Ασκήσεις ΣΔΕ (17/12/2018)

[A1] Να βρείτε ολοκληρωτικό παράγοντα $\mu(t,y) = t^{\alpha} y^{\beta}$ $(\alpha,\beta \in \mathbb{R})$ ώστε η εξίσωση: $\mu(ty^2 + 2t^2y)\,dt + \mu(t^2y - t^3y^2)\,dy = 0$ να είναι ακριβής. Στη συνέχεια να λύσετε την εξίσωση.

[A2] Να λυθεί το ΠΑΤ: $y'' + 4y = \cos\omega t$, $y(0) = y'(0) = 0$. Για ποιές τιμές της παραμέτρου ω είναι η λύση φραγμένη;

[A3] Να λυθούν οι εξισώσεις: (α) $ty' - y = \ln(t)\, y^2$ $(t>0)$, (β) $t^2y'' + 3ty' + 2y = 0$ $(t>0)$, (γ) $y'' + 5y' + 4y = e^{-t} + t^2$.

[A4] Να λυθεί το ΠΑΤ: $y'' + t^2y' + 2ty = 0$, $y(0) = 1$, $y'(0) = 0$ με την μέθοδο δυναμοσειρών (με κέντρο $t=0$).

[A5] Αν $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ να βρεθεί ο πίνακας e^{At}

[A6] Δείξτε ότι $y_1(t) = 1 - 2t^2$ είναι λύση της εξίσωσης: $y'' - 2ty' + 4y = 0$. Να βρεθεί δεύτερη λύση $y_2(t)$ γραμμικά ανεξάρτητη από την $y_1(t)$ (με την μορφή ολοκληρώματος μόνο).

[A7] Να βρεθεί η γενική λύση της εξίσωσης: $y'' + 4y' + 4y = t^{-2}e^{-2t}$ $(t>0)$.

[A8] Δείξτε ότι κάθε λύση της: $y' = ay + be^{\lambda t}$ $(a<0, \lambda<0)$ έχει την ιδιότητα $\lim_{t \to +\infty} y(t) = 0$.

[A9] Έστω το ΠΑΤ: $y' = y^3$, $y(0) = 1$. Βρείτε το μέγιστο διάστημα $t \in [0,\alpha)$ στο οποίο το Θεώρημα PL εγγυάται ύπαρξη και μοναδικότητα λύσης.

[A10] Σχεδιάστε το διάγραμμα φάσης της $y' = y(1-y^2)$ και χαρακτηρίστε τα σ.ι. ως προς την ευστάθεια.

ΓΧ 15/12/2018

Επανάληψη 17/12/2018

[A1] $\underbrace{(ty^2+2t^2y^3)}_{M}dt + \underbrace{(t^2y-t^3y^2)}_{N}dy = 0$

$M_y = 2ty + 6t^2y^2 \neq N_t = 2ty - 3t^2y^2$

Και εξίσωση δεν είναι ακριβής. Έστω $\mu = t^\alpha y^\beta$. Τότε

$\mu M dt + \mu N dy = \overbrace{(t^{\alpha+1}y^{\beta+2} + 2t^{\alpha+2}y^{\beta+3})}^{\tilde{M}}dt$

$+ \underbrace{(t^{\alpha+2}y^{\beta+1} - t^{\alpha+3}y^{\beta+2})}_{\tilde{N}}dy = 0.$

$\tilde{M}_y = (\beta+2)t^{\alpha+1}y^{\beta+1} + 2(\beta+3)t^{\alpha+2}y^{\beta+2}$

$\tilde{N}_t = (\alpha+2)t^{\alpha+1}y^{\beta+1} - (\alpha+3)t^{\alpha+2}y^{\beta+2}.$

Πρέπει να έχουμε:

$\beta+2 = \alpha+2 \Rightarrow \alpha = \beta.$ $\quad 2\beta+6+\alpha+3 = 0.$

$\Rightarrow 3\alpha = -9 \Rightarrow \underline{\alpha = \beta = -3}$

Άρα η εξίσωση γράφεται:

~~$t^{-2}y^{-2}dt + (-t^{-2}y^{-2}$~~

$\underbrace{(t^{-2}y^{-1} + 2t^{-1})}_{\tilde{M}}dt + \underbrace{(t^{-1}y^{-2} - y^{-1})}_{\tilde{N}}dy = 0.$

$\frac{\partial F}{\partial t} = \tilde{M} \Rightarrow F(t,y) = -t^{-1}y^{-1} + \overbrace{2\ln|t|}^{\ln t^2} + h(y).$

$\Rightarrow \frac{\partial F}{\partial y} = \cancel{t^{-1}y^{-2}} + h'(y) = \cancel{t^{-1}y^{-2}} - y^{-1} \Rightarrow h(y) = -\ln|y|$

$\therefore F(t,y) = -t^{-1}y^{-1} + \ln(t^2) - \ln|y| = c$

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$$\Rightarrow \ln\left(\frac{t^2}{|y|}\right) = c + t^{-1}y^{-1}$$

$$\frac{t^2}{|y|} = e^c \cdot e^{1/ty} \Rightarrow t^2 = \underbrace{e^c}_{c_1} |y| e^{1/ty}$$

[A2] $y'' + 4y = \cos\omega t$. $\quad y'(0) = y''(0) = 0$.

Ομογενής: $p(r) = r^2 + 4 = 0 \Rightarrow r = \pm 2i$

$$y_{o\mu} = c_1 \cos 2t + c_2 \sin 2t.$$

Αν $\omega \neq 2$: $y_c = A\cos\omega t + B\sin\omega t$. και γενική λύση της μορφής

$$y = c_1\cos 2t + c_2 \sin 2t + A\cos\omega t + B\sin\omega t$$

(ψευδοσυντονισμός). Αν $\omega = 2$

$$y_c = At\cos 2t + Bt\sin 2t$$

$$\Rightarrow y_c' = A\cos 2t - 2At\sin 2t + B\sin 2t + 2Bt\cos 2t$$

$$\Rightarrow y_c'' = -2A\sin 2t - 2A\sin 2t - 4At\cos 2t$$

$$+2B\cos 2t + 2B\cos 2t - 4Bt\sin 2t$$

$$= -4A\sin 2t - 4At\cos 2t + 4B\cos 2t - 4Bt\sin 2t$$

Άρα $y_c'' + 4y_c = -4A\sin 2t - \cancel{4At\cos 2t} + 4B\cos 2t$
$\quad - \cancel{4Bt\sin 2t} + \cancel{4At\cos 2t} + \cancel{4Bt\sin 2t}$
$\quad = \cos 2t$

$$\Rightarrow -4A\sin 2t + 4B\cos 2t = \cos 2t$$

$$\Rightarrow A = 0,\quad B = \frac{1}{4}.$$

Λύση:

$y(t) = c_1 \cos 2t + c_2 \sin 2t + \frac{1}{4} t \sin 2t$

που δεν είναι φραγμένη!

[A3] (α) $ty' - y = \ln(t) \cdot y^2$ $(t > 0)$.

Εξίσωση Bernoulli ($r=2$) Θέτουμε $u = y^{1-r} = y^{-1}$

$\Rightarrow u' = -y^{-2} y'$. Άρα.

$y' - t^{-1} y = t^{-1} \ln(t) y^2 \Rightarrow (-y^{-2}) y' - t^{-1}(-y^{-2}) y =$

$= t^{-1} \ln(t) (-y^{-2}) y^2$

$\Rightarrow u' + t^{-1} \underbrace{y^{-1}}_{u} = -t^{-1} \ln(t)$

$\Rightarrow u' + t^{-1} u = -t^{-1} \ln(t)$.

$\mu(t) = e^{\int t^{-1} dt} = e^{\ln t} = t$.

$\Rightarrow t u' + u = -\ln(t) \Rightarrow (tu)' = -\ln(t)$.

$\Rightarrow tu = -\int \ln(t) dt + c$

$\int \ln(t)\, dt = \int \underbrace{1}_{\frac{dv}{dt}} \cdot \underbrace{\ln(t)}_{u} dt = t \ln(t) - \int t \cdot \frac{1}{t} dt$

$= t\ln(t) - t$.

$v = t,\ u' = \frac{1}{t}$.

Apa: $tu = t - t\ln(t) \overset{+c}{} \Rightarrow u = 1 - \ln(t) + ct^{-1}$

$$\Rightarrow \cancel{\frac{1}{y} = 1 - \ln(t) + ct} \Rightarrow \cancel{y = \frac{1}{1-\ln(t)}}$$

$$\frac{1}{y} = 1 - \ln(t) + \frac{c}{t} = \frac{t - t\ln(t) + c}{t}$$

$$\Rightarrow y(t) = \frac{t}{t - t\ln(t) + c}$$

(β) $t^2 y'' + 3ty' + 2y = 0 \qquad (t > 0).$

$$y = t^r \Rightarrow y' = r t^{r-1} \Rightarrow y'' = r(r-1)t^{r-2}$$

$$\Rightarrow [r(r-1) + 3r + 2]t^r = 0.$$

$$\Rightarrow (r^2 + 2r + 2) = 0 \Rightarrow (r+1)^2 + 1^2 = 0$$

$$\Rightarrow r = -1 \pm i$$

$$\therefore y = c_1 t^{(-1+i)} + c_2 t^{(-1-i)} =$$

$$= t^{-1}[c_1 t^i + c_2 t^{-i}] = t^{-1}[c_1 e^{i\ln(t)} + c_2 e^{-i\ln(t)}].$$

$$= t^{-1}[c_1[\cos(\ln(t)) + i\sin[\ln(t)]]]$$

$$+ t^{-1}[c_2[\cos(\ln(t)) - i\sin(\ln(t))]] =$$

$$= \underbrace{(c_1 + c_2)}_{d_1} t^{-1}\cos\ln(t) + \underbrace{(ic_1 - ic_2)}_{d_2} t^{-1}\sin\ln(t)$$

$$= t^{-1}[d_1 \cos\ln(t) + d_2 \sin(\ln(t))]$$

(γ) $y'' + 5y' + 4y = e^{-t} + t^2$

$p(r) = r^2 + 5r + 4 = (r+1)(r+4) = 0 \Rightarrow r_1 = -1,\ r_2 = -4$

$\therefore\ y_{of} = c_1 e^{-t} + c_2 e^{-4t}$

$\psi_{c1} = \cancel{t}\ Ate^{-t} \Rightarrow \psi'_{c1} = Ae^{-t} - Ate^{-t}$

$\Rightarrow \psi''_{c1} = -Ae^{-t} - Ae^{-t} + Ate^{-t} = -2Ae^{-t} + Ate^{-t}$

$\therefore\ -2Ae^{-t} + \cancel{Ate^{-t}} + 5(Ae^{-t} - \cancel{Ate^{-t}}) + 4\cancel{Ate^{-t}} = e^{-t}$

$\Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3}$

$\psi_{P2} = A + Bt + \Gamma t^2 \Rightarrow \psi'_{P2} = B + 2\Gamma t \rightarrow \psi''_{P2} = 2\Gamma$

$\therefore\ 2\Gamma + 5(B + 2\Gamma t) + 4(A + Bt + \Gamma t^2) = t^2$

$\Rightarrow \underbrace{(2\Gamma + 5B + 4A)}_{0} + (\underbrace{10\Gamma + 4B}_{0})t + \underbrace{4\Gamma}_{1} t^2 = t^2$

$\Rightarrow \Gamma = \frac{1}{4} \Rightarrow \frac{10}{4} = -4B \Rightarrow B = -\frac{10}{16} = -\frac{5}{8} = B$

$4A = -\frac{1}{2} + \frac{25}{8} = \frac{21}{8} \Rightarrow A = \frac{21}{32}$

$\therefore\ y(t) = c_1 e^{-t} + c_2 e^{-4t} + \frac{21}{32} - \frac{5t}{8} + \frac{t^2}{4}$

[A4] $y'' + t^2 y' + 2ty = 0 \qquad y(0) = 1,\ y'(0) = 0$

$y(t) = \sum_{n=0}^{\infty} a_n t^n \Rightarrow y'(t) = \sum_{n=1}^{\infty} n a_n t^{n-1},\ y'' = \sum_{n=2}^{\infty} n(n-1) a_n t^{n-2}$

Επομένως:

$$\sum_{n=2}^{\infty} n(n-1)a_n t^{n-2} + \sum_{n=1}^{\infty} n a_n t^{n+1} + \cancel{\sum 2n(n-1)a_n t^{n+1}}$$
$$\sum_{n=0}^{\infty} 2 a_n t^{n+1} = 0$$

$$2a_2 + 6\underline{a_3 t} + 12 a_4 t^2 + \cdots$$

$$+ a_1 t^2 + 2a_2 t^3 + \cdots$$

$$+ \underline{2a_0 t} + 2a_1 t^2 + 2a_2 t^3 + \cdots$$

$$= 2a_2 + (6a_3 + 2a_0)t + (12a_4 + a_1 + 2a_1)t^2 + \cdots$$

Επομένως: $\cancel{\ }$ $n-2 = m+1 \Rightarrow \underline{m = n-3}$

$$\underline{n = m+3}$$

$$\cancel{\sum (m+3)(m+2)a_{m+3} t^{m+1}}$$

$$\sum_{n=2}^{\infty} n(n-1)a_n t^{n-2} = a_2 + \sum_{n=3}^{\infty} n(n-1)a_n t^{n-2}$$

$$= a_2 + \sum_{m=0}^{\infty} (m+3)(m+2)a_{m+3} t^{m+1}$$

Άρα.

$$a_2 + \sum_{n=0}^{\infty} \left[(n+2)(n+3)a_{n+3} + (n+2)a_n \right] t^{n+1} = 0$$

$$\Rightarrow a_2 = 0, \quad a_{n+3} = -\frac{a_n}{n+3} \quad n \geq 0.$$

Επομένως:

$\alpha_2 = 0 \Rightarrow \alpha_5 = 0 \Rightarrow \alpha_8 = 0 \Rightarrow \cdots \underline{\alpha_{2+3k} = 0}$ $(k \geq 0)$

$n=0.$ $\alpha_3 = -\frac{\alpha_0}{3} = -\frac{1}{3}$

$\alpha_6 = -\frac{\alpha_3}{6} = \left(\frac{1}{3}\right)\left(\frac{1}{6}\right)$

$$\Rightarrow \alpha_{3k} = \frac{(-1)^k}{(3k)(3k-3)\cdots 6 \cdot 3} = \frac{(-1)^k}{3^k \, k!} \quad k \geq 0$$

$n=1$ $\alpha_4 = -\frac{\alpha_1}{4} = 0 \Rightarrow \alpha_{3k+1} = 0$ $k \geq 0.$

$$\therefore \ y(t) = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k \, k!} t^{3k}.$$

[A5] $q(\lambda) = \begin{vmatrix} \lambda-1 & -2 \\ 2 & \lambda-1 \end{vmatrix} = (\lambda-1)^2 + 4 = 0 \Rightarrow \lambda = 1 \pm 2i$

$\lambda_1 = 1+2i$ $\begin{bmatrix} 2i & -2 \\ 2 & 2i \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \underline{0} \Rightarrow i\alpha = \beta$

$\underline{u} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ $\overline{u} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} 1+2i & 0 \\ 0 & 1-2i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}^{-1}$$

Έπωτε:

$$\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}^{-1} = \frac{1}{(-2i)} \begin{bmatrix} -i & -1 \\ -i & 1 \end{bmatrix} =$$

$$= \frac{1}{2i} \begin{bmatrix} i & 1 \\ i & -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$$

Άρα. $A = \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} 1+2i & 0 \\ 0 & 1-2i \end{bmatrix} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \frac{1}{2}$

$$e^{At} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ +i & -i \end{bmatrix} \begin{bmatrix} e^t \cdot e^{2it} & 0 \\ 0 & e^t e^{-2it} \end{bmatrix} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$$

$$= e^t \frac{1}{2} \begin{bmatrix} e^{2it} & e^{-2it} \\ i e^{2it} & -i e^{-2it} \end{bmatrix} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$$

$$= \frac{1}{2} e^t \begin{bmatrix} e^{2it} + e^{-2it} & -i e^{2it} + i e^{-2it} \\ i e^{2it} - i e^{-2it} & e^{2it} + e^{-2it} \end{bmatrix}$$

$$= \frac{1}{2} e^t \begin{bmatrix} e^{2it} + e^{-2it} & \frac{1}{i}(e^{2it} - e^{-2it}) \\ \frac{1}{i}(e^{-2it} - e^{2it}) & e^{2it} + e^{-2it} \end{bmatrix}$$

$$= e^t \begin{bmatrix} \cos(2t) & \sin(2t) \\ -\sin(2t) & \cos(2t) \end{bmatrix}$$

[A6] $y_1 = 1-2t^2 \Rightarrow y_1' = -4t \;, y_1'' = -4$

και $y_1'' - 2ty_1' + 4y = -\cancel{4} - 2t(-\cancel{4}t) + 4(\cancel{1}-2t^2) = 0$

$y_2 = vy_1 \Rightarrow y_2' = v'y_1 + vy_1' \Rightarrow y_2'' = v''y_1 + 2v'y_1' + vy_1''$

$\Rightarrow v''y_1 + 2v'y_1' + vy_1'' - 2t(v'y_1 + vy_1') + 4vy_1 = 0.$

$\Rightarrow v(\underbrace{y_1'' - 2ty_1' + 4y_1}_{0}) + y_1 v'' + (2y_1' - 2ty_1)v' = 0$

$\Rightarrow v'' + \left(\frac{2y_1'}{y_1} - 2t\right)v' = 0 \qquad v' = u$

$\Rightarrow u' + \left(\frac{2y_1'}{y_1} - 2t\right)u = 0.$

$\mu(t) = \exp\int\left(\frac{2y_1'}{y_1} - 2t\right)dt =$

$= \exp\{-t^2 + 2\ln|y_1|\} =$

$= \exp\{-t^2 + \ln(y_1^2).\} = \underline{e^{-t^2}y_1^2(t)}.$

$e^{-t^2}y_1^2(t)u' + e^{-t^2}y_1^2(t)\left(\frac{2y_1'}{y_1} - 2t\right).u(t) = 0$

$\left[e^{-t^2}y_1^2u\right]' = 0.$

$\Big[(e^{-t^2}y_1^2)' = e^{-t^2}2y_1y_1' - 2te^{-t^2}y_1^2 =$

$= e^{-t^2}y_1^2\left(\frac{2y_1'}{y_1} - 2t\right)\Big]$

$$\Rightarrow e^{-t^2} y_1^2\, u(t) = c$$

$$\Rightarrow u(t) = \frac{e^{t^2}}{y_1^2(t)} \qquad (c=1)$$

$$\Rightarrow v(t) = \int \frac{e^{t^2}\,dt}{y_1^2(t)} + \cancel{c_2}^{\,0} \qquad (c_2=0)$$

$$\Rightarrow y_2(t) = (1-2t^2)\int \frac{e^{t^2}\,dt}{(1-2t^2)^2}.$$

[A7]. $$y'' + 4y' + 4y = \underbrace{t^{-2} e^{-2t}}_{b(t)} \qquad (t>0).$$

$$p(r) = r^2 + 4r + 4 = (r+2)^2$$

$$\therefore\ y_{\text{ομ}} = c_1 \underbrace{e^{-2t}}_{\varphi_1} + c_2 \underbrace{t e^{-2t}}_{\varphi_2}.$$

Ειδική λύση της μορφής: $u_1(t)\underbrace{e^{-2t}}_{\varphi_1} + u_2(t)\underbrace{t e^{-2t}}_{\varphi_2} = y$

$$y' = \underline{u_1'\varphi_1} + u_1\varphi_1' + \underline{u_2'\varphi_2} + u_2\varphi_2'$$

$$\cancel{= u_1\varphi_1' +}$$

$$y'' = u_1''\varphi_1 + u_1'\varphi_1' + u_1'\varphi_1' + u_1\varphi_1''$$

$$+ u_2''\varphi_2 + u_2'\varphi_2' + u_2'\varphi_2' + u_2\varphi_2''$$

$$\Rightarrow y'' = u_1''\varphi_1 + 2u_1'\varphi_1' + u_1\varphi_1'' + u_2''\varphi_2 + 2u_2'\varphi_2' + u_2\varphi_2''$$

$$\Rightarrow \left(u_1''\varphi_1 + 2u_1'\varphi_1' + \underline{u_1\varphi_1''} + u_2''\varphi_2 + 2u_2'\varphi_2' + \underline{u_2\varphi_2''}\right) +$$

$$+ 4\left(u_1'\varphi_1 + \underline{u_1\varphi_1'} + u_2'\varphi_2 + \underline{u_2\varphi_2'}\right) + 4\left(\underline{u_1\varphi_1} + \underline{u_2\varphi_2}\right).$$

$\Rightarrow \cancel{u_1''\varphi_1 + 2(u_1'\varphi_1' + u_2'\varphi_2')}$

$\Rightarrow u_1''\varphi_1 + u_2''\varphi_2 + \cancel{4}(u_1'\varphi_1' + u_2'\varphi_2') + 4\underbrace{(u_1'\varphi_1 + u_2'\varphi_2)}_{0} = b$

from $\underline{u_1'\varphi_1 + u_2'\varphi_2 \equiv 0} \Rightarrow$

$\Rightarrow u_1''\varphi_1 + u_1'\varphi_1' + u_2''\varphi_2 + u_2'\varphi_2' \equiv 0.$

$\Rightarrow \underline{u_1'\varphi_1' + u_2'\varphi_2' = b.}$

$$\begin{bmatrix} \varphi_1 & \varphi_2 \\ \varphi_1' & \varphi_2' \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}.$$

$\cancel{u_1' =}$

$$u_1' = \frac{\begin{vmatrix} 0 & \varphi_2 \\ b & \varphi_2' \end{vmatrix}}{W}, \quad u_2' = \frac{\begin{vmatrix} \varphi_1 & 0 \\ \varphi_1' & b \end{vmatrix}}{W}.$$

$$W(t) = \begin{vmatrix} \varphi_1 & \varphi_2 \\ \varphi_1' & \varphi_2' \end{vmatrix} = \begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t} - 2te^{-2t} \end{vmatrix}$$

$$= e^{-4t} - 2t\cancel{e^{-4t}} + 2t\cancel{e^{-4t}} = e^{-4t}.$$

$$\Rightarrow u_1' = e^{4t} \begin{vmatrix} 0 & te^{-2t} \\ t^{-2}e^{-2t} & * \end{vmatrix} =$$

$$= e^{4t}(-t^{-1}e^{-4t}) = -t^{-1}$$

$\Rightarrow u_1'(t) = -\frac{1}{t} \Rightarrow \underline{u_1(t) = -\ln(t).}$ $(t>0)$

$$u_2'(t) = e^{4t} \begin{vmatrix} e^{-2t} & 0 \\ -2e^{-2t} & t^{-2}e^{-2t} \end{vmatrix} = t^{-2}$$

$\Rightarrow \underline{u_2(t) = -t^{-1}}$ ∴

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Άρα ειδική λύση:

$\tilde{\psi}_p = -\ln(t) \cdot e^{-2t} - \cancel{t^{-1}} t e^{-2t}$

από $\psi_p = -\ln(t) e^{-2t}$ και γενική λύση.

$$y(t) = c_1 e^{-2t} + c_2 t e^{-2t} - \ln(t) e^{-2t}$$

[A8] $y' = ay + be^{\lambda t}$ $(a<0, \lambda<0)$

$$y(t) = \underbrace{e^{at} y_0}_{\to 0} + \underbrace{\int_0^t e^{a(t-\tau)} b e^{\lambda \tau} d\tau}_{y_2(t)}$$

$$y_2(t) = be^{at} \int_0^t e^{-a\tau} e^{\lambda \tau} d\tau =$$

$$= be^{at} \int_0^t e^{(\lambda-a)\tau} d\tau.$$

Περίπτωση 1η. $a \neq \lambda$.

$$y_2(t) = be^{at} \left[\frac{e^{(\lambda-a)\tau}}{\lambda - a} \right]_0^t =$$

$$= \frac{b}{\lambda - a} e^{at} \left[e^{(\lambda-a)t} - 1 \right] =$$

$$= \frac{b}{\lambda - a} \left[e^{\lambda t} - e^{at} \right] \to 0 \quad (\lambda<0, a<0).$$

Περίπτωση 2η $a = \lambda$.

$y_2(t) = be^{at} t = bte^{at} \to 0$ καθώς $t \to \infty$

$$\left(\lim_{t\to\infty} \frac{t}{e^{-at}} = \lim_{t\to\infty} \left(\frac{1}{-ae^{-at}} \right) = 0 \right).$$

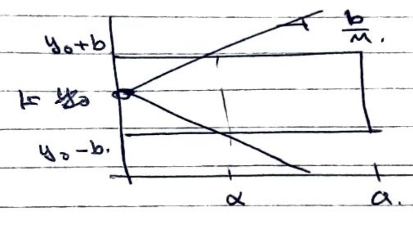
~~8ηλαση,~~

~~$\varphi(t) = 1 + \frac{1}{\sqrt{2}} \sin(\sqrt{2}t)$~~

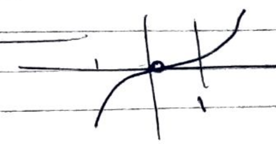
~~$\cos\sqrt{2}t$~~

~~$1+$~~

[Ag].



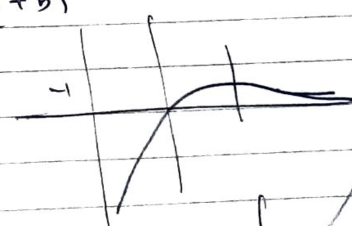
$y_0 = 1$.



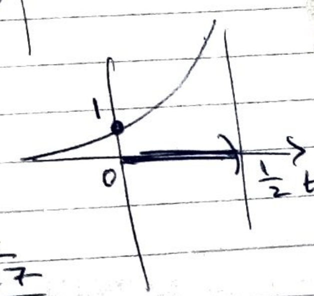
$$M = \max_{y\in[y_0-b,\, y_0+b]} |f(y)| = |y^3|^3 = (1+b)^3 \qquad \alpha = \min\left(a, \frac{b}{M}\right) =$$

$$g'(b) = \frac{1\cdot(1+b)^3 - b\,3(1+b)^2}{(1+b)^6} \qquad \left(= \frac{b}{(1+b)^3} = g(b) \right)$$

$$= \frac{\cancel{(1+b)^2}\,[1+b-3b]}{(1+b)^{\cancel{6}\,4}} = 0$$



$$1-2b=0 \qquad b^* = \frac{1}{2}$$



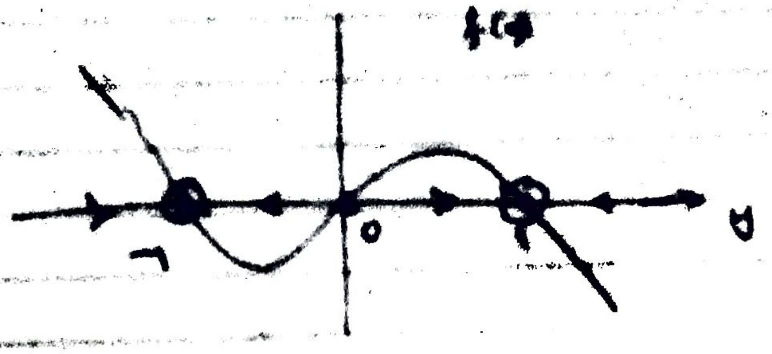
$$g(b^*) = \frac{\frac{1}{2}}{\left(\frac{3}{2}\right)^3} = \frac{\frac{1}{2}}{\frac{27}{8}} = \frac{4}{27}$$

$$\alpha_{max} = \frac{4}{27} \qquad \int\frac{dy}{y^3} = \int dt + c \Rightarrow \frac{y^{-2}}{-2} = t + c$$

$$\Rightarrow \frac{1}{y^2} = -2t + c \quad (c=1).$$

(13) $$y = \frac{1}{\sqrt{c-2t}} = \frac{1}{\sqrt{1-2t}}.$$

[A10] $f(y) = y(1-y)(1+y)$



$y = \pm 1$ ([illegible])

$y = 0$ ([illegible])