L1. Mathematical Logic

Homework Assignment #6.

<u>Exercise 1</u>: Ex. 1, p. 153.

<u>Exerise 2</u>: Ex. 4, p. 153.

<u>Exercise 3</u>: Ex. 6, p. 153.
(<u>Hint</u>: Work as we did to prove the existence of nonstandard models of arithmetic.)

<u>Exerise 4</u>: Let T_1, T_2 be sets of sentences (of the same first-order language). Prove that, if T_1, T_2 have no common model (i.e., there is no structure which is a model of both T_1 and T_2), then there exists a sentence φ such that $T_1 \vdash \varphi$ and $T_2 \vdash \neg\varphi$.

<u>Exerise 5</u>: Let $\mathcal{Z}$ be the structure for the first-order language of number theory which is defined as follows:

$|\mathcal{Z}| = \mathbb{Z}$ (set of integers)

$'^{\mathcal{Z}}$ is the function defined by $'^{\mathcal{Z}}(n) = n \overset{\mathbb{Z}}{+} 1$

$\oplus^{\mathcal{Z}}, \odot^{\mathcal{Z}}$ are the usual functions of addition and multiplication on $\mathbb{Z}$

$0^{\mathcal{Z}}$ is 0.

(a) Is it true that $\mathcal{N} \subseteq \mathcal{Z}$?
(b) Is it true that $\mathcal{N} \equiv \mathcal{Z}$?
(c) Is it true that $\mathcal{N} \cong \mathcal{Z}$? If not, is there an isomorphism of $\mathcal{N}$ into $\mathcal{Z}$?

Deadline. Friday, February 13.