$(A \supset \forall v B(\ldots v\ldots)) \equiv \forall v(A \supset B(\ldots v\ldots))$
$(A \supset \exists v B(\ldots v\ldots)) \equiv \exists v(A \supset B(\ldots v\ldots))$

However, note very carefully the following:

$(\forall v B(\ldots v\ldots) \supset A) \equiv \exists v(B(\ldots v\ldots) \supset A)$
$(\exists v B(\ldots v\ldots) \supset A) \equiv \forall v(B(\ldots v\ldots) \supset A)$

Extracting a universal quantification from the antecedent of a conditional (when the variable doesn't occur in the consequent) turns it into an existential quantification. This shouldn't be surprising when you recall that antecedents of conditionals are like negated disjuncts – remember $(A \supset B)$ is equivalent to $(\neg A \vee B)$ – and recall too that when quantifiers tangle with negation they 'flip' into the other quantifier. Consider, for example, the following chain of equivalences:

$$(\forall x Fx \supset Fn) \equiv (\neg\forall x Fx \vee Fn) \equiv (\exists x \neg Fx \vee Fn) \equiv \exists x(\neg Fx \vee Fn)$$
$$\equiv \exists x(Fx \supset Fn).$$

But having noted the various equivalences in this section for the record, do be very, very cautious in using them (careless application can lead to trouble)!

24.4 Summary

- Translation into QL can proceed semi-automatically, if (as a half-way house) we transcribe vernacular sentences into an augmented English deploying quantifier operators of the kind 'Every/some/no *A*, *v*, which is *B* is such that', and then use the schemas:

  (Every *A*, *v*, which is *B* is such that)$Cv \mapsto \forall v((Av \wedge Bv) \supset Cv)$
  (Some *A*, *v*, which is *B* is such that)$Cv \mapsto \exists v((Av \wedge Bv) \wedge Cv)$
  (No *A*, *v*, which is *B* is such that)$Cv \mapsto \forall v((Av \wedge Bv) \supset \neg Cv)$
  or $\neg\exists v((Av \wedge Bv) \wedge Cv)$

- The order of neighbouring quantifiers can be swapped if the quantifiers are of the same kind. And quantifiers inside conjunctions and disjunctions can, in certain circumstances, be 'exported' outside the conjunction/disjunction.

Exercises 24

A Suppose 'm' denotes Myfanwy, 'n' denotes Ninian, 'o' denotes Olwen, 'Fx' means *x is a philosopher*, 'Gx' means *x speaks Welsh*, 'Lxy' means *x loves y*, and 'Rxyz' means that *x is a child of y and z*. Take the domain of discourse to consist of human beings. Translate the following into QL:

1. Ninian is loved by Myfanwy and Olwen.
2. Neither Myfanwy nor Ninian love Olwen.
3. Someone is a child of Myfanwy and Ninian.
4. No philosopher loves Olwen.
5. Myfanwy and Ninian love everyone.
6. Some philosophers speak Welsh.
7. No Welsh-speaker who loves Myfanwy is a philosopher.
8. Some philosophers love both Myfanwy and Olwen.

9. Some philosophers love every Welsh speaker.
10. Everyone who loves Ninian is a philosopher who loves Myfanwy.
11. Some philosopher is a child of Olwen and someone or other.
12. Whoever is a child of Myfanwy and Ninian loves them both.
13. Everyone speaks Welsh only if Olwen speaks Welsh.
14. Myfanwy is a child of Olwen and of someone who loves Olwen.
15. Some philosophers love no Welsh speakers.
16. Every philosopher who speaks Welsh loves Olwen.
17. Every Welsh-speaking philosopher loves someone who loves Olwen.
18. If Ninian loves every Welsh speaker, then Ninian loves Myfanwy
19. No Welsh speaker is loved by every philosopher.
20. Every Welsh speaker who loves Ninian loves no one who loves Olwen.
21. Whoever loves Myfanwy, loves a philosopher only if the latter loves Myfanwy too.
22. Anyone whose parents are a philosopher and someone who loves a philosopher is a philosopher too.
23. Only if Ninian loves every Welsh-speaking philosopher does Myfanwy love him.
24. No philosophers love any Welsh-speaker who has no children.

B Take the domain of quantification to be the (the positive whole) numbers, and let 'n' denote the number one, 'Fx' mean *x is odd*, 'Gx' means *x is even*, 'Hx' means *x is prime*, 'Lxy' means *x is greater than y*, 'Rxyz' means that *x is the sum of y and z*. Then translate the following into natural English:

1. $\neg\exists x(Fx \wedge Gx)$
2. $\forall x\forall y\exists z Rzxy$
3. $\forall x\exists y Lyx$
4. $\forall x\forall y((Fx \wedge Ryxn) \supset Gy)$
5. $\forall x\forall y((Gx \wedge Rxyn) \supset Fy)$
6. $\forall x\exists y((Gx \wedge Fy) \wedge Rxyy)$
7. $\forall x\forall y(\exists z(Rzxn \wedge Ryzn) \supset (Gx \supset Gy))$
8. $\forall x\forall y\forall z(((Fx \wedge Fy) \wedge Rzxy) \supset Gz)$
9. $\forall x(Gx \supset \exists y\exists z((Hy \wedge Hz) \wedge Rxyz))$
10. $\forall w\exists x\exists y(((Hx \wedge Hy) \wedge (Lxw \wedge Lyw)) \wedge \exists z(Rzxn \wedge Ryzn))$

C Which of the following pairs are equivalent, and why?

1. $\forall x(Fx \supset Gx)$; $(\forall xFx \supset \forall xGx)$
2. $\exists x(Fx \supset Gx)$; $(\exists xFx \supset \exists xGx)$
3. $\exists x(Fx \supset Gx)$; $(\forall xFx \supset \exists xGx)$
4. $\forall x(Fx \supset Gx)$; $(\exists xFx \supset \forall xGx)$

The claim that, e.g., that a wff of the form $(A \vee \exists xFx)$ is equivalent to one of the form $\exists x(A \vee Fx)$ depends on our stipulation that the domain of quantification isn't empty. Why? Which other equivalences we stated in §24.3 above also depend on that stipulation?