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QUANTUM MECHANICS AND THE OBSERVER

The thing that strikes everyone who looks at quantum mechanics is 'superposition of states'. For example, one can have a hydrogen atom in such a condition that the probability that it is at one energy level is 25% and the probability that it is at the next higher energy level is 75%. Now, the problem is that one should not think of this as meaning that the atom is either at the first energy level or the second but we don't know which. *What then does it mean*? That is the question! That is what 'interpretations' of quantum mechanics are all about.

I shall not review the argument to show that one cannot think of it in a classical way, that one cannot think of it as meaning that the energy level is one or the other (nor can one think of it as meaning that the hydrogen atom is at an in between energy level). Physicists gave up that way of viewing it (which is, unfortunately, the only way of viewing it that one can 'explain to a barmaid', in Rutherford's phrase) long before there were more-or-less formal proofs that one cannot view it that way.

Formal proofs that there are no hidden variables are not, I think, what has played a role in the thinking of physicists; what physicists are more impressed by is the fact that if one tries to think of it that way then it doesn't square with any intelligible physical picture at all. Somehow the way of thinking that works is to think of the superpositions as a new kind of state, a new condition. Sometimes people try to picture it as the atom fluctuating between the two energy levels, but that doesn't seem to work either. It is, of course, the case that if one makes the appropriate measurement, which in this case would be a measurement of energy, then in 25% of the cases one will find the lower energy level and in 75% of the cases one will find the next higher energy level. But one shouldn't think of this as meaning that it is already at one or the other before one looks.

I am not necessarily saying what I believe, by the way, so far I am just repeating the conventional wisdom of physicists. Later on we can discuss whether any of this needs to be revised. Of course, every philospher of quantum mechanics does challenge the conventional wisdom at one point

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or another. But let's start with the conventional wisdom.

The famous two-slit experiment also involves superposition. Indeed, so does the orbiting of an electron around a nucleus. For the position of an electron in orbit is uncertain. Moreover, this uncertainty is, again, more than a mere ignorance on our part as to where the electron is; in some respects it is as if the electron were smeared out over its possible locations. In fact, this 'smeared out' behavior, the fact that the electron is in a superposition of its various possible position states and not in any one of them, is what accounts for the fact that atoms do not collapse to a mathematical point, as they should according to the laws of classical physics.

In the two-slit experiment, photons or electrons or any other quantum mechanical particles are released from a point source. Between the particles (say, photons) and the detector (say, a photographic plate) a barrier is placed in the form of a wall with two fine slits. The uncertainty in the position of the photon – more precisely, the fact that the photon is in a superposition of various position states – permits *each* photon to interact with *both* slits, so that what one gets on the photographic plate is not a simple sum of the patterns that one would obtain by just performing the experiment with the left slit open and just performing the experiment with the right slit open. Rather, it is as if, in the case of each photon that gets through, *half* the photon went through each slit and the two halves then intermingled and interfered (in the manner of waves – in fact, this phenomenon constitutes the celebrated 'wave aspect' of the photon). The final result is a system of visible interference fringes in the photographic picture. Yet, in spite of all this wave-like behavior, *each individual photon strikes the emulsion at one and only one definite point*. We never succeed in *demonstrating* that the photon is physically 'smeared out' by getting it to hit the emulsion in a way that leaves a smeared out crater or other proof that something spatially extended struck; it is only that the interference fringes force us to *infer* that the photon was spatially spread out *when we were not measuring its position*. (Reichenbach compares this to the Charles Addams cartoon of the skier who is skiing down a hill, whose tracks pass on opposite sides of a large tree. We do not see the skier pass through the tree – we have never seen such a thing, nor will we ever observe that – but the tracks seem to force the inference that the skier passed through the tree before we looked.)

So physicists have concluded that the two slit experiment is not a case in which we are dealing with a classical 'statistical mixture', i.e., the particle goes through slit *A* or it goes through slit *B* but we don't know which. The correct formula for the state of the particle is:

$$\Psi\,(x, y, z, t) = \frac{1}{\sqrt{2}}\,\Psi_A\,(x, y, x, t) + \frac{1}{\sqrt{2}}\,\Psi_B\,(x, y, z, t)$$

In this formula, Ψ_A, and Ψ_B represent *states* or *conditions* of the particle; they are (mathematically represented by) vectors in Hilbert space. Ψ_A is the state of a particle which definitely goes through slit *A* (i.e., the state of a particle which hits the screen when slit *B* is closed), Ψ_B is the state of a particle which definitely goes through slit *B*. Notice that the formula does not give the probability that the particle hits any region *R*, it rather gives a mathematical representation of the state of the particle after the interaction with the slits. From that we get the probability by an integration, i.e., the probability that it hits the region R on the photographic emulsion is the squared absolute value of Ψ integrated over the region R. So the probability calculation is not performed by considering two cases: 'either the true state is Ψ_A or the true state is Ψ_B', as we would if we were dealing with a merely *unknown* event (whether the particle went through *A* or *B*), but is rather performed by *superimposing* the two cases (mathematically; forming an appropriate vector sum of the two state vectors).

The dynamics is roughly as follows (I follow von Neumann's book): A closed system is thought of as starting out in either a pure state or a classical mixture. Since a classical mixture can be thought of as a situation in which there is some pure state but we don't know which, it is as if closed systems always started out in pure states. Then the state evolves according to the completely deterministic Schroedinger equation. This is the first model for change of state, and I shall refer to it as *time evolution*.

If, on the other hand, one disturbs the system by coming in from outside with a measuring device and measuring the value of an observable *O*, then the system 'jumps', and it jumps into a so-called eigenstate of the observable measured, i.e., a state in which the observable has a definite value. E.g., when I make an energy measurement upon an atom which is in a superposition of different energy states, the atom jumps into a definite energy level. The mathematical postulate which says what the probabilities are with which the system jumps into each one of the available eigenstates is called the Projection Postulate.

To summarize, there are two forms of change of state: there is the deterministic change of state in a closed system which we called *time evolution*, and there is the indeterministic, discontinuous (in general), jump which takes place upon measurement. Now the problem is obvious, isn't it?

If a system *M* comes along and performs a measurement upon a system *S* out in deep space, it looks as if I can view it in two ways: I can say *S* was a closed system undergoing time evolution until *M* came along and caused an intrinsically unpredictable quantum jump, or I can say, 'No, there was just one system all along, $M + S$', and that the entire interaction between *M* and *S* was just a physical interaction in a single closed system, in which case there should have been deterministic change of state. In the one interpretation (the one in which *M* performed a 'measurement' upon *S*) one will get a jump into *one* of the eigenstates of the observable measured; in the other (time evolution in the single closed system $M + S$) one will get a superposition of *all* the possible results of the measurement.

What von Neumann says about this[1] – and his ideas will be at the center of my discussion today – is that the two ways of looking at this can be reconciled. Consider, to be specific, a system consisting of a geiger counter, a tape recorder, and some radioactive material. The tape recorder records whether or not the geiger counter (which is at such a distance from the radioactive material that the clicks are not too frequent) does or does not click at a preset time. Von Neumann's view is that we can think of the system as just being the radioactive material, and in that case the geiger counter causes that system to jump into one of two states: beta particle or whatever emitted or beta particle not emitted at the preset time; or one can think of the system as being the geiger counter plus the radioactive material, in which case the system would go into a superposition of the states beta particle emitted and geiger counter clicking and no particle emitted (in the relevant direction) and geiger counter not clicking, if it weren't for the tape recorder; or one can think of the system as the radioactive material plus the geiger counter plus the tape recorder, and say that when I play back the tape the system jumps out of the superposition of geiger counter clicking and geiger counter not clicking at the preset time. So there is a certain relativity here; in fact, it seems reasonable to call quantum mechanics (in von Neumann's presentation of it, anyway), a 'theory of relativity' (a term Einstein never liked for *his* theory). On von Neumann's view there is a dependence of the truth upon one's perspective; there is no master truth.

In fact, one can push the 'relativity' farther. Von Neumann says one can put the cut between the observer and the system *inside the brain*. One can view the lateral geniculate nucleus as measuring the eyes and causing them to make 'quantum jumps'; or one can view the cortex as measuring the lateral geniculate nucleus and causing it to jump into one of a number of states; or ... (Perhaps the ultimate observer on von Neumann's view is the Kantian transcendental ego.)

I want to examine this thing more closely. Let me take a case that Schroedinger raised for the purpose of criticizing the orthodox interpretation of quantum mechanics; this is the famous case of 'Schroedinger's cat'. The idea is that one has a system, think of it as a satellite out in deep space, which contains a cat to which electrodes are attached (a repulsive idea, by the way!). The system includes an emitter that is preset to emit a photon at noon. When the photon is emitted it will be emitted in the direction of a half-silvered mirror. If the photon is reflected away, the cat will live; if the photon gets through the half-silvered mirror, it will strike a sensitive detector and the cat will be electrocuted.

Schroedinger's claim was that if von Neumann is right, the one can think of this closed system as being in a superposition of states of the form $\frac{1}{\sqrt{2}}$ live cat + $\frac{1}{\sqrt{2}}$ dead cat, and this is uninterpretable, isn't it? Something must be wrong with the theory. (In my one conversation with him, Einstein raised a very similar objection. Einstein said, 'If this theory is right, then my *bed* jumps into a definite state when I come into my room and look at it.') Well, I am not going to agree that the superposition is uninterpretable. I am going to take the heroic stance of assuming that the theory is right, and that one can have superpositions even of macroscopic states. Now, many people say that this is a tremendous extrapolation, an extrapolation to systems with many, many degrees of freedom, and this is where the theory is going to turn out to be wrong. Maybe they are right, but even if they are right there is still a possible world in which the present theory is true, and as a philosopher I am interested in giving the correct interpretation of quantum mechanics for that possible world.

Since I have taken this heroic stance, let me suppose that our technology becomes so perfect that we are able to manufacture macroscopic systems in pure states. So we actually bring into existence Schroedinger's cat-system, the satellite containing the cat and so on. We even manufacture large numbers of identical copies. ('Identical', of course, means the same pure state. Doubtless this is impossible, but I want to describe a certain thought experiment.)

So, let us suppose we manufacture a large ensemble of these Schroedinger cat-systems, and we examine each one at the preset time and see if the cat is alive or dead. Then classical physics and quantum mechanics both predict: you will find 50% of the cats are alive.

Now, suppose this is not the measurement we make. Suppose that instead we make a measurement to determine if the system is in a superposition. This means that we have to find some observable *O* (and there always is one) such that if the thing is in the superposition predicted by the theory then that observable has the definite value one, and instead of looking to see if the cat is alive or dead, what we do is measure *this* observable *O* in each one of these systems. If the system does not spontaneously go out of the superposition when a macro observable is involved, then we shall find the value *one in each case*. But, as I said a moment ago, assuming that the predicted value would be discovered is extrapolating quantum mechanics to systems with very very many degrees of freedom. If quantum mechanics ceases to hold when so many degrees of freedom are involved; if, for some reason, the ensemble spontaneously becomes a mixed one so that half the satellites are in one state and half the satellites are in a different state (or, perhaps, so that many, many different states are exhibited by members of the ensemble after the preset time), then *O*-measurement will not yield the single value *one* in every case, but instead we will find a smear of *O*-values, and that is how we will know that there is no longer a superposition.

If the prediction of quantum mechanics (as formalized by von Neumann) is correct, and when we measure the observable *O* we get the value *one* in every case, then after we have measured *O* we will not be able to find out whether the cat *was* alive or dead before the *O*-measurement. The *O*-measurement and the cat being alive or dead-measurement are measurements of incompatible observables, in the sense that making either measurement precludes making the other, on the standard interpretation of quantum mechanics. Perhaps the *O*-measurement disperses the entire system, so that the cat is *now* definitely dead, whether or not it was alive before the O-measurement, and so that there is no way to recover the information about the state of the cat's health prior to the O-measurement. And if we try to *first* measure the state of the cat's health and *then* measure O, we disturb the system by the first measurement, so that we cannot tell what the result of the O-measurement would have been if the system had not been disturbed, which was what we wanted to know.

The point to keep in mind is that on von Neumann's view one can *either* regard the cat as an observer, in which case half of the cats are alive at noon and half of the cats are dead – i.e., the ensemble is 'mixed' – or one can regard the outside person who measures O as the observer, in which case the ensemble is a 'pure' one, and all the systems are in a superposition of the form $\frac{1}{\sqrt{2}}$ live cat + $\frac{1}{\sqrt{2}}$ dead cat. It all depends upon where you put the cut between the observer and the system. There is no objective truth *about the meta-question* whether there is a determinate answer to the question 'Is the cat alive or dead?' It depends upon where you cut.

ANALYSIS OF QUANTUM MECHANICAL MEASUREMENT

According to the quantum mechanical definition of measurement, there should be a certain kind of correlation of properties between the system M and the system S after M performs a measurement upon S. This 'correlation' requirement is mathematically expressed as a requirement about the form of the state of the combined system $M + S$ after the measurement interaction – treating that interaction as time-evolution in a single closed system.

(Note how the 'relativity' we have been talking about plays a role in what was just mentioned: we look at the interaction between M and S as if it were *not* a measurement, i.e., we consider it from the 'outside' perspective, in which it is an interaction in the single closed system $M + S$. If this interaction results in a state of the kind I am about to describe, then we say that the same interaction may also be viewed from an 'inside' perspective, the perspective of M, as a measurement performed by M upon S. This moving back and forth between perspectives happens in every quantum mechanics text, but von Neumann is one of the few writers to make it explicit.)

The requirements that the state of $M + S$ must fulfill for the interaction to count as a measurement – rather, the requirements that the state of $M + S$ *in the perspective in which that state is the result of time evolution in a single closed system* must fulfill for it to be legitimate to 'put the cut between M and S' and view the state of S alone as the result of a quantum jump induced by the measurement performed by M – are easy to describe.

Imagine that we wish to measure a particular 'observable', i.e., a particular physical magnitude, say, total energy, in S. The measurement is to tell us which of a number of intervals $d_1, d_2, \ldots, d_n$ the energy lies in. The measuring system M contains a meter, or something of that kind, and we are going to find out which interval the measured observable in S (the energy) lies in by examining the meter and seeing which of the intervals $e_1, e_2, \ldots, e_n$ the meter needle lies in (of course the idea that the registering observable, as I shall call it, is a meter needle is only an example; any macro-observable whose values can be split up into the appropriate number of disjoint 'intervals' – mathematically, Borel sets – will do).

Let us call a possible state of $M + S$ a *determinate state* (from the point of view of this particular measurement, defined by the choice of an observable to be measured – energy, in the example, – a registering observable, a set of possible disjoint intervals that the value of the observable to be measured can lie in, and a set of corresponding disjoint intervals that the value of the registering observable can lie in) if the state is one in which the possible values of the observable to be measured lie in some one of the intervals d_i and the possible values of the registering observable lie in the *corresponding* interval e_i. (The 'possible values' of an observable when a system is in a state Ψ are the values that have a non-zero probability of being obtained if that observable is measured when the system is in the state Ψ. The mathematical postulate of quantum mechanics that we mentioned earlier, the Projection Postulate, determines which these are. If the possible values of an observable in Ψ are $r_1, r_2, \ldots, r_k, \ldots$ then Ψ itself can be thought of as a superposition of states or conditions in which the observable has the definite values $r_1, r_2, \ldots r_k$.)

Suppose that we start out with an M and an S which are not interacting, and we determine the state of $M + S$ at the end of an interaction by using quantum mechanics and treating the interaction as time evolution in the single closed system $M + S$. If the state of $M + S$, calculated in this way, is a *superposition of determinate states*, then the interaction can also be viewed as a measurement of the relevant observable in S by means of the registering observable in M; if the registering observable in M is found to have a value in the interval e_i (where "found" just means *by looking* – recall that the registering observable is a macro-observable), then, in this perspective, the interaction caused S to "jump" into a condition in which the possible values of the energy (or whatever the observable was that we were measuring) lay in the corresponding interval d_i.

Since a superposition of states is mathematically represented by a linear combination of vectors, we can also express this mathematically by writing

$$\Psi_{M+S} = \sum_i c_i \Psi_i \tag{1}$$

where for $i = 1, 2, \ldots, n$, Ψ_i is a determinate state in which the possible values of the two observables lie in d_i and e_i respectively.

In (1) the states are all states in the Hilbert space of $M + S$, the number of summands may be infinite or even continuous (so that the sum may actually have to be an integral); and there may have to be a "slop term", a small additional term of very low amplitude, corresponding to the fact that perfect correlation is strictly impossible.

The criterion I have just given for a measurement, represented by the formula (1), reflects the fact that one can look on a measurement as *either* throwing S alone into an eigenstate of the observable measured *or* as throwing $S + M$ from a superposition of determinate states into exactly one of them – yet another example of the pervasive "relativity" I have been describing.

INCOMPATIBILITY

Following von Neumann, we shall use the term *proposition* to mean any statement of the form: "observable so-and-so has such-and-such a value" (or a value in such-and-such an interval). (Von Neumann speaks of "*experimental* propositions", to emphasize that he is concerned with whether the propositions will be found correct or incorrect upon a measurement, not with whether they are true or false in some realist sense when we aren't looking). It can happen that there is only one state Ψ (up to multiplication by an arbitrary complex scalar, since, in quantum mechanics, vectors that are scalar multiples of each other have the same experimental significance) which assigns probability *one* to a particular proposition p, i.e., only one state such that it is *certain* that p will be found to be true if the system is in that state; such a proposition is called a *maximal* proposition (in quantum logic, which we shall describe below, the maximal propositions are the logically strongest contingent propositions). For any proposition p, the vectors Ψ which represent the states relative to which p has probability *one* (i.e., the states or conditions such that it is certain p will be found to be true if the system is in that state) form a *subspace* of the Hilbert space (the linear space used mathematically to represent states or conditions).

It can happen that two propositions p, q are such that no state Ψ simultaneously assigns probability *one* to both of them; i.e., if a state is such that either p or q is certain, then the other is uncertain. For example, according to the famous Uncertainty Principle,

$$\Delta q \cdot \Delta p \geqslant h/4\pi$$

where Δp and Δq are the uncertainties of momentum and position, respectively.

No state can be such that position and momentum are both determinate, in fact, if the position is extremely determinate (i.e., the possible values of position in that state lie in a very small interval Δq), then the Uncertainty Principle says that the momentum must be correspondingly indeterminate.

There are two cases in which it can happen that two propositions are related in this way; obviously it can happen if p and q are contraries or contradictories in classical logic. For example, no state can assign probability *one* to the statement that a particle is in one place and also assign probability *one* to the statement that it is in a different place. We shall refer to this as a *Boolean incompatibility*. But the incompatibility between the proposition that the particle has a definite momentum and the proposition that it has a definite position is evidently not of this classical, Boolean kind. (Mathematically, both kinds of incompatibility correspond to the fact that the intersection of the subspaces of Hilbert space corresponding to the two propositions is the O subspace. But in the case of "quantum mechanical incompatibility", as opposed to Boolean incompatibility, the projection operators onto these subspaces do not commute. For this reason, quantum mechanically incompatible propositions, such as the proposition that X has such-and-such a definite position and X has such-and-such a definite momentum, are also sometimes called "non-commuting".)

The received view about the experimental significance of quantum-mechanical incompatibility (i.e., of the relation between "non-commuting propositions") is that if p and q are incompatible, then they cannot both be *known* to be true by any measurement or combination of measurements that an observer could carry out. Incompatible propositions are such that their truth values cannot be simultaneously known. (Here and below we are only concerned with the non-Boolean kind of incompatibility; for in the Boolean case, if one knows one of the propositions is true, by a measurement, then one also knows the contrary proposition is *false*; so the truth-values of propositions of this kind can be simultaneously known.)

Another view, more of a minority view, but one which has been around for a long time, is that the truth values of incompatible propositions *can* be simultaneously known, but it is just that the knowledge cannot have predictive value. The classical argument for this minority view is the *time of flight* argument. As set out, for example, by Margenau[2], the argument runs as follows: emit a particle at a definite time t_0. Determine the time and place at which the particle hits a screen. From the distance between the positions at the two times, which is known, and the difference between the two times, which is known, one can determine the momentum of the particle while it was in flight. Also, by straight-line extrapolation, one can determine its position at times between t_0 and the time it hit, t_1. So one can know its *simultaneous* position and velocity; however, this knowledge has no predictive value, since the velocity is disturbed in an unpredictable manner when the particle hits the screen at t_1.

For many years I rejected the minority view just described because it seemed to me that the argument (the "time of flight" argument) offered in its defense imported too many assumptions from *classical* physics (e.g., that the particle has a straight line *trajectory*). (Although it is undeniable that time-of-flight measurements of velocity are in fact made.) Recently I have observed that *it follows from just the quantum mechanical criterion for measurement itself* that the "minority view" is right to at least the following extent: simultaneous measurements of incompatible observables *can* be made. That such measurements cannot have "predictive value" is true, because a measurement of any observable must disturb that observable unpredictably, according to quantum mechanics, unless the Hamiltonian of the interaction between M and S (i.e., the operator corresponding in quantum mechanics to the Hamiltonian function of classical physics) stands in an appropriate mathematical relationship to the operator corresponding in quantum mechanics to the observable to be measured, and the operators corresponding to observables which do not "commute" cannot *both* stand in the mathematical relationship in question to the Hamiltonian. Thus, a measurement which determined the values of two "non-commuting" observables would have to disturb at least one of them, and so could have predictive value with respect to only one of the two observables.

It is wrong, however, to use this fact as a reason for *dismissing* measurements of "non-commuting" observables, i.e., to think that measurements which disturb the value of the observable to be measured are of no scientific significance. Many measurements are made not to predict the future course of the system being examined, but to *test theories*; the system being examined is frequently *destroyed* in such experiments (so they certainly have no "predictive value" in the sense described); but such experiments are of great scientific importance. (The two slit experiment itself is one in which the particle being measured is destroyed by the experiment.)

MEASUREMENT OF NON-COMMUTING OBSERVABLES

Since the possibility of measuring non-commuting observables is of great importance for the interpretation of *quantum logic*, about which I shall speak below (it clears up a problem which hung me up for nearly twenty years), I shall describe a case in which one can show – without appeal to "time of flight", or any assumptions from classical physics – that incompatible propositions can be determined to be true.

Imagine a source of light inside an absorbing box. A very small shutter is opened at t_0 for a brief time. If a photon is emitted at t_0, then (since the space-time extent of the open shutter aperture is small) the position of the photon at t_0 is extremely definite. (Visualizing the photon as a wave bundle, one may say that the wave bundle is "packet like", i.e., concentrated in a tiny region, at t_0.) One may arrange the intensity of the light source so that the probability that a photon is emitted during the brief interval that the shutter is open is exactly one half. The state of the system at t_0 may, thus, be thought of as a superposition of two states: "photon emitted at t_0" and "no photon emitted at t_0". Mathematically, we can represent it in the form:

(2) $$\Psi_{M+S} = \frac{1}{\sqrt{2}}\,\Psi_{photon\ emitted} + \frac{1}{\sqrt{2}}\,\Psi_{no\ photon\ emitted}$$

(The states are all represented in the Hilbert space of $M + S$, where M – the measuring apparatus – is described below. Strictly speaking there is an additional term of small amplitude corresponding to the possibility that *two or more photons* are emitted at t_0: we shall regard this as part of the "slop" term that will appear in (1) anyway, when we apply the criterion (1) for measurement.)

In what follows I shall employ a conceptual trick due to Heisenberg; it is not essential to the argument, but it simplifies exposition. This is as follows: instead of saying that, in time evolution, the *state* changes, one can, alternatively, think of the state as staying the same, and the rule coordinating observables to states (i.e., determining the possible values of observables in states, the expectation values for the result of each measurement in a given state, etc.) as changing. This trick is known as "Heisenberg representation".

In Heisenberg representation, the form of the state Ψ_{M+S} does not change through the time evolution. (In the other, more conventional, representation, the states in the sum change, but the state of the system remains a sum of two states, one corresponding to the case in which no photon was emitted.)

As a measuring device we shall employ a *spherical* emulsion, with the shutter at its center. Let t_1 be the time at which a photon emitted at t_0 will hit such an emulsion, if it is emitted at the center of the sphere.

The state $\Psi_{no\ photon\ emitted}$ in the expansion (2) corresponds to the situation at t_1 in which the emulsion is blank (no "hits") and whatever photons were emitted by the light are still inside the absorbing box. The state $\Psi_{photon\ emitted}$ corresponds to a situation at t_1 in which there is a mark on the emulsion (a "hit") at an uncertain place.

I propose to regard this experiment as a measurement to determine whether or not a photon is emitted at t_0. (We might, for example, want to test our prediction that a photon will be emitted half the time.) The registering observable is the presence or absence of a mark on the emulsion after t_1.

Notice that this is, in fact, how any physicist would regard this experiment. Of course, the presence of a mark at t_1 shows a photon was emitted at t_0 (I assume there is no other source of light). I am not making a "proposal" here, but describing what the physicist actually does and the inferences he unhesitatingly makes.

A proposition, in the sense in which I am using the term (a statement to the effect that an observable has a value in a definite interval), can include such a statement as "A photon was emitted at t_0". (In fact, there is an "idempotent", or two-valued, observable which has the value 1 if a photon was emitted at t_0, and 0 if no photon was emitted.) With this in mind, we see that the expansion (2) has the form (1): $\Psi_{photon\ emitted}$ is a *determinate state* at t_1 in which the observable being measured (whether or not a photon was emitted at t_0) has the value 1 and the registering observable (whether or not there is a mark on the emulsion) has the value 1, and $\Psi_{no\ photon\ emitted}$ is a determinate state at t_1 in which the observable being measured has the value 0 and the registering observable has the value 0. What (2) says is that the state of the combined system $M + S$ is just a superposition of these two determinate states. By the quantum mechanical criterion for a measurement, it follows that we may view the situation at t_1 (when the mark has appeared or failed to appear on the emulsion – prior to t_1 the states are not determinate, because the registering observable does not have a value restricted to an interval excluding 0 in the state $\Psi_{photon\ emitted}$) as a *measurement*. If the mark is on the emulsion at t_1, then, in the perspective in which the "cut" is between M and S, a photon was emitted at t_0; if no mark is on the emulsion at t_1, then no photon was emitted at t_0. And, as already emphasized, this is how any physicist would interpret the presence or absence of a mark on the emulsion at t_1. If one does *not* accept (2) as an instance of the quantum mechanical criterion for a measurement (1), then one cannot give any *quantum mechanical* reason for concluding from the fact that a photon hit the emulsion at t_1 that a photon was emitted at t_0.

But the state $\Psi_{photon\ emitted}$ is itself a superposition. The relevant superposition is easiest to see if we think about the situation at t_1. Divide the emulsion into disjoint tiny regions $R_1, \ldots, R_n$. The uncertainty concerning where the mark is at t_1 corresponds to the fact that $\Psi_{photon\ emitted}$ is itself a superposition of states Ψ_i in which the mark is inside the region R_i and the photon hit inside the region R_i. Mathematically:

(3) $$\Psi_{photon\ emitted} = \sum_i c_i \ \Psi_i;$$

where the c_i are suitable complex numbers, and the $_i$ are as just described. Substituting this expression in (2) we get:

$$\Psi_{M+S} = \sum_i c_i \, \Psi_i + \Psi_{no\ photon\ emitted} \tag{4}$$

The expansion (4) *also* has the form (1); Ψ_i is a determinate state (if we think of ourselves as measuring the *position* of the impacting photon, if one struck). The registering observable is whether or not there is a mark on the photographic plate and which of the regions R_i the mark is in. In the state Ψ_i, the registering observable has a value in an interval which corresponds to there being a mark in R_i and the measured observable has a value which corresponds to the photon having hit in the region R_i. (4) says that the state Ψ_{M+S} may also be viewed as a superposition of determinate states in *this* way. If a mark is in the region R_i at t_1, then, in the perspective in which the "cut" is between M and S, a photon hit in the region R_i at t_1; if no mark is in any of the regions R_i at t_1, then no photon struck the emulsion at t_1. And again, any physicist would accept *this* interpretation of the experiment.

If we view the experiment in the first way, then finding a mark on the photographic plate throws S into the state 'photon emitted at t_0" (i.e., S is in this state prior to its interaction with M); if we view the experiment in the second way, then finding a mark on the photographic plate in, say, the region R_{17}, throws S into the state "photon at the position R_{17}" just prior to t_1. But these are incompatible states (as is easily seen from the Heisenberg representation in which they are just $\Psi_{photon\ emitted}$ and Ψ_{17}, respectively). (Stricktly speaking, $\Psi_{photon\ emitted}$ and Ψ_{17} are states of $M + S$ and not of S alone; but they correspond to *incompatible mixed states* in the subspace of just S.)

One might propose to rule out the first way of viewing the experiment on the grounds that the macro-observable used (presence or absence of a mark on the emulsion) does not "code" *all* the macroscopic information we have (we also know the *location* of the mark). But, besides being *ad hoc*, this leaves us with no way of knowing that a photon was emitted at t_0, which we clearly *do* know. There is nothing *wrong*, after all, with either interpretation of the experiment; all that is wrong is the orthodox remarks about it being impossible to measure incompatible observables in quantum mechanics.

Instead of using the Heisenberg representation, in which the states stay the same and the observables change their representation, we can also use the conventional representation, in which the states change. In this representation, the statement that a photon was emitted at t_0 does not correspond to the statement that the state is $\Psi_{photon\ emitted}$ at t_1, but rather to the statement that the state at t_1 is $U\ (\Psi_{photon\ emitted})$, where U is a certain unitary transformation. But $U\ (\Psi_{photon\ emitted})$, and $U\ (\Psi_{17})$ (this corresponds to a photon striking in the region R_{17}) are incompatible states in the conventional representation, since a unitary transformation preserves all relations of incompatibility.

We have written as if a measurement performed by M upon S throws S into a definite state of the observable measured *just prior* to the interaction; but this is not essential for our discussion. All that matters is that the two observables, *photon emitted at* t_0, and *photon located at the position* R_{17} *at* t_1 are incompatible by the criteria of incompatibility standardly used in quantum mechanics. And one and the same experiment can determine both of these incompatible propositions to be true.

It is, of course, perfectly correct that no *state* can assign the definite value 1 (or "true") to both of these propositions. There is no state in which both of these propositions have the probability one. But it does not follow, as is usually thought, that no *measurement* can assign truth to both of these propositions; the error lies in supposing that a measurement can throw a system into only one state. *Relative to a particular way of analyzing an experiment*, the experiment can throw S into only one state; but an experiment may admit of more than one analysis at the same time.

In my view, this extends rather than conflicts with the perspectival character of quantum mechanics so stressed by von Neumann. We had a case before (the ensemble of Schroedinger cat systems) in which the "inside" observer (the cat) assigned one state ($\Psi_{live\ cat}$) and the "outside" observer assigned a different state $\left(\frac{1}{\sqrt{2}}\Psi_{live\ cat} + \frac{1}{\sqrt{2}}\ \Psi_{dead\ cat}\right)$. This was the case in which the outside observer measured O.

If the outside observer chooses *not* to measure O, but rather to open the satellite at some time after the preset time and see if the cat is alive or dead, then he does not have to view the system as "jumping" from $\left(\frac{1}{\sqrt{2}}\ \Psi_{live\ cat}\right.$

$+ \frac{1}{\sqrt{2}} \Psi_{dead\ cat}\Big)$ into either *live cat* or *dead cat* when he looks (this is the interpretation that so distressed Einstein, in our conversation); rather, he can view the so-called "quantum jump" as not a *physical* jump at all, but simply another expression of the relativity of truth to the observer which it was von Neumann's concern to advocate. When we choose to measure the "mortality condition" of the cat (*alive* or *dead*), we choose to institute a frame *relative to which* the cat *has* a determinate property of being alive or a determinate property of being dead *and the measurement finds out which*; we are, so to speak, "realists" *about the property we measure*; but we are not committed to realism about properties *incompatible* with the ones we measure. Relative to *this* observer *these* properties are "real" (i.e., there to be discovered); but relative to a different observer different properties would be "real". There is no "absolute" point of view.

What made it seem as if there was a *physical* "jump" was the idea that we could not *retrodict* and say that the cat was alive *prior* to our looking. And it looked as if we could not say this because this would *conflict* with the assumption that prior to our looking the satellite was in the condition we prepared, which was (by hypothesis) $\left(\frac{1}{\sqrt{2}} \Psi_{live\ cat} + \frac{1}{\sqrt{2}} \Psi_{dead\ cat}\right)$. But if a system can have more that one state at a time relative to the same observer (provided only one has predictive value), then this argument collapses, and, indeed, the retrodiction that the cat was alive *before* we looked is just as correct as the retrodiction that a photon was emitted at t_0.

QUANTUM LOGIC

Von Neumann's book also contains the first hint of his idea of interpreting quantum mechanics with the aid of a non-standard logic[3]. Apparently he did not regard this as incompatible with the perspectival view I have been stressing; but it is my purpose to explore this connection today beyond the brief hint he gives in the book. (This is the remark that quantum logic has to do with the fact that propositions may not be *simultaneously* testable.)

The logic von Neumann proposes is based on orthomodular lattices (the ones of physical interest are isomorphic to the lattice of subspaces of the Hilbert space of the system being talked about).

I shall not give any technical details in the present paper. The key idea, in a contemporary formulation (e.g., Bub[4]) is that one is not allowed to conjoin "non-commuting" propositions. (Von Neumann himself did allow one to conjoin them, but treated all such conjunctions as identically false, i.e., like conjunctions of propositions whose incompatibility is Boolean. Today it seems more perspicuous to treat conjunctions of "non-commuting" propositions as not even well formed.)

Perhaps the best way to think of quantum logic is this: in quantum logic, the rule of conjunction-introduction (from p, q to infer the conjunction pq) is restricted to *compatible* propositions p and q.

In my example in the previous section, the two statements "The photon was emitted at t_0" and "The photon struck in the region R_{17}" are statements that have no conjunction. The two statements cannot be conjoined without violating the restrictions of quantum logic.

To recapitulate: in quantum logic there is a new semantical relation of *incompatibility* (over and above the classical notion of incompatibility, "p entails not-q"). One is not allowed to conjoin propositions which stand in this semantical relation to each other. The problem is, evidently: how is this relation, the quantum mechanical relation of incompatibility, to be understood?

When I wrote 'The Logic of Quantum Mechanics'[5], I shared von Neumann's view that incompatible propositions *cannot be known to be true* (by the same observer). In view of the example, this is wrong. If p is "The photon was emitted at t_0" and q is "The photon struck in the region R_{17}", then I can know both of them to be true; but within quantum logic there is no way I can conjoin those two pieces of information. What that *means* we shall discuss shortly. But it doesn't mean what I once thought it meant.

What I once thought it meant was based on the orthodox view that one cannot measure non-commuting observables at the same time. What turns out to be the case is that one can know that p and one can know that q (where p and q are the two statements in the example) but one is not allowed to have *a single text* in which one says both p and q.

What this means, of course, is that one is renouncing a certain cognitive ideal. The ideal is that one should be able to visualize knowledge, or, at any rate, *ideal* knowledge, as one text. (Ignoring error, ignoring the fact that people make mistakes:) all the things that any anyone knows anywhere should just be able to be conjoined.

So far we have two failures of this: there is the original failure pointed out by von Neumann, which I explained in terms of the population of Schroedinger cats; the statements by the cats (think of a milder version than Schroedinger's, in which the cats are not killed, but only tickled) and the statement by the outside observer who measures O cannot be conjoined; and in the case of the consequence that I pointed out of the quantum mechanical definition of measurement, two statements known to be true by one observer at one time cannot be conjoined.

Well, what *does* it mean that these two statements are incompatible if they can both be known to be true by one observer at one time? What, for that matter, is the interpretation of the Uncertainty Principle?

It does turn out, as we have already mentioned, that when one has two pieces of knowledge which "violate the Uncertainty Principle" in this way; or one has one interaction which can be read as a measurement in two ways, so that the two resulting pieces of knowledge cannot be conjoined; that they cannot both have predictive value.

But, as I pointed out above, many measurements in quantum mechanics have "no predictive value" in this sense; one doesn't know anything about the photon, or whatever, henceforth, because it is destroyed in the interaction. But, as I also pointed out, such measurements should not be dismissed; they are of great importance in physics.

So we have this situation: we can make measurements of non-commuting observables, but at least one of those measurements has no future value as far as that very system is concerned.

I have also mentioned how we represent this fact in quantum logic: we allow ourselves to conjoin statements that lie in a common Boolean sub-logic of the whole big quantum logic (that is what "commuting" comes to, in logical terms), but not to conjoin statements that do not lie in any common Boolean sub-logic. So we get a *lot* of texts. Even one observer may have a lot of texts. Only one of those texts at any given time will have predictive value. (That is, *direct* predictive value, predictive value about the objects upon which he performed the measurements.) Other texts may have predictive value for other observers.

So far I have only said that we do not *allow* the conjunction of certain statements in quantum logic. Is that just a perversity? Is it just that we have an idiosyncratic preference for writing p, q in certain cases and not p *and* q (pq)? Perhaps we just don't like the word "and"?

In effect, not allowing ourselves to conjoin all the statements we know to be true means that we have what amount to two different kinds of conjunction: one amounts to asserting statements in two different "frames", as I shall call them (different Boolean sub-logics); and the other, for which we reserve the *and*-sign, is conjunction of statements which lie in a common frame. Well, what is this ceremony for?

The fact that this isn't a classical logic because we *don't* allow all the statements that we know (or that anyone knows, or could know) to be conjoined doesn't show that this couldn't be *embedded* in a classical logic. But the question of whether this could be embedded in a classical logic was closed some years ago by Kochen and Specker.

The example Kochen and Specker gave to show the non-embeddability of the logic of quantum mechanics in classical logic was a very pretty one. The example they gave was the following: they describe a system (an orthohelium atom in its lowest excited state in a magnetic field with rhomboidal symmetry!) with the following weird property: the property that if you measure three spin components S_x^2, S_y^2, S_z^2, in any three mutually perpendicular directions you get 1, 1, 0 or 1, 0, 1, or 0, 1, 1. What makes this result weird is that it seems to directly contradict a theorem due to Gleason: that there is *no* way to assign zeroes and ones to *all* the points of a sphere so that for *every* orthogonal triple of points of the sphere two of the points are assigned ones and one of the points is assigned zero!

Kochen and Specker found that the paradox can be stated without reference to the theorem by Gleason mentioned. They succeeded in finding 117 directions in space (the reader may visualize these as 117 line segments of unit length meeting at a point) with the same relevant property as the whole sphere: that there is no way to assign 117 zeroes and ones (one to each line segment) so that for every triple of orthogonal segments (segments forming a "corner", or three right angles) contained in the 117 there are two segments in the triple which have been assigned a one and one segment which has been assigned a zero. (The 117 directions in question are rather rich in orthogonal triples; in fact, it is possible to form more than 60 different orthogonal triples from the 117 given line segments.) According to quantum mechanics, for every one of the 60-odd orthogonal triples that it is possible to form from the 117 directions, there are three squared spin components of which two are ones and one is a zero; yet according to classical *logic* this is impossible.

According to *logic*: for it is possible to think of the 117 directions in space as just sentential letters, $p_1, p_2, \ldots, p_{117}$. (Think of p_i as the proposition that the squared spin component in the ith direction is a one.) Then the 60-odd orthogonal triples correspond to certain triples from the collection of all triples it is possible to form using just these letters; and, if i_1, i_2, i_3 is the ith orthogonal triple, then the statement that two of the squared spin components in these directions are ones and one of them is a zero is just the proposition

$$(p_{i(1)}\, p_{i(2)}\, \bar{p}_{i(3)} \vee p_{i(1)}\, \bar{p}_{i(2)}\, p_{i(3)} \vee \bar{p}_{i(1)}\, p_{i(2)}\, p_{i(3)})$$

The *combinatorial impossibility* of assigning two ones and a zero to all 60-odd orthogonal triples is the same thing as the *tautological falsity* of the formula of the propositional calculus that is obtained by conjoining 60-odd formulas of the kind just illustrated.

The resolution of this paradox in quantum logic is extremely elegant (as Kochen and Specker pointed out). In von Neumann's logic ("quantum logic"), the formula of propositional calculus that asserts the combinatorial impossibility of assigning 117 ones and zeroes in such a way that every one of the relevant triples contains two ones and a zero is not valid. In other words, the conjunction of the 60-odd formulas $(p_{i(1)}\, p_{i(2)}\, \bar{p}_{i(3)} \vee p_{i(1)}\, \bar{p}_{i(2)}\, p_{i(3)} \vee \bar{p}_{i(1)}\, p_{i(2)}\, p_{i(3)})$ that is tautologically false in classical propositional logic is *consistent* in von Neumann's logic! The suggestion is that things which are *literally impossible according to classical propositional calculus* can and do happen, and that that is what we are observing in the case described by Kochen and Specker.

This paradox can also be resolved by following the ideas of the conventional interpretation of quantum mechanics (the so-called "Copenhagen Interpretation") due to Bohr and Heisenberg. According to this interpretation, quantum mechanics does not tell us what values physical parameters have when we are not measuring them; quantum mechanics only predicts the results measurements will have in well-defined experimental situations. So the formula $S_x^2 + S_y^2 + S_z^2 = 2$ which is involved in the experiment described by Kochen and Specker, for example, does not mean that the three squared spin components sum to two (and hence that two of them must be one and one of them must be zero, since these are the two permitted values), but only means that the sum *will be found to be two* if we make the measurement, and that two ones and a zero will be found *if we* *make the measurement*. But if you measure one triple, then you can't measure any other triple, due to the incompatibility relations; so there is no contradiction with classical logic, on this view: it is only a kind of miracle that the squared spin components always assume the right values *when we look*.

The view of quantum logic is not incompatible with the view of the Copenhagen Interpretation (and von Neumann himself seems to have accepted both). But it seems unsatisfactory to some that quantum mechanics should draw a distinction between "measured values" and "unmeasured values" if the latter are *physically meaningless*. If this distinction is only forced on us by classical logic, then, to some, this has seemed a good reason to *change the logic*. Indeed, although the quantum logical point of view is only accepted by a small minority of physicists, not to say philosophers and logicians of physics, its growing appeal is perhaps due to a certain reconciliation of operationism and realism; both operationalistically minded philosophers and realistically minded ones like the elimination of the distinction between "measured values" and "unmeasured values"; the former because "unmeasured values" that cannot be linked to the measured values by any theory are meaningless on even the most lenient operationalist view, and the latter because they like the idea that it is the "real" values (whether measured or not) that physical theories describe and relate.

Quantum logic itself has been interpreted both in verificationist and in "realist" ways by different authors. The interpretation which is ruled out is to think of quantum logic as a fragment of classical logic. Quantum logic is *essentially* non-classical. Each of the statements

$$(\bar{p}_{i(1)}\, p_{i(2)}\, p_{i(3)} \vee p_{i(1)}\, \bar{p}_{i(2)}\, p_{i(3)} \vee p_{i(1)}\, p_{i(2)}\, \bar{p}_{i(3)})$$

is true in the case described by Kochen and Specker, and their conjunction is likewise true (since the statements are compatible, it is legitimate to conjoin them); but the *distributed* form of this conjunction is a contradiction! Something which is a contradiction in classical logic is true in the quantum mechanical universe, namely, the above conjunction in its undistributed form.

The interpretation of quantum logic which I now favor is a "verificationist" one but not an "operationist" one.[6] The notion of *correctness* (idealized verification) in the logic on this interpretation is this: a statement is correct if it is verified by a measurement in the sense explained above. (Idealizing so that we can make perfect measurements and so on.) More precisely: A statement is verified by performing a measurement in which one of the determinate states lies in the subspace corresponding to the statement, under the canonical correspondence between propositions and subspaces of the Hilbert space provided by the von Neumann theory and observing a value of the registering observable that lies in the interval corresponding to that same determinate state. Notice how this differs from the "verificationism" associated with *Intuitionist* logic. In that "verificationism", the correctness of a compound statement is a function of the correctness of its parts in the case of disjunctions and conjunctions (though not of implications); in the "verificationist" interpretation of quantum logic, we specify a test condition for a disjunction as a whole in such a way that a disjunction may be correct even though neither disjunct is correct.[7] (For the relevant observer at the relevant time.)

Two statements are *incompatible* in quantum logic if no state is a determinate state which lies in the subspace corresponding to each or, more simply, if these two subspaces are disjoint. As pointed out above this means that no one measurement can verify both statements *under the same analysis of the measurement* (choice of a registering observable and of a correlation between values of the registering observable and values of the observable to be measured). Thus, in the case of conjunctions, there is a difference between verifying *each* conjunct and verifying the conjunction, which explains the fact that one can sometimes know incompatible propositions even though one has not "verified" their conjunction in the sense just explained. As mentioned previously, the decision not to conjoin statements which are incompatible is a way of making the distinction *in the logic itself* between cases in which both of the statements we know have predictive value and cases in which only one of the statements has predictive value after the measurement.

On this interpretation, quantum logic gives rise to an interpretation of quantum mechanics which resembles one version of the highly ambiguous "Copenhagen Interpretation".

Michael Gardner[8] has argued that one should not bother with quantum logic, or, indeed, with any of the other proposed interpretations of quantum mechanics, but one should just stick to what he calls "the minimal statistical interpretation". That amounts to saying, quantum mechanics just gives one the results of various measurements. It says nothing about how observables behave when we don't measure.

The interpretation of quantum logic and of quantum mechanics itself that I am proposing differs from this "minimal statistical interpretation", and from the Copenhagen Interpretation (if that can be distinguished from the minimal statistical interpretation) in several ways. First of all, I now believe that the only notion of truth that makes coherent sense is the so-called "non-realist" view that sees truth as an idealization of rational acceptability, rather than as "correspondence to reality", where correspondence is thought of as a *non-epistemic* relation (which is why whether a statement could be justified and whether it is true are regarded as *independent* questions by metaphysical realists). On a "non-realist" view, it is not unnatural, I think, to regard it as a deeply important question whether the verification of one statement never in principle precludes the verification of another (as was believed in Newtonian physics), or whether, on the other hand, the world is such that to verify one statement sometimes makes it impossible in principle to perform the experiment that would verify or falsify another (or makes it impossible to perform such an experiment without bringing it about that one or the other statement ceases to have any predictive import).

Someone who feels that truth should be linked to *verifiability* (or at least to idealized verifiability), might well be led on *a priori* grounds to consider quantum logic once he realized propositions might be "incompatible" in the sense just described. I don't mean that this is the only way in which one can be led to consider or even accept quantum logic; and it is empirical that there is such a relation of incompatibility in our world. But this illustrates the fact that even if we decide to accept quantum logic, we might be led to do so partly for *a priori* reasons.

In contrast, advocates of the Copenhagen Interpretation have always insisted upon classical logic. (I have seen a transcript of a discussion between von Neumann and Bohr in which Bohr said, "But the whole point of the Copenhagen Interpretation was to avoid changing the logic.") This is what makes the Copenhagen Interpretation so peculiar; on the one hand, the whole thrust is "don't talk about unmeasured phenomena"; on the other hand, the Copenhagen Interpretation *requires* a distinction between measured values and unmeasured phenomena, because classical logic is retained. This means that the Copenhagen theorist *has* to talk about unmeasured phenomena, if only to say, "we can't conceive them with our classical minds."

Secondly, the minimal statistical interpretation, and likewise the Copenhagen Interpretation, take measurements to yield information only about *present* values. Since they do not envisage or allow our "quantum mechanical time-of-flight" argument, they cannot say when the photon hits the screen that it must have succeeded in getting through the opening when the shutter was open at t_0. But then it seems that many inferences that physicists do make (e.g., in determining *velocities* of particles from the locations of successive *collisions* in a cloud chamber) cannot be accounted for at all. By broadening the notion of "measurement", I hope to have allowed and justified the practice of retrodicting past values from later data. This also allows us to regard observables we measure as having had values just *prior* to the measurement, and to say that the measurement *discovered* the value instead of *creating* it; although, as remarked above, the "quantum jump" reappears as a relativity of what is "real" in this sense to the particular observer or frame.

Thirdly, whereas the Copenhagen Interpretation and the minimal statistical interpretation seem to assume an operationalist view (according to which one doesn't need quantum mechanics, but only the so-called "pre-theories", to say what a measurement is), the interpretation I propose *loops back*: quantum mechanics is interpreted with the aid of the notion of measurement, and if one asks "what is a measurement?" the answer is given by quantum mechanics. One has to understand the theory in terms of verification, and one has to understand verification in terms of the theory.

The metaphysical realist interpretation of quantum logic, which I would no longer defend because I would no longer defend metaphysical realism, says, "No, this isn't a theory about measurement. This is a theory about what is true and false." I don't know how to account for the new relation of incompatibility on such a view, but I'm sure some metaphysical realist could find a way. "You just live in an Escher world," the metaphysical realist might say. "A world in which, because we have Boolean minds – i.e., the representations in our minds or on paper are themselves a commuting family of observables – we cannot possibly know how all the partial truths which Boolean minds can know can possibly fit together."

Although the interpretation I have proposed is not realistic in the sense of assuming a copy theory of truth (metaphysical realism), or even in the sense of assuming that all observables have determinate values, it is *internally* realistic in the sense that *within the interpretation* no distinction appears between "measured values" and "unmeasured values". Even if one interprets the logic in a verificationist way, that does not mean that one takes the theory to be *about* measurements. What discourse is about is a different question from whether the concept of truth associated with that discourse is realist or non-realist. I think that I am here in agreement with the theses of Professor Süssmann, especially with theses (6) and (8) which affirm that the micro-entities spoken of in quantum mechanics are as "real" as any entities knowable by us ("Quantum mechanics is the universal ontology"), while rejecting "Einstein's idea of the detached observer". There are real entities; *but which they are is relative to the observer.*

If one does not wish to stick one's neck out on such difficult and paradoxical questions, however, then it seems to me that the safest position is *not* the "minimal statistical interpretation", but one I might call the *minimal quantum logical interpretation.* This would be to accept quantum mechanics as reconstructed within quantum logic at face value while pushing the whole dispute about "metaphysical realism" back to *philosophy.* It is possible to have either a realist or a non-realist conception of truth in classical physics too; if either conception is untenable, it is for philosophical, not physical, reasons.

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NOTES

1 von Neumann, 1931.
2 Margenau and Park, 1968.
3 See also his article with G. Birkhoff (von Neumann and Birkhoff, 1936).
4 Bub, 1974.
5 Reprinted under this title in Putnam 1975. (Original title 'Is Logic Empirical'; first published in 1968 in *Boston Studies in the Philosophy of Science*, vol. 5, ed. R. Cohen and M. Wartofsky, D. Reidel, Dordrecht.
6 The use of the word "verificationism" here is in opposition to the use I made of this word in Putnam 1975 and its companion volume where it was virtually synonymous with *operationism.* Since the appearance of those volumes, Michael Dummett has convinced me that one may hold the theory that truth is (an idealization of) justification without being committed to the view that statements about sense data are more basic than statements about material objects, and without being committed to reductionism of any kind. Indeed, as Dummett points out, reductionists only renounce the correspondence theory of truth for the statements

they want to *reduce*; for statements in the *reducing* class they typically retain the views that (1) truth and justification are independent; (2) that truth is determinate and bivalent; (3) that there is, in the ideal limit anyway, just *one* true and complete description. In short, *reductionism is a form of subjective idealism* (when the reducing class is the class of sense datum statements); whereas the "verificationism" or "non-realism" espoused by Dummett and myself does not deny the reality of any of the objects of scientific or ordinary discourse, or construe some of these objects as constructions out of others, but consists rather in a renunciation of these three assumptions about truth itself. If there is a species of idealism here, it is a "transcendental" idealism and not a subjective idealism.

[7] This comes about because a state may be such that all possible values of a magnitude lie in either the interval D_1 or the interval D_2 although they do not all lie in D_1 or all lie in D_2. This happens whenever the vector representing the state lies in the span of the subspaces of the Hilbert space representing the statements "the value of M lies in D_1" and "the value of M lies in D_2". It is because the span of two subspaces is not their set-theoretic union that there can be states in which a disjunction is "correct" but neither disjunct is "correct".

[8] Cf. Gardner, 1971. On pp. 523–525 Gardner pointed out that the resolution or the paradoxes I offered in 'The Logic of Quantum Mechanics' doesn't work precisely for the reason that the statement that a particle has a definite position at t_0 is incompatible with the statement that it has a definite position at t_1 ("for any non-trivial source"). The present interpretation overcomes Gardner's objection by allowing one to *know* both of these statements without *conjoining* them. The two-slit paradox is now explained by the non-classical character of the conditional probability, which in turn is connected with the logic (Putnam and Friedman, 1978).

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