

27-4-2026

Δυσκολία

Ημικανονική μορφή πγπ.

Hk-max

$$\begin{aligned} \max \quad & c'x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

(δεν απαιτείται $b \geq 0$)

Hk-min

$$\begin{aligned} \min \quad & c'x \\ \text{s.t.} \quad & Ax \geq b \\ & x \geq 0 \end{aligned}$$

(δεν απαιτείται $b \geq 0$)

Μετατροπή $a'x \geq b \Leftrightarrow -a'x \leq -b$

$$a'x = b \Leftrightarrow \begin{cases} a'x \leq b \\ -a'x \leq -b \end{cases}$$

Έστω πγπ Hk-max

$$\begin{aligned} z_p = \max \quad & c'x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} x, c &\in \mathbb{R}^n \\ b &\in \mathbb{R}^m \\ A &\in \mathbb{R}^{m \times n} \end{aligned}$$

Έστω $L: \mathbb{R}^{n+m} \rightarrow \mathbb{R}$ οπν για $\underline{x} \in \mathbb{R}^n, \underline{w} \in \mathbb{R}^m, w \geq 0$

$$L(\underline{x}, \underline{w}) = c'x - \underbrace{w'}_{1 \times m} \underbrace{(Ax - b)}_{m \times 1}$$

$$= c'x - \sum_{i=1}^m w_i (a_i'x - b_i)$$

$$\begin{aligned} Ax - b &= \begin{pmatrix} a_1'x - b_1 \\ \vdots \\ a_m'x - b_m \end{pmatrix} \\ A &= \begin{pmatrix} a_1' \\ \vdots \\ a_m' \end{pmatrix} \end{aligned}$$

Lagrangian function $w_1, \dots, w_m$: ref/see Lagrange

Given
$$z_L(\underline{w}) = \max_{\text{v.t. } \underline{x} \geq 0} L(\underline{x}, \underline{w})$$

Lemma :
$$z_L(\underline{w}) \geq z_p \quad \forall \underline{w} \geq 0$$

Απόδειξη
$$z_L(\underline{w}) = \max \{ L(\underline{x}, \underline{w}) : \underline{x} \geq 0 \}$$

(καθάρων)
$$\geq \max \{ L(\underline{x}, \underline{w}) : \underline{x} \geq 0, A\underline{x} \leq \underline{b} \}$$

Ομως αν $\underline{w} \geq 0, A\underline{x} \leq \underline{b} \Rightarrow A\underline{x} - \underline{b} \leq 0 \Rightarrow$

$$\Rightarrow \underline{w}'(A\underline{x} - \underline{b}) \leq 0$$

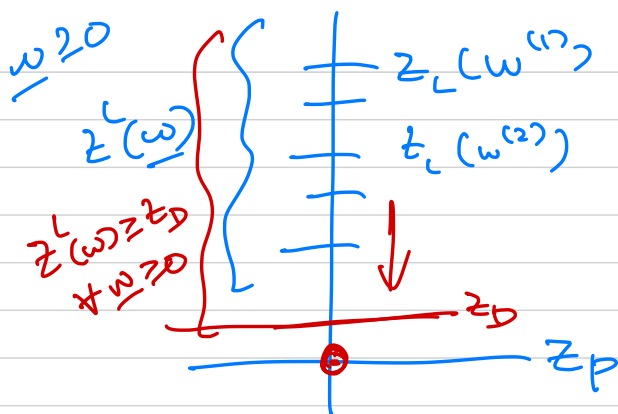
$$\Rightarrow L(\underline{x}, \underline{w}) \geq \underline{c}'\underline{x} \quad \forall \underline{x} \geq 0, \underline{w} \geq 0, A\underline{x} \leq \underline{b}$$

Επομένως
$$\max \{ L(\underline{x}, \underline{w}) : \underline{x} \geq 0, A\underline{x} \leq \underline{b} \} \geq$$

$$\geq \max \{ \underline{c}'\underline{x} : \underline{x} \geq 0, A\underline{x} \leq \underline{b} \} = z_p \quad \checkmark$$

$z_L(\underline{w})$ ανω φράξη του $z_p \quad \forall \underline{w} \geq 0$

$z_L(\underline{w})$: Λαβανίανι χαλάρων του z_p
(Lagrangian relaxation)



Εστω
$$z_D = \inf_{w \geq 0} z_L(w)$$

$$\Rightarrow (z_P \leq z_D)$$

z_D : Δύο πρόβλημα του z_P

Θεώρημα $z_P \leq z_D$ (ασθενή θεωρήματα δuality)
(weak duality)

Ερωτήσεις . 1) Ενισχύονται το $\inf_{w \geq 0}$?

συγ. $\exists w^0 \in \mathbb{R}^m, w^0 \geq 0 : z_D = z_L(w^0)$?

2) Αν ναι πως το βρίσκουμε;
πώς λύνεται το z_D ?

$$\text{"dual"} \quad w \quad z_D = \inf \{ z_L(\underline{w}) : w \geq 0 \}$$

$$z_L(\underline{w}) = \max_{\underline{x} \geq 0} \{ L(\underline{x}, \underline{w}) : \underline{x} \geq 0 \} =$$

$$= \max_{\underline{x} \geq 0} \{ \underline{c}'\underline{x} - \underline{w}'(\underline{A}\underline{x} - \underline{b}) \} =$$

$$= \max_{\underline{x} \geq 0} \{ \underline{w}'\underline{b} + (\underline{c}' - \underline{w}'\underline{A})\underline{x} \} =$$

$$= \underline{w}'\underline{b} + \underbrace{\max_{\underline{x} \geq 0} \{ (\underline{c}' - \underline{w}'\underline{A})\underline{x} \}}_{f_L(\underline{w})}$$

$$\underline{c}' = (c_1, \dots, c_n)$$

$$\underline{w}'\underline{A} = (w_1, \dots, w_m) (\underline{A}_1 \dots \underline{A}_n) = (w_1'\underline{A}_1, \dots, w_m'\underline{A}_n)$$

$$(\underline{c}' - \underline{w}'\underline{A})\underline{x} = \sum_{j=1}^n (c_j - w_1'\underline{A}_{1j}) x_j$$

$$f_L(\underline{w}) = \max_{x_1, \dots, x_n \geq 0} \sum_{j=1}^n (c_j - w_1'\underline{A}_{1j}) x_j$$

$$f_L(\underline{w}) = \begin{cases} +\infty & \text{αν } c_j - w_1'\underline{A}_{1j} > 0 \text{ για κάποιο } j \\ 0 & \text{αν } c_j - w_1'\underline{A}_{1j} \leq 0 \text{ για } j=1, \dots, n \Leftrightarrow \underline{c}' - \underline{w}'\underline{A} \leq 0 \end{cases}$$

$$\max \sum h_j x_j$$

$$x_j \geq 0, j=1, \dots, n$$

$$= \begin{cases} 0, & \text{αν } h_j \leq 0 \forall j \\ +\infty & \text{αν } h_j > 0 \text{ για κάποια } j \end{cases}$$

$$\max x_1 + 2x_2$$

$$x_1, x_2 \geq 0$$

$$\rightarrow +\infty$$

$$\Rightarrow z_{\underline{L}}(\omega) = w'b + f_{\underline{L}}(\omega) = \begin{cases} w'b, & \text{αν } c' - w'A \leq 0 \\ +\infty & \text{διαφορετικά} \end{cases}$$

$$z_D = \inf_{w \geq 0} z_{\underline{L}}(\omega) = \inf \{ w'b : c' - w'A \leq 0, w \geq 0 \} \quad \pi \chi \pi \cdot (w)$$

$$z_D = \min w'b$$

$$\text{υ.π. } w'A \geq C$$

$$w' \geq 0$$

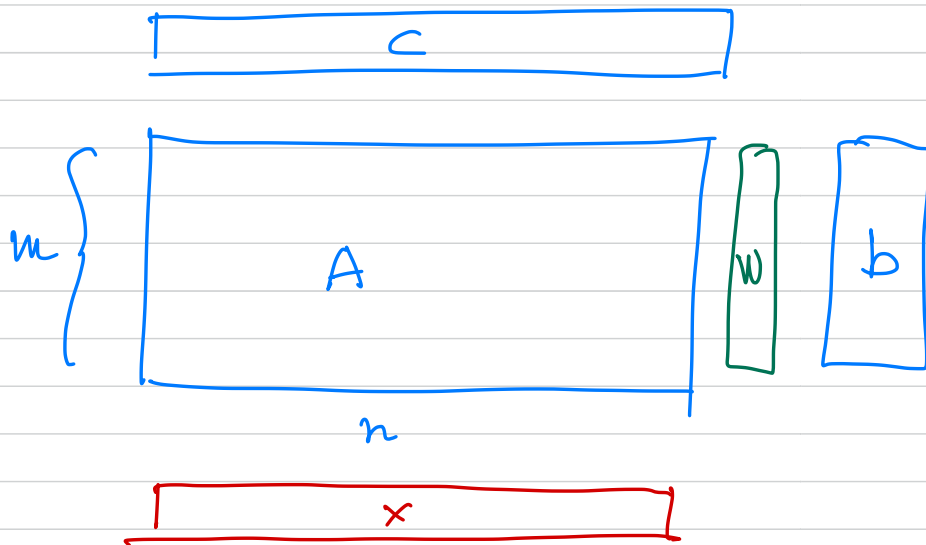
$$\begin{aligned} z_p &= \max_{(m \times n)} C'x \\ Ax &\leq b \\ x &\geq 0 \end{aligned}$$

$$\begin{aligned} z_D &= \min w'b \\ w'A &\geq C' \\ w &\geq 0 \end{aligned}$$

$$\begin{aligned} z_D &= \min_{(n \times m)} b'w \\ A'w &\geq C \\ w &\geq 0 \end{aligned}$$

$$\text{HK-max}(C, A, b)$$

$$\text{HK-min}(b, A', C)$$



Ευχαριστείτε $w_P \leq w_D$



$$\textcircled{(D)_D = P} \quad \textcircled{1}$$

$$\textcircled{2}$$

$$z_P = z_D \quad (\text{ισχύει δικόμωρα})$$