INCOMMENSURABILITY, INCOMPARABILITY, IRRATIONALITY[1]

Kindi, V., (1994) “Incommensurability, Incomparabillity, Irrationality”. *Methodology and Science*, 27, 41-55.

ABSTRACT

Since its introduction in the field of philosophy of science, *incommensurability* has been taken to imply, almost analytically, incomparability and irrationality. If two magnitudes are incommensurable, then, it is claimed, they cannot be assessed comparatively, likened or contrasted, and therefore they are not rationally accountable.

In this paper it is argued that the use of the term *incommensurable* in its original context of ancient Greek mathematics does not have the connotations of incomparability and irrationality. The use of the Greek word *ασύμμετρος* (incommensurable), *άρρητος* (ineffable), *άλογος* (irrational), which are all employed to refer to incommensurable magnitudes are investigated in order to contend that:

1. The lack of a common measure in the case of incommensurable magnitudes does not preclude an overall evaluation at a pre-theoretical level.
2. The contemporary identification of incommensurable and irrational should be attributed to the ambiguity of the Greek word *λόγος* and to the word *ratio* that translated *λόγος* into Latin. In mathematical contexts both *λόγος* and *ratio* have the sense of due relation between two similar magnitudes, whereas, in general and especially after the Enlightenment, one can take them to mean reason.

---

[1] Earlier versions of this paper were read at the Fiftheenth International Wittgestein Symposium, Kirchberg, Austria, August 1992, and the International Symposium on Early Greek Mathematics, Athens, Greece, August 1992.

I would like to thank Professors K. Gavroglu and V. Karasmanis and Dr. J. Christianidis for their comments and criticism.

INTRODUCTION

Kuhn introduced the term ‘incommensurability’ in the field of philosophy of science with his book *The Structure of Scientific Revolutions* (1962), inciting thereof a heated debate. The question was and is whether the presence of the disputed term in Kuhn's account of science infects it automatically with irrationality. That is, whether the affirmation of incommensurability, i.e., the affirmation of irreconcilable differences between consecutive or competing paradigms, renders any account of science arbitrary, and science itself a prey to the logically uncontrollable forces of tradition and personal idiosyncrasies. If paradigms are, as Kuhn claims, the bearers of reason, then, according to his critics, any attempt to transcend them, any attempt to bridge the gaps, to provide an account of their succession, or of any kind of a relation, is suspended over the dark abyss of irrationality. The presupposition that lays behind such reasoning is that incommensurability implies incomparability. Since there is no connecting line between any two paradigms, no common standard by which to judge them, then, the critics argue, the two paradigms cannot be compared.

Kuhn insisted from the very beginning that this is not the case. Incommensurability does not imply incomparability. Incommensurable paradigms can be talked about, translated, understood, juxtaposed, likened, contrasted, at least in the ordinary, pre-theoretical sense of the words. What it cannot be done, is a one to one correspondence, an evaluation based on shared or absolute, universal criteria. If the case is the incommensurability of concepts, then translation, Kuhn says, cannot be done by a manual which provides for every possible occasion or context. Some statements or linguistic expressions that belong to a paradigm may not correspond to anything in another, and therefore they may not be stated and translated to the language of this other paradigm.[2] In these cases, incommensurability, Kuhn maintains, implies ineffability.

In this paper we will not address the issue of incommensurability as such. We will not undertake to assess whether the history of the sciences supports the contention of incommensurability or its implications. Instead we will concentrate on a linguistic investigation of the word which, we expect, will lend support to Kuhn's idea that two things can be incommensurable and yet, at the same time, perfectly and reasonably accountable.[3] If we can establish that incommensurability does not necessarily,

---

2 Kuhn does not of course rule out *ad hoc* approximations. His point is rather that some statements which can have truth value in one community are simply unsayable in another.

3 I refrain from writing “rationally accountable” for fear of appearing to prejudge the issue at this early stage. If our linguistic investigation establishes readily and swiftly that incommensurability does not obstruct rational evaluation, then we run the risk of getting entangled from the start in the different senses of rationality. Kuhn has already suggested that we should change our concept of rationality, that we must rid it of the untenable attributes of absoluteness and unconditional universality. However important this issue is, it will not be discussed here. In this paper, we will not inquire

analytically, i.e., by virtue of its meaning, imply incomparability, then we open the ground for the substantial discussion of the issue of incommensurability in the sciences. The first objective of this paper then, is to argue linguistically that the thesis of incommensurability does not forbid juxtaposition and comparison. The second objective is to hint, linguisticly again, at the explicit or implicit line of reasoning that equates incommensurability and irrationality. It will be pointed out that the ambiguity of the term ferments this identification. Finally, it will also be suggested that it is not necessary to substitute ineffable for incommensurable since this nuance in meaning is already captured by the original word.

Before we proceed, a few words must be said to justify the adequacy of a linguistic investigation in the case of discussing the issue of incommensurability. The first thing to be noted is that the philosophical use of *incommensurability* is professedly owing to the use the term had in ancient Greek mathematics. It is then only natural to explore the senses of the word in that context. Second, admittedly Kuhn could have used an entirely different term (possibly the corresponding one from the language of a far away tribe) to signify the same problem, i.e., the relation of two competing or successive Paradigms. Yet, it is the contention of this paper that the original Greek term and especially the

---

into whether the possibility of rational evaluation, presumably implied by the linguistic interpretation we will attempt, is the one desired by Kuhn or demanded by his critics. So, we will confine ourselves to a linguistic investigation in view of assessing the compatibility of incommensurability and comparability. If this is established, then the compatibility of incommensurability and rationality is not precluded from the outset and remains to be enunciated.

connotations of its synonyms have contributed largely to the staging of the debate over the significance and the implications of the incommensurability thesis.

INCOMMENSURABILITY, INEFFABILITY, COMPARABILITY

Kuhn's view as regards the thesis of incommensurability, i.e., that it implies ineffability but not incomparability, find an unexpected, yet natural advocate in the corpus of ancient Greek mathematics from where Kuhn had originally borrowed the term.

The term *in-com-mensurable* corresponds exactly to the Greek term *α-σύμ-μετρος*. In ancient Greek mathematics those magnitudes which are measured by the same measure are said to be *commensurable (σύμμετρα)* and those that have no common measure are said to be *incommensurable (ασύμμετρα)* (Euclid, *Elements*, Book X, Definition 1). For example, if the side of a square is equal to the unit of length, then the diagonal (using Pythagorean theorem) is $\sqrt{2}$. No matter how much it defies common sense, the side, equal to one unit of length and the diagonal, equal to $\sqrt{2}$, do not have any common measure, any common divisor. Aristotle comments in his *Metaphysics* (Bk. I, Ch. 2, 983a 14ff):

> ... all men begin by wondering... at the incommensurability of the diagonal and side of a square. For it seems astonishing to all who have not yet seen the reason, that something cannot be measured even by the smallest measure. But they must come to the opposite and better conclusion at the end, as the saying has it, that is, only when they learn. For nothing would surprise the geometer more than if the diagonal should suddenly become commensurable.

Evidently, common sense cannot assimilate the concept of incommensurability. Its theoretical proof may reassure the geometer or anybody who is initiated into mathematical reasoning proper, but it cannot put away our empirical preconceptions which continue to puzzle us. In fact, in ancient Greek mathematics, incommensurable magnitudes are also described by words that reflect not the lack of a common measure (the literal meaning) but their perplexing and unintelligible character. Thus they are called *άρρητα* and *άλογα*.

Άρρητος is literally the ineffable, the inexpressible, the unutterable, the unspoken, the unsaid. It is compounded by the privative prefix 'α' and the adjective *ρητός* which is derived from the verb *είρω* (I say, I speak, I tell).[4] *Ρητός* is the stated, the specified. In mathematics *ρητοί αριθμοί* are the *rational numbers*, and *άρρητοι αριθμοί* the *irrational numbers*. Euclid, referring to the diagonal of the square calls it *"μήκει ασύμμετρον" (linearly incommensurable)* to its side, whereas Plato in his *Republic* (VIII, 546c 4-5) calls the same diagonal "*άρρητον*" (inexpressible or, as it is usually translated, irrational). The two words *ασύμμετρος* and *άρρητος* are used to express the same mathematical observation. The word *άρρητος* stresses specifically the impossibility of expressing a given magnitude by an utterable number (Note: to the Greeks only the integers were considered numbers).

[4] Actually, the future tense of the verb *είρω*, *ερώ*, is also used as the future tense of the verb *λέγω* (I say) from which as we will see the noun *λόγος* is derived. The word *ρήμα* (that which is said or spoken, word, a saying; in grammar, a verb), just like *ρητός*, is also derived from *είρω*.

Ἄλογος is an adjective meaning without speech or without reason.[5] It is compounded by the privative prefix 'α' and the noun *λόγος*. Definitely, one cannot exhaust the meanings of the noun *λόγος*. It is derived from the verb *λέγω* which means I say, I speak, I mean. This is the verb *λέγω* with the indication **(C)** in the Liddell and Scott *Greek-English Lexicon Greek-English Lexicon.* There are also *λέγω* (A) and *λέγω* (B). *Λέγω* (A) means to lay (the Latin *lex* and the English *law* are derived from it) and *λέγω* (A) means to gather, to choose, to count, to reckon up (the Latin *lego* is derived from the same root). Heidegger (1959, p. 124), commenting on the meaning of the word *λόγος*, allows one to infer that *λόγος* is derived from the other two verbs as well. "*Λέγω*", Heidegger says, "is to put one thing with another, to bring together, in short to gather, but at the same time the one is marked off against the other." [6]

---

[5] It also means the brute, the animal. Actually in modern Greek *άλογο* is the horse.

[6] Heidegger appeals to the first Heraclitian fragment to justify his suggestion. *λόγος*, he says, tells how things comport themselves. It is the permanent as well as the appearing collection of things, finally their *φύσις*. The fragment has been translated by Kirk (1970) as:

> Of the Logos which is as I describe it men always prove to be uncomprehending, both before they have heard it and when once they have heard it. For although things happen according to this Logos, they (men) are like people of no experience, even when they experience such words and deeds as I explain, when I distinguish each thing according to its constitution and declare how it is; but the rest of men fail to notice what they do after they wake up just as they forget what they do when asleep.

The derivation of the word *λόγος* from *λέγω* (B), which, as we have seen, means, among other things, to count, explains the fact that *λόγος* is used in connection with numbers. It features in the word *κατάλογος* which means catalogue, register, list, enumeration and it occurs in phrases like "*Συ γαρ εν ανδρών λόγω*" – "For you are amongst the ranks (or amongst the number) of men" (Herodotus III, 120). The strongest indication that *λόγος* has to do with numbers is its use in signifying the numerical ratio, proportion, analogy.[7] "*Ο αυτός λόγος*" is the phrase most frequently used to express sameness of ratio in Euclid's *Elements.* We also know that the Pythagoreans expressed musical intervals as numerical ratios. The octave was 12:6 (=2:1), the fourth 12:9 (=4:3), and the fifth 12:8 (=3:2).[8]

If the word *λόγος* does indeed signify a relationship between numbers, we are justified in inferring that the use of the word *άλογος* in regard to incommensurability, stresses the impossibility of assigning a numerical ratio, expressible in terms of integers, to the relation that holds between the side and the diagonal of a square. *Λόγος*, however, also

---

Kirk et al. (1983, p. 187) interpret *λόγος* in the same fragment as "perhaps...the unifying formula or proportionate method of arrangement of things, what might almost be termed their structural plan both individual and in sum."

7 *Λόγος* actually is another word for fraction in modern Greek.

8 '12' refers to the twelve parts into which a ruler, a canon, was divided. On this canon a string was stretched, parts of which were plucked to produce the three most important consonances. Reported by Gaudentius, 4th century A.D., in Szabó (1978, p. 115).

means saying, statement, speech, right of speech, power to speak. *Άλογος* then can be taken to be an exact synonym of the term *άρρητος*, that is a term signifying the impossibility of saying the number which normally would have corresponded to the diagonal. Indeed, in mathematical contexts, *άλογος* is used as a synonym of both *ασύμμετρος* and *άρρητος*. In the *Republic* (VII 534 d 5), Plato uses the proverbial, it seems, phrase "*άλογοι ώσπερ γραμμαί*" (irrational as the lines).[9] Also Democritus, as reported by Diogenes Laertius, had written a book with the title *Περί αλόγων γραμμών και ναστών (On incommensurable lines and solids).*[10]

From the above linguistic considerations it follows that the two words, *άρρητος* and *άλογος*, are synonymous emphasizing the astonishing and unanticipated result of the lack of the slightest common measure, of an effable, utterable, voiceable rational number to express the relation of a diagonal to the side of a square. This contention (i.e., that *άρρητος* and *άλογος* express ineffability) is corroborated by the fact that in Latin, the word *άλογος* was translated as *surdus* - that which is not heard, noiseless, silent, mute, dumb, according to the Lewis and Short *Oxford Latin Dictionary*. The English adjective *surd*, which derives etymologically from the same root, means, according to the *Oxford*

---

[9] The passage in which this phrase occurs discusses actually the appropriate nurturing and education of children. Here the reference purports to attest to the use of the adjective *άλογος* as another word for incommensurable in geometry. Any other connotation of *άλογος* will come forward in the next section of the paper.

[10] The reference is made in *Diogenes Laertius,* Vol. II, Book IX, 458. On the equivalence of the words *ασύμμετρος, άρρητος, άλογος,* see also Michel (1950, p. 414).

Dictionary, that which cannot be expressed in finite terms of ordinary numbers or quantities. In mathematics, the substantive *surd* is the irrational number or quantity, whereas in phonetics it signifies something voiceless, a speech-sound uttered without voice, as by a mute.

Unlike *άρρητος* and *άλογος*, the word *ασύμμετρος* stresses not ineffability but the lack of a common measure between two magnitudes. Each one of the two can be measured only by its own distinctive unit. Aristotle in his *Metaphysics* (1053a, 14-24) states:

> The measure is not always one in number - sometimes there are several; e.g., ... the diagonal of the square and its side are measured by two quantities. (...) The measure is always homogeneous with the thing measured.

It may appear that the disparity of the measures precludes any possibility of comparison. It surely precludes a specific kind of comparability: τhat which requires the commensurability of the incommensurable magnitudes, handling them, that is, by the same measure. However, the juxtaposition of the incommensurable magnitudes and an overall empirical comparative evaluation (not in terms of a common measure), as to their length for example, is not obstructed, at least when dealt with geometrically. Actually, in Book X of Euclid's *Elements* (Proposition 2) we are presented with a criterion of incommensurability, successive subtraction (*ανθυφαίρεσις*), which attests to what we are saying here. The proposition reads as follows:

> If, when the less of two unequal magnitudes is continually subtracted in turn from the greater, that which is left never measures the one before it, the magnitudes will be incommensurable.

Successive subtraction does not terminate in the case of incommensurable lines. There is always a residue. In the case of *ανθυφαίρεσις* then, one of the two incommensurable lines, the one with the residue, is seen as greater than the other.

In Plato's *Parmenides* (140c) we have a passage which discusses whether *το έν* (the one) can be equal or unequal to itself or other. There it is stated:

> If it is equal, it will have the same number of measures as anything to which it is equal. If greater or less, it will have more or fewer measures than things less or greater than itself, provided that they are commensurable with it. Or if they are incommensurable with it, it will have smaller measures in the one case (that is, when it is greater), greater in the other.

I believe that in this passage it is suggested that even in the case of incommensurable magnitudes one can speak of greater or lesser. One can even compare as to their length their respective measures.

In Aristotle's *Parva Naturalia* (439b, 19-32) the possibility of believing that there are more colours than just black and white is discussed:

Their number is due to the proportion of their components; for these may be grouped in the ratio of three to two, or three to four, or in any other numerical ratios (or they may be in no expressible ratio, but in an incommensurable relation of excess or defect), so that these colours are determined like musical intervals.

Here, the use of the word *incommensurable* clearly indicates that two magnitudes may stand in an incommensurable relation and yet be accountable as to their excess or defect, that is, assessed comparatively as to their quantity.

Finally, I should mention the expression '*διάμετρος ρητή'* (*Republic*, VIII, 546c) which is the rational approximation of the irrational diagonal in order to reinforce my claim that incommensurable magnitudes can be reasonably dealt with. It follows from the above that the thesis of incommensurability does not imply incomparability. In particular, the use of the word *ασύμμετρος*, unlike perhaps that of *άρρητος* and, rather than forbidding, it imposes some kind of a comparative procedure on the magnitudes under evaluation, emphasizing the relative character of incommensurability (Michel 1950, p. 414). Unless the diagonal and the side of a square are compared, they cannot be proclaimed incommensurable.

INCOMMENSURABILITY AND IRRATIONALITY

So far we have dealt with one aspect of the issue of incommensurability. Focusing on the possibility of comparison between two incommensurable magnitudes, we confined ourselves to the strictly mathematical meanings of the terms used. Thus we interpreted *άρρητος* and *άλογος* as that which, lacking a rational number (*α-ρητός*) and a mathematical ratio (*α-λόγος*) respectively, is finally ineffable. Up until now, we deliberately left out some other meanings of the same words that can take us to the second part of our investigation. These other meanings will help us trace the contemporary identification of incommensurability and irrationality.

The word *άρρητος* in ancient Greek texts (Herodotus, Xenophon, Sophocles, Euripides, etc.), besides meaning the ineffable, the unspoken, the mathematically irrational, verges on the occult (Liddell and Scott 1895; also Dodds 1951). In some non mathematical contexts it refers to things sacred, profane, religious, mysterious that are not to be spoken or divulged. This ambiguity of the word *άρρητος* provided the germ of a legend as regards the issue of incommensurability. It has been reported in the *Scholia* that accompanied Euclid's *Elements* and by Iamblichus, Plutarch, Pappus and others, that the discovery of the irrational in geometry had cost the early fifth century Pythagorean mathematician Hippasus his life (Burkert 1972, pp. 457-8). He was drowned at sea as a traitor for his impiety to disclose to the uninitiated and unworthy the mysterious processes of geometry. Burkert comments on this legend (ibid., p. 455) :

> The tradition of secrecy, betrayal and divine punishment provided the occasion for the reconstruction of a veritable melodrama in intellectual history. The realization that certain geometrical magnitudes are not expressible in terms of whole numbers is thought of as "une veritable scandale logique,"[11] bound to shake the foundations of the Pythagorean doctrine, which maintained "everything is number"; for the

[11] The reference is to Tannery (1930, p. 259).

> Greeks, number and irrationality are mutually exclusive.[12] Thus, one comes to speak of a *Grundlagenkrisis*... and to see in the tradition about the death of the 'traitor' a reflection of the shock and despair that this discovery must have brought.

Burkert (ibid., p. 462), challenging the allegation that a scandal had occurred, claims that the Pythagoreans were not at all upset by the discovery of the irrational. "The deep significance of the discovery, so dramatically expressed in the catchword *Grundlagenkrisis*, is not attested in the sources". He cites Kurt Reidemeister (1949, p. 30):

> Nowhere in the many passages about the irrational in Plato and Aristotle can we detect any reference to a scandal, though it would surely still have been known in their day.

Burkert (ibid.) also calls upon the testimony of B.L. van der Waerden ("not a philosophical problem, but one that arose within the development of mathematics itself") and Kurt von Fritz. Szabó (1978, p. 88) argues the same thing:

> Undoubtedly mystical-religious *άρρητα* were concerned with things that should no be expressed... Nonetheless the diagonal of a square was not called *άρρητος* for this reason, but just because a number could not be assigned to its length... It seems that the tradition (which views the discovery and even more so the public discussion of mathematical irrationality as "sacrilege") is just a naive legend which

---

[12] Burkert cites here Aristotle (*Metaphysics*, 1021a5): "ο γαρ αριθμός ρητός' – "for number is rational".

> sprang up later. This discovery was most probably never a "scandal" to mathematicians.

Burkert (ibid., p. 462) claims that “Pythagorean 'secrecy' was undoubtedly misused in later times, as a carte blanche to permit the publication of forgeries as newly discovered books, and brand the discoveries of later thinkers as plagiarism of Pythagoras.” In addition, he contends (ibid., pp. 462-3) that “the inherent connection of the problem of the irrational with Pythagorean speculation and philosophy, which some have supposed they saw, is doubtful. (...) Clearly Pythagorean number theory and deductive mathematics lie on two different planes; 'all things are number' never means 'all magnitudes are commensurable.'” Burkert makes here the distinction between Pythagorean cosmology and mathematics. The primary elements of the Pythagorean universe were numbers which were ascribed to things in the world.

Pythagorean number theory, Burkert remarks, interpreted only the relations of existing things. “The 'nonexistent' is left out of account.” So, the nonexistent rational number that was to account for the relation of the side to the diagonal of a square never posed a problem to the Pythagorean cosmology. It remained a mathematical problem which in fact, as Szabó (1978, p. 96) claims, was dealt with mathematically:

> ... the Pythagorean doctrine 'everything is number' was in no way shaken by the discovery of linear incommensurability. Although the length, which could not be assigned a numerical value, was described at first as an *άρρητον*, the initial surprise

gave way immediately to the realisation that the line whose length could not be given a numerical value should be measured by its square.[13]

Also Plato (*Laws* VII 820d), referring to the study of subjects like that of incommensurability, he says: “there is neither danger nor difficulty in them.”

So, if Burkert, Szabó and the others who claim that the word *άρρητος* in mathematical contexts does not have the connotations of mysteriousness, secrecy and profanity, are right, then one can conclude that incommensurability does not necessarily imply irrationality. It would have implied it, if the word *άρρητος* was meant to signify that which, by being unutterable, transcends reason and logical thought and thereby divulges something that shakes the whole rational cosmological edifice. However, this very ambiguity of the term may be regarded as contributing to the identification of incommensurability and irrationality.

The relationship of incommensurability and irrationality is better perhaps illuminated with the other synonym of incommensurable, the word *άλογος*. One may justifiably presume that the synonymity of *incommensurable* and *άλογος* completely vindicates those who equate incommensurability with irrationality. But, as we have already hinted at and we will discuss below, that is not necessarily the case. Certainly the word *λόγος* has the meaning of reason and naturally *άλογος* means that which is without reason, the irrational. Nonetheless, we have pointed out in the above linguistic discussion of the synonymous word *άρρητος*, that the mathematical problem of incommensurability did not present any real philosophical problem to the ancients. *Άλογος* in mathematical

[13] Cf. the methods developed to approximate the length of the diagonal cited above.

contexts means that which is without *λόγος*, i.e., in the double sense of a numerical ratio (x:y) and speech, therefore the inexpressible.[14] In the *Sophist* (238c) Plato, talking about the 'nonexistent'', 'that which is not', writes: "*αδιανόητόν τε και άρρητον και άφθεγκτον και άλογον είναι"*- "it is unthinkable, not to be spoken of or uttered or expressed."

One may assume that what Kuhn's critics mean by the word irrational in the context of discussing incommensurability, i.e., that which is groundless, contrary to reason, a prey or slave to passions, etc., is rooted in these philosophical doctrines that equate *λόγος* and *ορθός λόγος* (right reason). Already in the Sophists and in Plato's Dialogues one comes across the expression *ορθός λόγος* (right reason) in the sense of the correct argument, correct reasoning.[15] But it is in the Stoic philosophy that the doctrine of *ορθός λόγος* as a principle of morality assumes prominence. The Stoics, echoing Heraclitus, considered *λόγος* as the regulating principle of the universe and identified *λόγος* with God. Man has the privilege of *λόγος* (reason) and living according to reason takes him near God, makes him live according to nature. The same thought is echoed in St John's Gospel where Logos is identified with Christ. Later, from the 17th century onwards, the universal validity of the principle of sufficient reason was advanced and declared. There must always be some sufficient reason to explain and justify truth, validity, existence. Reason

---

[14] It is noteworthy that in modern Greek the neuter of the adjective *άλογος* is reserved for the horses (*άλογο* = horse), whereas the adjective for the irrational in the sense of contrary to reason is *παράλογος* (literally against reason). The word *λόγος* has been retained for the numerical ratios, while the irrational numbers are called *άρρητοι*.

[15] *Laws* 659d, 696c, 890d, *Phaedo* 93e, *Philebus* 43e, *Statesman*, 310c, etc.

was contrasted with experience, feeling, faith, tradition, and it was praised as the only reliable means of assessing truths about the world.[16]

One may claim that the identification of incommensurability and irrationality should be attributed to the ambiguity of the Greek word *λόγος* and especially to the ambiguity of the Latin and English word *ratio*. *Ratio*, which translates both *λόγος* and *ορθός λόγος*, is derived from the Latin verb *reor*[17] which, according to the *Dictionnaire Etymologique de la Langue Latine*, means *compter* (to reckon, to count), *calculer* (to calculate). In common language, *reor* took up the meaning of *penser* (to think), *estimer* (to estimate), *juger* (to judge). *Ratio*, according to the *Oxford Latin Dictionary,* means, a list, a roll, a sum, a number, a computation, but also "the faculty of the mind which forms the basis of computation and calculation, and hence of mental action in general, i.e., judgement, understanding, reason". In mathematical contexts, *ratio* is the exact equivalent of *λόγος*

---

[16] Heidegger (1962, 32ff) has a very interesting suggestion to make as regards the slide of meaning from *λόγος* in the primordial sense to *λόγος* as *assertion* and finally *ground*. Researching the etymology of Greek key words, he claimed that *λόγος* was primordially related to disclosedness (exhibiting the nature of things), and gradually it took up the meaning of assertion and discourse. *Λόγος*, as that which is exhibited (*λεγόμενον* - present participle of *λέγω*), became the ground because when I say something on something (*λέγω τι κατά τινός*), then the sub-ject (*υπό-κείμενο*) of my discourse (the *λεγόμενον*) already lies at the bottom.

[17] The etymology of the word is considered doubtful. Lewis and Short (1980) list for comparison the Sanscrit *rta*, correct, the Zend *areta*, complete, and the Greek *αρετή*, valour.

in the sense of due relation between two similar magnitudes. *Ρητοί αριθμοί* were translated in English as *rational numbers*, and *άρρητοι αριθμοί* as *irrational numbers*. It seems, therefore, that the various senses of *ratio* correspond fully to the senses of the Greek word *λόγος*. So, one may say that when we proclaim two magnitudes irrational the only thing we want to call attention to is simply the fact that these two magnitudes lack a common measure. Nevertheless, as we have very briefly stated above, *ratio*, *reason*, and rational ideas and ideals in general, assumed from the 17th century onwards and especially after the Enlightenment, a prominence that overpowered any nuance in meaning. Reason, contrasted with experience, tradition, faith, bias, personal idiosyncrasies and feelings, was appointed the ultimate judge and the ultimate ideal to which everything must look up to and gravitate towards. Anything that runs contrary to reason is, in a derogatory fashion, proclaimed irrational, i.e., worthy of condemnation and contempt. Irrationality, stripped of its mathematical connotations, becomes, just like in the Stoic philosophy, a moral concept, akin to appetite, impulse, the passions, the senses.[18]

---

[18] Chrysippus paralleled the irrational faculties of the soul to a runner's weight. Epicurus held that “all sensation is irrational”, (Long, A.A., Sedley 1989, Vol I, 84). Stobaeus (Long and Sedley 1989, Vol II, p. 410), reports that “they [the Stoics] say that passion is impulse which is excessive and disobedient to the dictates of reason, or a movement of soul which is irrational and contrary to nature”. Also, Galen (Long and Sedley, 1989, Vol I, p. 413), says that “irrationality must be taken to mean ‘disobedient to reason’ and ‘reason turned aside’; with reference to this movement we even speak in ordinary language of people ‘being pushed' and 'moved irrationally, without reason and judgement’.

The same ambiguity, characteristic of the word *ratio*, prevails over the word *surd*. In mathematics, it translated the word *άλογος* expressing the impossibility of uttering a rational number corresponding to the relation holding between two quantities. As we mentioned above, the Latin *surdus* means that which is not heard, silent, mute, dumb, and in that respect it corresponds also to the word *άρρητος*. It is said "of things that give out dull, indistinct sound" (*Oxford Latin Dictionary*). In this connection, it should be noted that from *surdus*, the word *absurdus* is compounded. *Absurdus*, which corresponds to the English word *absurd*, means "out of tune, irrational, incongruous, absurd, silly, senseless, stupid" (ibid.). So, we can conclude that both the words that render *incommensurable* into English (*surd, irrational*), just like the original words in Greek (*άρρητος, άλογος*), suffer from an ambiguity that engenders misunderstandings. A rigorously defined mathematical concept may be taken to mean something quite unintended. The mere inexpressibility, in the current framework, of a well-thought out relation between two magnitudes is turned into a threat to the foundations of our fastidiously organized world view.

In conclusion, we can restate the results of our linguistic review of the term *incommensurability*. First, it can be maintained that incommensurability, at least in its original use in mathematical contexts, does not preclude comparability. Actually it imposes it. Second, the identification of incommensurability and irrationality must be attributed to the ambiguity of the synonyms of incommensurable both in Greek (*άρρητος*, *άλογος*) and in English (irrational, surd/absurd). So, our linguistic review allows us to defend the thesis that incommensurability does not analytically imply incomparability and irrationality.

BIBLIOGRAPHY

Burkert, W., (1972) *Lore and Science in Ancient Pythagoreanism.* Cambridge, MA: Harvard University Press.

Diogenes Laertius, (1980) *Lives and Opinions of Eminent Philosophers in Ten Books***,** translated by R.D. Hicks, Cambridge, MA: Harvard Univeisty Press.

Dodds, E.R., (1951) *The Greeks and the Irrational*. Berkeley: The University of California Press.

Ernout, A., Meillet, A., (1959) *Dictionnaire Etymologique de la Langue Latine.* Paris: Librairie C. Klinck-sieck.

Heidegger, M., (1959) *An Introduction to Metaphysics,* trans. R. Manheim. New Haven: Yale University Press.

Heidegger, M., (1962) *Being and Time*. New York: Harper and Row.

Kirk, G.S. (1970) *Heraclitus. The Cosmic Fragments*. Cambridge: Cambridge University Press.

Kirk, G.S., Raven, J.E., and Schofield, M., (1983) *The Presocratic Philosophers* Cambridge: Cambridge University Press.

Kuhn, T.S., (1962) *The Structure of Scientific Revolutions*. Chicago: The University of Chicago Press.

Lewis and Short (1980 [1879]), *Oxford Latin Dictionary*, Oxford: Clarendon Press.

Liddell and Scott, (1990 [1889]), *Greek -English Lexicon*, Oxford: Clarendon Press.

Long, A.A., Sedley, D.N., (1989) *The Hellenistic Philosophers, Vol I, II* Cambridge: Cambridge University Press.

Michel, P.H., (1950) *De Pythagore a Euclide.* Paris: Belles Lettres

Reidemeister, K., (1949) *Das exacte Denken der Griehen*. Leipzig: Classen und Goverts

Szabó, A., (1978) *The Beginnings of Ancient Greek Mathematics*. Dordrecht: D. Reidel

Tannery, P., (1930) *Pour l'histoire de la science hellene: De Thales a Empedocle.* Paris: Gauthier-Villard.

REFERENCES