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The triangles in Plato's *Timaeus*

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Abstract

In the *Timaeus,* Plato associates the four *elements,* fire, air, water, and earth, with the regular pyramid, octahedron, icosahedron, and cube, and accounts for interchanges among the first three elements in terms of rearrangements of the faces of the corresponding solids. In 1937 Cornford argued that Plato's theory was based on the idea that the faces of the regular solids are composed from minimum atomic triangles. In this paper I argue that the text of the *Timaeus* does not sustain Cornford's interpretation and that Aristotle's discussions of the *Timaeus* strongly suggest that he knew nothing of the ideas Cornford attributes to Plato.

In this paper I want to discuss what I take to be the standard modern interpretation of the geometric theory of simple bodies in Plato's *Timaeus,* the interpretation advanced by Cornford [1937]. I shall first describe some of the relevant text of the *Timaeus* and men indicate why I think that Cornford's reasons are inadequate to justify his interpretation. I shall then argue that Aristotle's treatment of the *Timaeus* theory further undermines Cornford's position. I begin with some general discussion.

At 47e3 of the *Timaeus,* Timaeus announces a new beginning of his discourse on the coming to be of the natural world. He goes on to describe what we might call the provisional principles of his physics,[2] two

1. An earlier version it was read at the Los Angeles meeting of the Pacific Coast Divi- . sion of the *American Philosophical Association* in March, 1994.1 would like to thank my commentator Allan Silverman for his incisive (and skeptical) remarks. I would also like to thank the John Simon Guggenhein Memorial Foundation for a fellowship which provided me with the time to do the research leading to this paper.

2. Provisional because Timaeus explicitly denies that he will describe the ultimate principle or principles:

which he has already described, form and copy or being and becoming, and a third, the receptacle or space. At 53a there is a transition from the discussion of the three provisional principles of physics to the discussion of the geometric principles of simple bodies. Timacus is describing the pre-cosmic shaking of the receptacle:

> [...] At that time the four kinds were shaken by the recipient, which itself was in motion like an instrument for shaking, and it separated the most dissimilar kinds farthest apart from one another, and thrust the most similar closest together, so that these different things came to have different regions even before the universe made from them came to be ordered. Before that they were without proportion or measure. Fire, water, earth, and air showed indeed some traces of themselves, but they were altogether in such a condition as we should expect for anything when no god is present Such being their nature when the ordering of the universe was taken in hand, the god began to give them form using shapes *(etdS)* and numbers.
>
> That the god framed them with the greatest possible beauty and goodness from things which were not of this kind, must be taken above all as a principle we constantly assert. What I must now attempt to explain to you is the ordering and genesis of each of them. The account will be unfamiliar to you, but you share in the paths of education' which my explanations require and so will follow me. (i) In the first place, then, it is, of course, obvious to anyone that fire, earth, air, and water are bodies; but every kind of body has depth, (ii) Depth, moreover, must be bounded by what is plane *(epipedon)*. (iii) But every plane face *(basis)* which is rectilinear is composed of triangles, (iv) But all tiangles take their start *(archetai)* from two, each having one right angle and the other angles acute; of these triangles, one has on either side the half of a right angle, the division of which is determined by equal sides, and the other has unequal parts of a right angle allotted to unequal sides.

> **On this occasion, however, our contribution is to be limited as follows. We are not now to speak of the 'principle' or 'principles' or whatever they are to be called of all things if only on account of the difficulty of explaining what we think by our present method of exposition. You, then, must not demand this of me, nor could I persuade myself that I should be right in taking upon myself so great a task; but holding fast to what I said [at 27d6—29d3] originally about the power of plausible accounts, I will try to speak about all of these things in a way no less plausible than any other, indeed more plausible than any other, starting from the beginning In the same way as before [48c2-d4, translation here, and elsewhere, after Cornford 1937].**

3. μετέχετε των κατ& παίδευσιν οδών, understood by the Neoplatonists to mean that Socrates knows mathematics; sec Proclus, in *Euclid* 20,10-11 and *Platonic Theology* 1.2.11.4-6; and Simplicius, In *De Caelo* 641,26.

> We hypothesize this principle of fire and the other bodies, proceeding in accordance with the plausible account combined with necessity. The principles higher than these are known by god and such men as god loves [53a2-d7],

In this passage we get a quick 'analysis' of the four simple bodies, which proceeds as follows:

(i) The simple bodies are bodies, and bodies are three-dimensional. [53c4-6].

(ii) What is three-dimensional is contained by planes[4] [53c6-7].

(iii) Rectilinear plane surfaces[5] are divisible into triangles [53c7-8; see Figure 1],

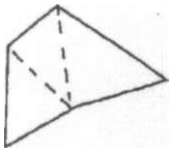


Figure 1

(iv) All triangles are divisible into right triangles, whether isosceles or scalene. These right triangles will be the 'principles' of the simple bodies [53c8-d6; see Figure 2].

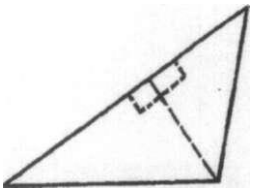


Figure 2

Timaeus says he will hypothesize the right-angled triangles as the *arche* of the simple bodies, but, as he had done before at 48c-d, he announces that

4. Cornford translates *epipedon* surface on the grounds that some solids have curved surfaces. I have translated *epipedon* 'plane' because that is the way it Is traslated in Aristotle's discussions of this part of die *Timaeus,* and I am going to argue that he means the bounding planes (faces) of the regular solids which Timaeus is going to describe.

5, Timaeus' use of the word *basis* shows that he is looking forward to the discussion of the simple bodies in terms of regular polyhedra and their races.

he is not going to talk about ultimate principles, ones known to god and anyone who might qualify as god's friend. I resist the temptation to discuss these higher principles, and turn to Timaeus' further development of his geometric account. Timaeus first says that his task is to find the four most perfect bodies 'such that some can come to be from one another by dissolution *(dialusis)',* a demand whose meaning becomes clear only when the task has been done, and the four simple bodies have been assigned to four of the five regular solids which Euclid constructs in the last book of the *Elements.*[6] The four solids are illustrated in Figure 3.

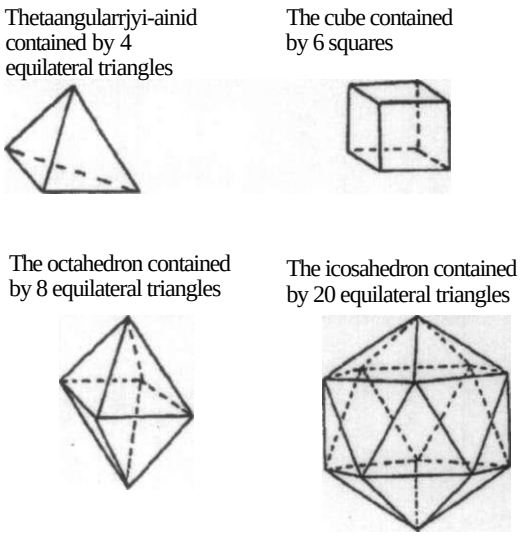


Figure 3

Timaeus gives a rough-and-ready construction of the first three of these by arranging their faces together at 54e3-55b3, and then a somewhat more complex construction of the fourth. At 55d6-56b6 Timaeus makes the following assignments of solids to simple bodies:

6.' Euclid also proves that these are the only five regular solids (XIII.17). In this paper I do not discuss Timaeus' curious remark at 55c4-6 about the dodecahedron or the curious discussion of the number of worlds which follows it. (The material in book XIII is generally thought to be based on work of Thcactetus).

triangular pyramid: fire,
octahedron: air,
icosahedron: water,
cube: earth.

I shall not discuss these correlations in any detail, since they seem to me to be a matter of rather arbitrary analogy. The cube is assigned to earth on grounds of stability (55d8-56a1). The remaining three are assigned on the assumption that fewer faces are correlated with more mobility, sharpness, lightness. As we shall see in the last part of this paper, the notion of lightness is of some significance since Aristotle focuses on it in some of his discussions of the *Timaeus.*

We now come to the most difficult parts of Timaeus' account of the simple bodies, for which we must return to 54a1, where Timaeus selects as the best of the scalene right triangles the half-equilateral. After saying that it would take too long to explain this choice, he concludes:

> So much, then, for the choice of the two triangles of which the bodies of fire and the rest have been fashioned, one isosceles, the other always having the greater side triple the lesser in square' [54b2-5; see Figure 4].

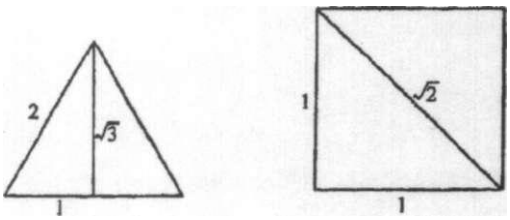


Figure 4

Timaeus immediately introduces an important clarification concerning the intertransformability of the four simple bodies:

> **It appeared that all four kinds could have genesis through one another and into one another. But this appearance is incorrect, since**

7. Or, **as we would put it, the right triangle with sides 1,2, and** Vf, **Here after I shall refer to these two triangles as the rudimentary right triangles,**

> four kinds come to be from the triangles we have chosen, and whereas three of them are constructed from the triangle with unequal sides, the fourth only is constructed from the isosceles triangle [54b6-c3].

When Timaeus procceeds to connect the constructions with the interchangeability of the simple bodies, we get a limited glimpse of his account of dissolution. In lact, according to Timaeus, the cube (earth) with its square faces will not transform into the other three regular solids (water, air, fire) with their equilateral triangular faces, although these three will transform into one another.' However, at this point the solids have not even been constructed, and so we get a very general description of what is at stake:

> Hence, it is not possible for all of them to dissolve into one another and become a few large things from many small ones and vice versa, but three of them can; for these are all composed of one triangle, and when the larger things are dissolved *(luthentdn),* many small ones will be formed from the same things and will receive the figures which are appropriate for them, and again when many small things are dispersed with respect to the triangles, they will become one in number and produce a single new form belonging to a sing'e body [54c3-**d2].**

This description is hardly transparent, since we do not know at wis point what Timaeus means by small and large things. However, Timaeus turns immediately to the construction of the solids and refers to what turns out to be the triangular pyramid as the smallest figure (55d5-6), and later he speaks of assigning the smallest of triangle-feced solids to fire, the largest to water, and the middle one to air [56a3-5], Thus, it is clear that in the present passage Timaeus is referring to the dissolution of water or air into air or fire and of fire or air into air or water.

We learn more starting at 56c8, alter Timaeus has described the four solids and correlated them with the simple bodies. He begins by describing the non-transformability of earth:

> Earth, when it meets with fire and is dissolved by its sharpness, would move around —whether when dissolved it is in a mass of air or of water—until its parts somewhere encounter one another, are

8. Aristotle *(De Caelo* III.7.306al-18) takes the separation of earth form the cycle of transformations as a clear example of preferring theory to 'facts'. I agree with him, but do not discuss the Issue further.

> fitted together again, and become earth; for they can never pass into another kind [56d 1-6].

That the parts in question are not segments of the cube but planes becomes clear from the more mathematical descriptions of transformations which follow. For the changes Timaeus describes can only be understood as simple matters of the rearrangement of the triangular faces:

- 1 water particle (20 triangular faces) => 1 fire particle (4) + 2 air particles (8+8) [56d6-56el]

1 air particle (8) => 2 fire particles (4+4) [56eI-2]

2 fire particles (4+4) => 1 air particle (8) [56c6]

2.5 air particles (8+8+4) =* **I** water particle (20) [56e6-7]

Changes are brought about by different kinds of particles coming together; changes cease when particles of one kind are together [56e7-57c6].[9]

One can see that these changes only require the breaking down of the triangle-faced solids into their faces and the reassembly of those faces. Similarly the non-transformability of earth could be explained by reference to the fact that its faces are square, while those of the other simple bodies are equilateral triangles. In other words, one might suppose that Plato believed in atomic faces (triangles and squares) or —to use Aristotle's term—- atomic planes. One could say the same about the earlier general description [56c3-d2] of the transformation of the three transformable simple bodies, but in that context Timaeus has just mentioned the two rudimentary triangles, and he goes on immediately to construct the equilateral face of the three triangle-faced solids from rudimentary triangles and shortly later does the same for the square face of the cube.[10] The constructions of the two kinds of faces are puzzling. For, as Figure 4 shows, the two faces

9. At 57h3, Timaeus says that air comes from fire and water from air; *CPA9b7-eS* (where, however, earth is included in the apparent changes). It appears that all the trasformations go up or down this path, that is, changes always take place as follows:

fire <-+ air «-> water

10.1 mention also that at 54d he calls the half-equilateral the element of the triangular pyramid and at 55a8-bl he mentions the number of half-equllaterals in the icosahedron and calls them its elements.

can be composed of pairs of rudimentary triangles, but Timaeus' actual constructions involve four triangles in one case and six in the other; see Figure 5. Timaeus says nothing to indicate why this construction is appropriate.

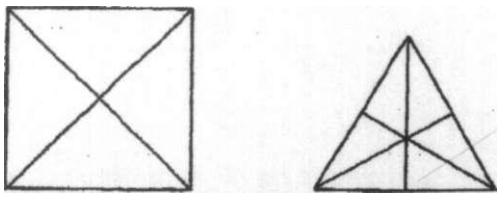


Figura5

Cornford offers a resolution of this difficulty based on a reading of the passage which brings an end to Timaeus' geometrization of nature:

> **All** the uncompounded and primary bodies came to be for reasons of this kind. The reason why there are several varieties within their kinds lies **in** the construction ***(sustasis)*** of each of the elements; the construction **in** each case originally produced its triangle not of one size only, but some smaller, some larger, the number of these differences being the same as that of the varieties **in** the kinds [57c7-d3].

Prior to Cornford, this passage was always taken to mean that the two rudimentary triangles come in different sizes; so the squares and equilateral triangles do; so the four regular solids do; so there is a variety of forms of the simple bodies. Cornford dislikes this interpretation because it means that Timaeus cannot allow for interchanges between bodies with faces of different sizes, *e.g.*, different varieties of air or earth. Clearly Cornford assumes that the Plato who saw no need to justify the claim that the 'apparent' transformation of earth into other things is illusory, would be worried about providing a mechanism for explaining other 'apparent' transformations. Cornford proposes that Timaeus really believes in *atomic* rudimentary triangles and mat he builds up larger rudimentary triangles of different sizes out of the atomic ones, and then triangular and square faces of different sizes out of the rudimentary triangles, and the regular solids out of the faces (see Figure 6).

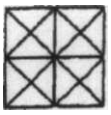


Figure 6

On Cornford's interpretation, interchanges among the varieties of the four simple bodies may involve not just breakdown into faces but also a breakdown into rudimentary triangles, and perhaps atomic rudimentary triangles. Cornford offered the following paraphrase of 56c8-d3:

> The reason why there are several varieties within their kinds lies in the way in which the elementary triangles *[i.e.,* the atomic rudimentary ones] of each of the two sorts are put together [/.«., assembled to form non-atomic rudimentary triangles]: in each case the elementary [atomic] triangles were put together in such a way as to produce the corresponding [non-atomic rudimentary] triangle not of one size only, but some smaller, some larger."

Cornford offers one 'philological' argument, based on the word *sustasis,* for his interpretation. I discuss and reject the argument in Appendix I," I am inclined to think in any case that if Cornford's paraphrase has a justification, it is to be found not in the Greek text, but in whatever interpretative advantages his construal of the *Timaeus* theory offers. Cornford spends some time arguing for the theoretical preferability of taking the rudimentary triangles rather than the equilateral triangle and the square as atomic." Pohle [1971] has shown that the alleged theoretical advantages of the Cornford interpretation are factitious: one can have just as satisfactory a theory of the interchange of bodies —if that is what one is seeking— by starting from atomic squares and equilateral triangles as one gets by starting from atomic rudimentary

11. Cornford [1937], 231-232 (material in braokets supplied by Ian Mueller).
12. In Appendix 2,1 discuss another passage from dw *Timaeus* [58d4-59a8] into which Cornford reads transformations among kinds of a single simple body.
13. For other argumentation of this kind see Bruins 1951.

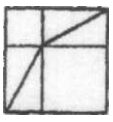
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triangles. If one is committed to the idea that Plato had an atomic physics which allowed transformations among various species of the simple bodies, the major reason one might incline to Cornford's [1937, 234] view is his explanation of Plato's construction of the equilateral and square faces from the rudimentary triangles:

> Why use 4 half-squares to construct a square when 2 would suffice? Why 6 hatf-equilaterals, when 2 would suffice? —a fact which Plato himself mentioned where the scalene element was originally described as 'such that a pair of them compose the equilateral triangle' (54a)? Evidently he was aware that there were at least two ways of composing an equilateral out of this element. The seemingly arbitrary procedure can be explained by supposing that, in the earlier construction of the four solids, Plato intended to describe solids of an intermediate size [...]. *[i.e.,* not the one with faces constructed out of two atomic rudimentary triangles]. He deliberately used more elementary triangles than would have been required, if he had had only one grade of solid in mind [...]. He chose to describe solids of a larger grade because he wanted to suggest that there are in fact several grades, and that when these larger solids are broken into elements, those elements can be recombined in several ways.

Clearly Cornford's argument here rests on the assumption that Plato's goal in the *Timaeus* is to provide a maximally plausible physical theory (within the constraints of some wildly implausible hypotheses). Given those assumptions I am inclined to think that Cornford's reconstruction is the best possible.[14] However, I am also inclined to think that Plato was not concerned about transformations among the species of the simple bodies, and that he envisaged no breakdown of the elementary solids beyond their faces, *i.e.,* the breakdown he explicitly describes. If an explanation of the 'seemingly arbitrary procedure' of constructing the faces be needed, men I would

14. Ahlvers [19S2,54 *ff.]* has pointed out that, If one allows breakdown of faces Into rudimentary triangles, It would be possible to allow interchanges between earth and the other simple bodies. The following figure (simpler than Ahlvers') shows a square divided into two squares and four half-equilaterals:



invoke the symmetry of the constructions, the fact that they construct the 'center' of the two kinds of faces *[Cp.* Taylor 1928, 373-374].

I hope I have said enough to undermine confidence in Cornford's interpretation of the triangles. I wish now to look at relevant parts of Aristotle's discussions of the *Timaeus* account of the simple bodies and Plato's 'atomism' in *De Caelo* and *On Generation and Corruption.* I shall argue that insofar as Aristotle gives any indication of what he thinks the *Timaeus* doctrine is —as opposed to what he thinks its implications are— he treats as Platonic atoms the faces of the regular polyhedra and not the rudimentary triangles. I shall also argue that most of Aristotle's references to Platonic triangles are to the triangular faces of the polyhedra of water, air, and fire, and that the one stretch of text in which Aristotle may refer to the rudimentary triangles *[De Caelo* IV] is not enough to support a contrary view.

Before turning to look at the passages, I give a general description of them:

(i) For the most part in *GC* and *De Caelo* Aristotle treats Platonic atomism alongside the somatic atomism of Democritus and Leucippus, and sometimes even treats them simultaneously.

(ii) In *GC* his principal target is somatic atomism, and except for one passage *[GC* 1.2.315b24-316al4] there are only passing references to Platonic atomism.

(iii) Aristotle's major discussions of Platonic physical theory occur in *De Caelo* III. 1,111,7-8, and IV.

(iiia) In book III of *De Caelo* Aristotle develops his own account of the simple bodies and eliminates alternatives. In **III. 1** he criticizes the *Timaeus* construction of the simple bodies out of planes, primarily on the grounds that planes are weightless, and cannot combine to produce something with weight; the discussion is complicated by Aristotle's introduction of the doctrine that lines are constructed out of points as something which Plato 'ought to hold'.

(iilb) In UI.7-8 Aristotle produces a long sequence of criticisms of the *Timaeus* construction.

(iiic) The topic of *De Caelo* IV is weight or heaviness and lightness. Aristotle's position is that fire and air are absolutely light, earth and water absolutely heavy, and that this difference reflects a difference of matters. He treats the *Timaeus* theory as recognizing

only one matter ('the triangles') and hence capable of explaining only relative heaviness, not absolute heaviness and lightness.

a. On Generation and Corruption

In *GC* 1.2 at 315b24, as Aristotle is beginning his discussion of change, he says:

> The starting point in dealing with all these difficulties is this: 'Do things come-to-be and alter and grow, and undergo the contrary changes because the primary things are indivisible magnitudes? Or is no magnitude indivisible?' For the answer we give to this question makes the greatest difference. And again, if the primary things are magnitudes, are these bodies, as Democritus and Leucippus maintain? Or are they planes, as is asserted in the *Timaeusl [GC* 1.2.3 l5b24-30].

Aristotle's main concern in 1,3 is Democritean somatic atomism, but he makes two criticisms of the dissolution *(dialusis)* of bodies into planes. The first seems to be that there is no point in stopping at planes *[Cp. De Caelo* III.1.299a6-8], the second that, whereas Democritus argued *pkysikos, they* argue *logikds.* Aristotle's illustration of the latter kind of argument is his one mention of triangles in *GC:* 'in me case of there being atomic magnitudes they say that (if there aren't) the *autotrigonon* will be many'. There may, of course, be some connection between the uniqueness of the *autotrigonon* and atomic triangles in particular, as opposed to atomic magnitudes in general, but Aristotle does not make the connection explicit, and I do not myself see how to make a connection between the *autotrigonon* and the *Timaeus.*

If we assume this reference to the *autotrigonon* is irrelevant to the *Timaeus,* all the references in *GC* to the *Timaeus* treatment of the simple bodies describe it in terms of planes, and usually talk about indivisible planes." There are also a number of texts in *De Caelo* which refer to the *Timaeus* or Plato by mentioning planes or indivisible planes.[16] Aristotle does not explain what he means by atomic or indivisible planes, but it seems to me reasonable to assume that he means the surfaces of the four regular solids, which are 'atomic' in Plato's account of change.

15. See 1.8J25b25-27 and 326a21-23,1.9.327a8, and II. 1329a21-24. The first three texts also mention indivisibility. Plato refers to the faces of solids as planes at 53c6 and 7, and55bl,c3, andefi.

16.I1.4.286b27-28, Ill.l.298b33-299a8, UI.7,30Sa35,305b30-3l, 306al, 3 08b35-309al. Only the last of these includes a reference to atomic planes.

b. De Caelo III.1

Aristotle never mentions atomic triangles, but he does sometimes assign other atomic magnitudes to 'those who construct bodies from planes', at least in the sense of saying that belief in such magnitudes follows from views they do express. One very complex passage in which he does this occurs in *De Caelo* III. 1, where Aristotle takes up the question of the existence of generation and destruction. He goes through a number of views and then mentions "those who make every body generable, composing them from planes and dissolving them into planes" [298b33-299a1]. Turning his attention to this position he says that "a glance will reveal many points in which it is contradiction to the findings of mathematics" [299a2-S], and adds that 'it is clear that it belongs to the same theory to compose solids out of planes, planes out of lines, and lines out of points', but the last of these means that a part of a line need not be a line, a view which he has investigated in his discussion of movement *[Physics* VI.I] and shown that there are no indivisible lines. It seems clear that Aristotle did not read a reduction to points (or, equivalently, indivisible lines) into the *Timaeus,* but thought of it as a 'logical' outcome of what does go on there.[17] Nevertheless, he goes on to use the reduction to points as a lever in his criticism of the *Timaeus* treatment of the simple bodies. The rhetorical considerations which lead him to do this are reasonably clear. Aristotle wants to use the notion of composition against what he takes to be the *Timaeus* account of weight found at 55c8 ff, where, having assigned the cube to earth, Timaeus says;

> Accordingly we shall preserve the probability of our ***account,*** if we assign this figure to earth; and of the remainder the ***least mobile*** to water, the most mobile to fire, and the intermediate figure to air. Again, we shall assign the smallest body to fire, the largest to water, and the intermediate to air; and again the sharpest to fire, the next to air, the

17. In this paper I do not discuss *Melaphisics* A.9.992a19-22, a passage which shows that Aristotle did have evidence for saying that Plato believed in Indivisible lines. I assume that this evidence was provided by what Plato said in discussion, not by the dialogues. For an attempt to discount this passage and for arguments for denying that Plato believed in atomic triangles of the standard kind see Drummond 1982.

I have found it best in treating Aristotle's remarks in *De Caelo* 111.1 to speak of indivisible lines, even though Aristotle usually speaks of points and mentions atomic lengths only once (299αI 1). My reason for doing so Is that the phrase 'atomic line' serves as a reminder that Aristotle has in view a theory according to which lines are composed of indivisibles.

> third to water. Now taking all these figures, the one with the fewest faces must be the most mobile, since it is sharpest and cuts best in every direc- · tion and,moreover the lightest as being composed of the smallest number of the same parts; the second must stand second in these respects, the third third [55e8-56b3],

It is clear that all Timaeus means is that in the case of the three regular solids bounded by triangular faces, weight is proportional to the number of bounding faces. Aristotle is presumably justified in assuming both that Plato intends to explain weight by reference to number of faces and that any such explanation is problematic. However, Aristotle wants (i) to fix on the terminology of composition (συνεστός ἑκ), (ii) to use the idea that solid is to plane, as plane to line, as line to point to infer that, on the view expressed in the *Timaeus,* physical bodies will be composed of points, and then (iii) to argue that one cannot explain the relative weight of bodies by reference to the number of composing points, since points do not have weight at all.

After developing his argument that differences of weight cannot be explained in terms of differences in number of composing points. Aristotle turns to the *Timaeus,* and first argues that there is no justification for restricting the combination of planes to the bounding of rectilineal solids:

> Furthermore, it would be strange if planes could only be joined together along a line. For just as a tine can be added to a line in two ways, by length and by breadth, a plan must be addible to a plane in the same way, However, a line can be joined to a line along a line if it is laid along it, but not if it is adjoined [at the end point]. But if it is possible to join a plane to a plane along a breadth, there will be [for Plato] a body which is neither an element nor composed of elements, namely the body composed of planes in this way [299b23-31].

There is a sense in which Aristotle's criticism here is a digression from his main goal of criticizing the *Timaeus* account of weight. But presumably he recognizes that he has no warrant for assigning to Plato the idea that particles of the simple bodies are literally composed of planes, and so he argues that Plato ought to recognize bodies literally composed of planes. Aristotle points out that such bodies would not be simple bodies or compounds of them, but I assume that he also has in mind that such bodies would be related to planes as lines to indivisible lines or points, and so would, in some

sense, make Plato subject to the criticism of theories of weight using the idea of composition from points.

At 299b32 Aristotle turns explicitly to apply his criticism of the atomic line theory of weight to what he takes to be the *Timaeus* theory:

> Furthermore, if one body is heavier than another because of the number of their planes, as is said in the ***Timaeus***, it is clear that the line and the point will have weight, since they are related to one another in the same way, as we said before [at 299a6-8]. But if bodies do not differ in this way, but because ***[e.g.]*** earth is heavy and fire light, then there will also be light and heavy planes, and similarly in the case of lines and points —for the earth plane will be heavier than that of fire [299b32-300*7].

It is reasonably clear that Aristotle's criticism succeeds only by blurring the bounding relation and the composition relation. There is no reason for Timaeus to ascribe weight to the bounding planes, just because their number determines the relative weight of the bounded solids. And if weight is taken as a primitive property of solids, there is no reason to ascribe weight to planes unless one thinks of the solids as literally composed of planes. Aristotle concludes his discussion" with general criticism which depends only on the idea of resolution and the assumption that solid is to plane as plane to line as line to point:

> In general it follows [for them] that either there is no magnitude ***[i.e.,*** nothing with a positive number of dimensions] at all or it can be eliminated entirely, if point is to line, as line to plane, and as plane to body. For since all resolve into one another, they will be resolved into what is primary; consequently, there might be nothing but points, and no body [300a7-12].

Obviously nothing Aristotle says in *De Caelo* II1.1 presupposes the rudimentary triangles of the *Timaeus* —indeed, there is no mention of triangles in 1II.1—, and nothing depends on the claim that there are any atomic magnitudes in the *Timaeus* itself: Aristotle introduces atomic lines as something Timaeus ought to be committed to.

!. Aristotle goes on to introduce two separate but related points, one about time and the now, the other about the Pythagorean composition of the heaven from numbers.

c. *De Caelo* ΠΙ.7-8

In this criticisms of the *Timaeus* III.7-8 Aristotle mentions atomic magnitudes in his fifth [306a26-3O6bl] and eleventh [307al9-24] objections;[19] in both cases the magnitudes in question are solids. The fifth is again a case of Aristotle arguing about what Plato is, in fact, committed to, whether or not he expresses such a commitment. We get an adequate paraphrase of Aristotle's argument by saying that since any part of fire is Fire, but not all parts of a pyramid are pyramids, either the pyramid has no parts, *i.e.,* is indivisible, or there are solids prior to the pyramid which do not correspond to any of the simple bodies. Aristotle apparently takes for granted that the second alternative is unacceptable to Plato.[10]

Objection 11 does nothing to clarify what Aristotle has in mind when he talks about atomic magnitudes, and seems to be directed as much at the somatic atomists as at Plato:

> At the same time it will follow that mathematical bodies bum and warm. For they possess angles and contain atomic spheres and pyramids —especially if there are atomic magnitudes, as they say. But if some magnitudes act and some do not, one ought to say what the difference is, but one should not talk without making distinctions in the way they do [307al9-24]. -

The only relevant mention of triangles in III.7-8 and the first relevant reference to them in *De Caelo* comes in objection 3 [306a20-23]. However, It is clear that here Aristotle means the triangular faces of the transformable solids. For he has just mentioned that earth is not dissolvable into another body, and continues:

> In the case of those which do dissolve the suspension[11] of the triangles is not reasonable. But this happens when they change into one another because they are composed of unequal numbers triangles.

19. I use Simplicius' numbering of the abjections [in *De Caelo* 642.3-670.25]. I discuss Simplicius' treatment of diem in a sequel to this paper. "Plato's mathematical chemistry and its exegesis in antiquity".

20. I believe Aristotle is right here: Plato could not admit that segments of a pyramid were fire, and It would not be easy for him to deny that the segments —if they exist— are prior to the pyramid.

21. *Paraior&sh,* the existence of plane figures in separation from the bodies to which they at times belong.

> In the case of those which do dissolve the suspension[23] of the triangles is not reasonable. But this happens when they change into one another because they are composed of unequal numbers of triangles.

d. De Caelo IV

Triangles play a larger role in book IV of Gael., where Aristotle discusses weight. In chapter 1 Aristotle complains that his predecessors have only spoken about relative weight, and sets out his own doctrine of absolute heaviness and lightness: heavy things move naturally toward the center of the cosmos, light things away from the center. Although Aristotle does not say so here, he will take for granted that the difference between a heavy and a light thing should be ascribed to a difference in matter. He will also invoke the following "phenomena":

> 1. fire and air are light, water and earth heavy; fire is lighter than air, earth heavier than water;
>
> 2. more of a light stuff is lighter than less of it and moves more quickly away from the center of the cosmos.

In chapter 2 Aristotle turns to criticizing the theories of weight of the Timaeus and of the somatic atomists. He begins with a brief explanation of what he takes to be the Timaeus theory:

> Some explain being lighter or heavier in the way in which it is written in the Timaeus: what is heavier is composed of more of the same things, what is lighter of fewer, just as more lead is heavier than less and more bronze than less. And similarly in the case of any other homogeneous thing, that one is heavier than the other is due to an excess of equal parts. (308b3-8)

Aristotle's account here is perhaps reasonable enough, although it is not clear that Plato would accept the analogy between more faces and more stuff. However, Aristotle goes completely beyond what is said in the Timaeus when he offers an account of difference in weight for things of different kinds:

> And they say that lead is heavier than wood in the same way; for all bodies are composed of certain identical things and of one matter, although it does not look that way. (308b8-12)

Here we get an reference to "certain identical things" and "one matter" of which everything is composed. That these things or matter are triangles becomes clear when, after complaining that Plato's account speaks only about relative heaviness or lightness, Aristotle offers a criticism:

> Consequently, fire does not move upward because of its small number of triangles of which they say each thing is composed; if this were the reason a greater quantity would move less and be heavier, since it would be composed of more triangles. But the opposite is what we see happening. (308bl5-18)

It is possible that Aristotle is here thinking of the rudimentary right triangles. However, he is simply mistaken when he implies that fire and earth are composed of the same triangles; and as we have seen, Plato does not even attempt to explain the weight of earth relative to the other simple bodies, but only the relative weight of the three intertransformable bodies. Whether or not Aristotle is thinking of the rudimentary triangles, it is clear that his major concern is to saddle Plato with a general theory of relative weight based on Timaeus' remark about the relative weight of water, air, and fire.[24] And it seems to me that Aristotle may not even be thinking about the rudimentary triangles at all. He may be conveniently forgetting that earth particles have square faces,

> and be heavier, since it would be composed of more triangles. But the opposite is what we see happening [308b 15-18].

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> In addition since they say that a homogeneous thing which has fewer [of the same pans] is lighter and that which has more is heavier, and that air and water and fire are composed of the same triangles but they differ only in the fewness and number of them (and this is why one is heavier, the other lighter),

22. It is clear that Simplicius recognizes Aristotle's apparent mistakes, although the relevant passages are unfortunately corrupt:

> **One should realize that Aristotle Is following his own hypotheses when he brings in the idea that fire and air arc absolutely light, water heavy; for having distinguished natural up and down in the universe, he distinguishes light and heavy by motion in these directions. But Plato does not distinguish light and heavy by the number of planes; for if he did water would be heavier than earth, since the icosahedron is composed of twenty equilateral triangles, f each of which is composed of four half-triangles t. (683.32-683.4. Επιστησαι δε χρή δη το πῦρ κα\ τῦν he'pet απλώς κοΟφα to fie 'όδωρ βαρι! τοις έαυτοϋ υποθεσεσιν επόμενος συνήγαγεν *6* Αριστοτέλης το γαρ &κω ma το κάτω κατα φυσιν διορίσας εν τφ παντΐ τη έπ> ταύτα κινήσει τδ κοϋφον τοϋ βαρέος διέστησεν. & 6έ Πλάτων ού τω πλήθε κοή τ(_6λιγοτητι των επιπέδων ούτω γαρ το 'ύδωρ βαρύτερον ήν τής γής εΐπερ. το. μεν εικοσάεδρον εξ ισοπλεύρων τριγώνων είκοσι συνεστηκεν ών εκασσαρα συνέστησεν ήμιτρίγωνα t)**

And at the end of his remark on this passage Simplicius says something about Aristotle's words 'since they say that a homogeneous thing which has fewer [of the same parts] is lighter'. After a lacuna the Simplicius passage ends with the words "since all things are not composed of homogeneous triangles, but only three, fire, air, water, he therefore makes his comparisons principally in terms of these three" [384.14-16. έπεί μή πάντα εξ ομοειδών τριγώνων σύγκειται αλλα τά τρίο μόνον πῦρ **αήρ \ίδωρ διδκοΛ εντούτοις μάλιστα την σύγκρισιν εποιήσατο].**

> there will be some quantity of air which is heavier than water; but what happens is just the opposite [308b21-27].

After this criticism Aristotle, turns his attention to the atomists, referring once to Plato as "those who say that bodies having weight are composed of primary and atomic planes" [308b35-309al], But towards the end of the chapter he speaks of those who postulate a single matter and compose things out of triangles [309b33-34]. Aristotle again speaks of triangles in IV.5 in summarizing the discussion which has proceeded and emphasizing the need to assume different kinds of matter to explain absolute lightness and heaviness. He first says: "For if mere is one matter for all things, such as the void or the full or magnitude or the triangles, everything will move up or everything will move down" [312b20-23]. Shortly thereafter he says that on the theories he has rejected some of the intermediate bodies will move down more quickly than earth; "for there will be more triangles or solids or particles *{mikra)* in a lot of air" [312b29-30].

De Caelo IV is the stretch of text which most strongly supports the claim that Aristotle assigned a role to the rudimentary right triangles of the *Timaeus,* but if he does he grossly distorts their character to foist a uniform theory of relative weight on Plato, who —it seems to me— went out of his way to avoid such a theory. Moreover, even if Aristotle is thinking about the rudimentary triangles, he connects them only with the question of weight. He never associates them with die question of the transformation of the simple bodies, the issue which is fundamental to the Cornford reading, Thus his testimony seems to confirm what 1 take to be the more straightforward, traditional understanding of the *Timaeus* rudimentary triangles as parts of surfaces which do not themselves break up in the course of transformations, that is to say, surfaces which are, from the point of view of physical theory, atomic.

Appendix 1. The word σύστασις at 57c9

Cornford [1937,231] writes:

> The editors take 'construction' (αύστασνν) in the passive sense, 'structure', which the word often bears elsewhere. They understand the whole statement to mean no more than that each of the two elementary triangles is of more than one size. If that were all, Plato would have wrapped this extremely simple statement in a singularly clumsy form. But the sentence cannot bear that meaning: it is impossible to say that the ***structure***

> of each elementary triangle *produced* the triangle which is the structure in question The word σύσταοις must have its active sense 'putting together'.

In a footnote Cornford adds that 'translators evade this absurdity [of saying that the structure of a triangle produces the triangle] by resorting more or less to paraphrase or by ignoring the fact that ἑκατόραν (σῦστασιν) is the subject of φυτεῦσαι'. Of the pre-Cornford translations I have consulted two,[23] are subject to something like Cornford's charge because they turn Plato's active grammatical construction into a passive one, but three either embrace Cornford's 'impossibility':

> Archer-Hind [1888]: [the structure of the rudimentary triangles] does not originally produce in each kind of triangle one and the same size only,
>
> Jowett [1892]: either structure did not produce the triangle of one size only [...] [retained in the revised edition (post-Cornford)],

or come very close to it Rivaud [1925]:

> chacun [des elements] na pas initialement engendre un triangle unique dc grandeur determinee.

These translations suggest that it is not as impossible, as Cornford claims, to understand Timaeus to be saying that the structure of the rudimentary triangles produced them in different sizes." Bury [1929] perhaps gives σύστασις an active sense, but, in any case, sees no problem in saying that the σύστασις produced the rudimentary triangles:

> Each [...] construction [of the two rudimentary triangles] produced not merely a triangle of one definite size [...]

The whole question whether συστασις has an active or passive sense seems to me quite nebulous. The pre-Cornford Liddell-Scott gave as an 'active' sense ***οἵσύστασις;***

23. Martin [1841] : "ces deux triangles, n'ayant pas toujours chacun la meme grandeur, ont forme [...]", Apelt [1922]: "[...] ward gleich zu Anfang das Dreicck nieht nur in einer Grossehervorgebracht..."

24, *Cp.* **assertions isuch as 'the form of a thing makes it what It is', where the form of η thing is understood as what the thing Is.**

A.I.1 *a putting together, composition, organization,* of the parts of an animal: Aristotle PA 646a20, *GA* 744b28, sq., al,; **ἡ** σ. τοῦ σώματος ἡ δια γυμνασίων Plato *Timaeus* 89A.

and among 'passive' senses:

B.II.1 *the composition, construction, structure, constitution* of a person or a thing [references omitted here],

BJI.2 *existence, origin* νόσων Plato *Timaeus* 89B, cf. C; **ἡ** ἐξ **αρχής** τών όλων σ. Diod. 1.7, *cf.* Plutarch 2.427A, B; σ. λαμβάνειν Arlst, *HA* 547bl4, Polyb..,etc.; of ariver, *Id.* 9,43.1,

and:

B.II.4 of bodies, *a becoming compact or solid,* **ἡ** τοῦ ὑγροῦ περί τήν ὑπερώην Hipp. 157 D, *cf.* Plutarch 2. 130C; ἑψειν μέχρι συστάσεως Galen.

In the explication of the 'active' sense Cornford cites several passages in the *Timaeus* in which the verb συνιστάναι is used for combining triangles, and then refers to the 89b occurrence of σύστασις cited in LS under me active A.I.1:

At 89A, συστασις means the 'bracing', 'pulling together' of the body by gymnastic exercise. Taylor [1928, 626] there compares *Laws* 782A πόλεων συστάσεις και φθοράς (ρ, 231).

Here is Taylor on 89 A:

The word [σῦστασις] is used here to denote the process antithetical to κόθρσις. κάθρσις removes the unwholesome and superfluous, συστασις brings firmly and compactly together constituents which ought to be united. The one is a liquification, the other a solidification. Hence L. and S. should have given the reference to this passage under B.ii.4, not under A.1, [...]. An exactly similar phrase is *Laws* vl. 782 a 5 πόλεων συστάσεις καῖ φθοράς 'consolidations and dissolutions of cities', *i.e.* 'the rise and fall of states', and not very dissimilar are σῦστασις νόσων and της περί τα νοσήματα συστάσεως, 'morbid formations' in b5, c4-5 *iitfra* [under B.Π.2 in LS]. Translate [the phrase in 89a]: Of all ways of purging and reforming the body*.

In the revised LSJ A.I.1 has been replaced by:

A.I.1 * *bringing together, introduction, recommendation,* under which

none of the LS references are included, the two Aristotle references being added to a new:

B.Π.1* *the composition, structure, constitution* of a person or a thing.

LSJ drop B.II.4 and its references, or replace it with

B.II.3* of bodies, *density* or *consistency*.

B.II.2 is replaced by

B,II.2* *coming into existence, formation*.

The references are pretty much the same except for the addition of *Laws* 782a, πόλεων συστάσεις και φθοράς.

I do not intend to try to sort out all these alleged senses of σύστασις. I take it to be clear that Cornford has to assume that if σύστασις has an 'active' sense it must mean something like 'put together by assembling'. I do not know of a *clear* case in the context of physical science where σύστασις (as opposed to συνίστημι or συνιστάνω) has the sense of assembling parts, but in any case I see no reason to suppose it has such a sence in the present passage." For there seems to me nothing impossible or even difficult about an interpretation which takes συστασις in the sense B.II.2*, 'coming into existence', 'formation', and takes Timaeus to be saying that the formation of the rudimentary triangles produced triangles of different sizes and is responsible for the different sizes.

Appendix 2. 58d4 — 59a8

I want to say a word about one other passage in the *Timaeus* which Cornford interprets in terms of transformations between different sizes of solids of the same kind. In the passage Timaeus distingishes between ordinary liquids and metals, which become fluid when fire melts them. Ordinary liquids are apparently said to flow because they contain small water particles of unequal size, (τό μεν οῦν ύγρόν δια τό μετέχον

25. Even if Cornford were right about the required sense of σόστασις, he has further to assume that it makes no sense for Timaeus to speak of the (atomic) rudimentary triangles being assembled. But it seems quite possible that Timaeus could have In mind an assembly of the triangles out of more ultimate principles, principles which, as I have mentioned, he refuses to discuss.

είναι των γενών τών υοατος δσα σμικρά άνισων δντων. κινητικών αΐιτδ τε καθ" αυτό και ύπ' ϋλλου δια τήν άνωμαλότητα κα\ τήν τοϋ σχήματος ίδέαν γέγονεν),

Metals in their solid state are characterized by large particles of a single size. Melting is said to involve the entrance of fire which breaks the particles up *(dialuein)* and makes the metal lose its uniformity. This description suggests that the loss of uniformity is due to the presence of fire particles among the water particles and the dissolution of some of the latter (and perhaps their recombination as air or fire particles). However, Timaeus goes on to describe the process of melting as a *kathairesis* of the *ongka,* which Cornford understands as a 'reduction in bulk' of the water particles.

Timaeus next describes the solidification of liquid metal as a matter of the departure of fire and the pressing together by the air of the liquid *ongkos* into the spaces abandoned by the fire, which makes the liquid uniform again. Timaeus characterizes the process opposite to melting, which he calls cooling *(psuxis),* as the departure of fire. This characterization of cooling suggests that the most important and perhaps only issue of uniformity in the case of metals is the presence or absence of fire particles. Cornford's [1937, 250] description of solidification is pure chutzpah:

> The thrust communicated to the air by the fire which is expanding it causes the metal to pass back from the liquid state to the solid. ***This means*** that the small icosahedra reform themselves as large icosahedra of their original grade [...] [my italics].

Cornford [1937, 250] seems to acknowledge the difficulty of his interpretation when he goes on to say:

> Some details of the two processes are not completely explained. Plato would not have vouched for them all; he only means that something of this sort must happen, and the mechanical explanation can at any rate be carried thus far.

The only real support for Cornford's interpretation seems to me to be the phrase *kathairesis* of the *ongka.* I am inclined to think that Cornford's understanding of it depends on his interpretation of Timaeus' chemistry, and that the reading of Taylor and Archer-Hind which makes it pick up the reference to the dissolution of the solidified metal or of its particles is more likely, [Archer-Hind translates 'dissolution of the mass', and Taylor speaks of the splitting up of icosahedra].



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