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Logan 5.6  $y'' + \lambda e^{4x} y = 0$ ,  $0 < x < 1$ ,  $y(0) = y(1) = 1$

$$\varepsilon = \frac{1}{\sqrt{\lambda}} \Rightarrow \varepsilon^2 y'' + e^{4x} y = 0 \quad k(x) = e^{2x}$$

$$\int_0^x k(\xi) d\xi = \int_0^x e^{2\xi} d\xi = \frac{e^{2x}}{2} - \frac{1}{2} = \frac{e^{2x} - 1}{2}$$

$$\text{Άρα } y_{\text{WKB}}(x) = \frac{1}{e^x} \left[ c_1 \sin\left(\frac{e^{2x}-1}{2} \sqrt{\lambda}\right) + c_2 \cos\left(\frac{e^{2x}-1}{2} \sqrt{\lambda}\right) \right]$$

$$y_{\text{WKB}}(0) = 1 \Rightarrow c_2 = 1$$

$$y_{\text{WKB}}(1) = 1 \Rightarrow \frac{c_1}{e} \sin\left(\frac{e^2-1}{2} \sqrt{\lambda}\right) + \frac{1}{e} \cos\left(\frac{e^2-1}{2} \sqrt{\lambda}\right) = 1$$

$$\Rightarrow c_1 = \frac{e - \cos\left(\frac{e^2-1}{2} \sqrt{\lambda}\right)}{\sin\left(\frac{e^2-1}{2} \sqrt{\lambda}\right)}$$

Δεν καταλαβαίνω τι να κάνω από εδώ και πέρα.