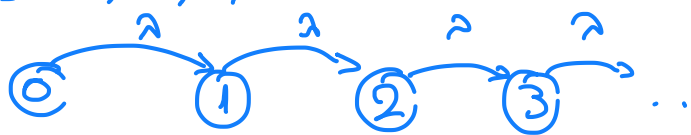


Παράδειγμα 2 (Διαδικασία Poisson - Διαδικασία Γέννησης)

$$S = \{0, 1, 2, 3, \dots\}$$



$$Q = \begin{bmatrix} 0 & \lambda & 0 & 0 & \dots \\ -\lambda & \lambda & 0 & 0 & \dots \\ 0 & -\lambda & \lambda & 0 & \dots \\ 0 & 0 & -\lambda & \lambda & \dots \\ 0 & 0 & 0 & -\lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$E[X(t)] = \sum_{j=0}^{\infty} j P_j(t) = j$$

$$\sum_{j=1}^{\infty} j P_j(t)$$

$$p'(t) = p(t) \cdot Q \Rightarrow$$

$$\begin{bmatrix} p_0'(t) & p_1'(t) & p_2'(t) & p_3'(t) & \dots \end{bmatrix} = \begin{bmatrix} p_0(t) & p_1(t) & p_2(t) & \dots \end{bmatrix} \begin{bmatrix} -\lambda & \lambda & 0 & 0 & \dots \\ 0 & -\lambda & \lambda & 0 & \dots \\ 0 & 0 & -\lambda & \lambda & \dots \\ 0 & 0 & 0 & -\lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$p_0'(t) = -\lambda p_0(t)$$

$$p_1'(t) = \lambda p_0(t) - \lambda p_1(t)$$

$$p_2'(t) = \lambda p_1(t) - \lambda p_2(t)$$

$$p_3'(t) = \lambda p_2(t) - \lambda p_3(t)$$

$$\vdots$$

$$p_j'(t) = \lambda p_{j-1}(t) - \lambda p_j(t), \quad j \geq 1$$

$$\left. \begin{array}{l} \sum_{j=0}^{\infty} j p_j'(t) \\ \Rightarrow \sum_{j=0}^{\infty} j p_j'(t) = -\lambda \sum_{j=0}^{\infty} j p_j(t) \\ \quad + \lambda \sum_{j=1}^{\infty} j p_{j-1}(t) \Rightarrow \end{array} \right\}$$

$$\frac{d}{dt} E[X(t)] = -\lambda E[X(t)] + \lambda \sum_{j=1}^{\infty} (j-1+1) p_{j-1}(t) \Rightarrow$$

$$\frac{d}{dt} E[X(t)] = -\lambda E[X(t)] + \lambda \underbrace{\sum_{j=1}^{\infty} (j-1) p_{j-1}(t)}_{E[X(t)]} + \lambda \underbrace{\sum_{j=1}^{\infty} p_{j-1}(t)}_{\sum_{j=0}^{\infty} p_j(t) = 1} \Rightarrow$$

$$\frac{d}{dt} E[X(t)] = -\cancel{\lambda E[X(t)]} + \cancel{\lambda E[X(t)]} + \lambda \Rightarrow$$

$$\frac{d}{dt} E[X(t)] = \lambda \Rightarrow$$

$$E[X(t)] = \lambda t + c$$

$$E[X(0)] = 0 \Rightarrow \lambda \cdot 0 + c = 0 \Rightarrow c = 0$$

Ans $\tau \varepsilon$ $E[X(t)] = \lambda t$.