both Huygens (cf. Stein, 1977) and Berkeley (cf. above) were aware that Newton's concepts of absolute acceleration and absolute rotation could dispense with the concept of absolute velocity in absolute space. But neither was in a position to imagine a spatio-temporal structure in which the distinction between uniform motion and acceleration, and between rotation and non-rotation, would be physically meaningful, while absolute velocity would not. Unlike Berkeley, Newton was convinced that the laws of motion, if they were true at all, were true of the universe itself, independent of anyone's perceiving it— and that such laws described motion in absolute space and time.

Only in the 20th century was the appropriate space-time structure defined, in which all and only the objective relations of Newtonian physics could be represented, without the excess structure represented by absolute space, and philosophers became aware of this mainly after Stein (1967).[9] In the later 19th century, however, physicists had already begun to see that absolute space could be dispensed with and replaced with an equivalence class of relative spaces— the "inertial systems" or "inertial frames"— any one of which is as good as any other for the de-

[9] Stein (1967) distinguished the four-dimensional affine structure required by Newton's laws from Newton's absolute space. That Newtonian mechanics corresponds to a four-dimensional space-time structure like the Minkowski space-time of special relativity, only with the Galilean symmetry group instead of the Lorentzian, was spelled out by Minkowski himself (1908). But the precise notion that "the [Newtonian] world is a four-dimensional affine space," as described by Stein, first appears in Weyl (1918, p. 130).

scription of a dynamical system. In effect, this development brought out the true import of Newton's use of Corollary V (or earlier, Law 3): there is no fact of the matter about whether a particular system is uniformly moving or at rest, and therefore no need to suppose that there is an encompassing immobile space with respect to which any such system has its true velocity. But this insight was hard won and only slowly digested.[10] In light of this, it is not so remarkable that Newton believed in absolute motion in immobile space. More remarkable is that he endeavoured to secure his dynamical reasoning from any doubt that might be thought to arise from our ignorance of the absolute motions.

Even Ernst Mach, known as one of Newton's sternest critics, came to a more sympathetic view of Newton's effort after the emergence of the concept of inertial frame (cf. DiSalle 2002). Absolute space had seemed to him a metaphysical answer to an empirical question: when we state the principle of inertia, relative to what do we describe the motion of a body as uniform and rectilinear? Mach's celebrated answer was that the law has no meaning except as a description of motion relative to the fixed stars and the earth's rotation. But he saw that the concept of inertial frame had placed this question in a different light. The laws of motion do not need a reference-frame relative to which they are meaningful; rather, they are themselves the principles by which an appropriate frame (an inertial frame) can be determined. Having identified one such frame, we know that any other frame in uniform motion with respect to the first is an equally suitable frame. Mach came by this means to appreciate how well Newton had grasped the physical equivalence of such frames, in spite of his belief in absolute space.

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[10] For the history of the concept of inertial frame, see DiSalle (2020a).

> It is very much the same whether we refer the laws of motion to absolute space, or express them abstractly, without express indication of the system of reference. The latter course is unproblematic and practical, for in treating particular cases the student of mechanics looks for a suitable system of reference. But owing to the fact that the first way, whenever there was any actual issue at stake, was nearly always interpreted as having the same meaning as the latter, Newton's error was much less dangerous than it would otherwise have been.
>
> ….Let us again emphasize that Newton's oft-mentioned Corollary V, which alone has scientific value, makes no mention of absolute space. (Mach 1933, p. 242.)

Yet the remark, though illuminating, may be misleading. For it seems to suggest that it was mere good fortune that Newton's theory of space and motion did not compromise the success of his physics. On the contrary, the success of his theory of motion, in spite of the assumption of absolute space, was the result of deliberate efforts by Newton himself. It was Newton who— aware of the empirical problem posed by absolute space— found the means to insulate his physical reasoning against it.

We can see this by comparing Mach's remark to Newton's actual reasoning. Surely Newton supposed that a body not subject to forces moves uniformly with respect to absolute space and time. But he knew as well as Mach did that absolute space does not serve as a reference-frame. As far as celestial motion was concerned, the empirical use of the laws of motion required reference to the fixed stars. This included, evidently, determining the "frame of the system of the world" by calculating its centre of gravity. It might appear to be a rather transparent bluff to claim to have distinguished, by reasoning on such a basis, the "true motions" from the apparent. Yet Newton recognized that to find a "suitable frame of reference" was a theoretical exercise for which the laws of motion provided empirical criteria. Taking the fixed stars to be at rest was only provisional; the dynamical analysis of planetary motion determined just how suitable a reference frame they are. If they were not a suitable frame— that is, not in some uniform state of motion— then the effects of its non-uniform motion would have to reveal themselves, in principle, to sufficiently precise measurement. By the third law of motion, every acceleration of any planet would have to be balanced by an equal and opposite reaction within the system. Any acceleration or rotation of the reference frame, therefore, ought to result in unbalanced forces, just as centrifugal forces exhibit the rotation of a frame of reference fixed to the earth. Newton himself discovered a more practical criterion: he could show that the orbits of the outer planets, as far as observation could determine, did not precess as, for example, the orbit of Mercury was later observed to do. That is, their apsides are approximately stable with respect to the fixed stars. But a relative space with respect to which these apsides are sufficiently stable is, Newton showed, a sufficient approximation to one that is at rest or in uniform motion (cf. Book III, Proposition XIV, 1687b, p. 420). On Mach's proposed empiricist foundation for the laws of motion, it would be meaningless to ask whether the fixed stars comprised a dynamically distinguished frame of reference; on Newton's abstract conception, it was an empirical question.

V. Newton's extended theory of relativity

Though he was unable to incorporate his relativity theory into an appropriate theory of space and time, Newton used the relativity principle to ensure that his reasoning about the forces at work in the system would not be undermined by ignorance about the state of motion of the system with respect to absolute space. In giving empirical criteria for establishing that the system is in (approximately) uniform motion or at rest, he came as close as anyone did to articulating the idea of an inertial frame, without giving up the idea of absolute rest altogether. Thus his theory of relativity was the most advanced account of physically equivalent states of motion until the 19th century, in spite of his conviction that such empirically indistinguishable states may be genuinely inequivalent with respect to absolute space.

Yet, sometime before the *Principia*, Newton saw the need to extend his theory, or more precisely, his treatment of physically equivalent systems, to consider physically inequivalent systems that may be practically indistinguishable—more precisely, uniformly accelerated systems that may be treated as if they are at rest or in uniform motion. This peculiar kind of accelerated system was described in the remarkable novel principle that became Corollary VI:

> Corollary VI: If bodies are moved in any way among themselves, and are urged by equal accelerative forces along parallel lines, they will all continue to move among themselves in the same way as if they were not acted on by those forces. (1687b, p. 20.)

A system of bodies acted upon by such a set of forces is in a dynamical state that is, necessarily, physically inequivalent to that of a uniformly moving system. For practical purposes, however, it may be treated as equivalent to the latter since, locally, it will be empirically indistinguishable. Of course, Newton was not especially interested in the ideal case described in the Corollary, of forces acting exactly equally and in parallel directions on all parts of a system. His interest was Galileo's discovery, carefully corroborated by Newton himself, that gravity actually does behave very much like such an accelerative force.

Newtonian gravity generally will not act exactly equally, or in parallel directions, on all parts of any interacting system of bodies: the forces will naturally vary with distance, and they will not be parallel, but converging on the centres of gravitational attraction. In principle, however, and in real examples, the actions of gravity may approximate the conditions of Corollary VI as nearly as may be imagined. Jupiter and its moons, for example, are not equally accelerated by the gravitational pull of the Sun, given their varying distances, and their accelerations are not parallel, but converging on the centre of the Sun. And Jupiter's elliptical orbit around the Sun does not, as a whole, approximate a uniform rectilinear motion. Yet the Jovian system may be treated as if it is in nearly uniform motion, over limited periods of time, because its immense distance from the Sun renders those differential accelerations negligible. Thus the system approximates the conditions of Corollary VI as closely as observation can determine. It should be emphasized, therefore, that Corollary VI as Newton understood it is strictly not an extended relativity principle. It does not enlarge the class of systems that are in principle equivalent. Any system undergoing such accelerations is, necessarily (by Newton's third law of motion), involved in an interaction with some other system, which must experience an equal and opposition reaction.

Corollary VI is, however, an extension of Newton's theory of relativity: it extends his power to make inferences about the forces acting within a system of bodies, even when we may be ignorant of their states of motion in a larger context. In particular, it protects our reasoning about those forces from our ignorance of possible larger interacting systems of which our local system may be a part. In the case of Jupiter and its moons, we may be sure that their accelerations toward the Sun affect their actions among themselves only negligibly. But by the same reasoning, we may trust our analysis of entire solar system, even if it is similarly bound in orbit around some distant and unknown gravitational source.

Newton himself drew these remarkable implications in *De motu Corporum liber secundus (*1687a).[11] He showed that his analyses of the forces at work among the Sun and the planets were not undermined by the possibility that the system as a whole might be accelerated toward some other system.

> It may be imagined that the sun and planets are impelled by some other force equally and in the direction of parallel lines; but such a force (by Cor. VI to the Laws of Motion) would not change the situation of the planets among themselves, nor would produce any sensible effect; but we are concerned with the causes of sensible effects. Therefore let every force of this kind be rejected as being precarious and having nothing to do with the phenomena of the heavens; then all the remaining force by which Jupiter is urged will tend (by prop. 3, corol. 1) toward the centre of the sun.(1687a, article 13).

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[11]This was Newton's original draft for a concluding section for the *Principia* written in a "popular" style, posthumously published as *The System of the World* (Newton 1728).

The last sentence indicates the methodological thought behind Newton's application of Corollary VI. Newton was explaining his calculation of the force acting on Jupiter, in order to show that it is directed toward the centre of the Sun— a calculation that was a crucial part of his argument for a quasi-heliocentric structure. Corollary VI allowed him to assert that such calculations could yield secure theoretical conclusions, even if the entire solar system is acted upon by some external force.

Newton's theory of relativity is integrated with his method of mathematical physics. It concerns not only equivalence of certain states of motion, based on physical quantities that remain invariant across such states; it also concerns the foundation of physical equivalence in the mathematical composition of motions. Galileo and Huygens had understood that the motions within a system could be composed with a common inertial motion of the whole system. Only Newton, however, extended this idea to the composition of accelerative forces. This then enabled him to grasp the idea of approximately equivalent states of motion, approaching by degrees the ideal case of a force acting precisely equally on all bodies in a system. Such an approach is exemplified by the passage from (first) a system such as that of the earth and its moon, in which differential accelerations toward the sun are significant enough to create a difficult three-body problem; to (second) a system like the Jovian system, in which differential accelerations are small enough for us to ignore them and to treat the system as practically isolated; to (finally) an ideal system such as is described in Corollary VI. The latter is essentially a limiting case of a system like Jupiter's, as the size of the system becomes small compared to its distance from the source of the external force (cf. Book I, Prop. LXV, Case 2). Thus Corollary V and Corollary VI belong together, as describing, first, in principle indistinguishable systems of bodies, and second, dynamically distinct systems that may be indistinguishable to any degree of approximation. Indeed, it is likely that this fact moved Newton to change the relativity principle from an independent assumption to a Corollary: historically, this change coincides with his first use of Corollary VI.

VI. Hypotheses, rules, and phenomena.

A final significant change in Newton's thinking appears in the re-organization of Book III, which connects Newton's methodology with the relativity of motion in a particularly striking way. The modern reader of the *Principia*, typically of Newton's third edition, might be surprised to learn that in the first edition, Book III begins with nine hypotheses: how is this to be reconciled with the later remark that "hypotheses have no place in experimental philosophy"? That remark first appears in the General Scholium, added to the second edition (1713, p. 484), but it can hardly be seen as a late development, since Newton had expressed something like it throughout his career. A change of principles is also implausible, since the principles designated as "hypotheses" in the first edition of Book III remain in the later editions, under different designations. Rather, the probable explanation is the simplest one: Newton was using the word "hypothesis" in two different senses. This explanation, moreover, is straightforward to document and to account for historically. When we consider its philosophical implications, however, and its effects on the arguments of Book III, we see that Newton's revision reflects important aspects of his thinking on absolute and relative motion.

A minor part of the history of Newton's revision is the publication, between 1702 and 1710, of John Harris's *Lexicon Technicum* (1708). This contained a noteworthy definition of "Hypothesis":

> When for the Solution of any Phenomena in Natural Philosophy, Astronomy, etc. some Principles are supposed as granted, that from thence an Intelligible and Plausible account of the Causes, the Effects of the proposed Phenomena may be given, the laying down or supposing such Principles to be granted, is called Hypothesis….Wherefore an Hypothesis is a Supposition of that which is not, for that which may be; and it matters not whether, what is supposed be true or not, but it must be possible, and should always be probable (Harris, 1708.)

The point of mentioning this definition is not that it influenced Newton's thinking. Indeed, the influence went primarily the other way, as Harris was a supporter of Newton's natural philosophy, and consulted him on scientific topics discussed in the Lexicon (see, for example, the entry on "Newtonian Philosophy"). Moreover, this definition of Hypothesis is quite consistent with one of Newton's long-established uses of the term. The important point is that this definition does not apply equally to all of the hypotheses in the first edition of Book III, for they are principles of several different kinds.

The nine hypotheses of Newton's first edition fall into four categories. The first two are the canons of inductive reasoning that, in the second edition, would become the first two "Regulae Philosophandi" (along with the new third Rule, with a fourth Rule added in the third edition).

The last five assert regularities observed by astronomers, chiefly that the planets in their orbits about the sun, and the known satellites in their orbits around their respective planets, obey (to a good approximation) Kepler's second and third laws of planetary motion; these would be designated "Phaenomena" in the second and third editions. Hypothesis III was a specific hypothesis about the nature of matter: "Every body can be transformed into a body of any other kind and successively take on all the intermediate degrees of qualities." (1687, p. 402.) In the later editions, this principle is not stated as a fundamental hypothesis, but is absorbed in the text as part of the argument for Proposition VI, Corollary 2. Among the nine Hypotheses, only Hypothesis IV remains a hypothesis in all editions: "That the centre of the system of the world is at rest. This is conceded by all, while some contend that the Earth, others that the Sun, rests in that centre." (1687, p. 402.) But its presentation changes significantly. Not only is it now Hypothesis I, by default; it is also moved from the beginning of Book III to just before Proposition XI. The shift emphasizes that this Hypothesis is no longer presented as a presupposition of Book III generally; rather, it is introduced specifically for the sake of the argument that follows, namely, the argument that determines "the frame of the system of the world." Indeed, Newton adds a sentence to the original Hypothesis that specifies its dialectical purpose: "Let us see what will follow from this." (1713, p. 373.)

Evidently, then, in describing all nine hypotheses as hypotheses, Newton was using the term in its logical sense: these were the principles assumed in the ensuing arguments. In revising the list, Newton changed his emphasis from their common logical role to their diverse sources of warrant. This does not suggest that Newton changed his mind about the hypothetical character of the claim that the centre of the system of the world is at rest, or about the status of "hypotheses, metaphysical or physical," in general. The revisions emphasize, rather, that Newton continued to regard Hypothesis IV as hypothetical, in a sense in which almost all of the other hypotheses were not. This is what likely resulted from Newton's reading of the Harris dictionary entry; in subsequent editions of the *Principia*, Newton ceased to use the word "hypothesis" in two senses. Instead, he used it exclusively to refer to principles assumed for the purpose of a particular argument, but, at best, more plausible than other possible alternatives. Newton was clearly aware that both hypotheses, that the centre of the world system is at rest and that it is in uniform motion, were possible. By Corollary V, either was compatible with all of his mathematical reasoning from the phenomena. Therefore neither hypothesis could be established as something more than a hypothesis, and he could defend the former hypothesis only as the more plausible of the two. In the *Liber secundus*, he had characterized the hypothesis that the system is in uniform motion as "hard" (1687a, section 28; or, in Motte's translation, "hardly to be admitted", 1728, p. 50); in the *Principia*, he says no more against it than that it is "against the fourth Hypothesis" (1687) or "against the Hypothesis" (1713).

Contrast this case with the cases of Hypotheses I and II, and V-IX. The former may perhaps be called hypothetical in a certain sense: we can do no more than suppose that nature is sufficiently simple and uniform to allow us to reject superfluous causes, or to assign the same effects to the same causes. At the same time, however, neither principle is, like Hypothesis IV, just one of two more or less plausible alternatives, of which the selection of one can make little difference to the progress of the subsequent reasoning. Evidently Newton renamed them "Regulae philosophandi," precisely because they guide the entire project of inductive reasoning about causes. Hypothesis I even has the form of a command: "no more causes… should be admitted than are true and sufficient to explain their effects." (1687, p. 402.) Hypothesis II is more nearly in the form of a hypothesis: "Therefore of effects of the same kind, the causes are the same" (ibid.), and remained so when renamed "Rule II" in the second edition. But in the third edition, Newton restated the second Rule as a command like the the first: "Therefore to the same effects, the same causes are to be assigned, as far as possible." (1726, p. 387.) This agrees with the forms of Rules III and IV: "the properties of bodies…are to be taken…," and "propositions drawn from the phenomena by induction…should be taken…." (1726, pp. 387-389.) The Rules are, in short, instructions on how to infer general features of nature from observation. Without such instructions, Newton's arguments from evidence could hardly proceed.

In the case of Hypotheses V-IX, the methodological distinction from Hypothesis IV is even more obvious, and Newton expressed it clearly enough by renaming them "Phaenomena." Again, they function logically as hypotheses, insofar as they are presupposed in subsequent reasoning. But their similarity to Hypothesis IV goes no further than this. They are clearly propositions derived from astronomical observation, indeed by Newton's comparison of results from several sources. Despite their similar logical function, the Phenomena evidently have an epistemic basis unlike any that could be claimed, even by Newton, for Hypothesis IV. The re-organization of the beginning of Book III, therefore, has a greater methodological significance than first appears. For, even though the contents of the original nine hypotheses remain in the later editions, and function more or less as they had in the first edition, their separation into distinct categories reveals their distinct epistemic foundations and methodological roles. In recasting most of his original Hypotheses in Book III, he emphasized their sources of strong warrant, distinguishing them radically from a hypothesis (Hypothesis I) that could be no more than merely plausible.

In light of all this, it is a sign of Newton's discernment that, in recasting his Hypotheses from the first edition, he maintained Hypothesis IV as a hypothesis. In presenting it as Hypothesis I, Newton was not only making it clear that he considered it to be hypothetical in a way that the Rules and the Phenomena were not. He also made it very explicit that it was being assumed for the purpose of argument, specifically in the argument that begins with Proposition XI, to determine "the frame of the system of the world". Newton spoke as clearly and carefully on this subject, and moreover with the same concern for possible sources of error, as he did on the other central aspects of his theory of gravity. His extraordinary clarity, in retrospect, is precisely what has drawn the attention of readers from Berkeley onwards to Newton's error regarding absolute space. He made it very clear that no physical argument depended on a knowledge of absolute velocities, or even on the existence of a physical distinction between uniform velocity and rest. The *Principia* contains no claim regarding the absolute velocity of any actual thing. That the system of the world— or any other physical system— is at rest, and not in uniform motion, can be no more than a matter of hypothesis, and either hypothesis is compatible with Newton's dramatic conclusion: that the centre of the system is its centre of gravity, and that it is nearly heliocentric only because the mass of the Sun, relative to the masses of the other planets, determines that it can never recede far from the centre of gravity. That is, neither the dialectical use of Hypothesis I, nor its character as a hypothesis, weakens the warrant for Newton's conclusion regarding the world-system. Against this background, however, the error stands out equally clearly: Newton had no physical or empirical grounds to regard either of the two hypotheses as any more plausible than the other— in retrospect, no grounds to suppose that there is a fact of the matter either way.

VII. Conclusion.

The theory of absolute space should be seen as an essential part of Newton's effort to protect his physical theory against arbitrariness and error. The notion of a background immobile space, with respect to which every body had a definite trajectory, seemed to give unambiguous meaning to the idea of true motion, as far removed as possible from the arbitrariness and uncertainty of Descartes' "motion in the proper sense." But to understand the theory of absolute space is to understand its position in the evolution of Newton's thinking, and therefore its role in the evolution of Newton's "theory of relativity." For an essential part of the latter was Newton's recognition that absolute space was itself a potential source of uncertainty, since motion with respect to space itself was inherently unknowable. Hence Newton's consistent use of the relativity principle to develop physical concepts, and a way of treating physical interactions, that could not be undermined by the problem of absolute space. Relativity for Newton was a physical principle that enabled him to treat the physical properties of any system of interacting bodies without regard to the motion of the whole system in space, except insofar as the system might exhibit rotation or non-uniform acceleration— either of which could be detected, in principle, by the physical means at Newton's disposal. The "general" principle of relativity advocated by Leibniz conferred a certain arbitrariness on the question of the system of the world; Newton's theory of relativity provided a methodological safeguard against arbitrariness. This is because for Newton, the principle of relativity was in fact a physical rather than a philosophical principle. Its basis was not epistemological equivalence, but the physical equivalence of states of motion, as identified by the laws of motion. Moreover, as a physical principle, it necessarily has an approximative dimension: it allows for states of motion that are nearly indistinguishable, and provides for a quantitative assessment of just how nearly indistinguishable they are. If philosophers have overlooked this aspect of Newton, it is perhaps because of the tendency of the last century to set Newton's "absolutism" against all of the philosophical insights that we associate with Einstein's theory of relativity. By understanding how Newton's conceptions of space, time, and relativity evolved together, we begin to see that, not unlike Einstein, Newton undertook a profound critical examination of the physical concepts with which he was working, seeking to separate their true physical content from what is merely relative, apparent, or arbitrary.

This leads me to a closing remark about George Smith. George has been a model for many people of the combination of philosophical rigour with attention to scientific and historical detail, and he has been among the most generous and inspiring colleagues and teachers that I have known. Beyond that, however, he has helped me to appreciate an aspect of Newton that I had not appreciated: that Newton's profound and unprecedented work of conceptual analysis was inseparable from his extraordinary attention to the details of evidential reasoning, and his adherence to the strictest standards for the grounding of theory in reliable evidence. I have tried to show how thoroughly Newton's thoughts about absolute and relative motion exemplified this empiricist methodology.

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