

22-5-26

Central Path Equations

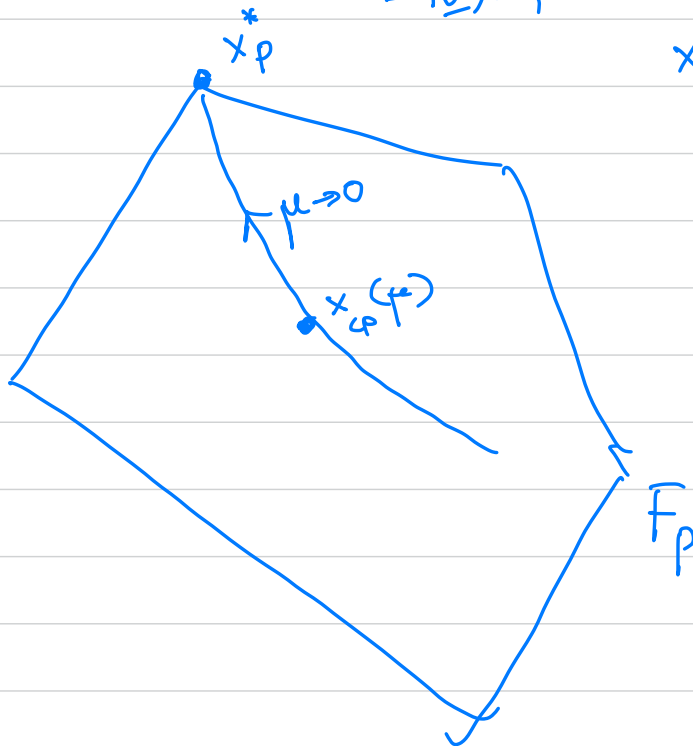
(*)
$$\begin{cases} Ax + w = b \\ A^T y - z = c \\ XZe = \mu e & x_i, z_i = \mu \\ YWe = \mu e & y_i w_i = \mu \\ x_i, y_i, w_i, z_i > 0 \end{cases}$$

$$X = \begin{pmatrix} x_1 & & \\ & \ddots & \\ & & x_n \end{pmatrix}$$

kfn

$$\underline{x}, \underline{y}, \underline{w}, \underline{z} > 0$$

$x_{cp}(\mu)$: vom zw *



Aok. 17.1 (Vanderbei)

$$z_p = \max -x_1 + x_2$$

$$y_1 \quad x_2 \leq 1$$

$$y_2 \quad -x_1 \leq -1$$

$$x_1, x_2 \geq 0$$

$$z_D = \min y_1 - y_2 = -\max y_2 - y_1$$

$$-y_2 \geq -1$$

$$y_2 \leq 1$$

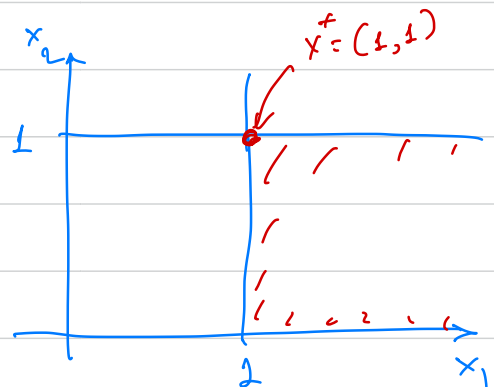
$$y_1 \geq 1$$

$$y_1 \geq 1$$

$$y_1, y_2 \geq 0$$

$$y_1, y_2 \geq 0$$

$$\left(\begin{array}{ll} \max & -x_1 + x_2 \\ & x_2 \leq 1 \\ & x_1 \geq 1 \end{array} \right)$$



$$z_p = \max -x_1 + x_2 + 0w_1 + 0w_2$$

$$x_2 + w_1 = 1$$

$$x_1 - w_2 = 1$$

$$z_D = -\max -y_1 + y_2$$

$$y_2 + z_1 = 1$$

$$y_1 - z_2 = 1$$

$$y_1, y_2, z_1, z_2 \geq 0$$

$$x_1, x_2, w_1, w_2 \geq 0.$$

Central Path Equations

$$x_1 - w_2 = 1$$

$$x_2 + w_1 = 1$$

$$y_1 - z_2 = 1$$

$$y_2 + z_1 = 1$$

$$x_1 z_1 = \mu$$

$$x_2 z_2 = \mu$$

$$y_1 w_1 = \mu$$

$$y_2 w_2 = \mu$$

$$x, y, z, w > 0$$

Ανί τα συμπέρασμα

$$x_1 = y_1$$

$$x_2 = y_2$$

$$z_1 = w_1$$

$$z_2 = w_2$$

Επομένως καταγράφουμε

$$\left. \begin{array}{l} x_1 - z_2 = 1 \\ x_2 + z_1 = 1 \\ x_1 z_1 = \mu \\ x_2 z_2 = \mu \end{array} \right\} \begin{array}{l} z_2 = x_1 - 1 \\ z_1 = 1 - x_2 \end{array}$$

$$x_1 z_1 = \mu \Rightarrow x_1(1 - x_2) = \mu \Rightarrow x_1 - x_1 x_2 = \mu$$

$$x_2 z_2 = \mu \Rightarrow x_2(x_1 - 1) = \mu \Rightarrow x_1 x_2 - x_2 = \mu$$

$$\underline{x_1 - x_2 = 2\mu \Rightarrow x_1 = x_2 + 2\mu}$$

$$\left. \begin{array}{l} x_1(1 - x_2) = \mu \\ x_1 = x_2 + 2\mu \end{array} \right\} \Rightarrow (x_2 + 2\mu)(1 - x_2) = \mu \Rightarrow$$

$$\Rightarrow x_2^2 + (2\mu - 1)x_2 - \mu = 0$$

$$\Delta = (2\mu - 1)^2 + 4\mu = 4\mu^2 + 1 > 0$$

$$x_2^* = \frac{1 - 2\mu + \sqrt{1 + 4\mu^2}}{2} \dots \dots \text{αντικ.}$$

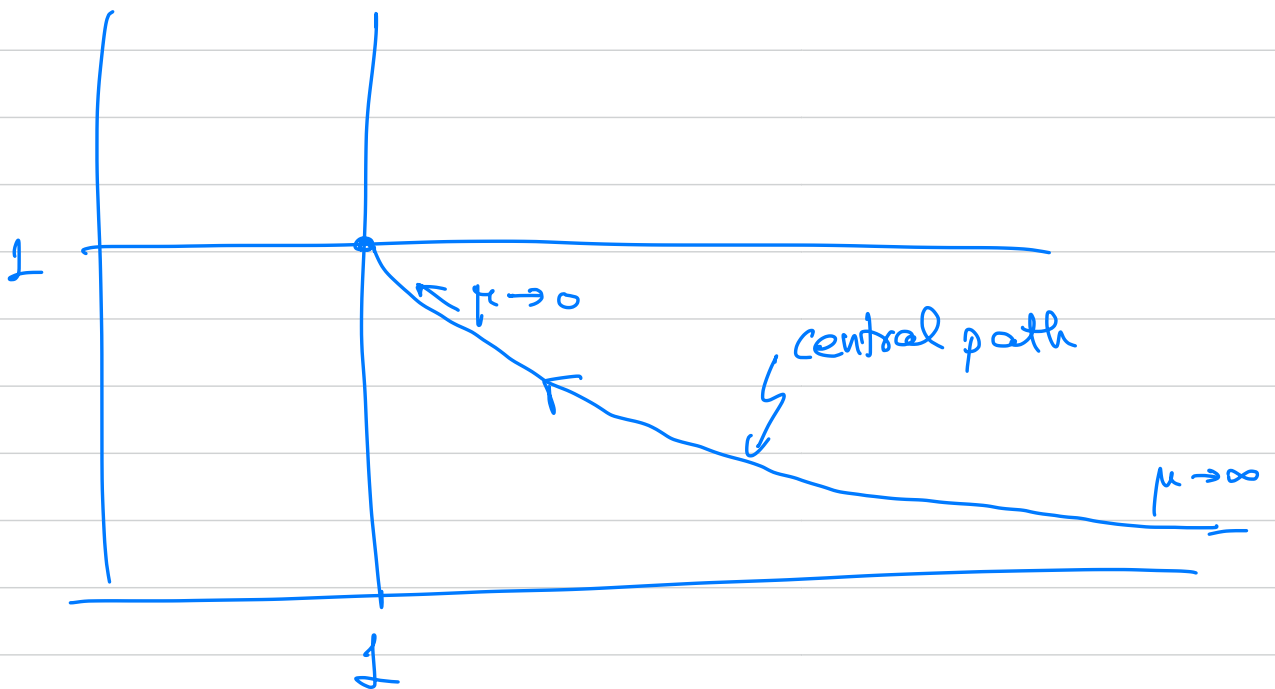
$$x_1^* = \frac{1+2\mu + \sqrt{1+4\mu^2}}{2} = y_1^*$$

$$x_2^* = \frac{1-2\mu + \sqrt{1+4\mu^2}}{2} = y_2^*$$

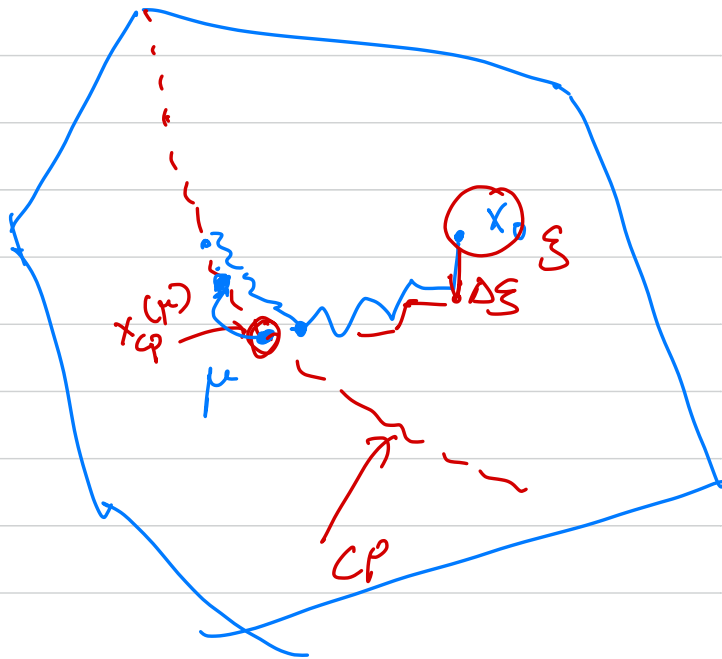
$$z_1^* = \frac{1+2\mu - \sqrt{1+4\mu^2}}{2} = w_1^*$$

$$z_2^* = \frac{-1+2\mu + \sqrt{1+4\mu^2}}{2} = w_2^*$$

$$x_{cp}(\mu) = \frac{1}{2} \begin{pmatrix} 1-2\mu + \sqrt{4\mu^2+1} \\ 1-2\mu + \sqrt{4\mu^2+1} \end{pmatrix} \xrightarrow{\mu \rightarrow 0} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



Αντιστάθμιση τύπου του Central Path



① Με δεδομένο μ πώς μπορούμε να βρούμε τις CP equations προσεγγιστικά

$$\left. \begin{array}{l} m \quad Ax + w = b \\ n \quad A^T y - z = c \\ n \quad xz e = \mu e \\ m \quad yw e = \mu e \end{array} \right\} \text{όπου } \left\{ \begin{array}{l} x \\ w \\ y \\ z \end{array} \right\} = \begin{pmatrix} x \\ w \\ y \\ z \end{pmatrix} \begin{array}{l} \in \mathbb{R}^n \\ \in \mathbb{R}^m \\ \in \mathbb{R}^m \\ \in \mathbb{R}^n \end{array} \in \mathbb{R}^{2(n+m)}$$

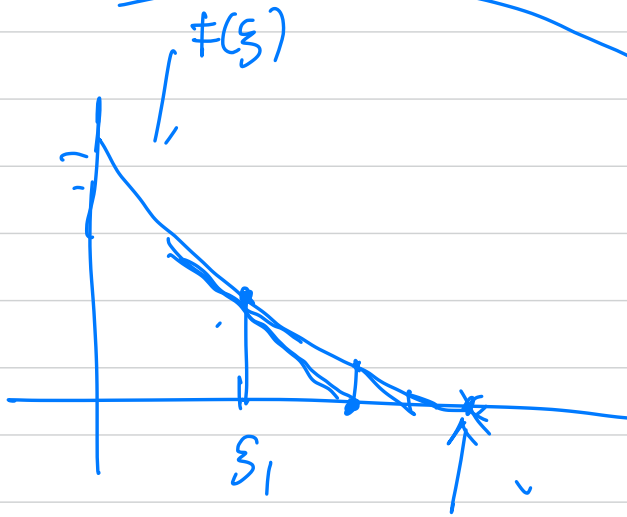
$$F(s) = 0$$

or $F(\xi) = \begin{bmatrix} Ax + w - b \\ A^T y - z - c \\ xze - \mu e \\ ywe - \mu e \end{bmatrix}$ $F: \mathbb{R}^{\begin{smallmatrix} 2(m+n) \\ 2(m+n) \end{smallmatrix}} \rightarrow \mathbb{R}$

$$F(\xi) = 0$$

Newton's method $\xi_1 \rightarrow F(\xi_1), F'(\xi_1)$

$$\xi_2 = \xi_1 + \delta \xi \Rightarrow F(\xi_2) = F(\xi_1 + \delta \xi) \approx F(\xi_1) + F'(\xi_1) \cdot \delta \xi = 0$$



$$\Rightarrow \delta \xi = - \frac{F(\xi_1)}{F'(\xi_1)}$$

$$\xi_2 = \xi_1 - \frac{F(\xi_1)}{F'(\xi_1)}$$

Εδώ $\xi, F \in \mathbb{R}^{2(m+n)}$

$$F'(\xi) = \begin{pmatrix} \frac{\partial F_1}{\partial \xi_1} & \dots & \frac{\partial F_1}{\partial \xi_{2(m+n)}} \\ \vdots & & \\ \frac{\partial F_{2(m+n)}}{\partial \xi_1} & \dots & \frac{\partial F_{2(m+n)}}{\partial \xi_{2(m+n)}} \end{pmatrix}$$

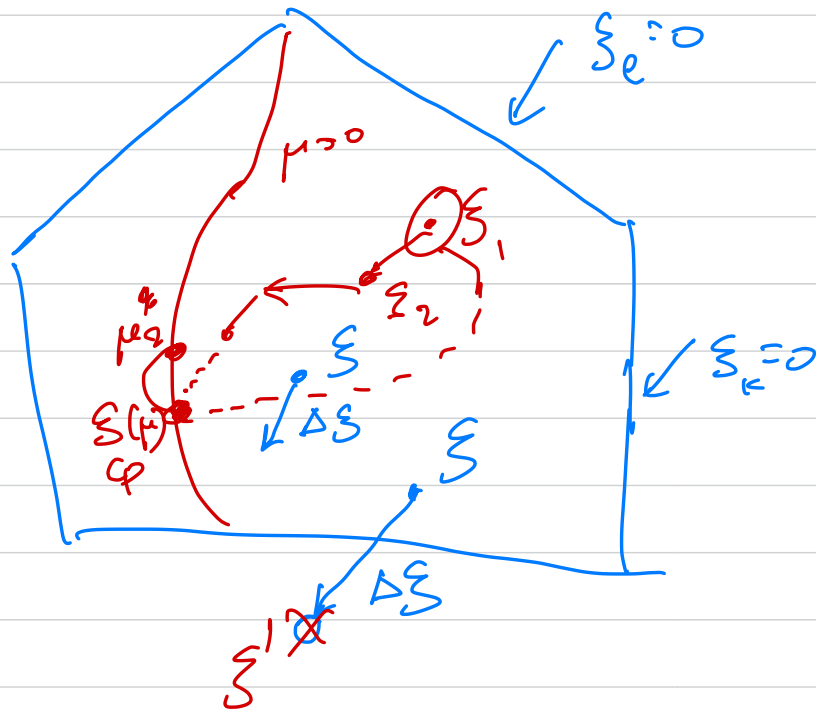
$\Downarrow$
 $\vdots$ πράξεις / παραγώγιση

$$F'(\xi) = \begin{pmatrix} x & y & w & z \\ A & I & O & O \\ O & O & A^T & -I \\ z & O & O & X \\ O & \gamma & w & O \end{pmatrix}$$

Για αρχικά ξ : υποβ $F(\xi), F'(\xi)$

$\xi' = \xi + \Delta \xi$ όπου $F'(\xi) \cdot \Delta \xi = -F(\xi)$
 (παραμεκὸ οἶσµα)
 ?

Πρόβλημα



$$\xi' = \xi + \theta \Delta \xi \quad \theta : \text{step-size}$$

$$\xi_i + \theta \Delta \xi_i > 0 \quad \forall i = 1, \dots, 2(m+n)$$

$$\theta = r \left(\max_{i,j} \left\{ -\frac{\Delta x_j}{x_j}, -\frac{\Delta z_j}{z_j}, -\frac{\Delta w_i}{w_i}, -\frac{\Delta y_i}{y_i} \right\} \right) \wedge 1$$

$0 < r < 1$ σταθερά

Μέχρι τώρα : επαναγ. αλγόριθμος
που συγκρίνει στο $\xi(\mu)$
για σταθερό $\mu > 0$

② Πώς επιλέγουμε μ ?

Ξεκινάμε με ένα $\xi_0 = \begin{pmatrix} x \\ y_0 \\ z_0 \end{pmatrix} > 0$

Αν ξ_0 ικανοποιεί τις CP $\Rightarrow$

$$\begin{array}{lcl} x_j z_j = \mu & \forall j & \\ y_i w_i = \mu & \forall i & \end{array} \Rightarrow \mu = x_j z_j = y_i w_i$$

Εδώ έχουμε

$$\begin{array}{c} x_1 z_1 \\ x_2 z_2 \\ \vdots \\ x_n z_n \\ y_1 w_1 \\ \vdots \\ y_m w_m \end{array}$$

διαφορετικά
μέλη ξ_j της
(> 0)

$$\mu = \frac{\sum x_j z_j + \sum y_i w_i}{n + m}$$

$$\xi_1 \rightarrow \tilde{\mu}_1 \rightarrow \Delta \xi_1: F'(\xi_1) \Delta \xi_1 = -F(\xi_1, \tilde{\mu}_1)$$

$$\rightarrow \xi_2 = \xi_1 + \theta_1 \Delta \xi_1 \rightarrow$$