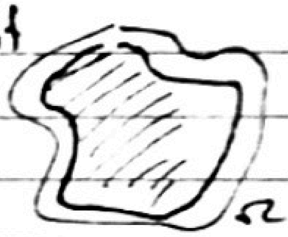
⑤

<u>Lemma</u>: If u satisfies MVP, u cont
then

$$\boxed{u(x) = u_\varepsilon(x) \in C^\infty(\Omega)}$$



$$u_\varepsilon(x) := u * \eta_\varepsilon = \int_{\Omega_\varepsilon} u(y)\,\eta_\varepsilon(x-y)\,dy$$

(Fix $x \in \Omega$, let $\boxed{\varepsilon < \operatorname{dist}(x, \partial\Omega)}$

<u>Pf</u>

$z = y - x$

$$\int_\Omega u(y)\,\eta_\varepsilon(y-x)\,dy = \int_{\mathbb{R}^n} u(y)\,\eta_\varepsilon(y-x)\,dy$$

$$= \int_{\mathbb{R}^n} u(z+x)\,\eta_\varepsilon(z)\,dz$$

$$= \frac{1}{\varepsilon^n} \int_{|z|<\varepsilon} u(z+x)\,\eta\left(\frac{z}{\varepsilon}\right)dz$$

$$= \int_{|z|<1} u(x+\varepsilon y)\,\eta(y)\,dy$$

$$= \int_{|z|<1} u(x+\varepsilon y)\,\eta(|y|)\,dy$$

continuity

$$= \int_0^1 \left(\int_{\partial B_1(0)} u(x+\varepsilon r w)\,\eta(rw)\,dS_w\right) r^{n-1}\,dr$$

$$= \int_0^1 r^{n-1}\left(\int_{\partial B_1(0)} u(x+\varepsilon r w)\,dS_w\right)\eta(r)\,dr$$

$$= \frac{1}{\omega_n} \int_{\partial B_1(0)} u(x+\varepsilon r w)\,dS_w = u(x)$$

□