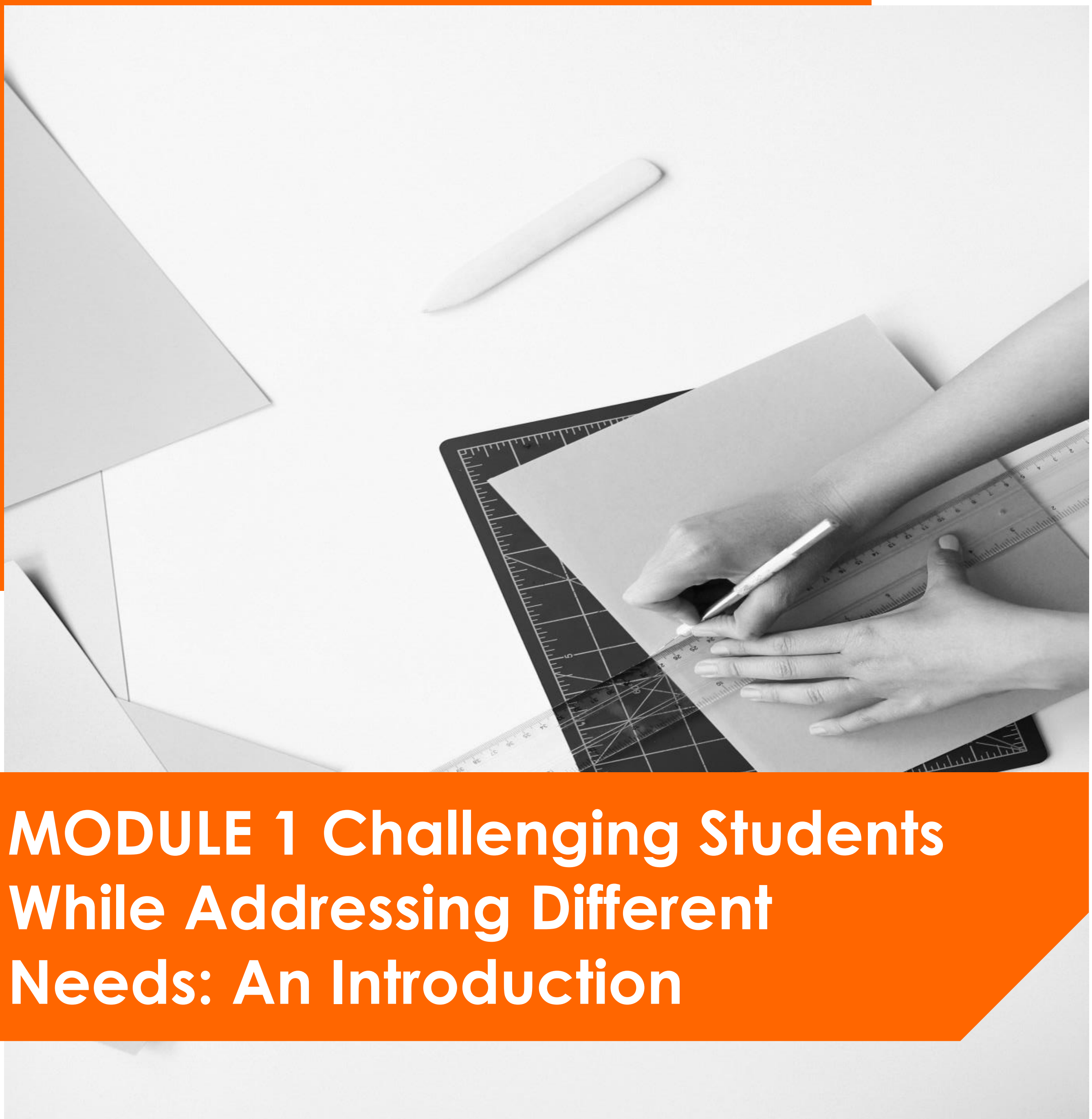


MODULE 1 Challenging Students While Addressing Different Needs: An Introduction

EDUCATE Project

Funded by the ERASMUS+
Programme of the
European Union

Educate
Erasmus+KA2

Booklet's publishing history, permissions, acknowledgements, and disclaimers.

This project, entitled "Enhancing Differentiated Instruction and Cognitive Activation in Mathematics Lessons by Supporting Teacher Learning (EDUCATE)" has been funded with support from the European Commission. This publication [communication] reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Organization	People[1]
University of Cyprus	Dr. Charalambos Y. Charalambous Chloe Georgiou Professor Constantinos Constantinou Professor Demetra Pitta-Pantazi Evridiki Kasapi Dr. George Olympiou Kassandra Georgiou Professor Mary Koutselini Dr. Stavroula Philippou
Instituid Oideachais Marino Institute of Education — An Associated College of Trinity College Dublin, the University of Dublin	Ann Concarr Dr. Ann Marie Gurhy Damien Burke Dr. Mark Prendergast Paul Timmins Dr. Seán Delaney Trevor Purtill
Hellenic Republic National and Kapodistrian University of Athens	Dr. Chryssavgi Triantafillou Professor Despina Potari Dr. Giorgos Psycharis Professor Theodossios Zachariades
U Lisboa Universidade de Lisboa — ie Instituto de Educação	Dr. Ana Henriques Professor Hélia Oliveira Dr. Henrique Guimarães Professor João Pedro da Ponte Professor Leonor Santos
Cyprus Pedagogical Institute	Dr. Elena Christofidou Dr. Nicos Papadouris Dr. Sofia Agathangelou
Υπουργείο Παιδείας και Πολιτισμού	Dr. Andreas Kythreotis Dr. Andreas Theodorides Christos Demosthenous Dr. George Yiallourides Dr. Marios Stylianides Rodoula Theodorou Stelios Ioannides Dr. Yiannis Savvides
	Dr. George Michaeloudes Savvas Nicolaou

[1] All names are listed in alphabetical order.

TABLE OF CONTENTS

INTRODUCTION TO EDUCATE AND ITS OBJECTIVES ____ 5
EDUCATE MODULES MAP ____ Error! Bookmark not defined.
CONCEPT MAP OF MODULE X ____ 7
SYMBOLS ____ 8
INTRODUCTORY ACTIVITY TITLE ____ 9
WORKING WITH CASES OF PRACTICE: (Pre-) Primary ____ 13
Title of the Case of Practice/Sub-unit ____ 13
WORKING WITH CASES OF PRACTICE: Secondary ____ Error! Bookmark not defined.
Title of the Case of Practice/Sub-unit ____ Error! Bookmark not defined.
EVIDENCE OF SUCCESS ____ 116
Basic Questions to Consider ____ 116
Reflection on Practice ____ 116
BRINGING IT ALL TOGETHER ____ 117
Takeaways from the Module ____ 117
SUPPLEMENTARY MATERIAL ____ 118
I. Additional literature/References/Further reading ____ 118
II. External Links ____ 118
III. Other Tools ____ Error! Bookmark not defined.

INTRODUCTION TO EDUCATE AND ITS OBJECTIVES

Dear Readers,

On behalf of all the members of the European ERASMUS+ project entitled, "**E**nhancing **D**ifferentiated Instr**u**ction and **C**ognitive **A**ctivation in Mathematics Lessons by Supporting **T**eacher L**e**arning (EDUCATE)", I would like to welcome you to our project's educational materials for teachers.

Sponsored by the ERASMUS+ programme of the European Union, the transnational project EDUCATE aims at integrating cognitive activation (i.e., working with challenging tasks) and differentiation, aspiring to promote both aspects in the subject of Mathematics, through the provision of appropriate support to teachers and teacher educators. Specifically, we have created these materials with the intention of supporting teachers in engaging all their students in challenging work on the one hand and scaffolding teacher educators in productively supporting teachers in doing so on the other hand.

Partners from four European countries, namely Cyprus, Greece, Ireland and Portugal have worked closely together to develop, pilot test, and refine these materials, as well as design a professional development process around them. Before producing these materials, a thorough review of the current literature and European/International and national documents in the four participated countries (i.e., top-down approach) was conducted; a needs-assessment analysis to identify prospective and practicing teachers' needs, challenges, and difficulties as they enact challenging tasks with all their students that cut across different European educational systems was also conducted by observing/analyzing a series of lessons in the four participating countries; reflective post-lesson discussions were also conducted with teachers (i.e., bottom-up approach). Moreover, we have solicitated practicing teachers' feedback on the clarity of the modules, their reasonableness, applicability and usefulness, in order to improve them.

The project in general, and the production of these materials, in particular, could not have taken place without the support of the teachers who volunteered to participate in the different phases of the project, whom we would like to thank for opening their classrooms and letting us explore their practice and think with them about the complexities inherent in working with challenging tasks with all students.

We would like to invite you to implement these materials either individually or in the context of video-club settings. Should you have any questions about the project or if you have any feedback for its rationale, activities, and publications, please visit our portal http://www.ucy.ac.cy/educate/en/ and explore it to get more information about the progress of the project and its outcomes. You could also directly contact the local coordinators from each partner country:

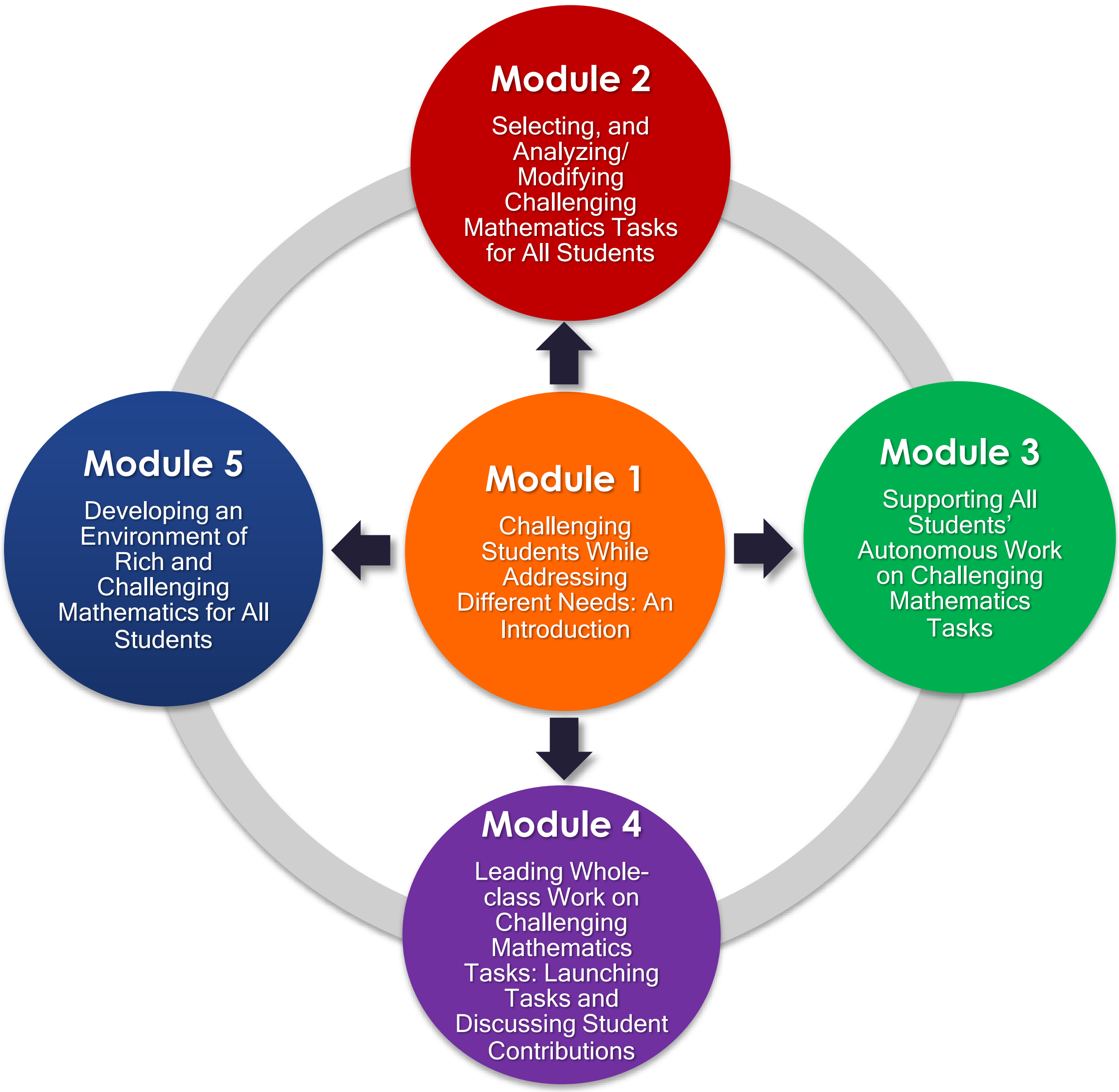
- University of Cyprus (Cyprus, Dr. Charalambos Y. Charalambous, *cycharal@ucy.ac.cy*)
- National and Kapodistrian University of Athens (Greece, Professor Despina Potari, *dpotari@math.uoa.gr*)
- Marino Institute of Education (Ireland, Dr. Seán Delaney, *sean.delaney@mie.ie*)
- Universidade de Lisboa (Portugal, Professor João Pedro da Ponte, *jpponte@ie.ulisboa.pt*)

On Behalf of the EDUCATE Project members,

The Project Coordinator,
Dr. Charalambos Y. Charalambous
Assistant Professor of Educational Research and Evaluation
Department of Education, University of Cyprus

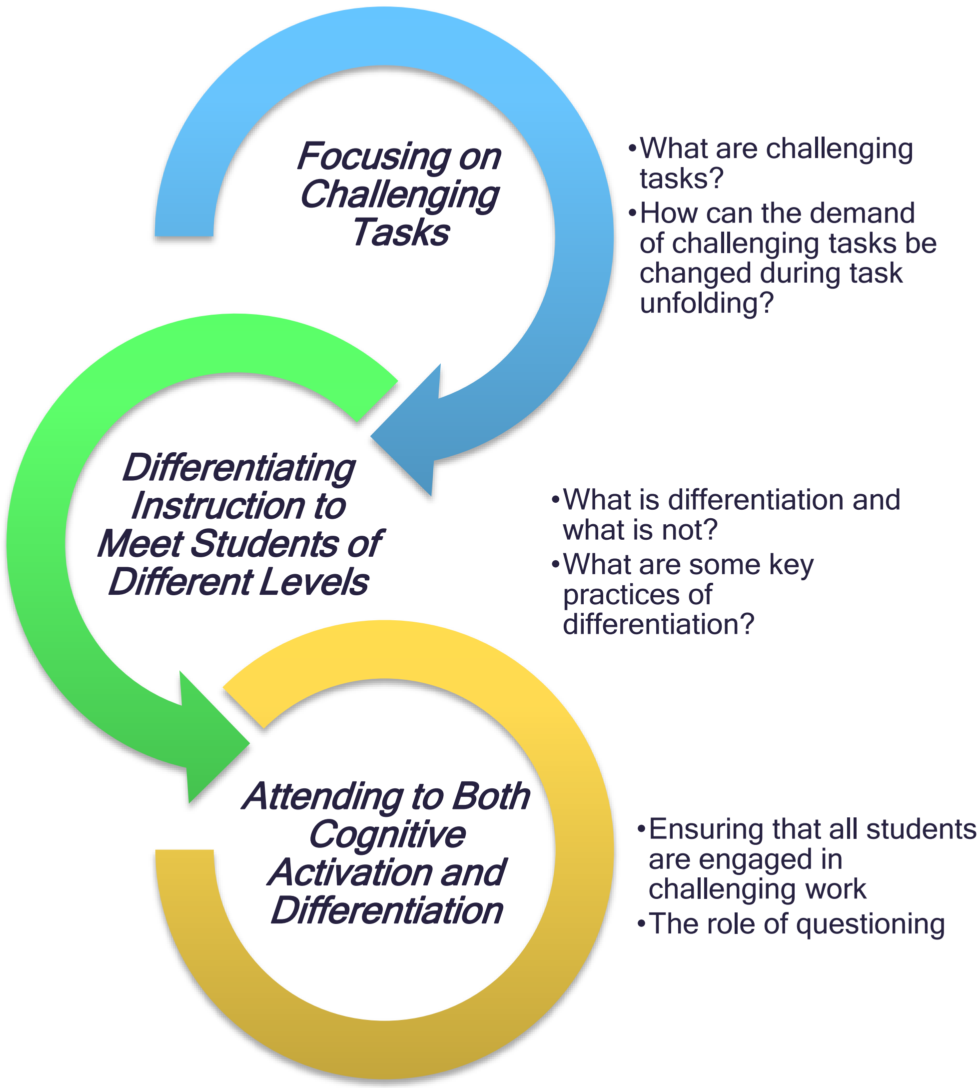
EDUCATE MODULES MAP

The EDUCATE materials consist of a series of five self-contained modules as illustrated by the five colored circles in the figure that follows. Each module focuses on different issues that are important for teachers to consider when planning and implementing challenging tasks with all their students. Module 1, the introductory one, is core to the work of all the other modules and hence it was placed in the middle to demonstrate its centrality as a compulsory module having an orientation to the other four modules to help you decide what to do next. However, we do not see the rest of the modules to be consecutively dependent on each other. Hence, although it would be beneficial for teachers to attend all five modules, we believe that there is also value in considering individual modules.



CONCEPT MAP OF MODULE 1

This is the introductory module to a set of five modules. Its purpose is to help you acquaint yourself with the goal of engaging *all* students in cognitively challenging mathematical work—a goal that can be decomposed into two parts: working with challenging tasks and differentiating instruction. Toward this end, we will first consider what it means and what is entailed in working with challenging mathematical tasks. Next, we will consider how, as teachers, we can differentiate our work to meet students of different readiness/ability levels. Finally, we will bring the two aspects of the aforementioned goal together and explain why the two goals can be thought to be synergistically related; we will use the example of questioning to illustrate the harmonious functioning of both goals.



SYMBOLS

Next to each activity there is one of the following symbols:

Individual work

Video-club setting

Read

Write or complete

Link-to-File

Watch

Reflect

Discuss

Learning Objectives

Plan

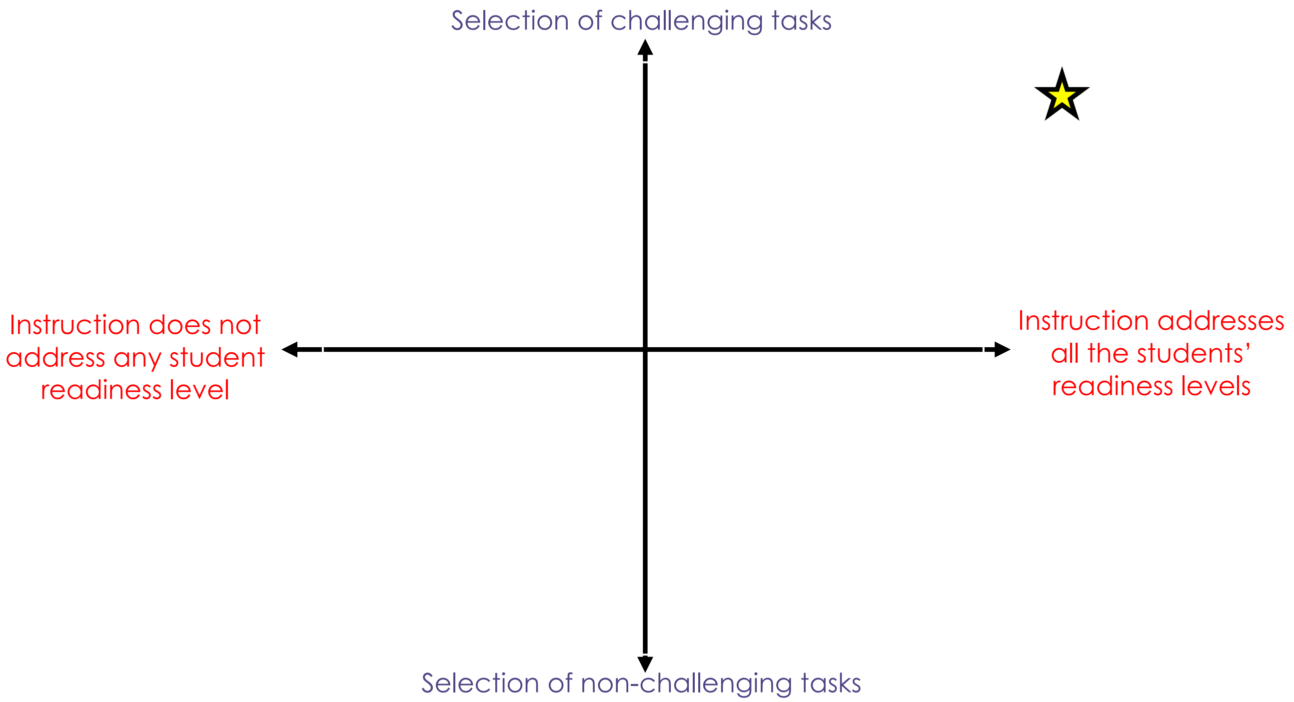
Assess

ENGANGING ALL STUDENTS IN COGNITIVELY CHALLENGING MATHEMATICS TASKS: MISSION IMPOSSIBLE?

Scenario

One of your colleagues attended a workshop on improving the quality of mathematics instruction. On coming back and debriefing her experience, she argued that **it is important to engage *all* students in mathematical activities that require mathematical thinking and reasoning–what she called challenging tasks**. A lively discussion ensued among the teachers, with several teachers doubting the extent to which this goal is realistic, especially for their mixed-ability level classes. Given how complex the work of teaching is and that for years teachers have been bombarded with several ideas like the one presented above, what is your take on this issue?

Think of where you would position your teaching in the two-dimensional space shown below, with the vertical axis representing the extent to which you select challenging tasks for use in your teaching and the horizontal axis corresponding to the degree to which your teaching addresses all the students' different readiness levels.



Discuss with your colleagues:

- Why have you positioned yourself and your teaching in this particular spot?
- What would you identify as the main challenges that you, as a teacher, or teachers in general, would face in selecting and using challenging tasks to productively engage all students in mathematical thinking and reasoning [this situation is shown with a star (*) in the diagram above]?
- Even if it were possible to engage all your students in such type of work, do you think that it would support students' learning? Why? Why not?

Hands on Activity for (Pre-)Primary

Watch the following video clips which refer to the tasks that appear below.

Videoclip 1

Context: In this clip, we will watch a Grade-5 class in Cyprus working on the "Mixed Juice" class. Before this clip, students worked on matching different recipes of mixed juice with different representations and identified the recipes that give the same taste of mixed juice. In this segment, the teacher uses a self-developed task that asks students to use one of these recipes and write down all the possible ratios that represent this situation.

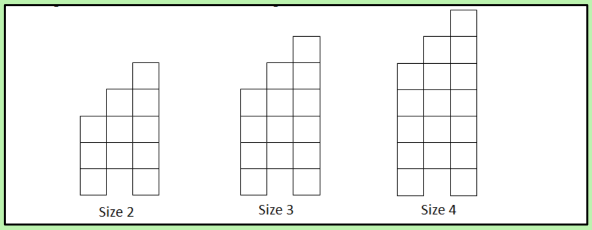
Task 1 ("The Mixed Juice" task):
George likes to drink a particular type of mixed juice that contains orange juice to apple juice with a ratio of 3:1. Write all the possible ratios that can be expressed based on the information given.

Link: Simone's lesson (28:55-31:40)

Videoclip 2

Context: In this clip, we will watch short snippets from the launching of the "Chairs" task and its enactment (autonomous work) in a fifth-grade class in Cyprus. The teacher decided to use the task to provide his students with opportunities for mathematical thinking and reasoning. The task took place during the end of the school year.

Task 2 ("The Chairs" task):
Alex uses identical tiles to make different-sized chair designs for a school art project. The pictures on your sheet show the first three designs created – size 2, size 3 and size 4.



Alex wanted to make a chair of Size 50. How many tiles would he need?

Link: Charalambos's lesson: 0.00 to 2:40, 34:14-35:08, 35:14-36:15, 36:20-36:59

Discuss:

- To what extent, do you think that students in these lesson video-clips are engaged in mathematical thinking and reasoning? Justify your thinking.
- If they are engaged, is that true for all students? Why? Why not?

Introduction to Module 1

This introductory module aims to familiarize you with the dual goal of selecting and using challenging tasks in your mathematics lessons, on the one hand, and immersing all your students—to the extent possible—in such tasks that have the potential to improve students' mathematical thinking and reasoning, on the other hand. Over the past decades, several research works internationally have documented the importance of providing students with opportunities to engage students in such tasks, since they were found to improve both the quantity and the quality of student learning. However, research has also shown that these tasks are not utilized that often and not with all students—apparently because of the complexity of this work. This module will engage you in activities that will help you start thinking more deeply around issues and challenges surrounding this dual goal. You will first have the opportunity to identify challenging tasks and determine their characteristics. Next, you will be exposed to different ideas related to differentiating your instruction to meet the needs and the readiness/ability levels of all your students. Finally, you will have the opportunity to consider the synergistic relationship between the two goals. You should bear in mind that, as an introductory module, Module 1, will expose you to several of the issues and challenges surrounding the dual goal of working with challenging tasks and ensuring that all your students are productively engaged in such tasks. You will have the opportunity to delve deeper in each of these issues and challenges in the remaining four modules.

	LEARNING OBJECTIVES (LO)	CASE OF PRACTICE ADDRESSING THE LO
LO1	Familiarizing teachers with cognitively challenging tasks and helping them identify aspects that render a mathematical task challenging	1
LO2	Identifying how the opportunities to engage students in cognitively challenging work can be modified during task presentation and implementation	1
LO3	Familiarizing teachers with the basic aspects of differentiation and different practices for achieving differentiation in today's classes	2
LO4	Explaining how working with challenging tasks can be in balance with trying to differentiate instruction and providing examples of instructional moves that can serve meet this dual goal	3

WORKING WITH CASES OF PRACTICE: (Pre-) Primary

CASE OF PRACTICE 1

Focusing on Challenging Tasks

Overview

CONTACT HOURS	2 hours
TYPE OF RESOURCES	Videoclips; Tasks; the MTF framework
EMPHASES	Discussing how task unfolding can offer different learning opportunities for students

Activities

Opening Activity

(1) Brainstorming Activity

- Based on the videos you observed in the introductory activity of this module, what do you think we can do, as teachers, to create a productive space for engaging students in mathematical thinking and reasoning? What might we do (often inadvertently) that might hinder such attempts?

(2) There are several ways in which we, as teachers, might craft or lessen students' opportunities to engage in mathematical thinking and reasoning. A group of U.S. researchers has proposed the *Mathematical Task Framework* (hereafter called MTF) to help us better classify these ways and through that make more deliberate and informed decisions about the opportunities we craft for our students' thinking. Look at the figure below and read the brief introduction to the MTF that appears below; then consider the question that follows.

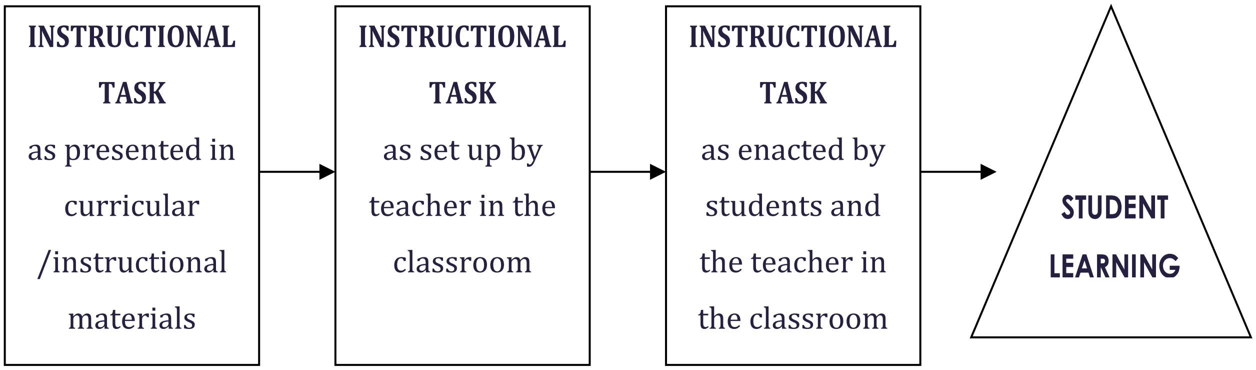


Fig.1. *The Mathematical Task Framework (adapted from Stein et al., 2000).*

About MTF: What Does it Tell Us and How Can it Be Used?

What does the MTF suggest? According to the MTF, instructional tasks pass through three stages: first, as they are presented in curriculum materials or in the handouts that the teacher prepares for her/his students; second, as they are set up by the teacher in the classroom during the launching (presentation) of the task; and third, as they are enacted/implemented during the lesson, while the students and the teacher interact while solving these tasks. Figure 1 captures these phases of task unfolding, emphasizing that what ultimate determines student learning is not only the *selection of cognitively challenging tasks*, but *how these tasks unfold during instruction*.

How can MTF be utilized? Over the past years, MTF has been used both as a research tool to examine instructional quality with respect to task unfolding but also as a professional development tool to sensitize teachers to the importance of attending to how the challenging aspects of a task might be altered during instruction, especially during the phases of task presentation and enactment/implementation.

Thinking of your previous lessons, in which area(s)–(a*) task selection*, (b) *task presentation*, and (c) *task enactment*–do you feel that you face more difficulties when trying to enhance your students' opportunities to engage in cognitively demanding work? Why do you think so?

The activities that follow will provide you with opportunities to discuss how different decisions we make as teachers, during the phases of task selection, presentation, and enactment can create different opportunities for student learning.

Activity 1 – Focusing on Task Selection

In this activity you will be exposed to different tasks. Read them carefully and then consider the questions that follow.

<u>Task 1 (Grade 4):</u>

Figure out the product in each case, using the approach shown in the worked out example below:

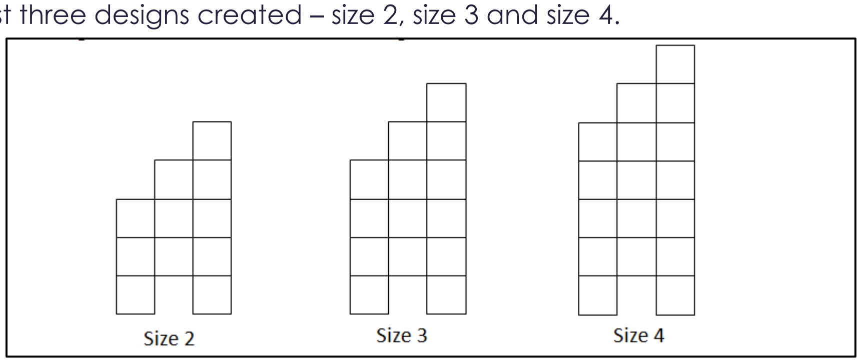
$$\begin{array}{r} {\scriptstyle 2} \\ 217 \\ \times \quad 3 \\ \hline 651 \end{array}$$

$$\begin{array}{r} 485 \\ \times \quad 5 \\ \hline \end{array} \qquad \begin{array}{r} 563 \\ \times \quad 7 \\ \hline \end{array} \qquad \begin{array}{r} 359 \\ \times \quad 4 \\ \hline \end{array}$$

Source: adapted from http://archeia.moec.gov.cy/sd/7/meros_2_enotites_4_6.pdf

<u>Task 2 (Grade 5):</u>

Alex uses identical tiles to make different-sized chair designs for a school art project. The pictures on your sheet show the first three designs created – size 2, size 3 and size 4.



Alex wanted to make a chair of Size 50. How many tiles would he need?

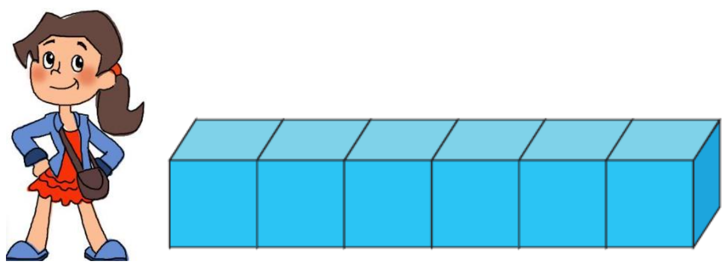
Task 3 (Grade 5):

Complete the following multiplications facts in one minute or less		
2X3=	5X4=	10X6=
4X7=	8X10=	8X4=
9X5=	3X4=	5X5=
6X8=	7X9=	2X6=
3X9=	8X7=	9X2=

Source: Task Sorting Activity (Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004, p. 57)

Task 4 (Grade 4):

The picture below represents 2/3 of Helen's train. How many cubes will Helen's train include in total?



Consider these four tasks and try to classify them according to how cognitively challenging they are (non-challenging vs. challenging), taking into account the corresponding target student audience.

Task	Level of Challenge (Low vs. High)
1	
2	
3	
4	

In your group, identify what features make the tasks challenging. List these features in the space provided below.

What challenges might you encounter in your practice in selecting such tasks for your teaching? How might you and/or your colleagues tackle these challenges?

Activity 2 – Focusing on Task Implementation

Almost twenty years ago, the National Council of Teachers of Mathematics (NCTM) in the USA has recognized the key role that teachers have not only in selecting cognitively challenging tasks (or using such tasks from their textbooks/curriculum materials), but mostly in how they interact with these tasks with their students. In particular, NCTM (2000) noted:

> "*Worthwhile tasks alone are not sufficient for effective teaching. Teachers must also decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them, and thus, eliminating the challenge*" (p. 19).

In this activity, we will consider how different teacher moves during task presentation and enactment might shape the opportunities that teachers have for student mathematical thinking and reasoning. Toward this end, consider we will consider two tasks and discuss their enactment in the class.

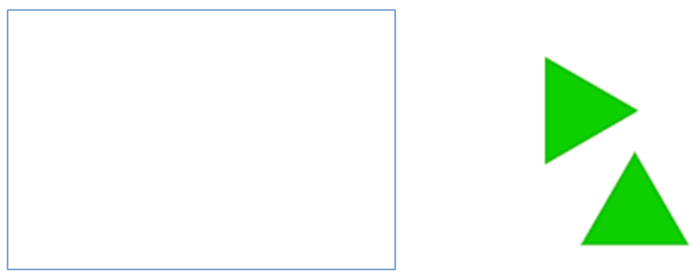
Read carefully the following task and determine its level of challenge (low vs. high).

MATHEMATICAL TASK

Geometry (Grade 2)

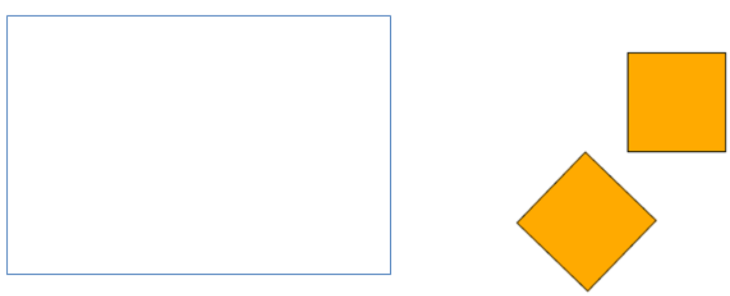
In this task, the teacher gives the students different pattern blocks (equilateral triangles, rhombi, squares, circles, trapezia) and asks them to figure out which can cover a given rectangular surface. An example of a teacher handout related to this task is shown below.

Use the green triangles of pattern blocks to cover the surface below.



What do you notice?

Now, use the orange squares of pattern blocks to cover the surface below.



What do you notice?

Watch the following video clips which refer to the launching and the enactment (student autonomous work and whole class discussion) of the task shown above.

Video clip

Context: We will observe a lesson from a Grade-2 class in Cyprus. The goal of this lesson was to help student to work with non-conventional units of area measurement to help them conclude that the best unit of measurement is the one that corresponds to a square with a size of one unit. The first task in this lesson asked students to compare two given surfaces, one made up of rhombi and another made up of triangles, to help them conclude that to make such a comparison a common unit of measurement was needed. The task shown above was the second in the row and it intended to help students understand that the square was the best unit of measurement for the given rectangular area. The next task was intended to help students investigate the suitability of squares of different sizes. We will watch three clips, one related to the teacher's launching of the task, one related to students' autonomous work, and a third one pertaining to the whole-class discussion.

Link: Isabel's lesson, Launching: 14:00-14:30, 15:00 – 16:30, Autonomous Work: 19:00-20:00, 25:00-26:45, Whole Class Discussion: 26:50 – 27:58, 31:40-32:20

Discuss with your colleagues:

- What is the level of challenge of the task, as presented in the teacher-made materials?
- Is the challenge maintained or does it get changed during the unfolding of this task?
- What are the teacher moves that contribute to the maintenance or the change in the cognitive challenge each time?

Based on your discussion above, working with your colleagues:

Identify some teacher's moves that contribute to presenting and enacting the task at a challenging level.

Launching	Autonomous Work	Whole Class Discussion

Connections to (my) Practice

For our next meeting:

Select a challenging task from your textbook/curriculum materials that is included in one of the lessons you're expected to teach.

Work on this task with your students and videotape its presentation and enactment (student autonomous work and whole class discussion).

Before our next meeting, watch your videotaped lesson, and consider the level of challenge during its presentation and enactment.

Select two short excerpts (from task launching, student autonomous work, or whole-class discussion) that you would like to share with your colleagues. These excerpts should be illustrating either instances in which the cognitive challenge was maintained or instances in which it changed.

Closing Activity

Revisit the four-quadrant diagram in the introductory activity of this module, and consider where you will see your teaching being situated ***during your next lessons***:

- If you situated it in a different spot compared to that of the introductory activity, jot down two things that you learned that helped you make this (even small) shift.
- If you situated it in more or less the same spot, jot down two things you would like to learn in next meetings that you envision helping you making a more substantive shift.

CASE OF PRACTICE 2

Differentiating Instruction to Meet Students of Different Levels

Overview

CONTACT HOURS	2.5 hours
TYPE OF RESOURCES	Videoclips; Tasks; Lesson plan extract; Narratives; Student artefacts
EMPHASES	What differentiation is and what is NOT? What are some key practices of differentiation?

Activities

Opening Activity

Video club Component

In the previous case of practice, you were asked to (a) **select** a challenging task from your textbook/curriculum materials that is included in one of the lessons you're expected to teach; (b) **work** on this task with your students and **videotape** its presentation and enactment (student autonomous work and whole class discussion); and (c) **watch** and **consider** the level of challenge during its presentation and enactment.

Share with your colleagues the two short episodes you selected from your videotaped lesson in which the level of challenge was either maintained or changed. Explain what this episode is about and your rationale for selecting it.

Discuss the shared episodes with your colleagues:

- How did the task you selected unfold?
- Were you able to maintain the challenge?

- If yes, how?
- If not, why?
- What challenges did you encounter in doing so?

Activity 1 – Considering Task Implementation

Below there is a scenario of the unfolding of a task in a sixth-grade classroom. Read the scenario and then discuss in your group the questions that follow.

Narrative (The "Orange juice Task" Episode)

Ms. Sofia is teaching ratios to a mixed-ability sixth-grade class of 18 students. This is the third lesson in the unit on ratios. In the previous lessons the teacher introduced the notion of ratios as a part-to-part and a part-to-whole relationship. The teacher also introduced the different notations used for expressing ratios (e.g., 1 to 3, 1:3, 1/3). In this lesson the teacher is using the following task to give students opportunities to reason mathematically, drawing on the notion of ratio and the various representations they encountered in previous lessons.

The Orange Juice task: Which recipe is the "most orangey"?[2]

Mix A 2 cups concentrate 3 cups cold water	**Mix B** 1 cup concentrate 4 cups cold water
Mix C 4 cups concentrate 8 cups cold water	**Mix D** 3 cups concentrate 5 cups cold water

Ms. Sofia starts the lesson by asking students to recall what was discussed in the previous lesson. About ten students raise their hands and refer to working on ratios. Mary, who is often quick to nominate her ideas, also rushes to clarify the three notations

[2] Source: Connected Mathematics Project

that were introduced on ratios. "Very good!" Ms. Sofia replies, "I am glad that you remember what we did yesterday." Based on this encouraging feedback from the students, Ms. Sofia feels that the class was now ready to move to the next lesson.

Knowing that each new concept needs to be connected to prior work, Ms. Sofia introduces the problem to be considered in today's lesson: "In our lessons so far, we have considered different ways of thinking about and discussing ratios. Today, we will work on a problem that will give you the opportunity to use ratios or other approaches we have considered in previous lessons to figure out the recipe that makes the juice most orangey. Ready? Go!" Ms. Sofia's tone of excitement seems to stir the interest even of the unmotivated students. Students start working on the problem in their groups and Ms. Sofia circulates around.

While circulating, and cognizant of the ways in which a teacher might lessen the demands, Ms. Sofia constantly encourages her students to think hard around the problem and discuss it in their groups: "I am sure that you can do it, if you draw on each other brains!" She is very supportive and keeps her enthusiastic tone, even when realizing that some of the students do not seem to make any real progress on the problem. To her satisfaction, some students seem to make significant progress, since they have compared the recipes using different approaches (part-to-part, part-to-whole or percentages). At the same time, however, other students seem to be stuck. She spends some time to ensure that these students are clear on what the problem is asking them to do and she makes sure that this is the case when she leaves the students to work on their own. In most cases, these students do not succeed in tackling the problem at hand. After allowing students to work for about 15 minutes on the problem she then organizes a whole-class discussion to let them share their ideas.

To illustrate three different ways of solving the problem, Ms. Sofia asks three students to share their work: Michael solves the problem using percentages. He first figures out the total cups in each recipe and then compares the four percentages. Helen addresses the problem by comparing fractions (2/5, 1/5, 4/12, and 3/8) after having converted them into fractions that share the same denominator. George proposes solving the problem by comparing part-to-part ratios (2:3, 1:4, 4:8, and 3:5). However, he is stuck, because he is not sure how to compare the ratios. Mary jumps in and proposes turning all the ratios into "ratios that have one in their first part", explaining that this would mean that the first ratio needs to be converted into 1:1.5. The bell rings and Ms. Sofia tells students that they will continue working on the problem during the next day.

Although exhausted, at the end of the lesson, Ms. Sofia is satisfied that the class has made significant progress since students were able to figure out different ways of working on the problem. In fact, she is even surprised by Mary's proposal to compare the ratios. "We will build on the ideas presented today and move on," she thinks, as she exits the classroom.

Guiding Questions

- To what extent was teaching, as described above, successful or not?
  - What elements do you think contributed to it being more or less successful?
- If you were the teacher, what would you do different, if anything, to make the lesson more successful?

Activity 2 – Teacher Beliefs and Perceptions about Differentiation

The teaching scenario, as described above echoes, an argument regarding undifferentiated or one-size-fits all instruction. Read this argument and then consider the activity that follows.

> "In many classrooms, the approach to teaching and learning is more unitary than differentiated. […] Most teachers (as well as students and parents) have clear mental images of such classrooms. After experiencing undifferentiated instruction over many years, it is often difficult to imagine what a differentiated classroom would look and feel like". Many teachers and teacher educators wonder how we can "make the shift from "single-size instruction" to differentiated instruction to better meet our students' diverse needs. To answer this question, we first need to clear away some misperceptions". (Tomlinson, 2017, p.2)

In the discussions we held with several teachers, the teachers shared with us some beliefs as to what differentiation is and how it can be materialized. Read the teacher beliefs which are provided below and then in your group discuss what your thoughts are on the issues raised by these beliefs, by also drawing on your own practice.

In differentiated instruction, all students are expected to achieve the basic learning objective of the lesson to the same extent.

At the end of the lesson, all students must have reached exactly the same point in achieving the lesson objectives.

All students should be assigned the same homework.

The way that I deal with mixed ability classes is to pair stronger with weaker students and ask them to think, pair, and share.

I know that often times we teach to an imaginary "average" student. I differentiate my approach by assigning harder problems to my more capable students.

One way that I try to differentiate my instruction is by giving students manipulatives to use.

For less-capable students, I am pleased if they answer at least one simple question, to experience some level of success. For average students, I want them to be able to apply what they learned since the lesson is more or less planned for these students. Capable students surely want another question or task to work on once they finish.

Part of differentiation also has to do with creating a classroom culture that accepts and celebrates errors or alternative student ideas.

Differentiation is a utopia, it is another 'fruit' that comes from above, another 'trend' in education that will fade away just like many other educational policies did. What do they [educational policymakers] know about teaching? How can we differentiate instruction for 20 different children? It's so unrealistic!

Differentiation is feasible if you think of different groups of students in your classroom instead of each individual separately.

Differentiating instruction for each student is not possible and that's why many of my colleagues consider it utopian, they are terrified of the idea of too much preparation at home.

Reflection Discussion

- Based on these ideas, what would you describe as differentiation? Why?
- What wouldn't you consider as differentiation? Why?
- What is your stance toward differentiated instruction?

Many teachers often misconstrue the notion of differentiation and/or consider it as another "hot topic" in education. However, teaching all levels of students in a single class can make one realize that differentiation is a sine qua non aspect of instruction. The more we understand what differentiated instruction is about and how it can be materialized, the better we can use it. In the text that follows, we draw on the literature, and specifically on Tomlinson's work, to define differentiation and emphasize its importance.

The WHAT and the WHY of Differentiation

Our memories from typical undifferentiated classrooms during our school years often make us unfamiliar with the notion of differentiation; hence we can hardly imagine how it looks like. Some consider differentiation as individualized instruction, or as a chaotic situation where the teacher "loses control" of students' behavior; others think of differentiation as another way of grouping similar-ability level students together and assigning hard/easy tasks and complicated/simple questions to high- or low- achievers, respectively. But is this really differentiation? To better understand what differentiation is, let's first define what it is not.

Misconceptions about differentiation. Differentiation should not be considered as individualized instruction. Imagine having to create a different customized lesson each day for each of the 20-plus students in a single classroom. Not only is this unrealistic, but it will also be exhausting for you. Moreover, differentiation is not just another way to provide homogeneous grouping; rather it is based on flexible grouping, recognizing that all students might have strengths in some areas and weaknesses in others. Thus, it is not just for students with identified learning challenges or for students who learn rapidly. There are also students in between who struggle in varying degrees and who also need differentiated instruction. By providing more work to some students or less to others does not mean that a teacher has really differentiated his/her instruction.

Then, what IS differentiation? Differentiation is a process of matching learning targets, tasks, activities, resources, and learning support to individual learners' needs, styles and rates of learning. When teachers differentiate their instruction, they do so in response to students' readiness levels, interests, and/or learning profiles. Teachers who differentiate, ***plan lessons proactively*** that provide multiple ways for students to make sense of ideas and knowledge and demonstrate what they learn, always trying to match their student needs. Thus, diagnostic and also systematic formative assessment throughout each unit helps teachers develop an understanding of their students' needs. In the activity that follows, we will learn some ways of designing and enacting a lesson plan in ways that attend to issues of differentiation.

Why do teachers need to differentiate? Regardless of some common features, students of the same age differ in the way they learn, as well as, their interests, personality, and preferences. To learn, each student needs to be challenged at an appropriate level and also feel successful. This cannot be achieved when ignoring student differences. Attending to these differences requires teachers to create several avenues to "get at" learning in an environment that recognizes and celebrates those differences.

Activity 3 – Planning for Differentiation

In a differentiated classroom, the teacher proactively plans to respond to student individual differences. In this activity, we will discuss how a teacher can plan for differentiation through his/her lesson plan. Toward this end, we will consider a lesson plan prepared by Ms. Barbara, a third-grade teacher in Cyprus. In her lesson plan, Ms. Barbara builds on the activities provided in the third-grade's textbook to engage students in developing and implementing algorithms for subtracting three-digit numbers with regrouping. The following lesson plan extract provides some general elements of the lesson. Read her lesson plan on subtraction of three-digit numbers with regrouping provided below and then identify elements/practices of differentiation that might be identified by this teacher's planning.

Lesson plan Extract for 'Subtraction of three-digit numbers with regrouping' task

Lesson plan "Subtraction of three-digit numbers with regrouping"

Tags

Topic: Subtraction with regrouping; numbers and operations
Target group: Grade 3 (lower primary)
Age range: 8 – 9 years old

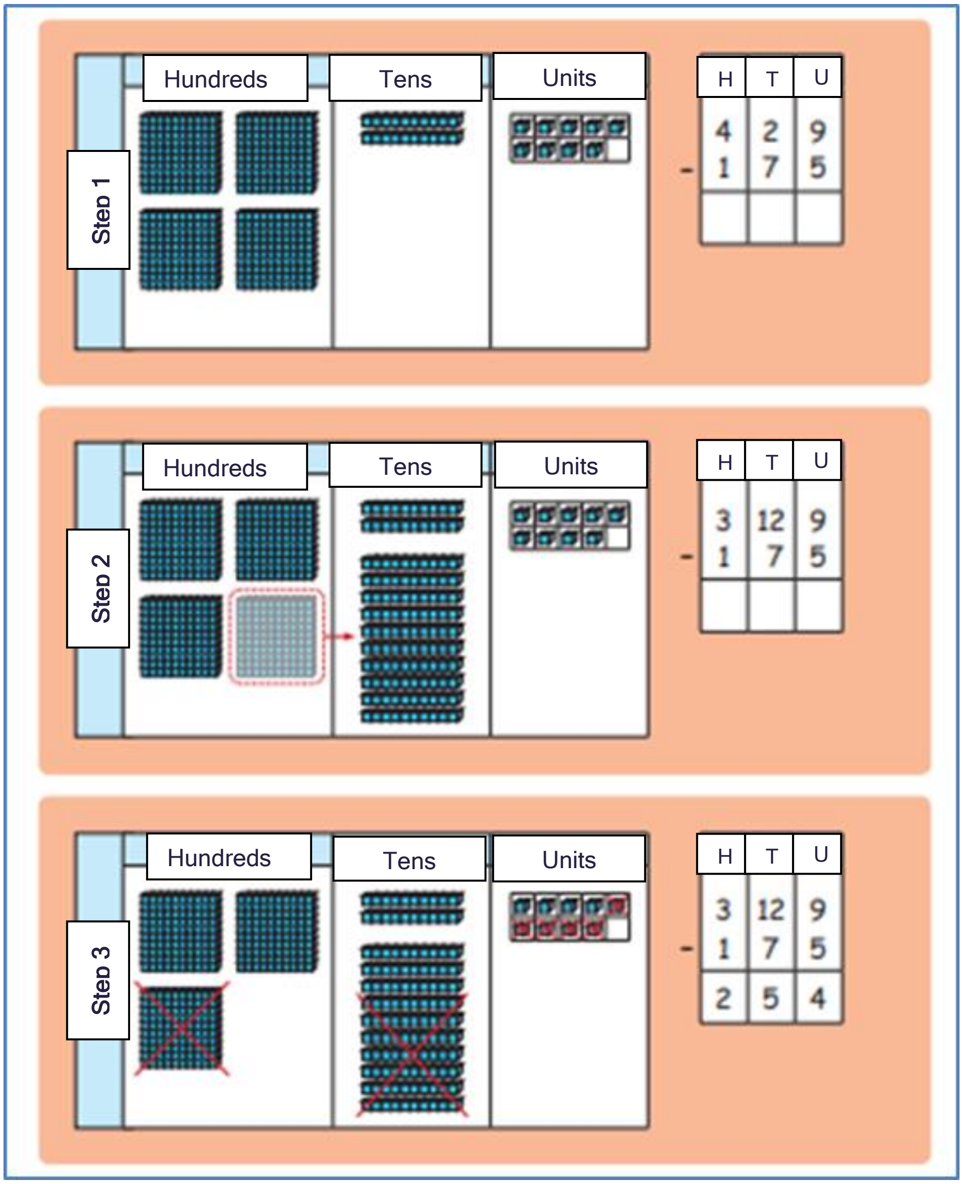
Tasks

Task 1

Using Dienes blocks, calculate the difference 539-184. Explain your thinking in the place provided below.

Task 2

Thalia worked with Dienes blocks, as shown below, to calculate the difference 429-175. Look at the figure below and try to figure out what she did.



Task 3

Calculate the difference, as in the example.

	H	T	U
	~~4~~ 3	~~5~~ ~~4~~ 14	3 13
-	1	8	7
	2	6	6

	H	T	U
	2	7	3
-		4	7

	H	T	U
	3	8	2
-	1	7	5

	H	T	U
	5	1	2
-		7	6

	H	T	U
	5	2	4
-		7	6

	H	T	U
	6	1	3
-	2	7	9

	H	T	U
	7	2	6
-	2	5	8

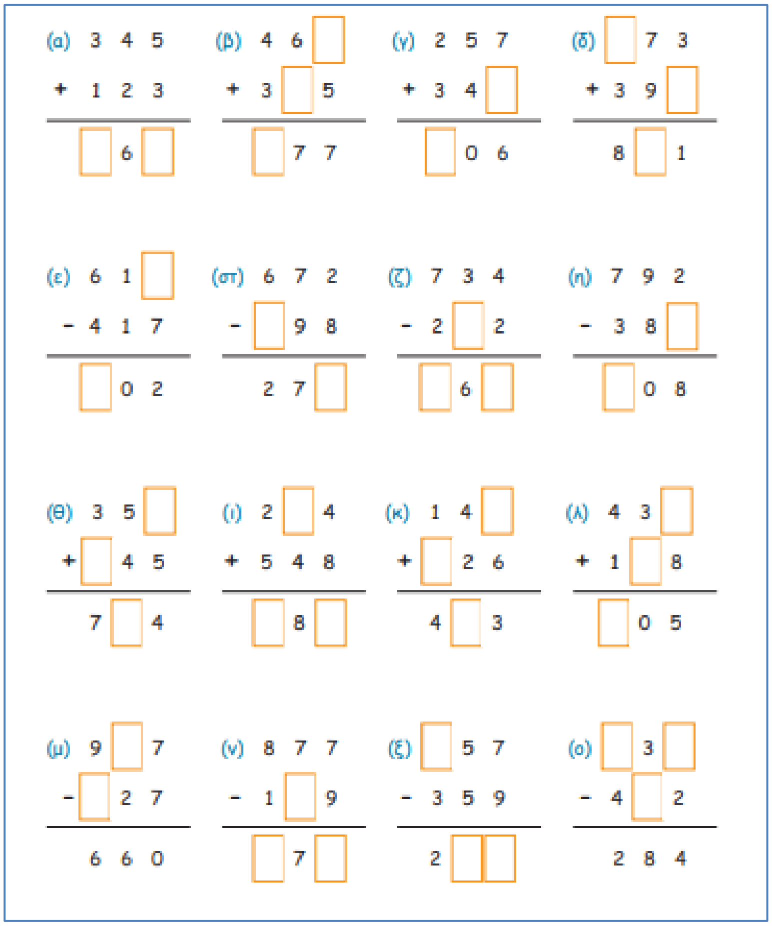
	H	T	U
	7	8	2
-	3	9	4

$$\begin{array}{r} 132 \\ -\ \ 49 \\ \hline \end{array} \quad \begin{array}{r} 627 \\ -348 \\ \hline \end{array} \quad \begin{array}{r} 461 \\ -287 \\ \hline \end{array} \quad \begin{array}{r} 354 \\ -\ \ 78 \\ \hline \end{array}$$

$$\begin{array}{r} 762 \\ -348 \\ \hline \end{array} \quad \begin{array}{r} 616 \\ -286 \\ \hline \end{array} \quad \begin{array}{r} 824 \\ -642 \\ \hline \end{array} \quad \begin{array}{r} 942 \\ -587 \\ \hline \end{array}$$

Task 4

Complete the missing numbers.



Learning Goals

By the end of this lesson, students should be able to:

a) Develop and implement algorithms for subtracting three-digit numbers with regrouping

b) Represent subtraction algorithms with regrouping (using Dienes blocks)

Prior Knowledge

Students are expected to be able to:

- Develop and implement algorithms for subtracting two-digit numbers with regrouping an three-digit numbers without regrouping
- Use Dienes blocks to represent subtraction algorithms with regrouping

Lesson unfolding

Tasks & Learning Activities	Expected Duration	Differentiation
Introduction/ Task launching	6 mins	*The lesson starts with providing a realistic scenario including subtraction without regrouping.* *"The Playtech Consoles store has a unique offer. The new mini Play Station 5 will have a discount of 124 euros, instead of its initial price of 545 euros. How much will it cost after the discount?"* *Students work individually for about 2 minutes in order to propose ways of working to calculate the result.* *Dienes are handed out to students who need the material. The teacher asks children who finish early to think about alternative ways of calculating the outcome. The teacher records the different ways proposed by the students.*
Autonomous work	8 mins	*The teacher introduces Task 1 of the textbook, using the following scenario:* *"The competitor of the Playtech Consoles store, that is the "Electronic" store, sells the same mini Play Station 5 console at the price of 539. It gives a discount of 184 euros. How much will the game console cost in this case?"* *Students are asked to work on Task 1 of their textbooks; they can work using Dienes blocks or any way they see fit. They are asked to explain their thinking.* *Differentiation:* *1. Dienes blocks are provided to students who need the material* *2. The teacher asks students who finish early to think about alternative ways of calculating the result and compare the result of the 2 subtractions. In particular, students are asked to do mental calculations without doing any vertical calculating to compare the two subtractions (alternative ways of finding the final cost). Students may follow different strategies (e.g., 545 and 539 have a difference of 6 while 124 and 184 have a difference of 60).* *3. The teacher scaffolds students during autonomous work if they need help. For example, if students have difficulties when exchanging hundreds and tens, some guiding questions in this case could be the following:* o *How many hundreds does the minuend have?* o *How many hundreds are there in the subtrahend?*
Whole-class discussion	12	*In the whole-class discussion, the teacher introduces the vertical subtraction algorithm presented in Task 2 by simultaneously connecting the Dienes blocks with the three steps of the algorithm. The teacher encourages the students to compare the way of subtraction presented in Task 2 with their own ways of subtraction (during autonomous work).*

		Guiding questions for students who are facing difficulties: o How many tens can we exchange with a hundred? o What makes this subtraction difficult to be solved? Why? What can we do to solve the subtraction?
Autonomous work	8	*The students are asked to work on Task 3 of their textbooks. All students are asked to apply the three steps to solve at least two subtraction tasks. They are then asked to explain their thinking by connecting the symbolic representation with its conceptual underpinnings.* *Differentiation:* 1. The teacher circulates around and identifies students who might face difficulties in connecting the two representations (Dienes blocks and symbolic representation). At the same time, she diagnoses any student difficulties that might occur in the application of the 3-step algorithm. 2. Children who solve more than two subtraction tasks early will be asked to proceed to at least 2 subtractions from Task 4.
Closure/ Assessment	6	*The teacher presents a vertical subtraction that contains an error. Students are asked to identify the error, by applying the 3-step process they've learned in this lesson.* *Students who finish early are asked to create their own subtraction task with a digit missing. Students exchange their tasks and solve them.*

Working in pairs, identify evidence of differentiation practices in the teacher's planning; list any other practices you might be using for differentiation when planning your lessons.

Activity 4 – Scaffolding Students During Autonomous Work

When students work autonomously teachers need to monitor students work and decide which practices to employ to scaffold students' work, without reducing the challenge of the task.

Watch the following video-clips; while watching them try to identify, discuss, and name teacher moves/practices which help in scaffolding students (record these practices at the table included at the end of this activity).

Video-clip 1

Task: Students are asked to symbolically show the subtraction 539-184 with different materials (e.g., Dienes blocks, a small board with magnets, beads on an abacus presenting units, tens and hundreds)

Context: The lesson takes place in a third-grade class in Cyprus. Students have already worked on subtraction of three-digit numbers with regrouping. The teacher asks students to work in pairs to symbolically show the subtraction 539-184.

Transcript (Nicole's Lesson 28:46 – 29:18, differentiating materials; text in red will go into the teacher ed module):

T: Ok, I'll assign each pair a different way to work on the task. Ok? I want each pair to use what I will give them and show in symbols what we've just did [subtraction 539-184]. The subtraction: how many we had; how many we subtracted and what was left. Ok?

S: Miss, in what way shall we show that?

T: Using your hundreds, tens, and units.

Note: During the interview, the teacher says that different ways were assigned to different students based on their ability levels. She specifies that she asked more capable students to work on two different ways of figuring out the subtraction. Less capable students were asked to use Dienes blocks, which is more "tangible, to help them understand the subtraction better". For students in the middle, she gave them the magnets that looked like the Dienes blocks, asking them to show their thinking process on small white boards. In this way, she thought that she "included all learning levels".

Video-clips 2a and 2b

Task: "I want to buy the new mini Playstation 5 for my godson, which costs 415 euros. Today there is a discount of 104 euros. How much will I have to pay for it?" (subtraction without regrouping)

Context: These clips come from a different third-grade class in Cyprus and they are taken from a lesson on subtraction with regrouping. We will be first watch a clip from the very beginning of the lesson, in which the teacher launches an introductory task to her students that involves a subtraction without regrouping. The second clip pertains to students' autonomous work on this task.

Transcript: (Savvia Christou: 1:10-1:41; 2:37-3:55 asking students to solve the task in whatever way they see fit and use whatever materials they want; second clip: differentiating approach according to students' level).

First clip:
T: I'll give each of you a handout that has the word problem that I've just told you, and you have to read it again. Let me remind you: use a straight line to underline the givens, and a curved wavy line to underline the question [of the problem]. Each of you can work using whatever way you prefer to carry out the operation and to find the final price [of the Playstation]. If there is somebody who needs the manipulatives [refers to the Dienes blocks], please let me know, and I will give you the blocks to work with. Ok?

Second clip:
Peter (one of the more capable students of the class): I'm done.

T: Please explain to me how you worked on this task.

Peter: Five minus four is one. One minus zero is one. Four minus three is one [executes the subtraction vertically.]

T: Nice! What are these that you found? Kilograms? Grams? Kilometers?

Peter: Money, dollars.

[Another student interrupts and asks if they can use Dienes blocks.]

T: Michelle, do you find it difficult working without the [Dienes] blocks? If you have difficulties, you can use the manipulatives. […] you can find it next to your desk: the box with the blocks. Whoever has difficulties can use it. [Teacher returns to Peter and Kisa sitting next to him, who is also a very capable student]; I want you to find another way, a different way to figure out the price [of the Playstation].

[Teacher then moves to one of the least capable students and asks him:]

T: Have you read the problem? What was the original price?

Student: 415.

T: And how much was the discount?

Student: 104.

T: Bravo! So, what it the final price? How should I go about finding that? What operation should I carry out? It costs 415 [euros] and I will pay 105 [euros] *less* [with intonation]. Think about it.

Video-clip 3

Task: Students are given six recipes for making a mixture of orange and apple juice and are asked to figure out which of them have the same taste. To support them, the teacher also gives them eight different representations and asks them to match the representations with the give recipes (two of the representations are left unmatched).

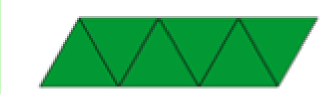
Context: The lesson takes place in a fifth-grade class in Cyprus. In particular, this is the second lesson in a row that introduces students to ratios and it concerns the part-to-part and the part-to-whole relationship. Before the short excerpt to be observed, the teacher asked students to work in pairs to match the given representations with the recipes.

Transcript (Simone's Stavrou (12:24-12:42, differentiating time, extra task)

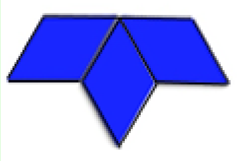
T: Yes, I will give you an extra minute, because I want to help for some more. Those of you who finished with the representations and you're left with some that are unmatched, please turn your paper and write a recipe that matches those representations. Ok? Which recipe [they represent]: either in fractions or in ratios; whatever you prefer.

Video-clip 4

Task: Students are given the following two surfaces and they asked to determine if there is one that is bigger than the other.



Surface A



Surface B

Context: The lesson takes place in a second-grade class in Cyprus. In the clip we will be observing the teacher has already launched the task, giving students the opportunity to explore this problem using different materials (pattern blocks, papers, scissors, etc.) we enter the class when the students work in their groups on the problem and the teacher circulates around, observing their work and scaffolding students. Notice that in the previous lesson, the students work on perimeter, using their rulers.

Transcript (Mary Papamichael: scaffolding, asking questions, providing feedback, 14:10-16:30; and then 17:13- 1734).

S1: I did it using my ruler.

T: Using your ruler? What did you do with your ruler?

S1; I counted how much [inaudible].

T: But are we focusing on what is around, today [implying the perimeter] or the insider?

S1; The inside.

T; Nice, what's inside. How are we going to study what's inside?

S1; If we join many of those (points to the triangles that he holds) and stick them together, it would be as if [inaudible].

T; Show me what you're thinking. Figure out a way to show this to me.

S2; Shall we find how many triangles this [surface] has?

T; Oh! Do we have triangles in both shapes?

S2; In this one [Surface A].

T; What do we have here?

S2; That shape.

T; How do we call it?

S2; [does not respond.]

T; Rhombus. Can we compare these two surfaces as they are right now? [Teacher moves to a different desk.] Do you need any help? What have you done so far? George, you need to use these [not clear what she is point to.]

S3; Miss, I've calculated the perimeter.

T; why do you calculate the perimeter, Christos?

S4; To figure out which is bigger. This is 24. And this is [inaudible].

T; Christos, today we're studying what is around the shape or inside it?

S4; Inside.

T; The area that it occupies, the surface. So, which surface is bigger?

S5; [has placed the one surface on the other]. This one [not clear to which surface the student refers to], because it goes beyond [this other surface] on this side.

T; But this [the other surface] also goes beyond the first one from that side.

S5; In this one [Surface A].

T; [as she walks around] You have some materials on your desks. Make use of them! You also have something else in your green box. Would it help us in any way? [Teacher approaches the students in the desk with whom she worked at the beginning of the clip.] Tell me. What have you done?

Ss; [Inaudible.]

T; you can choose whatever way you think is more appropriate to figure out if one surface is bigger or smaller than the other.

Teacher continues circulating and talking to students.

S6; Miss, I noticed that for Surface B, if we use different shapes, we can still get it.

T; So, you've covered Surface [B] using triangles. How many triangles have you counted?

S6; Six.

T; Six. Take this other surface now and check how many triangles you need for it.

Teacher again moves to a different group.

Record the practices that you identified in the table that follows

Number of Video-clip	Practices on Differentiating Instruction
Video clip 1	
Video clips 2a and 2b	
Video clip 3	
Video clip 4	

Activity 5 – Holding a Productive Whole-Class Discussion

Let's get back to the lesson plan of Ms. Barbara. While students were working autonomously on "Task 1", Ms. Barbara was monitoring her students' progress and making decisions about her next instructional moves. Quite pleased, she noticed that different students developed different solution strategies to solve the task, but there were also several students who had difficulties. She now has to make a very important decision: how to organize the whole-class discussion in a way that allows for differentiation. Read the narrative and student solutions that follow and consider the questions that follow.

Ms. Barbara noticed that 10 out of the 23 students provided a correct solution. Underlying these responses there was substantial variation in terms of how students actually dealt with the problem at hand. In particular, six students performed a vertical subtraction and arrived at the correct response (two of these students explicitly indicated in their response, as shown in Figure 1, while the remaining four did not provide such information, see Figure 2). Four students performed the subtraction horizontally, as shown in Figures 3 and 4. Thirteen of the students gave a non-valid response. Again, Ms. Barbara noticed substantial variation in terms of the underlying reasoning that led to these responses. In particular, three of students added the two numbers (Figure 5). Four other students also arrived at a wrong solution (i.e., 455), though through different paths (see Figures 6-8). Four student responses illustrated a minor slip on the part of students (Figure 9) whereas two students argued that the subtraction cannot be solved (Figure 10).

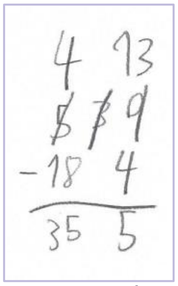


Figure 1

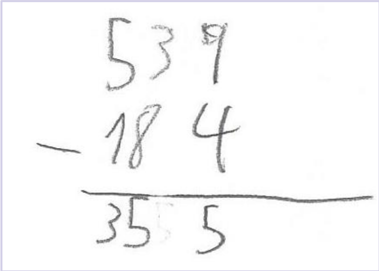


Figure 2

Figure 3

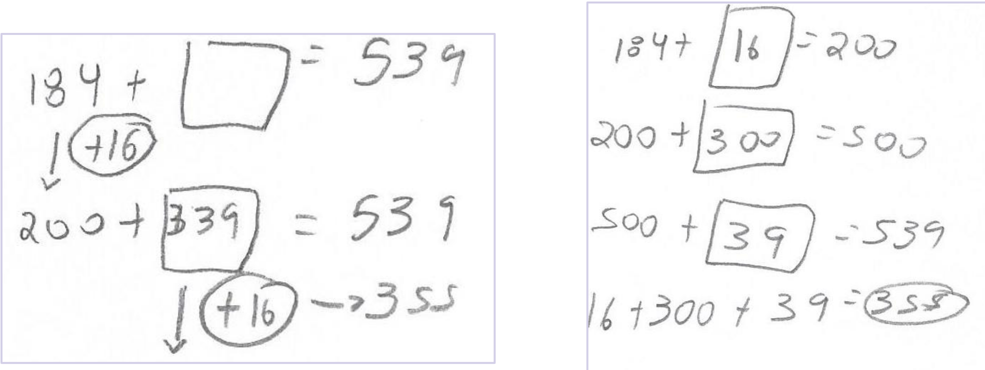


Figure 4

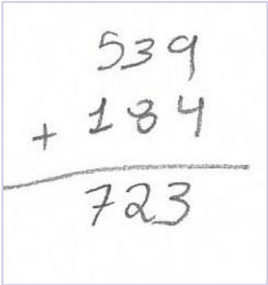


Figure 5

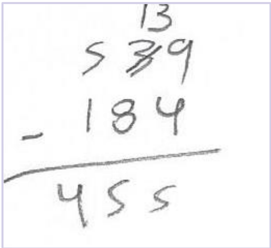


Figure 6

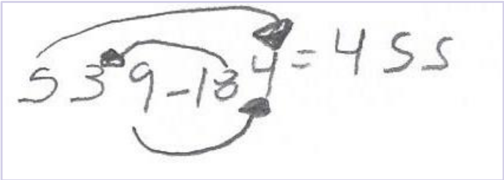


Figure 7

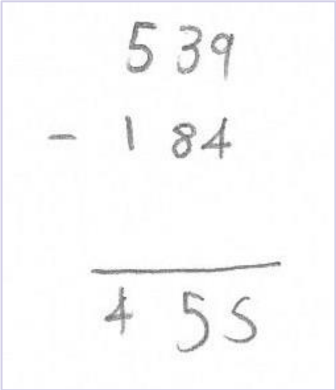


Figure 8

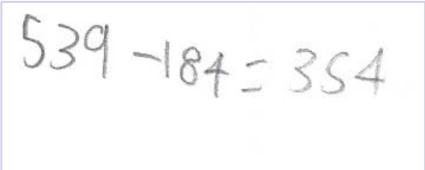


Figure 9

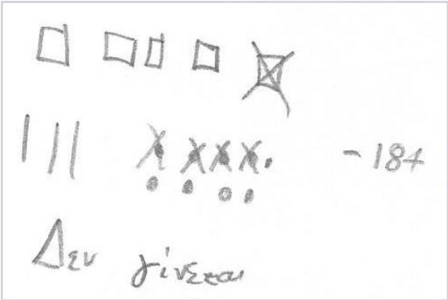


Figure 10

Guiding Questions

- How would you conduct whole class discussion to allow for differentiation?
- Which solutions would you use? Why?
- Would you omit any solutions? Why? Why not?
- How would you organize the whole-class discussion?
- What strategies would you use to differentiate your approach during whole-class discussion?

Based on your answer to the last question, identify the practices you would use for whole-class discussion.

To manage complexity, so far, we have broken working on differentiation into different phases, starting with lesson planning, then considering how to differentiate one's approach while student autonomous work on an assigned task, and ending with how to organize a whole-class discussion that takes into consideration issues of differentiation. You will learn more about differentiation in each of these phases in Modules 2, 3, and 4, respectively.

Connections to (my) Practice

Design a lesson that takes into consideration differentiation practices as discussed and codified in our meeting (at least one from planning, autonomous work, and whole-class discussion).

Teach the lesson and select two video clips one from student autonomous work and another from whole-class discussion that are illustrative of your attempts to differentiate your approach (regardless of how successful these attempts were).

Closing Activity

Discuss with your colleagues:

- Which from the practices identified above do you consider the most important for supporting your work?
- How would you incorporate them in your next teaching attempts?

CASE OF PRACTICE 3

Concurrently Attending to the Work around Challenging Tasks and Issues of Differentiation

Overview

CONTACT HOURS	3 hours
TYPE OF RESOURCES	Videoclips; Tasks; Narratives; Interview excerpts
EMPHASES	• When and how working on challenging tasks and differentiation might work harmoniously, ensuring that all students are engaged o The Role of Questioning

Activities

Opening Activity

Video club Component

In the previous case of practice, you were asked to (a) **design** a lesson that takes into consideration differentiation practices as discussed and codified in our previous meeting (at least one from planning, autonomous work, and whole-class discussion), (b) **teach** the lesson, and (c) **select** two video clips, one from student autonomous work and another from whole-class discussion, that are illustrative of your attempts to differentiate your approach (regardless of how successful these attempts were).

Share with your colleagues the two short episodes you selected from your videotaped lesson. Explain what these episodes are about and your rationale for selecting them.

Discuss the shared episodes with your colleagues:

- How did the task you selected unfold?
- How successful was your attempt to differentiate your instruction?

- Were you able to maintain the cognitive challenge for all students?
  - *If yes, how?*
  - *If not, why?*
- What difficulties did you encounter in doing so?

In case of practice 1 of this module, we focused on challenging tasks – what it means and what it entails – recognizing the importance of engaging students in challenging mathematical tasks for advancing their problem solving and reasoning skills. Then, in the second case of practice, we considered how, as teachers, we can differentiate our instruction and structure learning environments that take into account diverse student needs and readiness levels in order to maximize the probability for *all* students being engaged in challenging work. In the third and final case of practice of this module we bring these two constructs, *working with challenging tasks,* on the one hand, and *differentiating instruction,* together.

Activity 1 – Working on Challenging Tasks and Differentiation

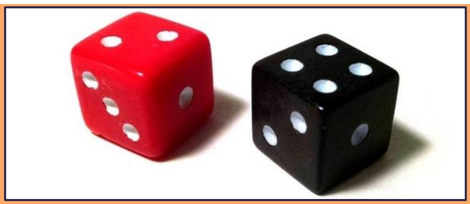
Watch the following video-clips. Each presents a different classroom episode centered on the enactment of a different task. The tasks and the context associated with their enactment is described in the boxes below. While watching the video clips focus on how *differentiation* and *students' engagement in cognitively challenging work* manifest themselves in the classroom. In doing so, consider the following guiding questions.

Guiding Questions

- What differentiation practices did you identify?
  - *Were they implemented effectively?*
- What was the cognitive level at which students were expected to work?
  - *What evidence is there about the cognitive level at which students actually worked?*
- Did you identify any issues or problems related to the teacher's attempt to work on both fronts: challenging tasks and differentiation?

Video-clip 1a and 1b

Task: Roll the 2 dice. Look now. Look away. How many dots did you see? Look again. What is the sum of the numbers? Repeat 3 times.



1. What other sums can you make by rolling the two dice?
2. What is the smallest sum you can make with the dice?
3. What is the largest sum you can make with the dice?
4. What are all the possible sums you can make with the dice? How do you know?

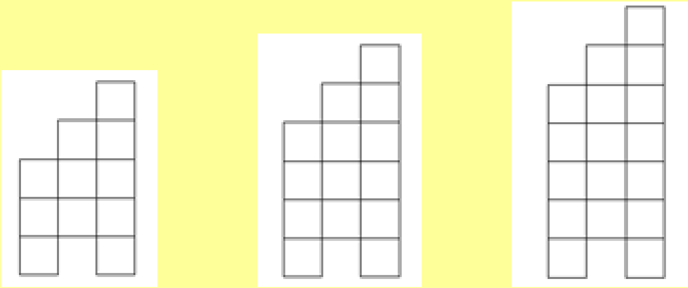
Context:
(1a) This segment is from the introductory part of the lesson. The teacher is going through what each part of 'The Dice' task involves with the whole class group. This videoclip specifically relates to parts three (finding the smallest possible sum of two dice) and four (finding the largest possible sum of two dice) of the pre-primary task. Following on from this videoclip the teacher goes through the remaining parts of the task before the children begin to work on the task autonomously.
(1b) Children are working autonomously on the task. The teacher is going around to students individually/in pairs and engaging them in math talk about what they are doing. In this particular videoclip the teacher is asking the students about what all the possible sums they can make might be. When she gets back a number sentence from each of two children she asks one student if she could do 1 + 2 in a different way. The child physically moves the dice around and says " 2 + 1 altogether makes three".

Transcript: XXX

Link: 1a) Carole's lesson 12:26 - 13:30 (smallest sum) and 13:30 - 14:25 (largest sum) 1b) Carole's lesson 20:40 - 21:34

Video-clip 2

Task: "**Making ... Chairs**"
Christopher plays with his LEGO and makes "chairs" of different sizes, as shown in the figure below.



Size 2 **Size 3** **Size 4**

(a) What form will the chair size 5 have? How many cubes will he take to make it?
(b) What form will the chair size 6 have? How many cubes will he take to make it?
(c) What form will the chair size 1 have? How many cubes will he take to make it?
(d) What form will the chair size 100 have? How many cubes will he take to make it?

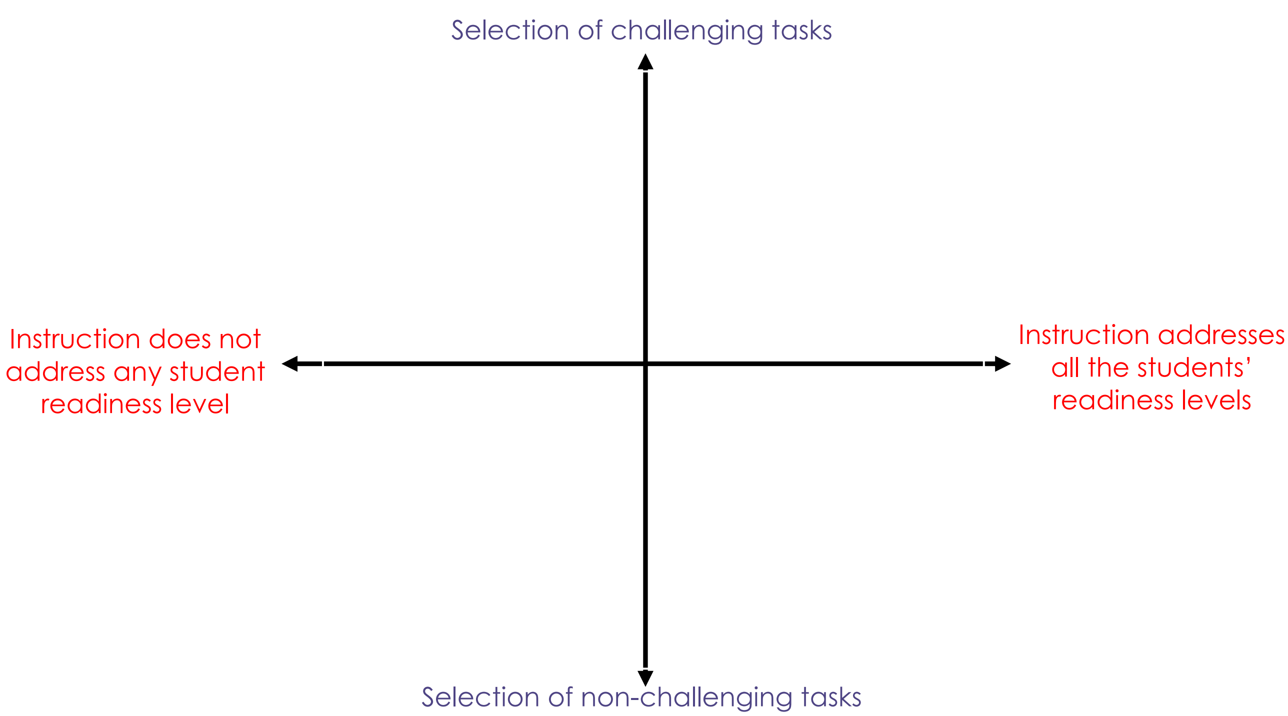
Context: This clip comes from a 5th-grade lesson that was conducted by Mr. Antoniou towards the end of the school year. In this lesson, the students worked on the 'Chairs' task from the algebra unit. In this clip, we are watching an excerpt from the whole-class discussion that was held after the students had autonomously worked on question (d) of the task. The students are asked to share their answers and then they discuss how their solutions emerged. The discussion unfolds around the fixed and changing parts of the pattern. Before the end of the lesson, the teacher encourages the students to think of a 100,000-size chair and consider how many cubes it will have and also, explain how they found it out.

Transcript: XXX

Link: Charalambos's lesson 37:00 -mexri to telos

Activity 2 – Teachers' Challenges and Difficulties When Trying to Engage All Students in Cognitively Demanding Work

Let's now focus on video-clip 2. Think of where you would position Mr. Antoniou's teaching in the two-dimensional space shown below, with the vertical axis representing the extent to which challenging tasks are selected for use in the lesson and the horizontal axis corresponding to the degree to which instruction addresses all the students' readiness levels.

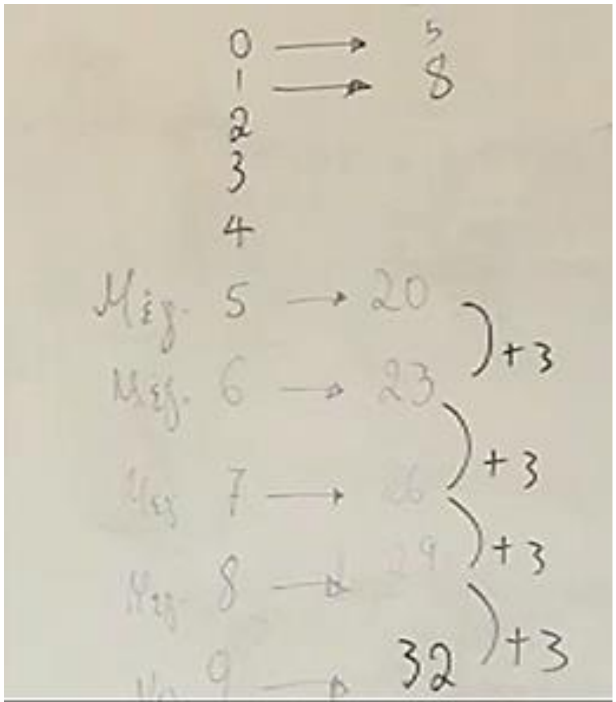


Guiding Questions

- Why have you positioned this teacher's teaching in this particular spot?

When discussing with Mr. Antoniou, we asked him to comment on this particular episode. He shared with us some thoughts about the difficulties and challenges he faced while trying to concurrently work on challenging tasks and differentiation. Read the teacher thoughts in the interview excerpt that follows and try to identify specific difficulties and challenges he faced during the planning and/or the enactment phase of the lesson; then in your group discuss your thoughts on the issues raised by this teacher, by also drawing on your own experience.

"The different levels of students were an issue. Some [kids] were more advanced than the rest of the class, others were far behind, and others faced too many difficulties. At some point I was afraid that I would not be able to manage these different levels and 'speeds'. Indeed, the different pacing that students follow based on their capabilities makes working with challenging tasks with all the students a very demanding task for us [teachers]. Surprisingly, some students finished earlier than I expected… So, I assigned them something else to do; for example, I asked them to consider the next subtask or think of a different way for solving a subtask. Fortunately, I was prepared [for multiple student solutions] … I had thought different solutions from the previous night so I could attend to the students' thoughts and follow their reasoning. Towards the end of the lesson [during whole-class discussion] because I realized that we had very little time left, I did some of the thinking for the students. I did not realize that this was in issue while teaching; but having then watched the video of this lesson, I realized that this was an issue! When I wrote the numbers of the first five terms of the pattern on the classroom board [see Figure below], I saw that some students, who previously [during autonomous work] had difficulty with thinking of how many cubes there will be in chair [term] 1, were very eager to answer... When I watched the video, I realized that I fall into the 'trap' of somehow telling them the answers".



Snapshot from the classroom board

<u>Translation</u>:
Μέγ.: Size

Guiding Questions

- What difficulties and challenges did this teacher face in selecting and using challenging tasks to productively engage all students in mathematical thinking and reasoning?
- Did you experience similar challenges/difficulties during your teaching?

In the discussions we held with several teachers, the teachers shared with us some difficulties and challenges they face while trying to plan or enact a lesson that promotes both student cognitive engagement and differentiation. Read the teacher thoughts in the interview segments provided below and then in your group consider the questions that follow.

Teacher Difficulties and Challenges

> I had to think of the time that I would let kids to work on their own, so that no one would end up being bored and those who needed a great deal of time to finish the task would have enough time to think and work. For how long should I let students struggle? I wasn't sure how long I needed to stay with each student. I'm always in a hurry and I am afraid that sometimes I end up telling [students] the answers.

> I do not understand very well what the task is about, what the goal for its inclusion in the textbooks is.

> As a teacher you also need to have a very good understanding of the content and of how to teach certain things. You have to know very well what is behind each mathematical concept [to effectively work on such tasks].

> I had a difficulty figuring out the right questions that could lead students to the targeted generalization. Or how to guide them without telling them what to do, or 'closing up' the discussion, and [thus] minimizing their thinking?

> Students completely went wild from the moment they got the materials. I tried to get them back by using routines, but I didn't manage to do it. So, we had frequent interruptions, the flow of the lesson was not smooth, students did not pay attention, we didn't focus on the critical points of the task, and we went out of time.

Reflection Discussion

- How do you see differentiation and working with challenging tasks go together?
- Which do you think are the biggest challenges and difficulties that a teacher might face in planning and implementing a lesson that aims to work at both fronts? Why?

Differentiating mathematics instruction so that the task complexity remains to an appropriate level for all students can develop and consolidate students' understanding. The constructs of *Ensuring that students engage with challenging tasks* and *Differentiating teaching,* work in tandem. These two constructs bear an interactive connection; teacher's moves or actions intended to promote each of these constructs necessarily reflect on the other as well. Any attempt to ensure that all students are engaged in challenging tasks requires anticipating the diversity in the class in terms of levels of readiness and planning differentiated instruction to accommodate this diversity. In a similar manner, it is not possible to effectively enact differentiated instruction unless the tasks used in the classroom are purposefully selected to ensure appropriate fit between the level of the cognitive challenge incurred by the tasks, on the one hand, and the corresponding level of student readiness, on the other. Read the following text which elaborates more on this idea using the gear metaphor to explain how these constructs function together and then, comment on this idea.

Working on Challenging Tasks and Differentiation: The Gears Metaphor

Working with challenging tasks and differentiating instruction can be parallelized to a set of gears – each construct corresponding to a gear shape – that work together to enhance student learning (interlocking constructs), as shown in the figure that follows. The function of this set of gears depends on the coordination or the balance between the two gears; if the one does not work well, we cannot have reasonable expectations for the promotion of the other. If the one gear has a malfunction and stops moving then the other gear will stop working as well.

These two constructs work in tandem; teacher's moves or actions intended to promote each of these constructs necessarily reflect on the other as well. Any attempt to ensure that all students are engaged in cognitively challenging tasks requires anticipating the diversity in the class in terms of levels of readiness and planning differentiated instruction to accommodate this diversity. In a similar manner, it is not possible to effectively enact differentiated instruction unless the tasks used in the classroom are purposefully selected to ensure appropriate fit between the level of the cognitive challenge incurred by the tasks, on the one hand, and the corresponding level of student readiness, on the other. By giving undifferentiated hints to students and more or less degrading the level of challenge for students, the teacher fails in promoting both constructs.

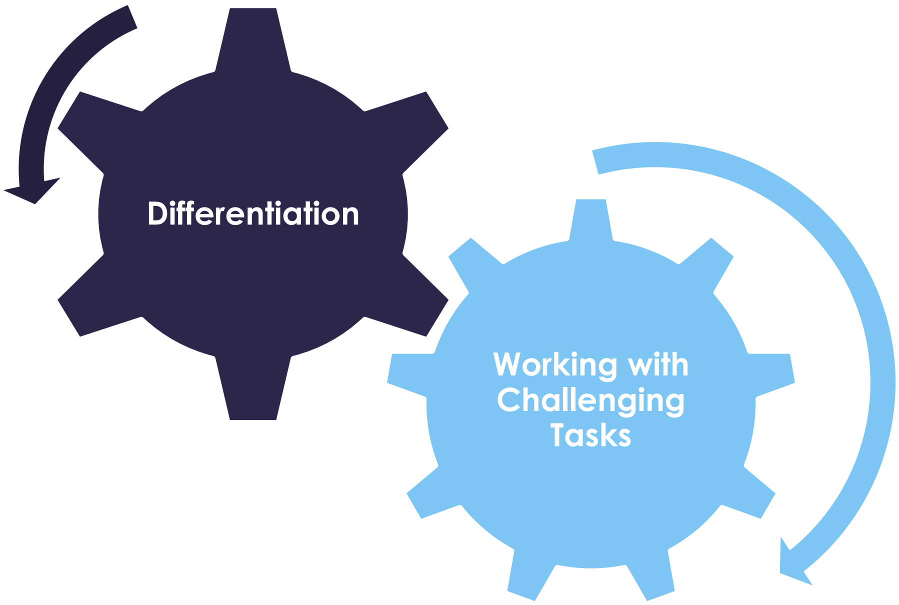


Figure 1. *Working with Challenging Tasks and Differentiation: Illustrating the Synergy*

Reflection Discussion

- Can this representation fully describe the relationship between working with challenging tasks and differentiation?
  - *Which aspects of this relationship does it describe successfully, and which does it not?*

Every set of gears may not be able to move smoothly due to several reasons, such as tooth wear and premature failure. Similarly, as discussed in activity 2, teachers appear to face various difficulties and challenges when trying to simultaneously work on cognitively challenging tasks with all their students. At the same time, just like good oiling of the gears helps the system works much better and more functionally, there are certain instructional aspects that can support teachers' attempts to work on both fronts, if executed appropriately. In the rest of this case, we will work on one of these aspects that both research and our work with teachers have suggested to be important for promoting either of the two goals considered herein–and thus, we envision to play a pivotal role to the smooth and appropriate functioning of this gear system.

Using Questioning to Facilitate the Gear System Function

Scaffolding is another metaphor borrowed from the field of construction, where a scaffold is a temporary structure that helps with the building of another structure. In education, it refers to the temporary support provided for the completion of a task that students otherwise might not be able to complete. This support can be provided in a variety of manners, such as modeling, feedback providing, or questioning.

Questioning is a key instructional aspect related to scaffolding. For example, in every lesson, teachers ask students several questions. Simply posing questions, however, does not automatically lead to cognitively challenging students, let alone all of them. Some of the questions ***can either facilitate or (temporarily) block the movement of both gears*** depending on their characteristics and what they require from students to do.

Teachers who participated in other phases of the EDUCATE project, pointed to ***the difficulty they have in asking good questions which cognitively challenge students and differentiate instruction at the same time***; such difficulties are reported more explicitly or tacitly in the literature, as well. Hence, a teacher could reasonably wonder, "What questions should I ask to stimulate student mathematical thinking when working on challenging tasks? How can I do this for all my students?" The activities that follow aim at investigating how questioning can support the movement of the 'gear system' considered above and thus, help concurrently work at both fronts.

Activity 3 – Using Questioning to Scaffold Students' Work

In this activity, we will peer inside Ms. Nicole's classroom, a fourth-grade teacher at a small provincial primary school in Ireland. In today's lesson, she is using the task 'Fractional Triangles' to explore with her students the relationship between the part and the whole using a spatial representation of fraction. You will be provided with some narratives; in these narratives, we are observing the interactions among Ms. Nicole and her students at different phases of this lesson. At different junctures, you will be asked to consider the teacher' interactions with students and her questioning or write what questions you would pose to cognitively challenge the students and differentiate instruction. Read the learning objectives, the task and the narratives; and each time consider the questions that follow.

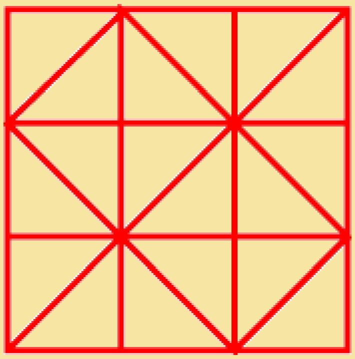
Learning Objectives:

By the end of this lesson, students should be able to:

- Recognize the part of a unit/shape when the fraction of the part is given.
- Divide shapes/units into equal parts when the fraction of the part is given.

Task 'Fractional Triangles':

1. Use the lines on this figure to show how the pattern of triangles can be used to divide the square into two halves, three thirds, six sixths and nine ninths.



2. More lines are needed to divide it into four quarters. What is the least amount of lines needed to do this if the quarters are in one piece and all the same shape?
3. How many ways can you divide it into halves using just the lines given?

Source: https://nrich.maths.org/2124/note

Narrative 1a (Launching the task 'Fractional Triangles')

Ms. Nicole thought that this problem could be used as part of a lesson on finding fractions of various shapes. It was a good opportunity to enable children explore fractions using an open-ended and challenging task to develop an understanding of the relationship between the part and the whole. She was sure that her students were all familiar with the notion of fraction, as well as, with naming and sorting fractions. However, students' visualizations greatly varied and she thought that this task and its representation may prove very difficult for some and yet readily accessible to others.

The school bell rang and all students came into the classroom and took their seats. The lesson began by displaying the design to the entire class on an interactive whiteboard and giving out a worksheet (with the task on it) and then, inviting students to "think-pair-share", that is to talk to their neighbor about what they saw on the first page for a minute and then share their thinking in a whole-class discussion. After about a minute, she went through of initiating a discussion around the task by asking a couple of questions.

(a) If you were in Ms. Nicole's shoes, what questions would you pose at this point to engage all students in challenging work and differentiate instruction at the same time? Why?

- *In which order would you pose these questions? Why?*

Below is how Ms. Nicole decided to handle this situation.

Narrative 1b (Launching the task 'Fractional Triangles')

Ms. Nicole: OK class. First, it is important to make sure that we all share a common understanding of the task. Also, we should see how this relates to what we have been doing in previous sessions on fractions. So… What are we asked to do here?

Alicia: We need to divide the square into halves, sixths and ninths.

Ms. Nicole: Let us assume that we were asked to divide it into thirds. How would we do go about dividing the square into thirds?

Alicia: That is easy. We can just use the four vertical lines. They help us in dividing the square into three thirds.

Ms. Nicole: What do the rest of you think?

The majority of the class: Agree.

Ms. Nicole: Any other way of doing this?

Maria: We can also use the four horizontal lines. Same result.

Ms. Nicole: Great! Can you count the number of triangles in each of the three pieces?

John: There are two triangles in each of the small squares. This gives us six triangles in total.

Ms. Nicole: So. Each third contains six triangles. Can you infer without counting the number of triangles in the large square?

Melanie: There should be eighteen triangles.

Ms. Nicole: How come?

Melanie: It is just three times six.

Ms. Nicole: Great! Provided that one third equals six, three time six yields eighteen. It is what we have learnt about fractions.

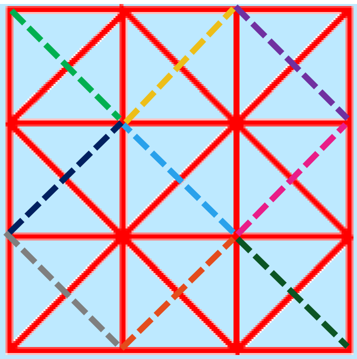
Guiding Questions

- To what extent do you believe that Ms. Nicole managed to cognitively engage the majority of the students in this episode? Why?
- What would you do differently in order to increase the number of students who became cognitively engaged with this task?
  Ms. Nicole stated that she would like to address two goals: (a) ensure common understanding of the task by all students and (b) elicit prior knowledge. Assess the appropriateness of the questions she posed with respect to this dual goal.

The class then continued working on the task.

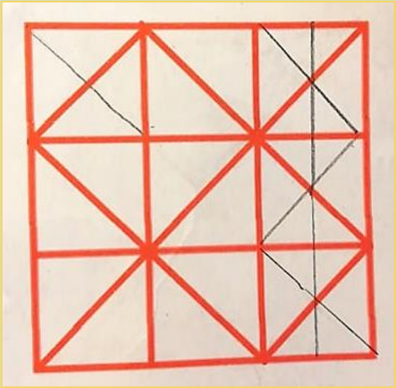
Narrative 2a (Student Autonomous work: Different approaches for solving the task)

Students were working autonomously on solving question 2, while Ms. Nicole was circulating around monitoring what they were doing. In the 'Dolphins' group, a smaller group of students, Melanie, Maria and David, seemed more confident on what they were doing: they decided to work with the thirds and further divide each into four quarters, to find the minimum number of lines needed to divide the square into fourths; by drawing nine lines they would have twelve pieces in each third and they could further group the twelve pieces in four groups of three.



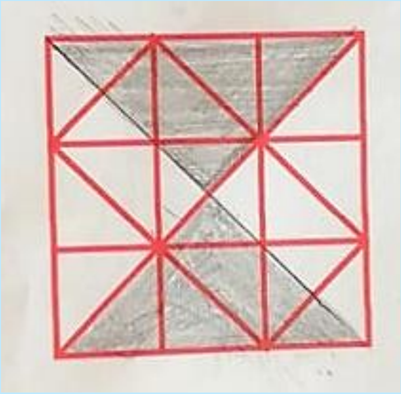
Melanie, Maria, and David's solution

On the other hand, another pair of students, Elena and Mario, who were seated in the same group with Melanie and Maria, seemed to be lost. They were drawing lines working in an unsystematic way and they didn't know what to do.



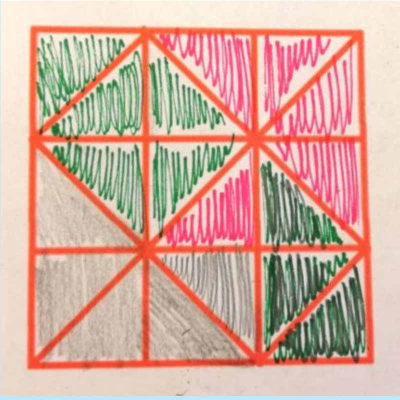
Elena and Mario's solution

In the next group, the 'Jellyfish' group, Connor and Sophie had already finished with question 2: they created the quarters by drawing the second diagonal of the square.



Connor and Sophie's solution

Two children of the same group, Adam and Seán, tried to group the small triangles into four groups. One of the groups they created had fewer triangles than the rest of the groups.



Adam and Seán's solution

Ms. Nicole decided to head to these groups and have a discussion with the students.

(a) What questions would you pose to engage these group of students in challenging work and differentiate instruction at the same time? Why?

- *In which order would you pose these questions? Why?*

This is how Ms. Nicole decided to manage student autonomous work.

Narrative 2b (Student Autonomous work: the students seek to identify the least amount of lines needed to divide the square into four quarters)

Group 1 ('Dolphins' group: Melanie, Maria, and David): the students who divided the design into quarters but did not use the least amount of lines

Ms. Nicole: So, what have you done so far?

Melanie: We agreed that we cannot do this using the lines already in the square. We thought that we should work with the thirds and further divide each into four quarters.

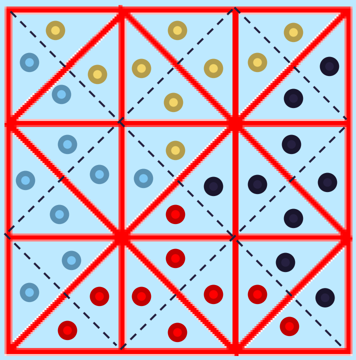
Ms. Nicole: How would that be of any help?

Maria: This will give us three times four; that is twelve pieces. Twelve is a multiple of four and we can divide these twelve pieces in four groups of three.

Ms. Nicole: That is really nice! You drew nine additional lines. Not bad. Though, the goal was to come up with the least amount of lines. Could we do this with fewer lines? I would like to leave you with this question and get back to you in a while. Think about it. Way to go.

Pair 2 ('Dolphins' group: Elena and Mario): this pair includes some of the students in the lowest level of readiness.

Ms. Nicole: So, what have you done so far?
Elena: We are really struggling with this. No clue whatsoever.
Ms. Nicole: What is the task about?
Mario: We need to find the least amount of lines needed to divide the square into four quarters.
Ms. Nicole: Well I am sure you can do this. Though, as the time presses I will offer some help. There is an easy way to do this. There are 18 triangles. Draw diagonals to divide each triangle into two small triangles. This will give you 36 triangles. Dividing these into quarters will get nine small triangles in each quarter. There you go! No just count the lines you had drawn.



Teacher's suggestion to Elena and Mario

Pair 3 ('Jellyfish' group: Connor and Sophie): this pair includes some students who finished question 2 early

Ms. Nicole: Hi, Connor and Sophie! What are you doing here?
Sophie: We have already finished with the second question. It was sooooo easy! Can we work on question 3?
Ms. Nicole: Oh! You've finished already! How did you work on that, Connor?
Connor: We drew the second diagonal of the square. So, we divided the square into four quarters which are "in one piece and all the same shape".
Ms. Nicole: Nice work. Was there anything that made it difficult for you?

Connor: Yes… at first, we thought that it was impossible [to divide it into quarters].
Ms. Nicole: Why?
Connor: Because the triangles were eighteen which is not a multiple of four.
Sophie: And then we decided to draw the diagonal to make the shape symmetrical.
Ms. Nicole: Very good ideas. Bravo! So, now go to question 3.

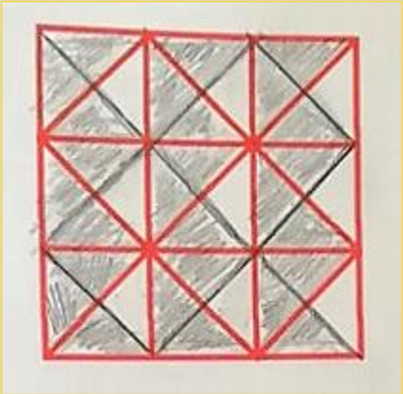
Pair 4 ('Jellyfish' group: Adam and Seán): the students who got stuck
Seán: Ms. Nicole, could you please come?
Ms. Nicole: Yes, tell me, Seán. Is there a problem?
Seán: We do not know what to do… We got stuck.
Adam: Here is what we did… [pointing to their solution]
Ms. Nicole: Did you group the triangles into four groups?
Seán: Yes.
Ms. Nicole: How many triangles are there in each group?
Seán: There are five triangles, but there is one [group] which has only three triangles.
Ms. Nicole: And all the triangles are…?
Adam: Eighteen!
Ms. Nicole: So, why don't you make the groups equal and then see what's left over?
Seán: Hmm… if we delete some we can have four groups of four triangles…
Ms. Nicole: Is there a way to divide the two triangles that excess?
Adam: I don't know…
Ms. Nicole: Think of it and we will discuss it together in two minutes.

Returns to first group
Ms. Nicole: Any progress on this?
Melanie: Not really.
Ms. Nicole: Just a tip. So far you have worked with the thirds. How about working instead with the smallest possible unit that you can discern in the various shapes comprising the square.
Maria: Yes. Let's think about the smallest triangles.
David: How many triangles are there? There are two triangles per small square. There are nine squares so there should be eighteen triangles.
Maria: This does not sound helpful. It is not a multiple of four.
Ms. Nicole: What do you think Melanie?

Melanie: I agree with David. However, like we did before we can now divide each--imagine splitting each triangle into two smaller triangles. This would give us 36 small triangles. We could work with that. It is a multiple of four.

Ms. Nicole: Very nice. Now think about the extra lines you should draw to divide each of the original triangles into two smaller equal triangles. Let us hope you will come up with a number smaller than nine.

The students eventually come up with a set of five lines that affords to divide the square into four quarters (see the figure below).



Melanie, Maria, and David's final solution

(a) The teacher worked with two different grups of students. Compare her interaction with these two groups in terms of promoting cognitive engagement.

(b) Obviously, the students in the second group were not well positioned to address the task. How, would you change Ms Nicole's approach to helping these studengts? Come up with a set of questions that could somehow scaffold students' attempt to eventually tackle the task.

At this point, the teacher decided to have a whole-class discussion.

Narrative 4 (Initiating a Whole-class Discussion)

Ms. Nicole realized that the time has passed and they should now have a whole-class discussion to share their solutions. She noticed that during autonomous work on question 2 students worked in various ways: some considered a very complex solution -they had worked with the thirds and further divide each into four quarters; others divided the triangles into four unequal groups; other students were struggling with the task, whereas some students finished early. Ms. Nicole wanted to engage all students in a meaningful whole-class discussion so that they all have the opportunity to discuss mathematics with one another, refining and critiquing each other's ideas and understandings. How should the discussion be organized and what questions should she posed to her students?

(a) What questions would you pose at this point to engage all students in challenging work and differentiate instruction at the same time? Why?

- *In which order would you pose these questions? Why?*

Which characteristics should good questioning have to cognitively challenge students and differentiate instruction at the same time? Discuss these characteristics in your group and record them below.

Connections to (my) Practice

Optional:

Design a lesson that takes into consideration the role and the characteristics of questions that promote or hinder both differentiation and cognitive activation at the same time as discussed and codified in our meeting.

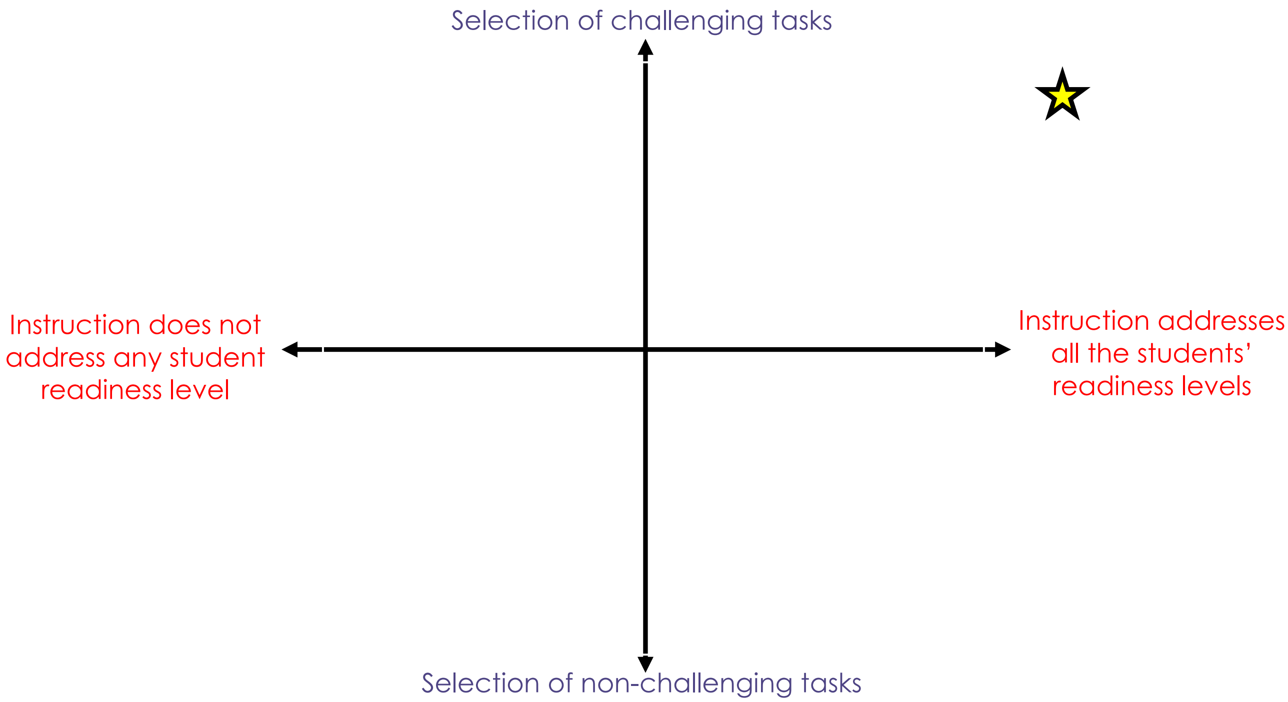
Teach and videotape the lesson and select two video clips one from student autonomous work and another from whole-class discussion that are illustrative of your attempts to pose appropriate questions to promote both differentiation and cognitive activation (regardless of how successful these attempts were).

ENGANGING ALL STUDENTS IN COGNITIVELY CHALLENGING MATHEMATICS TASKS: MISSION IMPOSSIBLE?

Scenario

One of your colleagues attended a workshop on improving the quality of mathematics instruction. On coming back and debriefing her experience, she argued that **it is important to engage *all* students in mathematical activities that require mathematical thinking and reasoning–what she called challenging tasks**. A lively discussion ensued among the teachers, with several teachers doubting the extent to which this goal is realistic, especially for their mixed-ability level classes. Given how complex the work of teaching is and that for years teachers have been bombarded with several ideas like the one presented above, what is your take on this issue?

Think of where you would position your teaching in the two-dimensional space shown below, with the vertical axis representing the extent to which you select challenging tasks for use in your teaching and the horizontal axis corresponding to the degree to which your teaching addresses all the students' different readiness levels.



Discuss with your colleagues:

- Why have you positioned yourself and your teaching in this particular spot?
- What would you identify as the main challenges that you, as a teacher, or teachers in general, would face in selecting and using challenging tasks to productively engage all students in mathematical thinking and reasoning [this situation is shown with a star (*) in the diagram above]?
- Even if it were possible to engage all your students in such type of work, do you think that it would support students' learning? Why? Why not?

Hands on Activity for Secondary

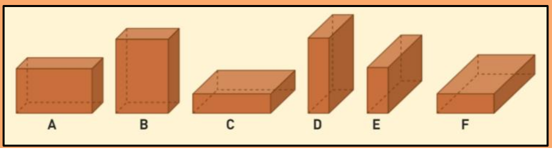
Watch the following video clips which refers to the tasks above.

Videoclip 1

Context: In this clip, we will watch a Grade-10 class in Portugal working on the "Boxes" task. Before this clip, students had worked autonomously in solving the first part of the task (part a). This segment concerns the interaction of the teacher with a group of 3 students, who had solved the task by computing the volume of the container and the volume of the box and then figuring out how many times the volume of the box fits in the container without paying attention to the restrictions put by the linear dimensions of the boxes. The teacher faces the issue of leading the students to realize this restriction as they formulate their strategies for solving the problem.

Task:

Consider one container with a width of 2m, a length of 4m and a height of 2,5m to transport boxes with the shape of rectangular parallelepipeds and with the following dimensions: length 70cm; width 50cm and height 30cm. Suppose that the boxes can be introduced in the container in any position, as the figure shows:



Videoclip 2

Context: We will watch short snippets from the launching of this task and its enactment with a group of students in an 8th-grade mathematics class in the USA. The video was drawn from the TIMSS 1999 video study. The task was enacted in the context of an introductory lesson on exponents. In the first part of the lesson (which lasted for about 38 minutes), the teacher worked with the students in deductively figuring out the basic rules for exponents shown in the task below. In the episode we will be watching, the teacher asked the students to draw on these rules to prove the two identities shown in Task 2.

Task:

Given the following rules of exponents

$$a^m . a^n = a^{m+n}$$

$$(a^m)^n = a^{m.n}$$

$$(ab)^m = a^m b^m$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

If all boxes are packed as in position C, investigate the maximum number of boxes that it is possible to put into the container. Show how you got your answer. **Link:** L11 – Portugal (07:10-10:14)	prove that $a^0 = 1$, showing all your work. **Link:** http://www.timssvideo.com/us3-exponentshttp://www.timssvideo.com/us3-exponents (38:30-39:00. 47:46-50:34)

Discuss:

- To what extent, do you think that students in these lesson video-clips are engaged in mathematical thinking and reasoning? Justify your thinking.
- If they are engaged, is that true for all students? Why? Why not?

Introduction to Module 1

This introductory module aims to familiarize you with the dual goal of selecting and using challenging tasks in your mathematics lessons, on the one hand, and immersing all your students—to the extent possible—in such tasks that have the potential to improve students' mathematical thinking and reasoning, on the other hand. Over the past decades, several research works internationally have documented the importance of providing students with opportunities to engage students in such tasks, since they were found to improve both the quantity and the quality of student learning. However, research has also shown that these tasks are not utilized that often and not with all students—apparently because of the complexity of this work. This module will engage you in activities that will help you start thinking more deeply around issues and challenges surrounding this dual goal. You will first have the opportunity to identify challenging tasks and determine their characteristics. Next, you will be exposed to different ideas related to differentiating your instruction to meet the needs and the readiness/ability levels of all your students. Finally, you will have the opportunity to consider the synergistic relationship between the two goals. You should bear in mind that, as an introductory module, Module 1, will expose you to several of the issues and challenges surrounding the dual goal of working with challenging tasks and ensuring that all your students are productively engaged in such tasks. You will have the opportunity to delve deeper in each of these issues and challenges in the remaining four modules.

	LEARNING OBJECTIVES (LO)	CASE OF PRACTICE ADDRESSING THE LO
LO1	Familiarizing teachers with cognitively challenging tasks and helping them identify aspects that render a mathematical task challenging	1
LO2	Identifying how the opportunities to engage students in cognitively challenging work can be modified during task presentation and implementation	1
LO3	Familiarizing teachers with the basic aspects of differentiation and different practices for achieving differentiation in today's classes	2
LO4	Explaining how working with challenging tasks can be in balance with trying to differentiate instruction and providing examples of instructional moves that can serve meet this dual goal	3

WORKING WITH CASES OF PRACTICE: Secondary

CASE OF PRACTICE 1

Focusing on Challenging Tasks

Overview

CONTACT HOURS	2 hours
TYPE OF RESOURCES	Videoclips; Tasks; the MTF framework
EMPHASES	Discussing how task unfolding can offer different learning opportunities for students

Activities

Opening Activity

(1) Brainstorming Activity

- Based on the videos you observed in the introductory activity of this module, what do you think we can do, as teachers, to create a productive space for engaging students in mathematical thinking and reasoning? What might we do (often inadvertently) that might hinder such attempts?

(2) There are several ways in which we, as teachers, might craft or lessen students' opportunities to engage in mathematical thinking and reasoning. A group of U.S. researchers has proposed the *Mathematical Task Framework* (hereafter called MTF) to help us better classify these ways and through that make more deliberate and informed decisions about the opportunities we craft for our students' thinking. Look at the figure below and read the brief introduction to the MTF that appears below; then consider the question that follows.

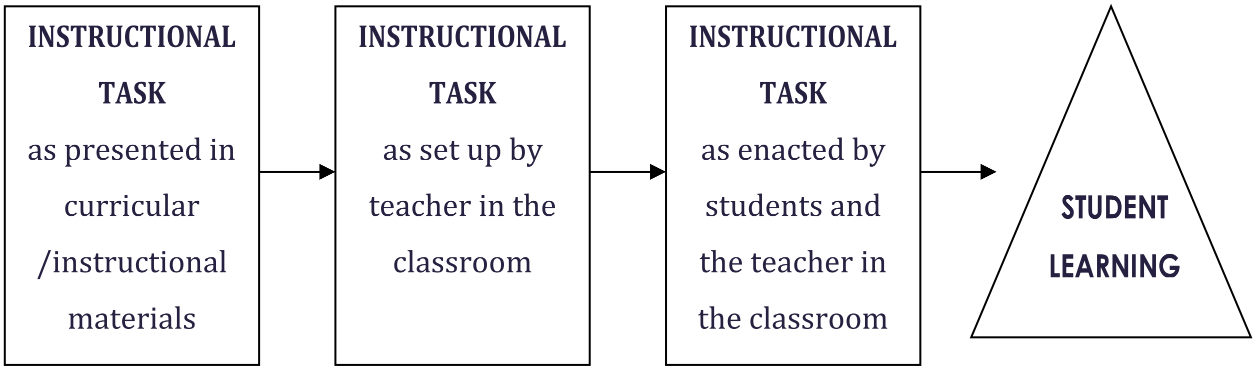


Fig.1. *The Mathematical Task Framework (adapted from Stein et al., 2000).*

About MTF: What Does it Tell Us and How Can it Be Used?

What does the MTF suggest? According to the MTF, instructional tasks pass through three stages: first, as they are presented in curriculum materials or in the handouts that the teacher prepares for her/his students; second, as they are set up by the teacher in the classroom during the launching (presentation) of the task; and third, as they are enacted/implemented during the lesson, while the students and the teacher interact while solving these tasks. Figure 1 captures these phases of task unfolding, emphasizing that what ultimate determines student learning is not only the *selection of cognitively challenging tasks*, but *how these tasks unfold during instruction*.

How can MTF be utilized? Over the past years, MTF has been used both as a research tool to examine instructional quality with respect to task unfolding but also as a professional development tool to sensitize teachers to the importance of attending to how the challenging aspects of a task might be altered during instruction, especially during the phases of task presentation and enactment/implementation.

Thinking of your previous lessons, in which area(s)–(a*) task selection*, (b) *task presentation*, and (c) *task enactment*–do you feel that you face more difficulties when trying to enhance your students' opportunities to engage in cognitively demanding work? Why do you think so?

The activities that follow will provide you with opportunities to discuss how different decisions we make as teachers, during the phases of task selection, presentation, and enactment can create different opportunities for student learning.

Activity 1 – Focusing on Task Selection

In this activity you will be exposed to different tasks. Read them carefully and then consider the questions that follow.

Task 1 (Grade 9):

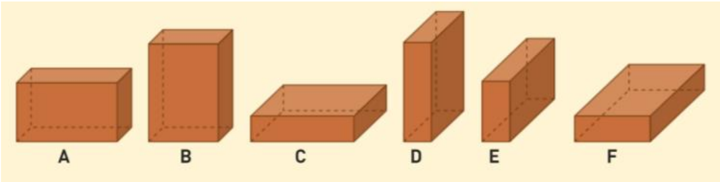
Match the following rule to its correct name:

1.	$a + b = b + a$	a.	Identity property for multiplication
2.	$(a + b) + c = a + (b + c)$	b.	Commutive property of addition
3.	$a(b + c) = ab + ac$	c.	Transitive property
4.	$a + 0 = a$	d.	Associative property of addition
5.	$a(1) = a$	e.	Identity property for addition
6.	If $a = b$, and $b = c$, then $a = c$	f.	Distributive property

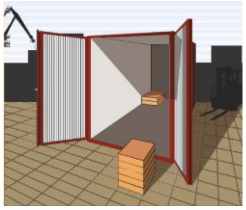
Source: Task Sorting Activity (Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004, p. 71)

Task 2 (Grade 10):

Consider one container with a width of 2m, a length of 4m and a height of 2,5m to transport boxes with the shape of rectangular paralelipeds and with the following dimensions: length 70cm; width 50cm and height 30cm. Suppose that the boxes can be introduced in the container in any position, as the figure shows:



(a) If all boxes are packed as in position C, investigate the maximum number of boxes that it is possible to put into the container. Show how you got your answer.

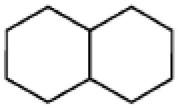


(b) If all boxes are packed in the same position inside the container, investigate which of the given positions A, B, C, D, E or F should we chose to transport the maximum number of boxes. Show how you got your answer.

Task 3 (Grade 7):

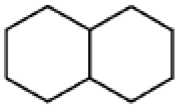
Figure out the product of each multiplication *using the given pattern blocks*.

Find 1/2 of 1/3. Use pattern blocks. Draw your answer.



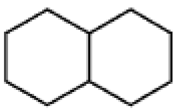
1/2 of 1/3 or 1/2 × 1/3 = ________

Find 1/3 of 1/4. Use pattern blocks. Draw your answer.



1/3 of 1/4 or 1/3 × 1/4 = ________

Find 1/4 of 1/3. Use pattern blocks. Draw your answer.



1/4 of 1/3 or 1/4 × 1/3 = ________

Source: Task Sorting Activity (Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004, p. 67, adapted)

Task 4 (Grade 12):

Given that

$$\int a\,dx = ax + c, \qquad \forall a \in \mathbb{R}$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c\,, \qquad \forall r \in \mathbb{R} - \{-1\}.$$

Figure out the following

$$\int 4\,dx$$
$$\int -\pi\,dx$$
$$\int x^4\,dx$$
$$\int x^{1000}\,dx$$
$$\int x^{-3}\,dx$$

Source: http://archeia.moec.gov.cy/sm/271/mathimatika_c_lyk_kk_a_tefchos.pdf (adapted)

Consider these four tasks and try to classify them according to how cognitively challenging they are (non-challenging vs. challenging), taking into account the corresponding target student audience.

Task	Level of Challenge (Low vs. High)
1	
2	
3	
4	

In your group, identify what features make the tasks challenging. List these features in the space provided below.

What challenges might you encounter in your practice in selecting such tasks for your teaching? How might you and/or your colleagues tackle these challenges?

Activity 2 – Focusing on Task Implementation

Almost twenty years ago, the National Council of Teachers of Mathematics (NCTM) in the USA has recognized the key role that teachers have not only in selecting cognitively challenging tasks (or using such tasks from their textbooks/curriculum materials), but mostly in how they interact with these tasks with their students. In particular, NCTM (2000) noted:

> "*Worthwhile tasks alone are not sufficient for effective teaching. Teachers must also decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them, and thus, eliminating the challenge*" (p. 19).

In this activity, we will consider how different teacher moves during task presentation and enactment might shape the opportunities that teachers have for student mathematical thinking and reasoning. Toward this end, consider we will consider two tasks and discuss their enactment in the class.

Read carefully the following task and determine its level of challenge (low vs. high).

MATHEMATICAL TASK

Geometry: The 'Quadrilaterals from Midpoints' Task

Task 1

In Figure 1, the points D, F and E are midpoints of the sides of the triangle ABC.

(a) Investigate what type of quadrilateral is the BDFE.
(b) Study how the quadrilateral BDFE changes when the triangle ABC changes. That is how the type of quadrilateral is connected to the type of the triangle.

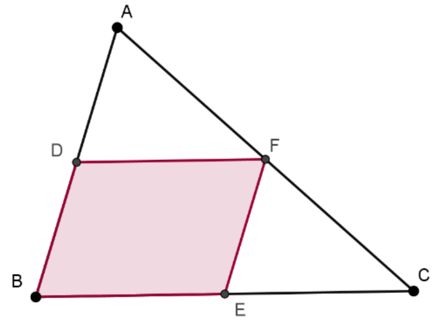


Figure 1

Watch the following video clips which refer to the launching and the enactment (student autonomous work and whole class discussion) of the task shown above.

Video clip
Context: We will observe a lesson from a Grade-10 class in Greece. In this lesson students are asked to draw on a specific theorem they studied in a previous lesson (i.e., the straight line segment that connects the midpoints of two sides of a triangle is parallel to the third side and equal to its half), to determine the relation between the types of the triangle and its circumscribed quadrilateral (see Figure 1 above). In the first part, students identified that based on the theorem, BDFE ought to be a parallelogram. In the second part, which is the focus of the video clips we will be observing, students are engaged in the investigation of the relation between the types of the triangle and its circumscribed quadrilateral. We will watch three clips, one related to the teacher's launching of the task, one related to students' autonomous work, and a third pertaining to the whole-class discussion.
Link: Launching: A) 2:49 – 4:47 (for both parts of the task), A) 18:43 – 20:33 (for the second part) Autonomous Work: A) 28:47 – 29:44, B) 00:00-00:50, B) 04:47-5:38 (transcript)+05:38-06:04 Whole Class Discussion: 16:41 – 18:22

Discuss with your colleagues:

- What is the level of challenge of the task, as presented in the teacher-made materials?
- Is the challenge maintained or does it get changed during the unfolding of this task?
- What are the teacher moves that contribute to the maintenance or the change in the cognitive challenge each time?

Based on your discussion above, working with your colleagues:

Record some teacher's moves that contribute to presenting and enacting the task at a challenging level.

Launching	Autonomous Work	Whole Class Discussion

Connections to (my) Practice

For our next meeting:

Select a challenging task from your textbook/curriculum materials that is included in one of the lessons you're expected to teach.

Work on this task with your students and videotape its presentation and enactment (student autonomous work and whole class discussion).

Before our next meeting, watch your videotaped lesson, and consider the level of challenge during its presentation and enactment.

Select two short excerpts (from task launching, student autonomous work, or whole-class discussion) that you would like to share with your colleagues. These excerpts should be illustrating either instances in which the cognitive challenge was maintained or instances in which it changed.

Closing Activity

Revisit the four-quadrant diagram in the introductory activity of this module, and consider where you will see your teaching being situated ***during your next lessons***:

- If you situated it in a different spot compared to that of the introductory activity, jot down two things that you learned that helped you make this (even small) shift.
- If you situated it in more or less the same spot, jot down two things you would like to learn in next meetings that you envision helping you making a more substantive shift.

CASE OF PRACTICE 2

Differentiating Instruction to Meet Students of Different Levels

Overview

CONTACT HOURS	2.5 hours
TYPE OF RESOURCES	Videoclips; Tasks; Lesson plan extract; Narratives; Student artefacts
EMPHASES	What differentiation is and what is NOT? What are some key practices of differentiation?

Activities

Opening Activity

Video club Component

In the previous case of practice, you were asked to (a) **select** a challenging task from your textbook/curriculum materials that is included in one of the lessons you're expected to teach; (b) **work** on this task with your students and **videotape** its presentation and enactment (student autonomous work and whole class discussion); and (c) **watch** and **consider** the level of challenge during its presentation and enactment.

Share with your colleagues the two short episodes you selected from your videotaped lesson in which the level of challenge was either maintained or changed. Explain what this episode is about and your rationale for selecting it.

Discuss the shared episodes with your colleagues:

- How did the task you selected unfold?
- Were you able to maintain the challenge?

    - If yes, how?
    - If not, why?
- What challenges did you encounter in doing so?

A Activity 1 – Considering Task Implementation

Below there is a scenario of the unfolding of a task in a twelfth-grade classroom. Read the scenario and then discuss in your group the questions that follow.

Narrative (The "Ryan's Birthday" Episode)

Mr. O'Connor is teaching probabilities to an honors twelfth-grade class of 15 students. In this lesson he wants his students to draw on the idea of independent events introduced in yesterday's lessons to solve a challenging problem. He enters the class excited and, once students are settled down, he challenges them: "Do you know that we can use mathematics to make a lot of money?" The teenagers in his class argue that he exaggerates, but Mr. O'Connor, continues by introducing the Ryan's Birthday problem shown below[3]:

Ryan's Birthday Task

Ryan works in a big computer company. Last night there was a gathering of the company and several of its employees attended the event. At some point they started discussing about birthdays. Ryan thought that he could challenge his colleagues and even use this opportunity to make some money! After quickly counting how many people there were in the gathering, he offered odds of 30 to 1 to anybody who wanted to bet that there is no shared birthday among the people in the room. What was the minimum number of people in the room so that Ryan's offer is reasonable?

Excited that the problem attracts students' attention, Mr. O'Connor gives them some time to read it and understand what it is asking them to do. He then initiates a brief discussion. "In previous lessons," he says, "we've discussed several ideas that might be useful for helping you solve this problem. Because I don't want to be doing the thinking for you, we will have a brief brainstorming activity during which you can nominate ideas that might be useful for approaching the problem. I'll be jotting all these ideas on the board, but I won't be telling you whether they are helpful or not. It is up to you to decide which ones, if any, you are going to draw upon in your group to solve this problem." Different ideas are then offered. Several students talk about

[3] Inspired by The Birthday Fact (http://www.j-bradford-delong.net/movable_type/archives/001393.html)

probability; a few link odds to ratios and fractions. Mary, who is often quick to nominate her ideas, rushes to throw in the idea of independent events. Without giving any hint to students as to which ideas are helpful, Mr. O'Connor gives students 20 minutes to work on the problem. Satisfied that the idea of independent events discussed in yesterday's class has been brought up in the discussion, he circulates around and keeps track of the groups' work.

While circulating, and cognizant of the ways in which a teacher might lessen the task demands, Mr. O'Connor constantly encourages his students to think hard around the problem and discuss it in their groups without, however, giving them any mathematical hints: "I am sure that you can do it, if you draw on each other brains!" He is very supportive and keeps his enthusiastic tone, even when realizing that some of the students do not seem to make any real progress on the problem: "Come on guys, I am sure that you're smarter than Ryan! If you give it a shoot, you will figure it out." To his satisfaction, some students seem to make significant progress: they figure out the probability of two people in the room not sharing the same birthday date (364/365 = 0.99726), and then continue with three (364/365 x 363/365 = 0.99180), four (364/365 x 363/365 x 362/365 = 0.98364), and five people (364/365 x 363/365 x 362/365 x 361/365 = 0.97286), building on the idea of independent events. At the same time other students seem to be stuck and are ready to give up. He spends some time to ensure that these students are clear on what the problem is asking them to do and that they understand that it is about probability. He also encourages them to remember some of the strategies they discussed in previous lessons for dealing with complex problems, without revealing however, that they could simplify the problem starting with two people and then moving on to more people. After allowing students to work on the problem for about 25 minutes, he organizes a whole-class discussion to let them share their ideas.

Different groups of students offer their ideas about approaching the problem. The class first agrees that it is a probability problem. Three of the five groups also suggest translating Ryan's odd into a probability of 0.03. Two groups propose starting with just two people, crediting Mr. O'Connor for suggesting the strategy of simplifying complex problems in previous lessons. Once the probability of two people not sharing the same birthday is figured out, Mary's group offers the idea of independent events, upon which the class then draws to figure out the probabilities of people not sharing the same birthday for a number of cases, ranging from three to ten people. At this point, the bell rings. Releases the class, Mr. O'Connor assigns the rest of the problem as homework, remarking that now "it's become just a piece of cake."

Although exhausted, at the end of the lesson, Mr. O'Connor is satisfied that the class has made significant progress since students were able to figure to successfully work on the problem. "I need to thank Carla [one of his colleagues] for suggesting this problem to me. It has been really instrumental for helping my class work on and apply the idea of independent events," he thinks, as he exits the classroom.

Guiding Questions

- To what extent was teaching, as described above, successful or not?
  - What elements do you think contributed to it being more or less successful?
- If you were the teacher, what would you do different, if anything, to make the lesson more successful?

Activity 2 – Teacher Beliefs and Perceptions about Differentiation

The teaching scenario, as described above echoes, an argument regarding undifferentiated or one-size-fits all instruction. Read this argument and then consider the activity that follows.

> "In many classrooms, the approach to teaching and learning is more unitary than differentiated. […] Most teachers (as well as students and parents) have clear mental images of such classrooms. After experiencing undifferentiated instruction over many years, it is often difficult to imagine what a differentiated classroom would look and feel like". Many teachers and teacher educators wonder how we can "make the shift from "single-size instruction" to differentiated instruction to better meet our students' diverse needs. To answer this question, we first need to clear away some misperceptions". (Tomlinson, 2017, p.2)

In the discussions we held with several teachers, the teachers shared with us some beliefs as to what differentiation is and how it can be materialized. Read the teacher beliefs which are provided below and then in your group discuss what your thoughts are on the issues raised by these beliefs, by also drawing on your own practice.

In differentiated instruction, all students are expected to achieve the basic learning objective of the lesson to the same extent.

At the end of the lesson, all students must have reached exactly the same point in achieving the lesson objectives.

All students should be assigned the same homework.

The way that I deal with mixed ability classes is to pair stronger with weaker students and ask them to think, pair, and share.

I know that often times we teach to an imaginary "average" student. I differentiate my approach by assigning harder problems to my more capable students.

One way that I try to differentiate my instruction is by giving students manipulatives to use.

Part of differentiation also has to do with creating a classroom culture that accepts and celebrates errors or alternative student ideas.

For less-capable students, I am pleased if they answer at least one simple question, to experience some level of success. For average students, I want them to be able to apply what they learned since the lesson is more or less planned for these students. Capable students surely want another question or task to work on once they finish.

> Differentiation is a utopia, it is another 'fruit' that comes from above, another 'trend' in education that will fade away just like many other educational policies did. What do they [educational policymakers] know about teaching? How can we differentiate instruction for 20 different children? It's so unrealistic!

> Differentiation is feasible if you think of different groups of students in your classroom instead of each individual separately.

> Differentiating instruction for each student is not possible and that's why many of my colleagues consider it utopian, they are terrified of the idea of too much preparation at home.

Reflection Discussion

- Based on these ideas, what would you describe as differentiation? Why?
- What wouldn't you consider as differentiation? Why?
- What is your stance toward differentiated instruction?

Many teachers often misconstrue the notion of differentiation and/or consider it as another "hot topic" in education. However, teaching all levels of students in a single class can make one realize that differentiation is a sine qua non aspect of instruction. The more we understand what differentiated instruction is about and how it can be materialized, the better we can use it. In the text that follows, we draw on the literature, and specifically on Tomlinson's work, to define differentiation and emphasize its importance.

The WHAT and the WHY of Differentiation

Our memories from typical undifferentiated classrooms during our school years often make us unfamiliar with the notion of differentiation; hence we can hardly imagine how it looks like. Some consider differentiation as individualized instruction, or as a chaotic situation where the teacher "loses control" of students' behavior; others think of differentiation as another way of grouping similar-ability level students together and assigning hard/easy tasks and complicated/simple questions to high- or low- achievers, respectively. But is this really differentiation? To better understand what differentiation is, let's first define what it is not.

Misconceptions about differentiation. Differentiation should not be considered as individualized instruction. Imagine having to create a different customized lesson each day for each of the 20-plus students in a single classroom. Not only is this unrealistic, but it will also be exhausting for you. Moreover, differentiation is not just another way to provide homogeneous grouping; rather it is based on flexible grouping, recognizing that all students might have strengths in some areas and weaknesses in others. Thus, it is not just for students with identified learning challenges or for students who learn rapidly. There are also students in between who struggle in varying degrees and who also need differentiated instruction. By providing more work to some students or less to others does not mean that a teacher has really differentiated his/her instruction.

Then, what IS differentiation? Differentiation is a process of matching learning targets, tasks, activities, resources, and learning support to individual learners' needs, styles and rates of learning. When teachers differentiate their instruction, they do so in response to students' readiness levels, interests, and/or learning profiles. Teachers who differentiate, ***plan lessons proactively*** that provide multiple ways for students to make sense of ideas and knowledge and demonstrate what they learn, always trying to match their student needs. Thus, diagnostic and also systematic formative assessment throughout each unit helps teachers develop an understanding of their students' needs. In the activity that follows, we will learn some ways of designing and enacting a lesson plan in ways that attend to issues of differentiation.

Why do teachers need to differentiate? Regardless of some common features, students of the same age differ in the way they learn, as well as, their interests, personality, and preferences. To learn, each student needs to be challenged at an appropriate level and also feel successful. This cannot be achieved when ignoring student differences. Attending to these differences requires teachers to create several avenues to "get at" learning in an environment that recognizes and celebrates those differences.

Activity 3 – Planning for Differentiation

In a differentiated classroom, the teacher proactively plans to respond to student individual differences. In this activity, we will discuss how a teacher can plan for differentiation through his/her lesson plan. Toward this end, we will consider a lesson plan prepared by Mr. Demetris, a tenth-grade teacher. In his lesson plan, Mr. Demetris included two tasks through which students would investigate the different types of parallelograms created by connecting the consecutive midpoints of the sides of a triangle/quadrilateral. The following lesson plan extract provides some general elements of the lesson but focuses mainly on Task 1. Read his lesson plan provided below and then identify elements/practices of differentiation that might be identified by this teacher planning.

Lesson plan Extract for 'Quadrilateral from Midpoints' task

Lesson plan "Quadrilaterals from midpoints"

Tags

Topic: Parallelograms, rectangular, rhombs, squares; Euclidean geometry
Target group: Upper secondary
Age range: 15-16

Tasks

Task 1

In Figure 1, the points D, F and E are midpoints of the sides of the triangle ABC. Investigate what type of quadrilateral is the BDFE. Study how the quadrilateral BDFE changes when the triangle ABC changes. That is how the type of quadrilateral is connected to the type of the triangle.

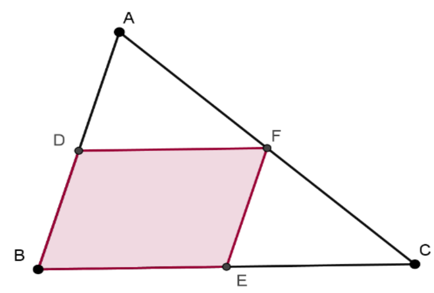


Figure 2

Task 2

In Figure 2, the points E, F, G, H are midpoints of the sides of ABCD. Investigate what type of quadrilateral is the EFGH? Study how the quadrilateral EFGH changes when the quadrilateral ABCD changes. That is how the type of the EFGH is connected to the quadrilateral ABCD).

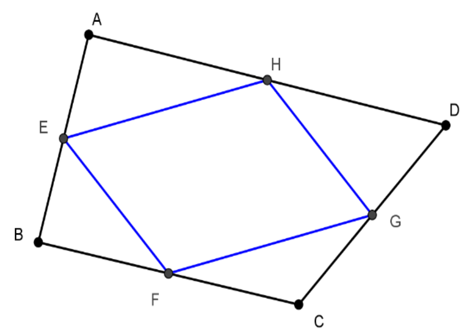


Figure 3

Learning Goals

By the end of this lesson, students should be able to:

c) Apply the midsegment theorem in order to investigate the quadrilateral formed when connecting the midpoints of sides of triangles/parallelograms.

d) Make conjectures and experiment with how changing certain angles of the triangle/quadrilateral results in certain quadrilaterals.

e) Classify/prove the quadrilateral formed when connecting midpoints of sides of triangles/quadrilaterals.

Prior Knowledge

Students should be able to:

- Recognize and name the shapes of parallelogram, rectangular, rhombi, and square.
- Identify properties of the shapes listed above.
- Explain the **Midsegment Theorem** which states that the midsegment connecting the midpoints of two sides of a triangle is parallel to the third side of the triangle, and the length of this midsegment is half the length of the third side.

Lesson unfolding

<table>
<tr><th>Tasks & Learning Activities</th><th>Expected Duration</th><th>Differentiation</th></tr>
<tr><td>Introduction/ Task launching</td><td>2-3 mins</td><td>Provide clarification about the requested investigations. Move points A, B etc. in geogebra to show the dynamic nature of the shapes.</td></tr>
<tr><td>Autonomous work</td><td>10-15 mins</td><td>Students use the geogebra as they wish, moving the figures. Different students are expected to have different approaches and different justifications as they work on this problem. Student difficulties are expected to increase as they move from task 1 to task 2 and from conjectures to justifications and proves in task 2. In task 1, students may see that DF is half and parallel to BC. The identification of the type of BEFD is more demanding.

<u>FOR TASK 1</u>

For students who do not have an effective strategy for completing the proof the following questions might be posed to elicit their thinking:
1. What are you asked to show in this problem?
2. What data are available to you for solving this task? Use the following table and fill it with the givens and what you're expected to find.

<table><tr><th>Givens</th><th>Wanted</th></tr><tr><td></td><td></td></tr></table></td></tr>
</table>

3. What do you observe when you move points A, B etc. of the triangle ABC in geogebra? Use the table below to write your observations *about how the quadrilateral BDFE changes when the triangle ABC.*

Observations

Shape: BDFE	*Shape: ABC*

4. As you move points A, B etc., what did you observe regarding:
   - The type of the quadrilateral?
   - The sides of the quadrilateral?
   - The angles of the quadrilateral?
   - The diagonals of the quadrilateral?

   Can you draw a conclusion based on your observations?

5. Problematize students what different triangles we have and have them think of all possible situations of different triangles: scalene, right triangle, isosceles, equilateral, right isosceles triangle. Possible question:
   - How do you know that you have thought of all the possible cases? Consider if it matters which angle of the triangles (angle B or a different angle) we consider to be, for example, the right angle. You can use the following table to record down your observations.

Table 1

Type of triangle *ABC*	Type of angle *B*	Type of quadrilateral *BDFE*

   - Do you think joining the midpoints of any triangle will create a figure that is a e.g. parallelogram? How could you test this idea?

		Pose the following questions to students who correctly find the relationship between the quadrilateral BDFE and the triangle ABC and provide work to support the conclusion that the figure is a parallelogram but do not explicitly draw this conclusion. • Can you explain how you can show that the figure is a parallelogram? • What is your conclusion? Is this quadrilateral a parallelogram? **For students who do not use mathematical terminology or notation correctly.** • What is the difference between $\overline{EH}$ and EH? **Some students will provide complete and correct responses to all components of the task. Possible questions and tasks for extending their work and thinking:** • How would shape DFE change as the triangle ABC changes if we draw segments connecting all the consecutive midpoints D, F and E? • Early finishers can start working on task 2. o Do you think joining the midpoints of any quadrilateral will create a figure that is a parallelogram? How could you test this idea?

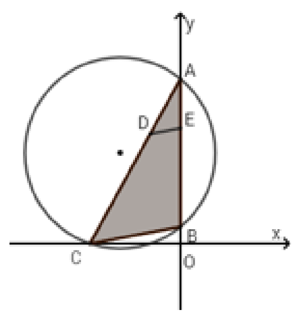
Working in pairs, identify evidence of differentiation practices in the teacher's planning; list any other practices you might be using for differentiation when planning your lessons.

Activity 4 – Scaffolding Students During Autonomous Work

When students work autonomously teachers need to monitor students work and decide which practices to employ to scaffold students' work, without reducing the challenge of the task.


Watch the following video-clips; while watching them try to identify, discuss, and name teacher moves/practices which help in scaffolding students (record these practices at the table included at the end of this activity). Please note that these clips come from three Grade-10 classrooms, a Greek classroom in which students work on the "Quadrilaterals from midpoints" tasks discussed in Activity 3 above, and two Portuguese classrooms, in which students are working on the Tasks 1 and 2 that follow.

In the plane with a Cartesian coordinate system, consider the circle with equation $(x+2)^2+(y-3)^2=10$ and the triangles [ABC] and [AED], as depicted in the figure below.



We know that:

- Points A and B belong both to axis *Oy* and to the circle;
- Point C belongs both to axis *Ox* and to the circle;
- $[DE]$ and $[CB]$ are parallel line segments and $\overline{DE}=\frac{1}{3}\overline{BC}$.

1.1. Determine the area of the triangle [ADE].

Show how you have found the answer: you should explain your reasoning and present all computations you have made.

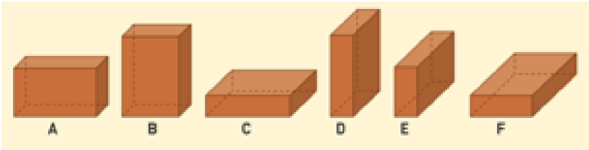
1.2. Define analytically the triangle [ADE].

Show how you have found the answer: you should explain your reasoning and present all computations you have made.

Task 1

1. Consider one container with a width of 2m, a length of 4m and a height of 2,5m to transport boxes with the shape of rectangular prisms and with the following dimensions: length 70cm; width 50cm and height 30cm.

   a) Suppose that the boxes can be introduced in the container in any position, as the figure shows:



   If all boxes are packed as in position C, investigate the maximum number of boxes that it is possible to put into the container. Show how you have got your answer.

   b) If all boxes are packed in the same position inside the container, investigate which of the given positions A, B, C, D, E or F should we chose to transport the maximum number of boxes. Show how you have got your answer.

Task 2

Videoclips 1a and 1b

Context: The first clip comes from the Greek lesson. Students are working on the second task (see Activity 3 above). The teacher tells students that they can experiment with modifying the triangle using GeoGebra. The second clip comes from a Portuguese class, in which students work on Task 2. Most students began solving the task by computing the volume of the container and the volume of the box and then figuring out how many times the volume of the box fits the container without paying attention to the restriction put by the linear dimensions of the boxes. This clip illustrates the interaction of a teacher with a group of three students and recognizing their difficulties, the teacher uses physical models as active representations.

Transcripts: (Greek lesson: 20:30-21:12, Portuguese lesson: L11): Both clips pertain to using representations to scaffold students' work)

Videoclip 2

Context: This clip come from a lesson occurring in a Portuguese Grade-10 class during which students are working on Task 1. During the clip to be watched, a pair of students calls the teacher because they do not know how to delineate a strategy to solve the task. Nothing is written in their worksheet by then. They explain to the teacher that they are not able to find a strategy to solve the problem because they cannot foresee what to do with the triangle. The teacher conducts a productive dialogue with the group, asking for explanations and at the same time validating some of their ideas in order to help them to progress in solving the problem.

Transcripts: XX (L01 on questioning, clarifying, and handling incorrect solutions).

Videoclips 3a and 3b

Context: All these clips come from the Greek classroom in which students are working on the tasks shown in Activity 3. In the first clip, students are working on the first task, trying to prove the mid-point theorem, which they actually proved in a previous lesson. The teacher approaches one of the groups, and in interacting with them tries to help them realize that proving the theorem was not necessary. In the second clip, students are again working autonomously on the task and the teacher approaches a group of them. A student makes speculations as to when the parallelogram is a rectangle or a square or rhombus. The teacher encourages this student and his team to work more systematically on the task and make more concrete conjectures as to how different types of triangles are linked to different types of parallelograms.

Transcript: XX (Greek lesson: 7:25-8:45 and 28:45-30:45): questioning and providing feedback.

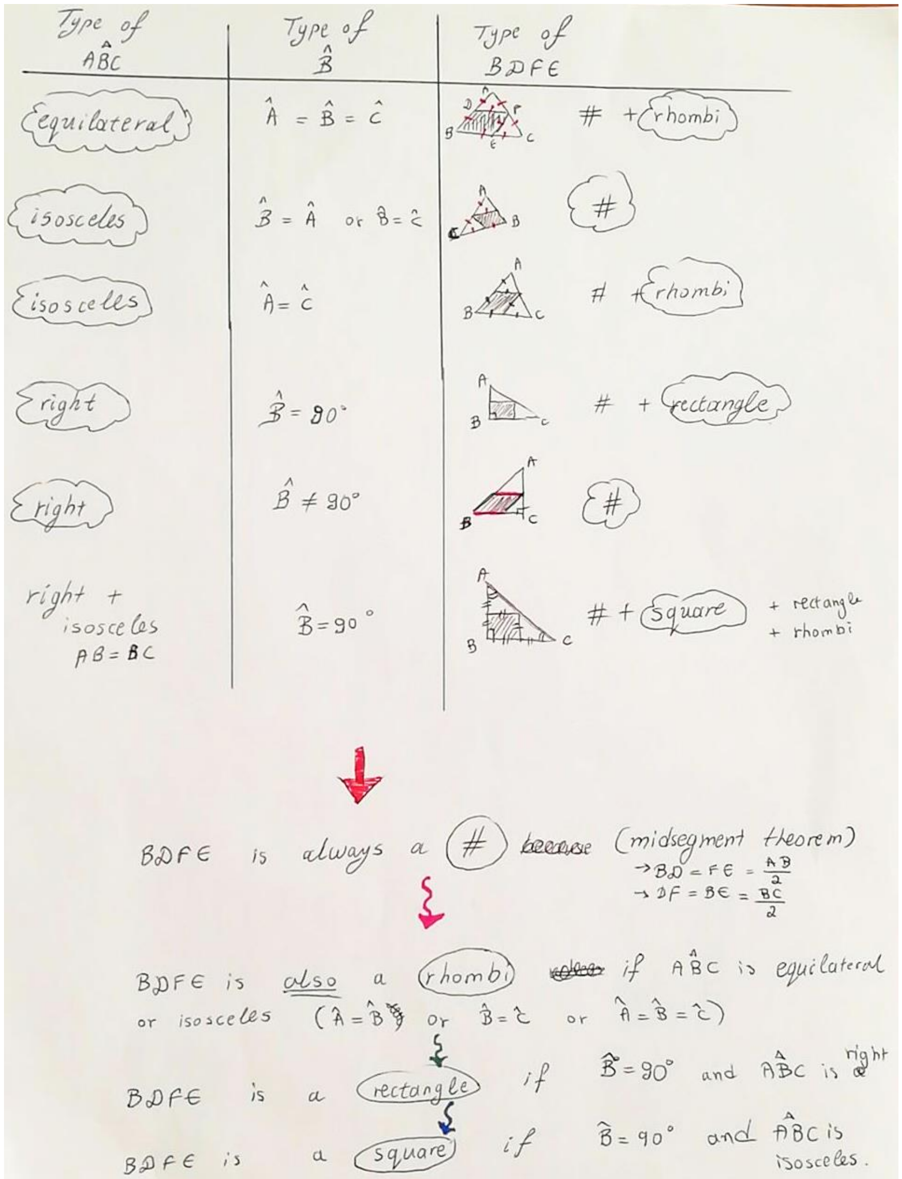
Record the practices that you identified in the table that follows

Number of Video-clip	Practices on Differentiating Instruction
Video clips 1a and 1b	
Video clip 2	
Video clips 3a and 3b	

Activity 5 – Holding a Productive Whole-Class Discussion

Let's get back to the lesson plan of Mr. Demetris. While students were working autonomously on the second question of Task 1 of the "Quadrilaterals from midpoints" lesson, Mr. Demetris was monitoring his students' progress and making decisions about his next instructional moves. Quite pleased, he noticed that different students developed different solution strategies to solve the second question of Task 1 (that is, "Study how the quadrilateral BDFE changes when the triangle ABC changes"), but there were also some students who had difficulties. He now has to make a very important decision: how to organize the whole-class discussion in a way that allows for differentiation. Read the following short narrative and student solutions of Task 1 below from Mr. Demetris lesson and consider the questions that follow.

While Mr. Demetris was monitoring students' autonomous work, he noticed that 11 out of the 19 students were successful in providing a correct response. Underlying these responses there was substantial variation in terms of how students actually dealt with the problem at hand. In particular, six of the students provided solutions similar to Figures 1 and 2. Eight of the students gave a non-valid response or did not consider all the possible triangle cases: a pair of students attempted to prove the midsegment theorem as shown in Figure 3 instead of using it as a given; six students offered solutions as shown in Figures 4-8. Mr. Demetris now wonders how he should structure the whole-class discussion based on all these different student responses in a way that allows for differentiation.



Type of $\hat{ABC}$	Type of $\hat{B}$	Type of BDFE
equilateral	$\hat{A} = \hat{B} = \hat{C}$	# + rhombi
isosceles	$\hat{B} = \hat{A}$ or $\hat{B} = \hat{C}$	#
isosceles	$\hat{A} = \hat{C}$	# + rhombi
right	$\hat{B} = 90°$	# + rectangle
right	$\hat{B} \neq 90°$	#
right + isosceles $AB = BC$	$\hat{B} = 90°$	# + square + rectangle + rhombi

BDFE is always a # ~~because~~ (midsegment theorem)
→ $BD = FE = \frac{AB}{2}$
→ $DF = BE = \frac{BC}{2}$

BDFE is also a rhombi ~~when~~ if $A\hat{B}C$ is equilateral or isosceles ($\hat{A} = \hat{B}$ or $\hat{B} = \hat{C}$ or $\hat{A} = \hat{B} = \hat{C}$)

BDFE is a rectangle if $\hat{B} = 90°$ and $A\hat{B}C$ is right

BDFE is a square if $\hat{B} = 90°$ and $A\hat{B}C$ is isosceles.

Figure 1

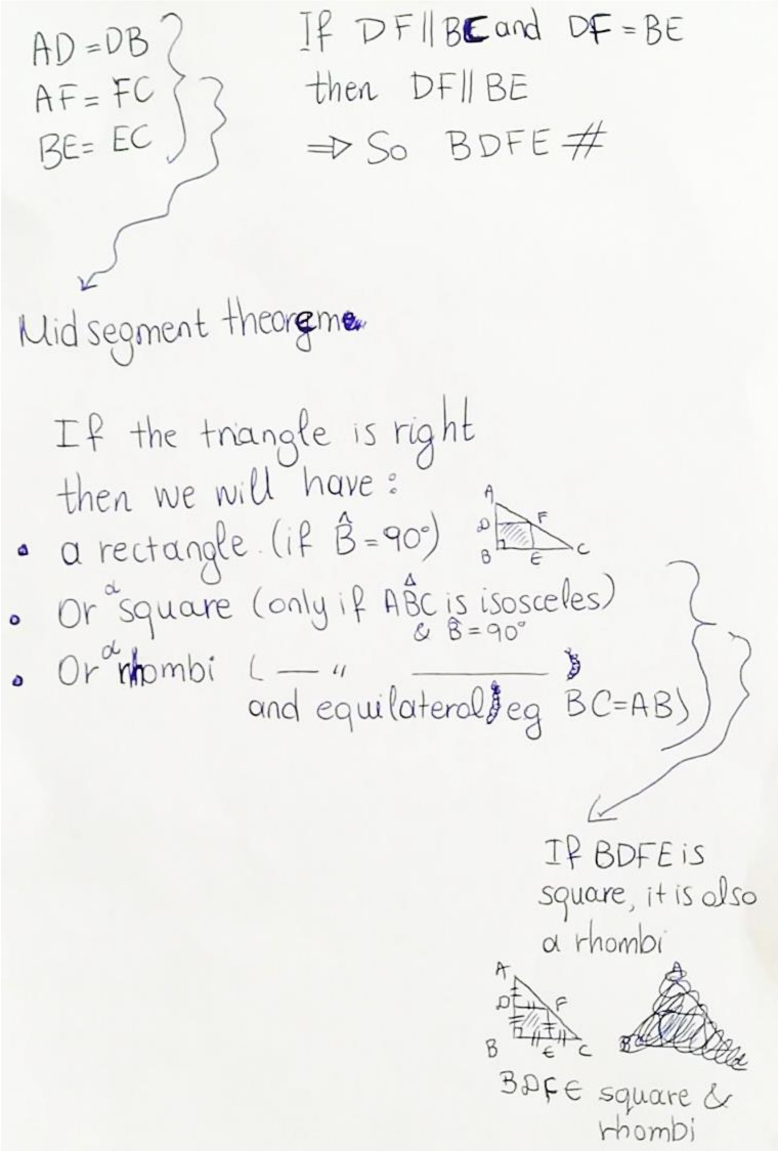


Figure 2

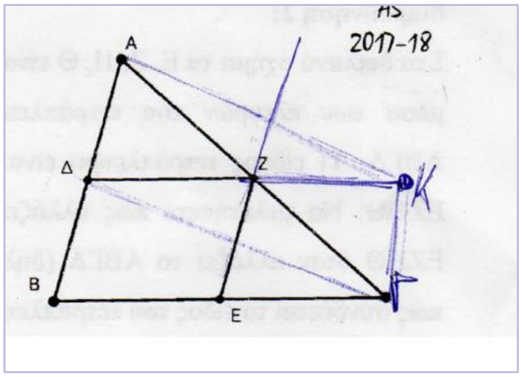


Figure 3

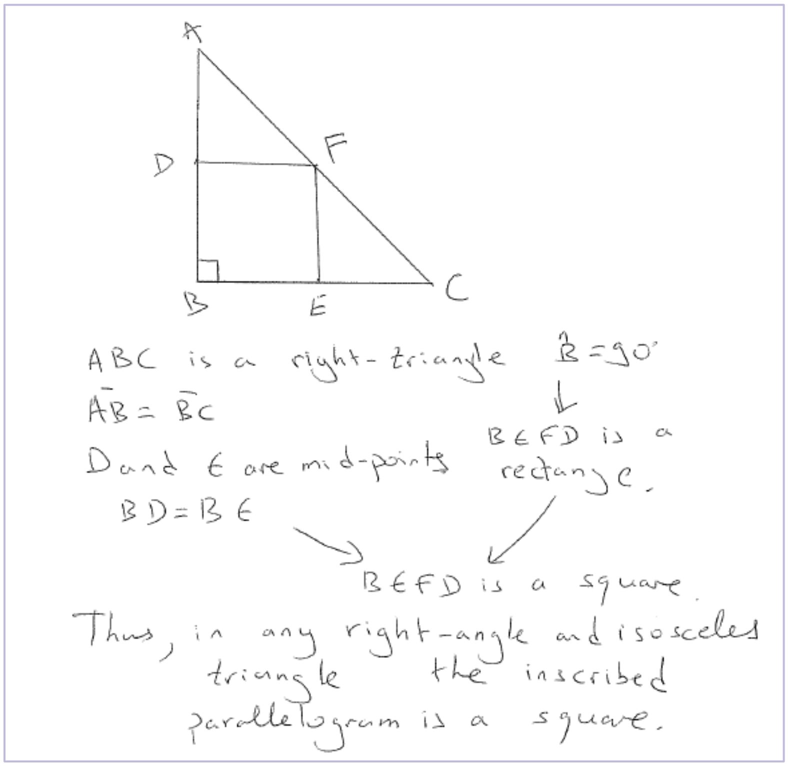


Figure 4

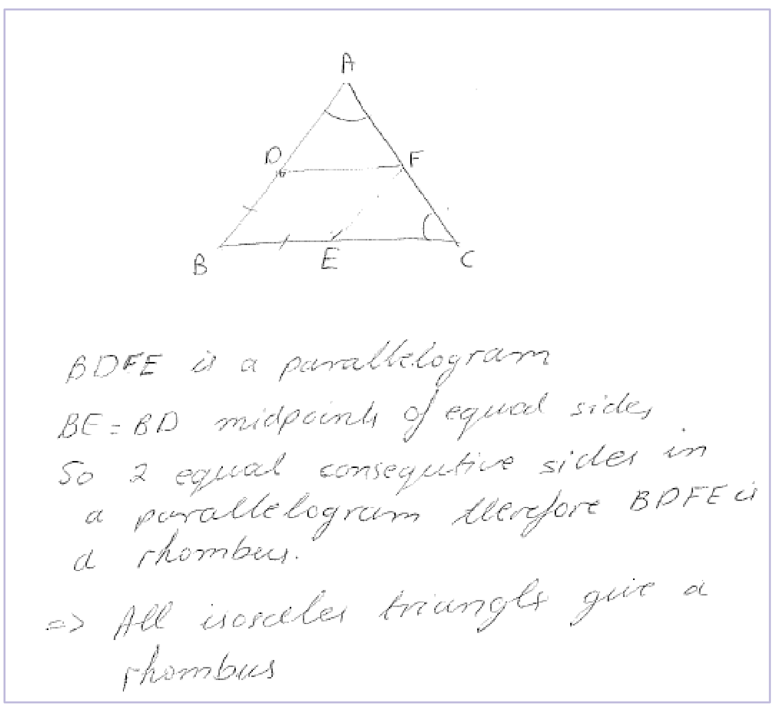


Figure 5

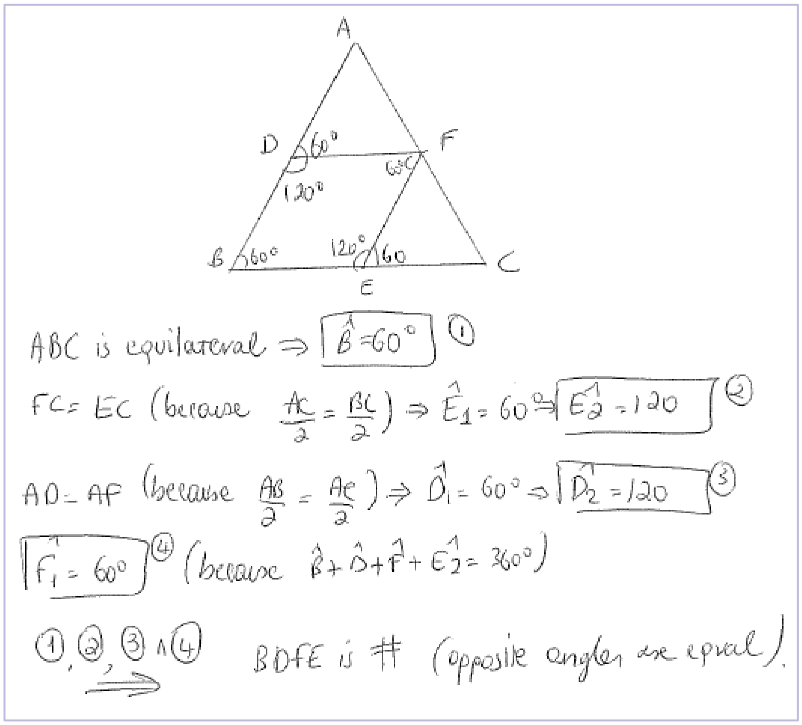


Figure 6

Guiding Questions

- How would you conduct whole class discussion to allow for differentiation?
- Which solutions and difficulties of students would you use? Why?
- Would you omit any solutions? Why? Why not?
- How would you organize the whole-class discussion?
- What strategies would you use to differentiate your approach during whole-class discussion?

Based on your answer to the last question, identify the practices you would use for whole-class discussion.

To manage complexity, so far, we have broken working on differentiation into different phases, starting with lesson planning, then considering how to differentiate one's approach while student autonomous work on an assigned task, and ending with how to organize a whole-class discussion that takes into consideration issues of differentiation. You will learn more about differentiation in each of these phases in Modules 2, 3, and 4, respectively.

Connections to (my) Practice

Design a lesson that takes into consideration differentiation practices as discussed and codified in our meeting (at least one from planning, autonomous work, and whole-class discussion).

Teach the lesson and select two video clips one from student autonomous work and another from whole-class discussion that are illustrative of your attempts to differentiate your approach (regardless of how successful these attempts were).

Closing Activity

Discuss with your colleagues:

- Which from the practices identified above do you consider the most important for supporting your work?
- How would you incorporate them in your next teaching attempts?

CASE OF PRACTICE 3

Concurrently Attending to the Work around Challenging Tasks and Issues of Differentiation

Overview

CONTACT HOURS	3 hours
TYPE OF RESOURCES	Videoclips; Tasks; Narratives; Interview excerpts
EMPHASES	• When and how working on challenging tasks and differentiation might work harmoniously, ensuring that all students are engaged o The Role of Questioning

Activities

Opening Activity

Video club Component

In the previous case of practice, you were asked to (a) **design** a lesson that takes into consideration differentiation practices as discussed and codified in our previous meeting (at least one from planning, autonomous work, and whole-class discussion), (b) **teach** the lesson, and (c) **select** two video clips, one from student autonomous work and another from whole-class discussion, that are illustrative of your attempts to differentiate your approach (regardless of how successful these attempts were).

Share with your colleagues the two short episodes you selected from your videotaped lesson. Explain what these episodes are about and your rationale for selecting them.

Discuss the shared episodes with your colleagues:

- How did the task you selected unfold?
- How successful was your attempt to differentiate your instruction?

- Were you able to maintain the cognitive challenge for all students?
    - *If yes, how?*
    - *If not, why?*
- What difficulties did you encounter in doing so?

In case of practice 1 of this module, we focused on challenging tasks – what it means and what it entails – recognizing the importance of engaging students in challenging mathematical tasks for advancing their problem solving and reasoning skills. Then, in the second case of practice, we considered how, as teachers, we can differentiate our instruction and structure learning environments that take into account diverse student needs and readiness levels in order to maximize the probability for *all* students being engaged in challenging work. In the third and final case of practice of this module we bring these two constructs, *working with challenging tasks,* on the one hand, and *differentiating instruction,* together.

Activity 1

Watch the following video-clips. Each presents a different classroom episode centered on the enactment of a different task. The tasks and the context associated with their enactment is described in the boxes below. While watching the video clips focus on how *differentiation* and *students' engagement in cognitively challenging work* manifest themselves in the classroom. In doing so, consider the following guiding questions.

Guiding Questions

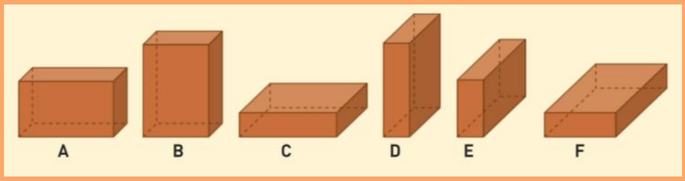
- What differentiation practices did you identify?
    - *Were they implemented effectively?*
- What was the cognitive level at which students were expected to work?
    - *What evidence is there about the cognitive level at which students actually worked?*
- Did you identify any issues or problems related to the teacher's attempt to work on both fronts: challenging tasks and differentiation?

Video-clip 1

Task:

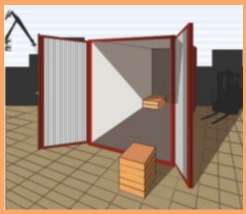
1. Consider one container with a width of 2m, a length of 4m and a height of 2,5m to transport boxes with the shape of rectangular prisms and with the following dimensions: length 70cm; width 50cm and height 30cm.

a) Suppose that the boxes can be introduced in the container in any position, as the figure shows:



If all boxes are packed as in position C, investigate the maximum number of boxes that it is possible to put into the container. Show how you have got your answer.

b) If all boxes are packed in the same position inside the container, investigate which of the given positions A, B, C, D, E or F should we chose to transport the maximum number of boxes. Show how you have got your answer.



Context: This episode occurs when a group of three students is working on 'the Boxes' task involving tri-dimensional geometry, about packing boxes in a container. The task requires the students to visualize the situation as well as to mobilize their knowledge about the volume of a parallelepiped. It presents students with two questions, the second being an extension of the first. This episode concerns the interaction of a teacher with the students. Looking at the work done by the students, the teacher realizes that they did not understand an important condition of the problem. In order to help the students to understand this condition and visualize the situation, the teacher uses physical models as active representations.

Transcript: XXX

Link: Summary of Episode L11 (Portuguese videoclip)

Video-clip 2

Task: Dividing money based on certain ratios (e.g., divide $200 into a ratio of 1:3; divide $210 into a ratio of 2:5).

Context: This excerpt is taken from an eight-grade class in Australia in which there are 30 students. The lesson focuses on developing student understanding of ratios. It is the fourth lesson in a seven-lesson unit on ratio and rates. The lesson starts with the teacher, Ms. Page, giving students 12 unifix cubes and asking them to divide them into different ratios. The class then focuses on the ratio 1:3. The teacher asks students to think how many cubes they will need to represent the ratio 1:3 if one cube represents a pile of things. Students build up this ratio using four cubes, one on their right-hand side and three on their left-hand side. The teacher emphasizes that they used four cubes/blocks. We enter the classroom at the point at which the class transitions to considering these ratios in the context of money problems.

Transcript:

T: I want you to divide 200 dollars in the ratio one to three. So, create for me the ratio one to three again. Right. So, you have? One box on the left, three boxes on the right. Now the whole idea- Dylan. The whole idea about dividing things up in a ratio is that you have equal piles in each of your four-equal amounts in each of your four boxes. So, what will go in one box if you have to fill four boxes equally, Ashley?

A: Fifty dollars.

T: Fifty dollars. So, four shares is equal to 200 and so one empty box or share has 50, so the ratio one to three will equal 50 to 150. Yes? Okay. So, you write this down please. It's best if you write the question since it's an example, so write divide 200 dollars. Divide 200 dollars. Right. Two hundred dollars. Divide 200 dollars in the ratio one to three. I (inaudible) forgotten your (inaudible). Mm. Then copy down four shares equals 200 dollars. One share is 50. One to three is 50 to 150. Four shares is equal-Now if we're going to be very good, people, with this, then we ought to write an English statement at the end of it, so we should say something like the first person gets 50 and the second-That's presuming that was a story about people, the second person could have been companies I suppose, gets 150. Okay, so you really ought to put some English in at the end to actually answer what the question was. Right? Finished? Okay, next one then was... the next one that we had, we recreate a ratio of two to five. So, recreate your ratio of two to five. Which was? How many blocks required?

Ss: Seven.

T: Seven. It's starting to seem really easy now, isn't it? Okay, and I actually want you to divide 210 dollars. Two hundred and ten dollars between those seven boxes in the ratio two to five. I want you to divide 210 dollars into those seven boxes but in the ratio two to five. So firstly, you have to figure out how much is in one box. Just a minute, girls, we'll just let the others have a think first. Danny. You don't need a calculator to do that.

D: You told me to get it out Miss.

T: Well you don't need a calculator to do that. What is it Tanya? One box in each box. Would it be equal to 30? I think it would. How would you find 30?

T: Seven divided by (inaudible).

T: Seven. Twenty-one divided by? Seven is three. Two hundred and ten divided by seven is 30. Yep. Okay. So, two boxes, 60, five boxes, 150, so the first person gets? Sixty dollars and the second person gets- 150 dollars. Write that down.

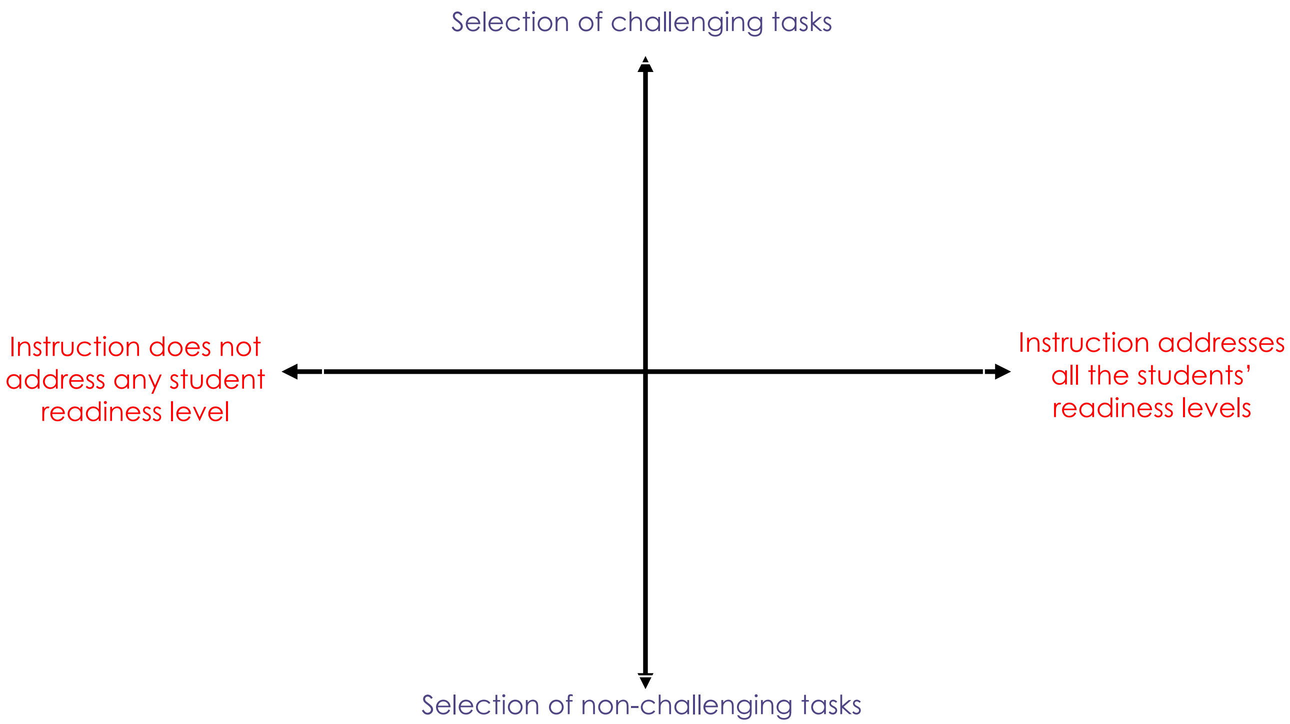
S: Write what down?

T: Write down the English. So, the first person 60 and the second person gets? A hundred and fifty again.

Link: http://www.timssvideo.com/au4-ratios (9:58-16:00)

Activity 2 – Teachers' Challenges and Difficulties When Trying to Engage All Students in Challenging Work

Let's now focus on video-clip 2. Think of where you would position Ms. Page's teaching in the two-dimensional space shown below, with the vertical axis representing the extent to which challenging tasks are selected for use in the lesson and the horizontal axis corresponding to the degree to which instruction addresses all the students' readiness levels.



Guiding Questions

- Why have you positioned this teacher's teaching in this particular spot?

When discussing with Ms. Page, we asked her to comment on this particular episode.[4] She shared with us some thoughts about the difficulties and challenges she faced while trying to concurrently work on challenging tasks and differentiation. Read the teacher thoughts in the interview excerpt that follows and try to identify specific difficulties, and challenges she faced during the planning and/or the enactment phase of the lesson; then in your group discuss your thoughts on the issues raised by this teacher, by also drawing on your own experience.

"The different levels of students were an issue. Some [kids] were more advanced than the rest of the class, others were far behind, and others faced too many difficulties—

[4] This is an imaginary interview for the purposes of this module.

as was the case with Dylan who wanted to use his calculator to figure out how much was 210 divided by seven. At some point I was afraid that I would not be able to manage these different levels and 'speeds'. Plus, the use of the cubes, the blocks, made it harder for me, since some kids were just playing with them without really using them to figure out the ratios. As expected, some kids did not really need the cubes; they could figure out the problem immediately. But I wanted to use the cubes to help the class understand that in order to solve such problems, one should not just be thinking of the part to part but of the part to the whole. So, that's why I spent quite some time working on the ratio of 1:3, because I wanted them to really understand that the entire set of blocks, or cubes if you wish, was four. Once this was clear, once we discussed that, we had 4 piles of cubes, then solving the money problem was a piece of cake. In hindsight, if I were teaching the lesson again, I would think of additional problems for students who were done earlier. I think that would keep these students engaged and motivated, as well."

Guiding Questions

- What difficulties and challenges did this teacher face in selecting and using challenging tasks to productively engage all students in mathematical thinking and reasoning?
- Did you experience similar challenges/difficulties during your teaching?

In the discussions we held with several teachers, the teachers shared with us some difficulties and challenges they face while trying to plan or enact a lesson that promotes both student cognitive engagement and differentiation. Read the teacher thoughts in the interview segments provided below and then in your group consider the questions that follow.

Teacher Difficulties and Challenges

> I had to think of the time that I would let kids to work on their own, so that no one would end up being bored and those who needed a great deal of time to finish the task would have enough time to think and work. For how long should I let students struggle? I wasn't sure how long I needed to stay with each student. I'm always in a hurry and I am afraid that sometimes I end up telling [students] the answers.

> I do not understand very well what the task is about, what the goal for its inclusion in the textbooks is.

> As a teacher you also need to have a very good understanding of the content and of how to teach certain things. You have to know very well what is behind each mathematical concept [to effectively work on such tasks].

> I had a difficulty figuring out the right questions that could lead students to the targeted generalization. Or how to guide them without telling them what to do, or 'closing up' the discussion, and [thus] minimizing their thinking?

> Students completely went wild from the moment they got the materials. I tried to get them back by using routines, but I didn't manage to do it. So, we had frequent interruptions, the flow of the lesson was not smooth, students did not pay attention, we didn't focus on the critical points of the task, and we went out of time.

Reflection Discussion

- How do you see differentiation and working with challenging tasks go together?
- Which do you think are the biggest challenges and difficulties that a teacher might face in planning and implementing a lesson that aims to work at both fronts? Why?

Differentiating mathematics instruction so that the task complexity remains to an appropriate level for all students can develop and consolidate students' understanding. The constructs of *Ensuring that students engage with challenging tasks* and *Differentiating* *teaching,* work in tandem. These two constructs bear an interactive connection; teacher's moves or actions intended to promote each of these constructs necessarily reflect on the other as well. Any attempt to ensure that all students are engaged in challenging tasks requires anticipating the diversity in the class in terms of levels of readiness and planning differentiated instruction to accommodate this diversity. In a similar manner, it is not possible to effectively enact differentiated instruction unless the tasks used in the classroom are purposefully selected to ensure appropriate fit between the level of the cognitive challenge incurred by the tasks, on the one hand, and the corresponding level of student readiness, on the other. Read the following text which elaborates more on this idea using the gear metaphor to explain how these constructs function together and then, comment on this idea.

Working on Challenging Tasks and Differentiation: The Gears Metaphor

Working with challenging tasks and differentiating instruction can be parallelized to a set of gears – each construct corresponding to a gear shape – that work together to enhance student learning (interlocking constructs), as shown in the figure that follows. The function of this set of gears depends on the coordination or the balance between the two gears; if the one does not work well, we cannot have reasonable expectations for the promotion of the other. If the one gear has a malfunction and stops moving then the other gear will stop working as well.

These two constructs work in tandem; teacher's moves or actions intended to promote each of these constructs necessarily reflect on the other as well. Any attempt to ensure that all students are engaged in cognitively challenging tasks requires anticipating the diversity in the class in terms of levels of readiness and planning differentiated instruction to accommodate this diversity. In a similar manner, it is not possible to effectively enact differentiated instruction unless the tasks used in the classroom are purposefully selected to ensure appropriate fit between the level of the cognitive challenge incurred by the tasks, on the one hand, and the corresponding level of student readiness, on the other. By giving undifferentiated hints to students and more or less degrading the level of challenge for students, the teacher fails in promoting both constructs.

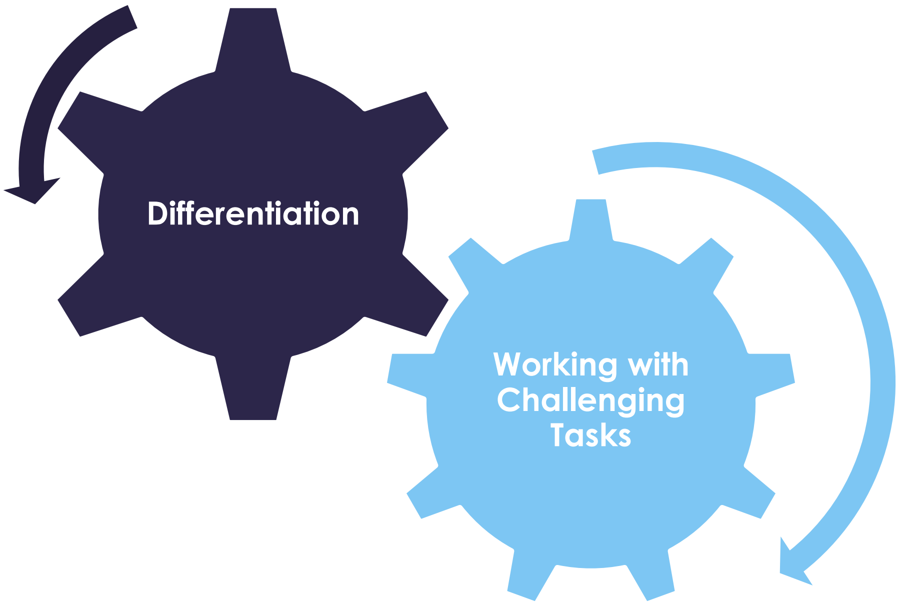


Figure 1. *Working with Challenging Tasks and Differentiation: Illustrating the Synergy*

Reflection Discussion

- Can this representation fully describe the relationship between working with challenging tasks and differentiation?
  - *Which aspects of this relationship does it describe successfully, and which does it not?*

Every set of gears may not be able to move smoothly due to several reasons, such as tooth wear and premature failure. Similarly, as discussed in activity 2, teachers appear to face various difficulties and challenges when trying to simultaneously work on cognitively challenging tasks with all their students. At the same time, just like good oiling of the gears helps the system works much better and more functionally, there are certain instructional aspects that can support teachers' attempts to work on both fronts, if executed appropriately. In the rest of this case, we will work on one of these aspects that both research and our work with teachers have suggested to be important for promoting either of the two goals considered herein–and thus, we envision to play a pivotal role to the smooth and appropriate functioning of this gear system.

Using Questioning to Facilitate the Gear System Function

Scaffolding is another metaphor borrowed from the field of construction, where a scaffold is a temporary structure that helps with the building of another structure. In education, it refers to the temporary support provided for the completion of a task that students otherwise might not be able to complete. This support can be provided in a variety of manners, such as modeling, feedback providing, or questioning.

Questioning is a key instructional aspect related to scaffolding. For example, in every lesson, teachers ask students several questions. Simply posing questions, however, does not automatically lead to cognitively challenging students, let alone all of them. Some of the questions ***can either facilitate or (temporarily) block the movement of both gears*** depending on their characteristics and what they require from students to do.

Teachers who participated in other phases of the EDUCATE project, pointed to ***the difficulty they have in asking good questions which cognitively challenge students and differentiate instruction at the same time***; such difficulties are reported more explicitly or tacitly in the literature, as well. Hence, a teacher could reasonably wonder, "What questions should I ask to stimulate student mathematical thinking when working on challenging tasks? How can I do this for all my students?" The activities that follow aim at investigating how questioning can support the movement of the 'gear system' considered above and thus, help concurrently work at both fronts.

Activity 3 – Using Questioning to Scaffold Students' Work

In this activity, we will peer inside Ms. Jenny's classroom, a seventh-grade teacher at a small provincial lower secondary school in Ireland. In today's lesson, she is using the task 'Fascinating Fractions' to explore with her students some spatial representations of fractions where area, rather than just length, is concerned. You will be provided with some narratives; in these narratives, we are observing the interactions among Ms. Jenny and her students at different phases of this lesson. At different junctures, you will be asked to consider the teacher' interactions with students and her questioning or write what questions you would pose to cognitively challenge the students and differentiate instruction. Read the learning objectives, the task and the narratives; and each time consider the questions that follow.

Learning Objectives:

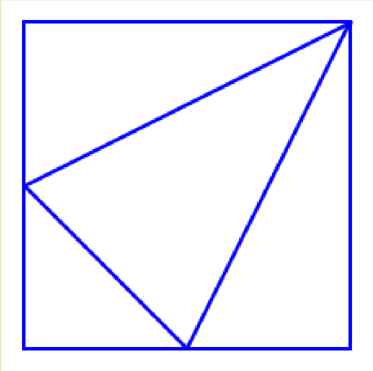
By the end of this lesson, students should be able to:

- Calculate triangular areas and expressing them in fractional forms based on geometrical relationships

Task 'Fraction Fascination':

Subtask 1

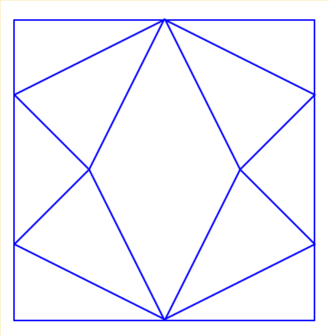
I drew this picture by drawing a line from the top right corner of a square to the midpoint of each of the opposite sides. Then I joined these two midpoints with another line.



1. Can you see four triangles in the square?
2. What fraction of the area of the square is each of these triangles?

Subtask 2

Then I drew another picture:



3. How is this made using the first square?
4. What is the shape that has been created in the middle of this larger square?
5. What fraction of the total area of the large square does this shape take up?

Source: https://nrich.maths.org/5061

Narrative 1a (Launching the task 'Fractions Fascination')

Ms. Jenny was not sure how her students would respond to this task; they were all familiar with the notion of fraction, partitioning the whole/unit into equal parts, simplifying fractions, finding the area of different geometrical shapes, and handling algebraic expressions. However, students' visualizations greatly varied and she thought that this task and its representations may prove very difficult for some and yet readily accessible to others. Although she spent significant time thinking about these issues during her planning last night, Ms. Jenny was still pondering about these issues when the school bell rang.

All students came into the classroom and took their seats. The lesson began by projecting the first image and giving out a worksheet (with the first image on the first page and the second on the second page) and then, inviting students to talk to their neighbor about what they see on the first page for a minute. After about a minute, she through of initiating a discussion around the task by asking a couple of questions.

(b) If you were in Ms. Jenny's shoes, what questions would you pose at this point to engage all students in challenging work and differentiate instruction at the same time? Why?

- *In which order would you pose these questions? Why?*

Below is how Ms. Jenny decided to handle this situation.

Narrative 1b (Launching the task 'Fractions Fascination')

Ms. Jenny: OK class, let's share our thinking about the first image. Remember that we have to stop talking and pay attention to what our classmates would like to share with us. Did we all see the triangles? Who would like to start? [Some pairs are raising their hands] Alicia and John [who were also raising their hands], the floor is yours.

Alicia: We noticed that this is a square [pointing to the border of the first image].

Ms. Jenny: In how many triangles is this square divided?

John: Yes, and it is divided into [counting silently] … four triangles.

Ms. Jenny: Do we all see the four triangles?

All students: Yes. No [A student, Paul, answered negatively].

Ms. Jenny: Paul, can you come to the board and show us how many triangles do you see? [Paul, goes to the board and shows only the central triangle]. What kind of triangle is this?

Paul: Iso…sceles?

Ms. Jenny: Nice. What other types of triangles do you see on this image?

Paul: Hmmm…Oh! I can see one, two… three triangles! I didn't consider them as triangles at the beginning.

Ms. Jenny: Thanks, Paul. How do we call the triangles which have a 90^{o} angle?

Paul: Right-angled.

Ms. Jenny: Great! So, guys, does anyone know how I drew this picture? If I tell you to copy this picture what would you do?

Penny: I would draw a big triangle starting from the upper right corner of the square… to the... [inaudible]

Ms. Jenny: How do we call the center of a segment?

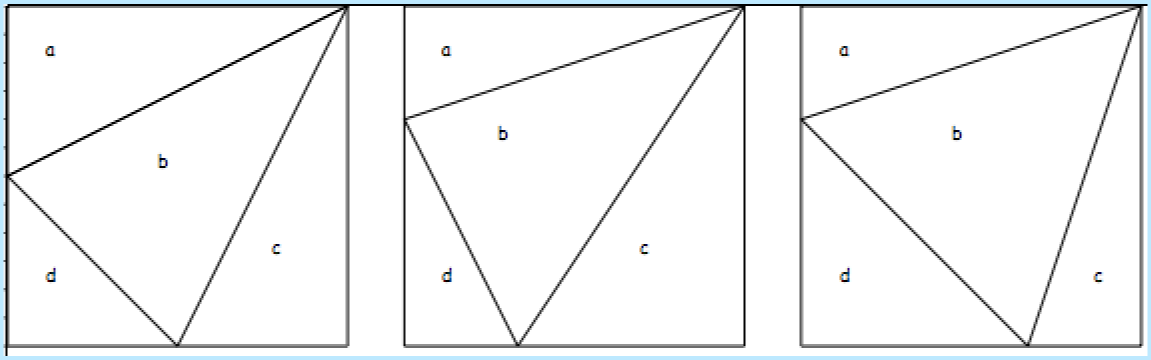
Alicia: Midcenter…

Ms. Jenny: Nice try. It is called 'midpoint'. Thanks, Penny and Alicia. I drew this picture by drawing a line from the top right corner of a square, as Alicia said, to the midpoint of each of the opposite sides. Then I joined these two midpoints with another line. So, what do I know now about the length of these segments [pointing to the two equal parts of the square sides]?

George: They have half the size of the side of the square.

Ms. Jenny: Nice. Now, I will give you this worksheet [with the subtasks 1 and 2 on it] and you will try to solve subtask 1 in your groups. Decide how you would like to work; you can do anything you would like on the worksheet using the printed image that I gave you at the beginning. If you have any questions you can raise the red cards and I will come. Those who finish earlier can raise the green cards and when I see them they can consider the extended task that is displayed on the interactive board.

These images came about by drawing a line from the top right corner of the square to the one third of the opposite sides rather than the midpoints as in the first, original one. What do you notice about the four areas in each of these three examples?



The extended subtask.

Ms. Jenny was thinking that the resulting discussion was a great assessment opportunity to understand what her students could grasp from the representation used in the first subtask.

Guiding Questions

- Is differentiation and cognitive engagement of the students simultaneously achieved in this episode?
  - *If yes, how? In what respects?*
  - *If no, why?*
- Assess the appropriateness of the questions posed by the teacher in regards to cognitively challenging students and differentiating her approach simultaneously.

The class then continued working on the task.

Narrative 2a (Student Autonomous work: The Group with the Scissors and the Ruler)

Students were working autonomously, while Ms. Jenny was circulating around monitoring what they were doing. A group of three students (Mary, Michael, and Daniel) decided to cut the image to explore the area of the four triangles to solve the second question of subtask 1. Michael took the two scalene right-angled triangles, put them on top of each other and noticed that they were equal-sized. Then, Mary suggested that they could take each triangle and put it over the large square to see how many identical triangles fit into the large square. Both boys agreed with Mary's idea and each of them took a different triangle and started exploring the area of each triangle compared to the area of the large square. Alicia and John, who were also in this group, were working on their own; they were using their ruler to measure the height and the base of each triangle and then use this information to calculate the area of each triangle. Ms. Jenny surprisingly noticed that this student group used the scissors and the ruler and decided to head first to that group and have a discussion with them.

(c) What questions would you pose to engage these three students in challenging work and differentiate instruction at the same time? Why?

- *In which order would you pose these questions? Why?*

<u>**Narrative 2b**</u> (Student Autonomous work: The Group with the Scissors)

Michael: Hi, Ms. Jenny!

Mrs. Jenny: Hi guys. How are we doing on this task?

Michael: We are working on subtask 1 [laughing].

Ms. Jenny: I know that you are working on that [smiling]! What was subtask 1 asking you to do?

Michael: The first question was asking us to say how many triangles we see. We answered that before. Then, question 2 was asking us to figure out the fraction that shows the area of the triangles.

Ms. Jenny: And what did you decide to do?

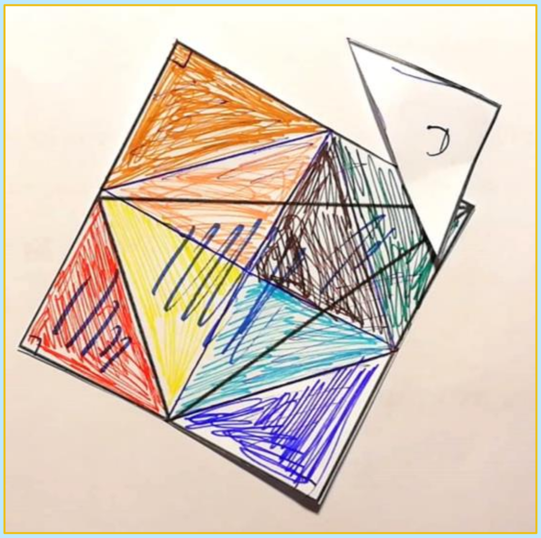
Michael: We cut the triangles of the first worksheet to see how many same triangles fit into the square of the second worksheet. Mary had this idea!

Ms. Jenny: Nice idea, Mary. [Addressing Mary:] What have you discovered so far?

Mary: The area of the small triangle is the 1/8 of the area of the square.

Ms. Jenny: How did you find that out?

Mary: I took the small right-angled triangle and I draw 8 triangles of this size into the square. So, it is the 1/8. Mike worked the same way.



Mary's solution

Michael: I found that the big right-angled triangle is the ¼ of the square, do you see? [Pointing to his cut shapes]

Ms. Jenny: Good. Daniel, tell me about your work.

Daniel: I used what Michael and Mary discovered.

Ms. Jenny: Can you please elaborate more on this?

Daniel: I took the central and bigger triangle and I used Mary and Mike's answers.

Ms. Jenny: What made you decide to do that?

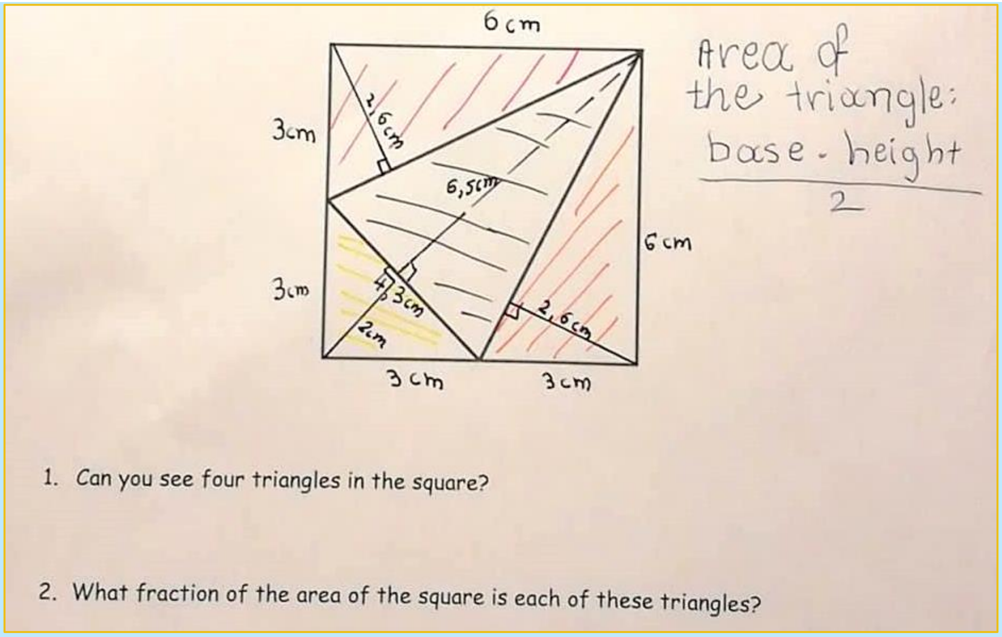
Daniel: I could not fit it into the square… Mary found that this triangle is the 1/8 of the square, Mike found that the two equal triangles are the 2/4 of the square. So, then I added 1/8 + 2/4 = 1/8 + 4/8 = 5/8. The area of the big triangle is the remaining area, it is 3/8 of the square.

Ms. Jenny: I am really impressed. Keep up the good work! You can move to subtask 2 and I will come back to you in about five minutes. [Addressing Alicia and John] Hey, guys! What shapes do we have here?

Alicia: Triangles.

Ms. Jenny: Oh, you used your ruler. Do you know the formula of calculating the triangle area?

John: Yes [pointing on their worksheet]. We are measuring the base and the height of the triangles.



Alicia and John's solution

Ms. Jenny: Are you going to apply the formula using these decimals? Are you sure that this is going to give you the fraction of each triangle? Be careful.

Alicia: Emmm… they [pointing to the group who were using their scissors] cut the shapes…

Ms. Jenny: Yes, you can work in any way you would like. Continue your work and I will be with you in a couple of minutes.

Guiding Questions

- What do you think of the way Ms. Jenny interacted with her students?
  - Was she able to work on both fronts (both challenge her students and differentiate her approach)?
  - *If yes, how?*
  - *If no, why?*
- Assess the appropriateness of the questions posed by the teacher in regards to cognitively challenging students and differentiating her approach simultaneously.

Here is how the teacher interacted with another group of students.

Narrative 3a (Student autonomous work: The Group with the Algebraic Expressions)

George, Paul and Richard decided to work in a totally different way. They considered that the length of the square is equal to X. So, the two sides of the square are divided into two equal parts (X/2). George, made use of the formulas for calculating the triangle area and the square area; this way he calculated the area of each triangle and the large square, as shown in Figure 1. Richard, followed a similar solution path, but instead of subtracting the areas of the three triangles from the area of the square as George did, he used the Pythagorean theorem to find the length of the sides of the central triangle, as shown in Figure 2. Paul copied George's way but at some point, he got lost and did not know how to proceed. Ms. Jenny went to their group and initiated a discussion with them.

(a) If you were in her shoes, what questions would you pose to engage all students in challenging work and differentiate instruction at the same time? Why?

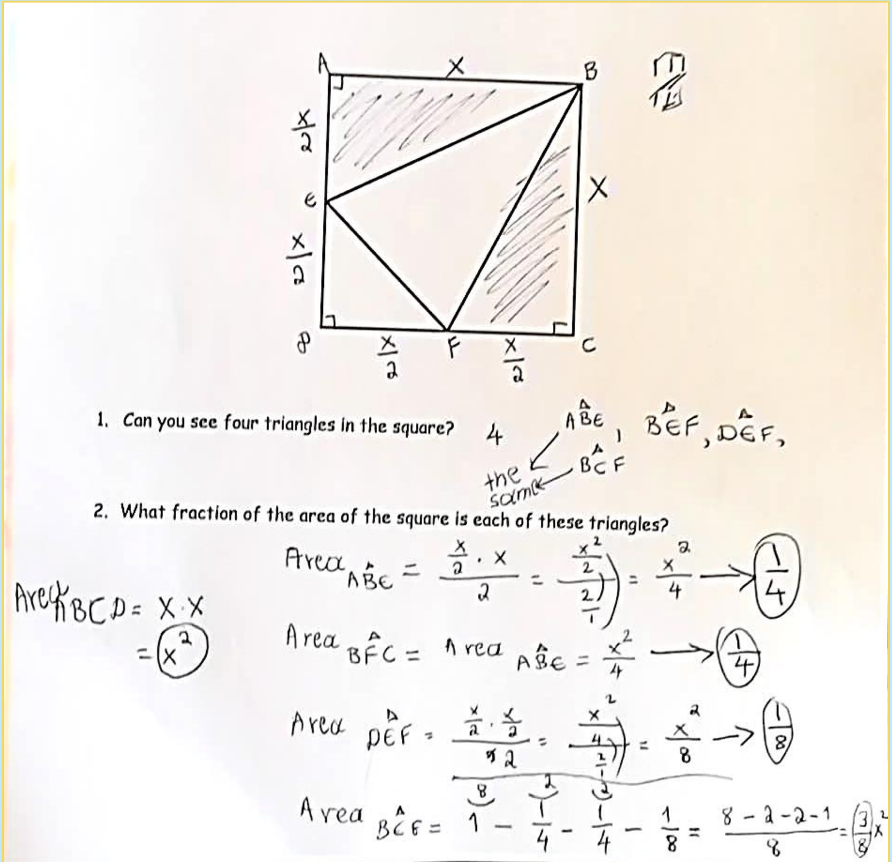
- *In which order would you pose these questions? Why?*

This is how the teacher interacted with this group of students.

__Narrative 3b__ (Student autonomous work: The Group with the Algebraic Expressions)

Ms. Jenny: Tell me about your solution. How our discussion at the beginning of the lesson informed your work?

George: Let's say that this side is equal to X. At the beginning we said that these are the midpoints of these two sides [pointing to the sides of the square] … so… they are the half [in size], X/2.



George's solution

Ms. Jenny: Bravo! What is the area of the square?

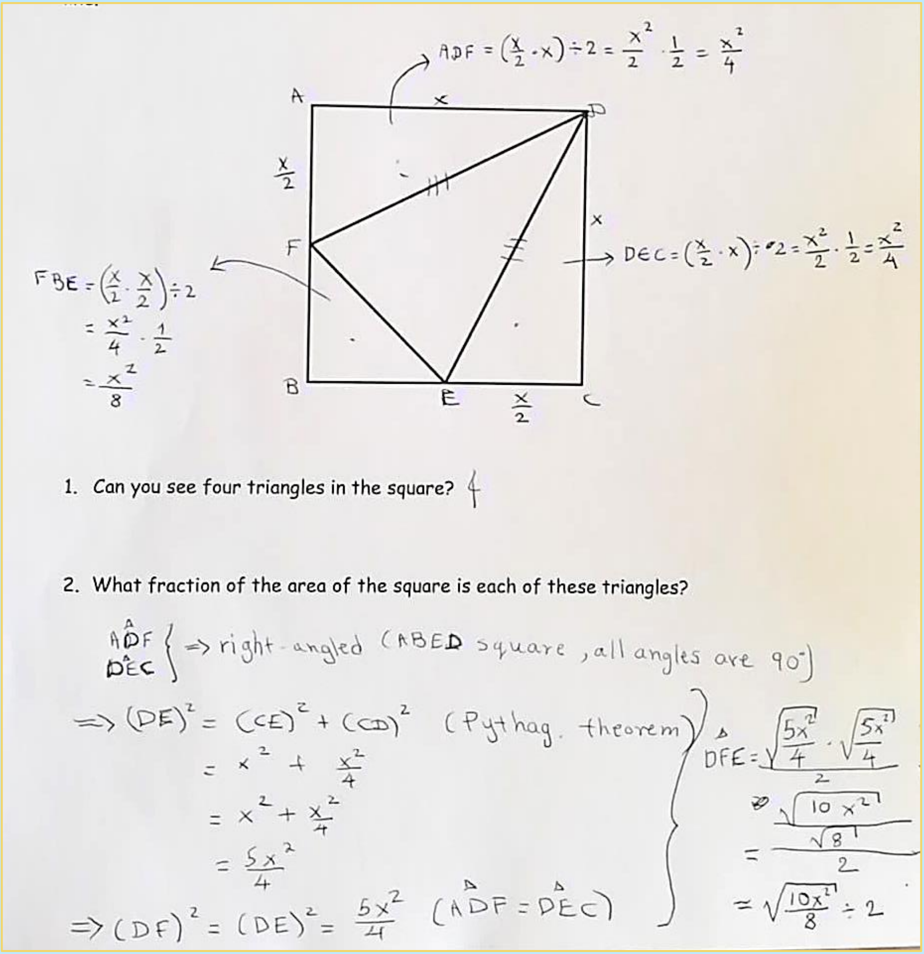
George: X^2.

Ms. Jenny: [What is] The area of the triangles?

George: The first one is $X^2/4$ [the scalene right-angled triangle], the other one is $4X^2/8$ [the isosceles right-angled triangle], and the last one is $3X^2/8$ [the isosceles acute-angled triangle].

Ms. Jenny: Good, what fraction of the area of the square is each of these triangles, if we know that the area of the square is X^2? Write the area of each triangle as a fraction. [The teacher now turns to Richard] Richard, have you thought of another way that this could be done? Oh, how did you use the Pythagorean theorem?

Richard: I tried to find the length of the sides [of the central triangle] … I found that this is $\sqrt{\frac{10X^2}{8}}/2$.



Richard's solution

Ms. Jenny: OK, how do we find the area of the triangle?

Richard: We multiply the base by the height, and then divide by 2.

Ms. Jenny: Nice. Did you find the length of the base and the height of the triangle or did you find something else?

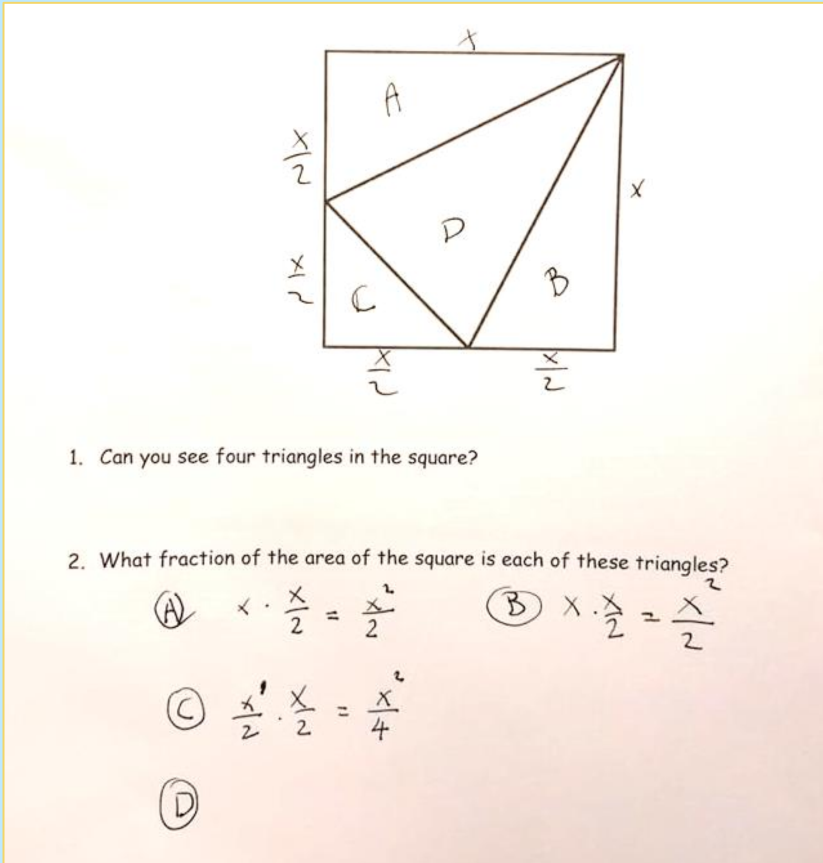
Richard: I don't know… I am lost.

Ms. Jenny: You used a really smart way. I liked the idea of utilizing the Pythagorean theorem. Can you identify the base and the height of the triangle? Do this and I will come back in a while.

Paul: Ms. Jenny…. I am not a math person. Could you please give me a hint?

Ms. Jenny: What have you done so far?

Paul: I did what George did. Can you check if I am right?



Paul's solution

Ms. Jenny: What exactly did you do? Please, explain to me why you decided to do it this way.

Paul: I don't know how to explain it...

Ms. Jenny: If this way does not serve you, try to think of something else.

Richard: Ms. Jenny, I can't find the base and the height…

Ms. Jenny: OK, wait and we will discuss it with the whole class in a couple of minutes.

Guiding Questions

- Is differentiation and cognitive activation simultaneously achieved in this episode?
  - *If yes, how?*
  - *If no, why?*
- Assess the appropriateness of the questions posed by the teacher in regards to cognitively challenging students and differentiating her approach simultaneously.

At this point, the teacher decided to have a whole-class discussion.

Narrative 4 (Initiating a Whole-class Discussion)

Ms. Jenny realized that the time has passed and they should now have a whole-class discussion to share their solutions. She noticed that during student autonomous work on subtask 1, the solutions described above had emerged: Some students had cut the triangles with their scissors or ruler to explore their area, other students had used algebraic expressions, while some students utilized the Pythagorean theorem to calculate the length of the central triangle. Ms. Jenny wanted to engage all students in a meaningful whole-class discussion so that they all have the opportunity to discuss mathematics with one another, refining and critiquing each other's ideas and understandings. How should the discussion be organized and what questions should she posed to her students?

(b) What questions would you pose at this point to engage all students in challenging work and differentiate instruction at the same time? Why?

- *In which order would you pose these questions? Why?*

Which characteristics should good questioning have to cognitively challenge students and differentiate instruction at the same time? Discuss these characteristics in your group and record them below.

Connections to (my) Practice

<u>Optional</u>:

Design a lesson that takes into consideration the role and the characteristics of questions that promote or hinder both differentiation and cognitive activation at the same time as discussed and codified in our meeting.

Teach and videotape the lesson and select two video clips one from student autonomous work and another from whole-class discussion that are illustrative of your attempts to pose appropriate questions to promote both differentiation and cognitive activation (regardless of how successful these attempts were).

EVIDENCE OF SUCCESS

Basic Questions to Consider

- Why are the tasks used in a lesson so important for student learning?
- What makes a mathematical task challenging? Identify at least three attributes that contribute to this challenge.
- Propose two tasks that you think are challenging. In each case clarify what makes the task challenging for students (you could also specify the student population which you have in mind).
- How would you define differentiation to a colleague who would be interested in learning what it is?
- What strategies would you name for materializing differentiation in your own class?
- In this module, an argument was made that differentiation and working with challenging tasks work synergistically. Explain how you understand this synergistic relationship. Give a couple of examples to make your argument more concrete.

Reflection on Practice

- Thinking of your previous lesson, was there a task that was planned as demanding but its demands were adjusted during the enactment? Why did this happen?
- Select one task for your videotaped lessons (different from those you might have chosen during this Module). Specify the level of the task as it was presented in the curriculum materials, as it was presented, and as it was enacted. Explain how you would classify the cognitive challenge of the task at these different levels.
- The following clip which comes from an 8th-grade lesson on ratios taught in Australia (http://www.timssvideo.com/au4-ratios, 05:04-16:18). In this segment, the students have already been given piles of cubes, which they use to solve problems on sharing different amounts based on certain ratios. Watch carefully this lesson segment and consider the following:
    - Try to identify the cognitive level at which the task was presented and enacted in this class. If the task was presented/enacted at a low level, what would you do differently in order to present it/enact it with students at a higher cognitive level
    - Did the teacher attempt to differentiate her approach? If so, please provide concrete examples of her doing so. If not, please identify instances in which she could have done so and explain why you think it would be important to do so at those particular lesson junctures.
- Prepare a lesson for a topic you will teach next week. Briefly annotate your lesson explaining how you took into consideration the dual task of engaging all students in challenging mathematical tasks. Please provide concrete examples to justify your answers.
- In Case 3 of this module, questioning was presented as an instructional aspect in which teachers engage quite frequently, which can contribute to concurrently working at both fronts under consideration: working with challenging tasks and differentiation. Reflecting on your practice, what other instructional aspects/teacher moves would you name as concurrently serving this dual goal? Explain your thinking.

BRINGING IT ALL TOGETHER

Takeaways from the Module

- Planning lessons in which challenging tasks are included can increase students' opportunities for quality learning:
    - Can you identify which tasks are challenging and which are not?
    - Can you name what makes a task more/less challenging?
    - Can you identify which learning goals challenging/less challenging goals serve?
- Selecting challenging tasks is not enough in and of its own for increasing students' learning opportunities:
    - Have you thought of ways in which the cognitive challenge of a task can be modified during instruction?
    - Have you thought of particular factors that might contribute to the modification of challenge during task presentation and enactment?
- Differentiating your teaching is necessary for ensuring that all students are productively engage in the lesson and by the end of it they learn worthwhile mathematics:
    - Have you thought of concrete ways in which you can differentiate your instruction to meet students of different readiness/ability levels?
    - Can you identify certain decisions you made which might run against the goal of differentiation?
- Working on challenging tasks and differentiation can work synergistically to each other:
    - Can you identify certain episodes from your teaching that illustrate this harmonic/synergistic relation?
    - What decisions might a teacher make that might run counter to both these goals?

SUPPLEMENTARY MATERIAL

I. Additional literature/References/Further reading

Arbaugh, F., & Brown, C. A. (2004). What makes a mathematical task worthwhile? Designing a learning tool for high school mathematics teachers. In R. N. Rubenstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics: Sixty-sixth yearbook* (pp. 27-41). Reston, VA: NCTM.

Arbaugh, F., & Brown, C. A. . (2006). Analyzing mathematical tasks: A catalyst for change? *Journal of Mathematics Teacher Education*(8), 499-536

Boston, M. D., & Smith, M. S. . (2009). Transforming secondary mathematics teaching: increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal of Research in Mathematics Education, 40*(2), 119-156.

Boston, M. D., & Smith, M. S. (2011). A 'task-centric approach' to professional development: enhancing and sustaining mathematics teachers' ability to implement cognitively challenging mathematical tasks. *ZDM*, *43*(6-7), 965-977.

Doyle, W. (1988). Work in Mathematics Classes: The Context of Students' Thinking During Instruction. *Educational Psychologist, 23*(2), 167 – 180.

Smith, M. S., Stein, M. K., Arbaugh, F., Brown, C. A., & Mossgrove, J. . (2004). Characterizing the cognitive demands of mathematical tasks: A task-sorting activity. In G. W. Bright & R. N. Rubenstein (Eds.), *Professional development guidebook for perspectives on the teaching of mathematics: Companion to the sixty-sixth yearbook* (pp. 45-72). Reston, VA: NCTM.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319-369). Charlotte, NC: Information Age Publishing.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2016). *Implementing Standards-Based Math Instruction: A Casebook for Professional Development*. Τόπος: Teachers College Press.

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2015). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*, *18*(2), 123-140.

Tomlinson, C.A.(2001). *How to differentiate instruction in mixed-ability classrooms* (2nd ed.) *Alexandria*, VA: Association for Supervision and Curriculum Development.

Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners* (2nd ed.). VA: Association for Supervision and Curriculum Development.

II. External Links

https://www.youtube.com/watch?v=mVRYSC8YyYA

https://www.youtube.com/watch?v=EOPe_cJ67No

5 different modules

Module 1: Title

Module 2: Title

Module 3: Title

Module 4: Title

Module 5 Title

Partner Organizations

University of Cyprus

CYPRUS PEDAGOGICAL INSTITUTE

Υπουργείο Παιδείας και Πολιτισμού

Institiúid Oideachais Marino Institute of Education

An Associated College of Trinity College Dublin, the University of Dublin

HELLENIC REPUBLIC
National and Kapodistrian University of Athens

U LISBOA Universidade de Lisboa

ie Instituto de Educação