top as it is customary in computer science. The main reason is that a tree obtained in failing to falsify a given proposition can be viewed as a formal proof of the proposition. The proposition at the root of the tree is the logical conclusion of a set of inferences, and it is more natural to draw a proof tree in such a way that each premise in a rule occurs above its conclusion. However, this may be a matter of taste (and perhaps, aesthetics).

In the rest of this section, it will be assumed that the set of connectives used is $\{\wedge, \vee, \supset, \neg\}$, and that $(A \equiv B)$ is an abbreviation for $(A \supset B) \wedge (B \supset A)$, and $\perp$ an abbreviation for $(P \wedge \neg P)$.

**Definition 3.4.2** The Gentzen system $G'$. The symbols $\Gamma$, $\Delta$, $\Lambda$ will be used to denote arbitrary sequences of propositions and $A$, $B$ to denote propositions. The *inference rules* of the sequent calculus $G'$ are the following:

$$\frac{\Gamma, A, B, \Delta \rightarrow \Lambda}{\Gamma, A \wedge B, \Delta \rightarrow \Lambda} \ (\wedge : left) \qquad \frac{\Gamma \rightarrow \Delta, A, \Lambda \quad \Gamma \rightarrow \Delta, B, \Lambda}{\Gamma \rightarrow \Delta, A \wedge B, \Lambda} \ (\wedge : right)$$

$$\frac{\Gamma, A, \Delta \rightarrow \Lambda \quad \Gamma, B, \Delta \rightarrow \Lambda}{\Gamma, A \vee B, \Delta \rightarrow \Lambda} \ (\vee : left) \qquad \frac{\Gamma \rightarrow \Delta, A, B, \Lambda}{\Gamma \rightarrow \Delta, A \vee B, \Lambda} \ (\vee : right)$$

$$\frac{\Gamma, \Delta \rightarrow A, \Lambda \quad B, \Gamma, \Delta \rightarrow \Lambda}{\Gamma, A \supset B, \Delta \rightarrow \Lambda} \ (\supset : left) \qquad \frac{A, \Gamma \rightarrow B, \Delta, \Lambda}{\Gamma \rightarrow \Delta, A \supset B, \Lambda} \ (\supset : right)$$

$$\frac{\Gamma, \Delta \rightarrow A, \Lambda}{\Gamma, \neg A, \Delta \rightarrow \Lambda} \ (\neg : left) \qquad \frac{A, \Gamma \rightarrow \Delta, \Lambda}{\Gamma \rightarrow \Delta, \neg A, \Lambda} \ (\neg : right)$$

The name of every rule is stated immediately to its right. Every rule consists of one or two upper sequents called *premises* and of a lower sequent called the *conclusion*. The above rules are called *inference rules*. For every rule, the proposition to which the rule is applied is called the *principal formula*, the propositions introduced in the premises are called the *side formulae*, and the other propositions that are copied unchanged are called the *extra formulae*.

Note that every inference rule can be represented as a tree with two nodes if the rule has a single premise, or three nodes if the rule has two premises. In both cases, the root of the tree is labeled with the conclusion of the rule and the leaves are labeled with the premises. If the rule has a single premise, it is a tree of the form

$$\begin{array}{ll} (1) & S_1 \\ & | \\ (e) & S_2 \end{array}$$