

PROBLEM SOLVING ACTIVITY IN THE WORKPLACE AND THE SCHOOL: THE CASE OF CONSTRUCTING SOLIDS

ABSTRACT. The purpose of the present study is to document, compare, and analyze the nature of spatial reasoning by practitioners (plumbers) in the workplace and students in the school setting while constructing solids, with given specifications, from plane surfaces. Data were collected from a plumbing workshop and five high school students while constructing a cylindrical container of capacity one-liter and height of 20 cm. The results confirm the power of activity theory and its methodology in explaining and identifying the structural differences between the two activities in the two different cultural settings. Students and plumber activity structures differed in the operational aspect (actions) and the means and concrete conditions (operations) under which such a goal is carried out. Activity theory has the potential to explain the differences between the two activities in terms of differences in the motive, social-cultural settings, the tools that were available and accessible which resulted in different actions, and the constraints (operations) under which the task was executed. Theoretical and pedagogical implications were identified.

1. INTRODUCTION

Extant research on work-related mathematical practices was motivated by several purposes, aimed at understanding the use of different mathematical concepts, and using a variety of interpretive and methodological frameworks.

The majority of work-related studies that aimed at understanding the use of arithmetical ideas in the workplace were motivated by social-cultural purposes without attempting direct comparisons to school settings (Scribner, 1986; Lave, 1988; Saxe, 1991; Pozzi et al., 1998). Other studies were motivated by both pedagogical and cultural purposes and focused on contrasting the use of arithmetical ideas in workplace and school settings (Reed and Lave, 1981; Carraher et al., 1985; Nunes et al., 1993; Jurdak and Shahin, 1999). In general, work-related studies using arithmetical ideas have demonstrated that informal and idiosyncratic computational strategies were predominant in the workplace whereas formal school-like strategies were predominant in school settings. They also provided support to the hypothesis that the problem solving strategies were closely related to the setting in which they had been learned and to the requirements of the context in which they are applied. Moreover, these studies demonstrated that, un-



like school mathematics, computations were almost error-free in the workplace. The work-related studies used a variety of conceptual frameworks including situated learning, cognition in practice, and ethnomathematics. Only one study (Pozzi et al., 1998) used activity theory.

Two work-related studies focused on spatial and measurement ideas in the workplace (Millroy, 1992; Masingila, 1996). Millroy (1992) studied the mathematical ideas of a group of carpenters in South Africa. She documented the valid mathematical concepts and processes that were embedded in their everyday working activities such as congruence, symmetry, spatial visualization, straight and parallel lines, and proportion as well as processes such as problem solving, reasoning, comparing, and measuring. On the other hand, Masingila (1996) studied the measurement concepts embedded in the daily activities of carpet layers. She concluded that their intentions in solving problems of assessing and installing carpets were very different from what we find in mathematics textbooks and that the problems in the carpet-laying context were mostly optimization problems. In both studies activity theory (Leont'ev, 1981) was used as a conceptual framework.

Illuminating as the studies of Millroy and Masingila are of the issues that concern the mathematics used by practitioners in these workplaces, these studies have not addressed a direct comparison of mathematical tasks carried out by practitioners, and the 'same' tasks done by students in a school setting. The purpose of the present study is to document, compare, and analyze the nature of spatial reasoning by practitioners (plumbers) in the workplace and students in the school setting while constructing solids, with given specifications, from plane surfaces.

The points of departure of the present study from other work-related studies include the purpose of the study, the mathematical ideas it addresses, and the conceptual and the methodological frameworks it adopts. As for the purpose of the study, to our knowledge, there has been no study that attempted to document and contrast the use of space geometrical ideas by practitioners in the workplace and students in the school setting. Though they dealt with spatial and measurement ideas, the studies of Millroy (1992) and Masingila (1996) did not attempt direct comparisons of workplace and school settings. Moreover, the present study deals with a core mathematical idea of transforming two-dimensional surfaces into three-dimensional solids and vice versa – an idea that has not been dealt with before. This idea has become increasingly important in recent curriculum reforms and is an essential part of mathematical literacy for developing spatial sense (NCTM, 1989 for example). Moreover, this study uses activity theory as a conceptual framework and the systematic-structural

analysis (Zinchenko and Gordon, 1981) that is based on activity theory as conceptualized by Leont'ev (1981) and which has not been used in any work-related mathematics study.

The reason for using activity theory as a conceptual model is that this study makes the assumption that thinking and doing are inseparable – a central assertion of activity theory which assumes that our knowledge of the world is mediated by our interaction with it, and thus, human behavior and thinking occur within meaningful contexts as people conduct purposeful goal-directed activities. This theory strongly advocates socially organized human activity as the major unit of analysis in psychological studies rather than mind or behavior. Leont'ev (1981) has defined activity as: "... the unit of life that is mediated by mental reflection. The real function of this unit is to orient the subjects in the world of objects. In other words, activity is not a reaction or aggregate of reactions, but a system with its own structure, its own internal transformations, and its own development" (p. 46). Thus we feel that activity theory is best capable of capturing the totality of the activities of plumbers and students while engaged in constructing solids.

The method of structural analysis (Zinchenko and Gordon, 1981) was used because it puts forward an operational analytic method derived from activity theory itself. It provides a way for representing the structure of activity as a system of interconnected units with potential relationships among them and among types of connections. In the systematic-structural approach, it is assumed that the structure of actions and operations, the internal transitions from one action to another, and their sequential organization depend on the objective content of activity. Thus the identification of the organization and sequence of the actions of the objective content of an activity provides a characterization of its level, form, and type (Zinchenko and Gordon, 1981).

Activity theory and methodology

Human activity can be realized in two forms: 'mental' activity or internal activity and practical objective or external activity (Leont'ev, 1981). The fundamental and primary form of human activity is external and practical. This form of activity brings humans into practical contact with objects thus redirecting, changing and enriching this activity. The internal plane of activity is formed as a result of internalizing external processes. "Internalization is the transition in which external processes with external, material objects are transformed into processes that take place at the mental level, the level of consciousness" (Zinchenko and Gordon, 1981, p. 74).

Leont'ev (1981) identified several interrelated levels or abstractions in a theory of activity. Each level is associated with a special type of unit. The first most general level is associated with the unit of *activity* that deals with specific real activities such as plumbing (labor activity) and problem solving (instructional activity). The second level of analysis focuses on the unit of a *goal-directed action* that is the process subordinated to a conscious goal. The third level of analysis is associated with the unit of *operation* or the conditions under which the action is carried out. Operations help actualize a general goal and make it more concrete.

Three types of actions in mental activities were identified: perceptual, mnemonic, and cognitive (Zinchenko and Gordon, 1981). Perceptual actions are those by which the human being maintains contact with the environment. They are initiated by stimuli from the environment and enriched on the basis of prior experience. Mnemonic actions refer to actions, which involve recognition, reconstruction, or recall (Piaget and Inhelder as cited in Zinchenko and Gordon, 1981). Cognitive actions involve thinking in terms of images of real objective processes (Gal'perin cited in Zinchenko and Gordon, 1981).

2. METHOD

Subjects

The participants in this study consisted of two groups. One group included five tenth-grade students (two girls and three boys) from two schools in Beirut. The students were chosen by their mathematics teachers as being the top in their class in mathematics. The other group consisted of five adult plumbers in a plumbing workshop with little or no school experience but who have been practicing plumbing for a long time. In this plumbing workshop, there was the owner, Abou Jamil, the artisan master, his two sons and two other laborers.

The craft of traditional plumbing (referred to as 'Arabic' plumbing) has become almost extinct and is limited to a few workshops in old quarters of cities and towns. After visiting a number of workshops in different areas, the only workshop that met the needed specifications was Abou Jamil's shop. Abou Jamil was certified by the Ministry of Economy to make standard measurement containers that were used to check on and monitor the correctness of meters of gasoline distributed by the gas stations.

The task

The same task was given to the plumber and the students and it asked for constructing a cylindrical container, with a bottom, of one-liter capacity and height of 20 cm. This task was a breakdown in the normal behaviors of the students and the plumber. Though the students have had experience in solving problems involving the volume of a cylinder, they did not have experience in actually constructing cylinders with given specifications. Similarly, the plumber had an extensive experience in constructing cylindrical containers, but these were of standard dimensions that were different from those of the task.

Context

The plumbing activity was observed over a 6-week period in the Ouzaii area on the road towards the south of Beirut in Abou-Jamil's workshop which is a medium-sized room with a long corridor leading to a small backyard. In the entrance, you can see metal works, a small bench, and a table on which the plumber works. Next to it there is a heater with a gas bottle and a small wooden cabinet with drawers placed against the wall and containing small hand tools. On the other side of the shop, pieces of leftover iron sheets are placed next to the wall. Electrically or manually operated machines for bending metal are placed in the middle of the shop. The students were interviewed in their own school but outside their classrooms.

Data collection techniques

Three methods of collecting data were used: Participant observation, interviewing, and collection of artifacts. The three methods were used concurrently during fieldwork and the investigator (second co-author) wrote detailed field notes and sketched the sequence of the actions done by the plumbers' and students after every encounter with them. The observations included the following:

- Descriptions of the sequence of the actions when involved in an activity;
- The math demonstrated in their actions;
- Links to academic mathematics that could be recognized in the setting;
- The role and use of tools;
- Problems encountered while doing the task;
- Interaction between the subjects, and, distribution of job tasks.

Interviews with the plumber were done in Arabic and ranged from informal conversation to semi-structured to structured interviews. The informal conversations and semi-structured interviews were written-up, while all the structured interviews were video taped and transcribed immediately after each encounter. The problem solving sessions with students were tape-recorded. The investigator collected sketches that exemplified the executive actions devised by both the plumbers and the students while they worked on the task and video recorded the plumbers at work. In the video, the investigator introduced the plumbers and then asked them to explain what they were doing while the camera recorded their actions to get a clear demonstration of their work.

Procedure

After establishing entry to the plumbing workshop in Ouzaii area towards the south of Beirut, the investigator explained briefly to the owner, Abou Jamil, and his assistants about the research and asked if she could come to observe what and how they perform their job. On the second visit, Abou Jamil showed her around the workshop and he talked about his life as an apprentice and how he started out a long time ago. Also he explained how this craft is no longer appreciated and he expressed great fears about the disappearance of his trade. He showed works that he had been collecting for years, bunches of iron sheet cuttings like templates, which he used when he wanted to construct objects with similar dimensions. The investigator agreed with Abou Jamil from the very beginning that she could stay as long as her presence will not hinder their work in any way. Also, the investigator took their approval for audio and video taping their interviews. The structured interviews consisted of previously prepared questions while working on specific tasks. To establish rapport, the structured interviews were started by giving a familiar task (constructing a box with given dimensions), followed by the main task (constructing a cylindrical container of one-liter capacity and 20 cm high). After the investigators studied the field notes of the first structured interview, they decided to give to the plumbers two variations of the same task (constructing a cylindrical container of one-liter capacity and 15 or 30 cm high) in a follow-up session. Appointments were made for these interviews in order not to disrupt the plumbers' work.

The interviews with the plumbers and the students were made concurrently but separately. The investigator took appointments with teachers to interview their students at their own convenience. Some interviews with the students occurred the same day the investigator interviewed the plumbers. When first interviewed, the students were introduced to the research

and their cooperation was sought. They were given the task orally and were asked to construct a cylindrical container of one-liter capacity and of 20 cm height in whichever way they saw convenient. They were provided with paper and pencil, scissors, scotch tape and straight edge rulers. In their own school but outside their classrooms and in groups of two or three, they worked out loud on the task for about 15 minutes, writing on their own paper, and the session was tape-recorded. The data produced by each student consisted of written work plus the transcripts of think- a loud protocols. Throughout the session, the investigator asked them questions to explain their problem solving approach.

Since the plumbers in the workshop were headed and directed by Abou Jamil, henceforth we shall refer to the five plumbers as 'plumber'. Also, since the students worked in groups, we shall refer to the students as 'students' to reflect their collective group work.

Data analysis

The data consisted of: (a) descriptive and reflective field notes (b) transcribed taped interviews, and (c) students' problem solutions. Using the analytic-structural framework of Zinchenko and Gordon (1981), data were analyzed to reconstruct the internal planes of the activities of the plumber and the students in an attempt to identify the best structure that is supported by the three sources of data. The process of examining and re-examining the three sources of data was done in iterative steps as follows:

1. The field notes, interview transcriptions, and problem solutions were read and compared by the two researchers in an attempt to identify possible general scenarios for each of the two activities.
2. Once a plausible general scenario for each of the activities of the plumber and students was agreed on, the actions in the internal plane of each were identified in an iterative manner. An action was hypothesized and support for it was sought from the three sources of data. For example the action of "perceiving the 3-dimensional cylindrical container as a 2-dimensional sheet" in the internal plane of the plumber was supported by what he said (interview) and what he did (field notes).
3. Using the three sources of data, each action in the internal plane was classified into one of three categories (perceptual, cognitive, mnemonic).
4. The sequence and interconnections between and among the actions of the plumber and students were identified from the three sources of data.
5. A schematic diagram of the structure of each of the plumber's and students' internal activity was constructed in stages until the final diagram

was judged by the researchers as capturing each of the two activities (Figure 1).

3. RESULTS OF STRUCTURAL ANALYSIS

In the following two sections, a documentation of the structural analysis of each of the plumber's and students' internal activity will be given by identifying the actions involved, their type, their sequence, interconnections, and citing the evidence from the operations performed to support the descriptions offered. A schematic diagram of the structure of each of the plumber's and students internal activity is given in Figure1.

Plumber's internal activity

(1) *Perceiving the 3-dimensional cylindrical container (C) as a 2-dimensional sheet (S) (perceptual action).* When asked to construct a cylindrical container of height 20 cm and capacity one liter, the plumber paused for a while as if consciously trying to understand the task before doing it. He then looked at his shop corner where he had placed leftover pieces of plane metal sheets of different dimensions cut in a variety of shapes and selected a plane metal sheet of a convenient size. By choosing an object already available in his environment, the plumber consciously transformed the image of a 3-dimensional cylinder into a 2-dimensional rectangular metal sheet with specific measurements.

(2) *Recognizing the perimeter (P) of the base of (C) as the width (W) of (S) and the height (H) as the length of (S) (perceptual action).* The evidence for this perceptual action comes from the external actions of the plumber including what he said. When given the target task, the plumber started to refer to a standard container to measure olive oil by *occa* (according to him an '*occa*' is one kilo and 200 grams). He said that the '*occa*' container has a height of 17.5 cm and perimeter of 43 cm. He went further to explain how he makes '*occas*' from a standard metal sheet (1 m $\times$ 2 m). He would divide the width of the sheet i.e. 100 cm into five equal parts each of 17.5 cm width and cuts 43 cm across the length of the sheet i.e 200 cm. Since the 200 cm can take 43 cm four times (allowing for material to make the circular bottoms of the cylinder), four sets (each set has five 17.5 cm $\times$ 43 cm sheets) can be made out of each standard sheet (Figure 2). Through direct physical contact with his environment, the plumber established a correspondence between the perimeter of the base and the height of the

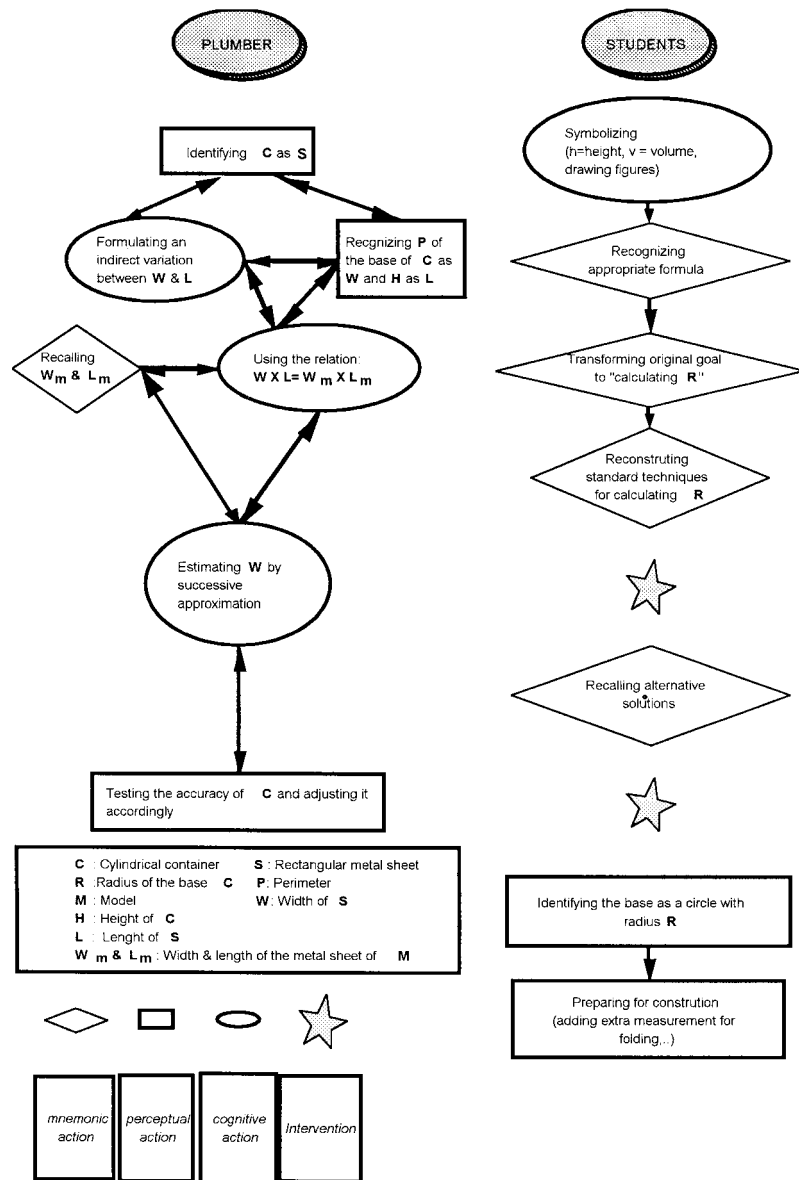


Figure 1. The structures of the internal activity of each of the plumber and the students.

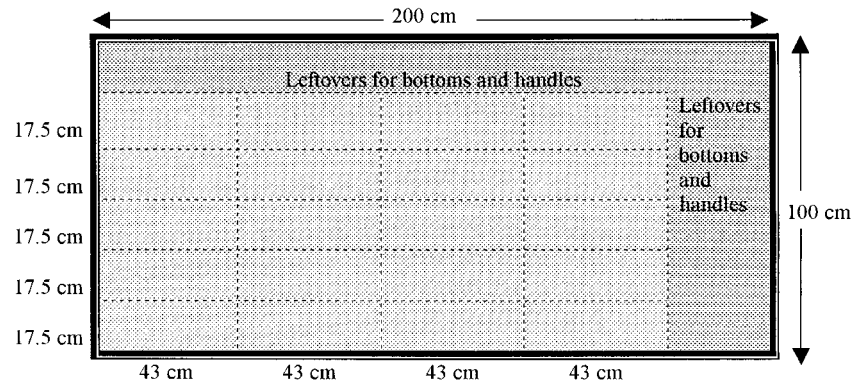


Figure 2. A sketch of the design the plumber said he would follow to make cylinders 17.5 cm high and 43 cm in perimeter from a standard 100 cm $\times$ 200 cm standard iron sheet.

container and the width and length of the metal sheet, respectively.

(3) *Formulating an indirect variation between W and L* (cognitive action). When he embarked on the task of constructing a 20 cm high cylinder, the plumber immediately referred to a one-liter cylinder (henceforth we shall refer to it as model container (M)) of perimeter 27cm and height of 17.5cm (he himself recalled these measurements). He further said that “if we have to increase the height, we have to decrease the perimeter”. He asked his assistant to give him a 25 cm by 20 cm sheet of metal and commented that he allowed about 7.5 mm for the folding. He used his tools to make a hollow cylinder of 25 cm perimeter and 20 cm height. He then placed the hollow cylinder on a left over sheet and traced the base and used his tool to cut the circular bottom, allowing 4 mm for the folding. He used his tool to fix the bottom. To check the accuracy of the capacity of the container, he filled the one-liter model with water and poured it in the container he constructed and found that it just fills it.

At this stage it was not clear from the actions of the plumber how he arrived at the dimensions i.e. 25 cm $\times$ 20 cm needed to make the container. We thought at this stage that he added to the height (from 17.5 cm to 20 cm) what he subtracted from the perimeter (27 cm to 25 cm). So two variations of the task were given to test this hypothesis. In analyzing the actions of the plumber, we could recognize a web of four inter-related inseparable cognitive actions and one mnemonic action. These are:

(4) *Using the relation $W \times L = W_M \times L_M$ (W_M and L_M are width and the length of the model container)(cognitive action).*

(5) *Recalling width and length of the model (mnemonic action).*

(6) *Estimating the needed (W) width of the sheet (if the height of the cylinder given) by successive approximation (cognitive action).*

(7) *Checking the accuracy of (C) and adjusting it accordingly (perceptual action).*

To substantiate the above four actions, two episodes resulting from two variations of the task will be described in some detail.

Episode 1

The investigator asked the plumber to make a container 30 cm high of one-liter capacity. The plumber paused for a while, and recalled the measurements of the previous container (25 a sheet by 20 cm). He asked his assistant to give him a sheet 30 cm by 20 cm. He said: “the basic rule is that when we increase the length (height), we decrease the width (perimeter)”. The investigator asked how he arrived at a width of 20 cm. He said “the 30 cm is 20 plus 10. Divide the 10 by 2 to get 5 and add it to the 25 (the width of the previous model) along 20 (the height of the previous model) to give you a 30 cm by 20 cm”.

After rolling it and filling it with one liter of water, he realized that he did not allow for the folding so he added 1 cm to the length to become 31 cm and to the width to become 21 cm. He refilled it again, and was satisfied that it is was one liter.

At this stage we conjectured that the plumber was trying to cut the sheet of the model (25 cm by 20 cm) to another sheet with approximately the same area (20 cm by 30 cm). To test this hypothesis we gave a third variation of the task.

Episode 2

The investigator asked the plumber now to make a one-liter container 15 cm high. The following conversation took place:

P: “To get this one, we look at the first container (17.5 cm high by 27 cm wide). We can take these dimensions and ‘invert them’ ...” ...

He paused for a while and continued. . .

P: "If we take the width to be 35 times 15, it must be equal to 31 times 21 (the dimensions of the second container. . . so calculate 35 and 15 as well as 31 and 21".

I: "Shall I divide, add, multiply?"

P: "Just calculate"

I: "Multiply?"

P: "Yes"

The Investigator multiplied 31×21 to get 651 and 35×15 to get 525.

P: "To have the same capacity 31×21 , we add to the length what we decrease from the width"

He then took the difference $561 - 525 = 126$ and said let us add 1 cm to the 35 to get 36.

P: "Calculate now 36 width for 15 cm height"

when the investigator did and got 540 the plumber said:

P: " $540 - 525 = 15$ cm. So we can say now a one cm in width will give 15 cm in capacity. If we say now 40 width by 16 length then we calculate. . ."

investigator calculated $40 \times 16 = 640$.

P: "Compared to 565, that is closer. . ."

At this point, he said "let us go back to the original container 17.5×27 and take from it one cm. He asked the investigator to check $17.5 \times 27 = 472.5$. He said "that is reasonable, perhaps we made a mistake in the measurements". He measured the original container and found it to be $28.5 \times 18 = 513$.

Students' internal activity

(1) *Transforming information into symbols* (cognitive action). All five students proceeded by using symbols to represent the given: (V) represented the volume of the task container and (h) stood for its height. They also used a sketch to represent the cylinder. The students gathered information from the task and built up a symbol-mediated mental model. The problem of constructing a container of 20 cm in height and volume of one liter seemed to them as something that "goes on in the head" without intimate physical interaction with the surroundings. This could be explained by the fact that students traditionally learn by transforming the elements of a problem into symbols to be manipulated without manipulating the objects in the learning environment.

(2) *Recognizing the appropriate formula* (mnemonic action). The formula for calculating the volume of a cylinder was understood by the students as a rule for calculating one of the variables (V,R,h) if the other two are given. This action was classified as mnemonic action because it was a simple recall of a learned rule.

(3) *Reducing the original task to 'calculating R'* (mnemonic action). The students reduced the problem of constructing the container into a classroom-like exercise of finding the value of the radius R in the formula $V = \pi R^2 h$ once V and h are given. Calculating R became the substitute goal of actually constructing the cylinder.

(4) *Reconstructing relevant competencies to find the variable R* (mnemonic action). Setting up necessary calculations for calculating R was an automatic recall of appropriate techniques to solve an exercise of a type known to students. Reducing the problem into finding the value of the variable R in the equation, students lost track of the goal of the task and attended only to the execution of the steps to calculate R.

The commitment of students to calculate the radius R for a full 20 minutes time confirmed that they were unable to construct the container as is required. This pursuit of a particular goal i.e. calculating the radius R, which they were unable to justify when asked, blocked other problem solving strategies and limited the students' options for finding alternative solutions to the situation at hand. Locking themselves in a single approach and losing sight of their problem-solving repertoire, students were unable to reason or justify the outcomes of their solution. Two out of the five students failed to calculate the value of the variable R due to mistakes in calculation or unit transformation (liter into cm^3) and thus claimed that the problem cannot be solved, they said: "there can't be any cylindrical container with a length of 20 cm and a capacity equal to 1 liter". The others who managed to calculate R stopped at writing the value of R as the ultimate solution to the problem.

In an attempt to redirect the students' attention towards the task of constructing the container, the investigator intervened probing the students with questions such as: "Where is the container? Can you construct the container now that you have the value of R?"

(5) *Recalling alternative solutions* (mnemonic action). Deciding on what other subgoal to pursue after intervention, the students proceeded by searching for alternative solutions. This was done by recalling other formulas in which the calculated value of the radius R can be used. Given R, the peri-

meter of a circle can be easily calculated through the formula $\text{perimeter} = 2\pi R$. Two of the three students who calculated R calculated the value of the perimeter of a circle (none mentioned the term 'base of cylinder'). At this point, it was clear that the students' attention was diverted and the original goal was lost. For the second time the investigator intervened by asking the same probing questions to re-focus the students' attention to the task requirements.

(6) *Identifying the base as a circle* (perceptual action). The students started by constructing the base of the cylinder by utilizing tools from their environment or from those provided by the investigator. They selected a point as the center of the circle and using a ruler, they located several points each at a distance from the center equal to the calculated R . Without using (or asking for) a pair of compasses, they sketched a circle that passes through the points. Using scissors, they cut the circle along the circumference.

(7) *Preparing for construction* (perceptual action). Once the base of the cylinder was constructed, and as they were working from the properties of the cylinder and the constraints of the tools that existed in their environment, the students realized that extra measurements must be added. At that point, the students reworked the actions in step 6 and this time allowed extra measurement for folding. They folded the paper in the form of a hollow cylinder trying to fit it to the circular base they produced earlier. Using scotch tape they attached both ends of the paper and attached the base to the cylinder. There was no attempt to validate their construction by empirical means because they were convinced of the validity of their mathematics.

4. COMPARISON BETWEEN STUDENTS' AND PLUMBER'S ACTIVITY STRUCTURES

Based on the structural analysis of plumbers' mental activity and the associated actions, we can identify a 3-phase cyclic web structure (Figure1). This structure consisted of the following phases:

1. *Perceptual-cognitive phase*: (a) perceiving the 3-dimensional cylindrical container (C) as a 2-dimensional sheet (S) (*perceptual action*), (b) recognizing the perimeter (P) of the base of (C) as the width (W) of (S) and the height (H) as the length of (S) (*perceptual action*), and (c) formulating an indirect variation between W and L (*cognitive action*).

2. *Cognitive- mnemonic phase*: (a) using the relation $W \times L = W_M \times L_M$ (W_M and L_M are width and length of the model container) (*cognitive action*), (b) recalling width and length of the model (*mnemonic action*), (c) estimating the needed (W) width of the sheet (if the height of the cylinder given), by successive approximation (*cognitive action*).
3. *Perceptual phase*: testing the accuracy of the container (C) and adjusting it accordingly (*perceptual action*).

On the other hand, the structural analysis of students' mental activity revealed a 3-phase linear structure marked by two interventions (Figure 1):

1. *Cognitive- mnemonic phase*: The first phase represented the period before the first intervention and it consisted of: (a) transforming information into symbols (*cognitive action*), (b) recognizing the appropriate formula (*mnemonic action*), (c) reducing the original task to 'calculating R ' (*mnemonic action*), and (d) reconstructing relevant competencies to find the variable R (*mnemonic action*).
2. *Cognitive phase*: recalling alternative solutions (*cognitive action*).
3. *Perceptual phase*: (a) identifying the base as a circle (*perceptual action*), and (b) preparing for construction (*perceptual action*).

The differences in the activity structures between plumbing and schooling are apparent in the types and sequence of their actions as well as in the degree of complexity in their respective internal plane. The activity structure of the plumber starts with perceptual actions, moves to cognitive-mnemonic actions, and ends with perceptual actions, whereas, the activity structure of the students starts with cognitive-mnemonic actions, goes back to cognitive action, upon the intervention of the investigator, and ends in perceptual actions only following the second intervention from the investigator.

The type and sequence of actions reflect the kind of interaction between the plumber and the students with their respective environments. While the plumber's attention was always directed towards the object being represented, students' focused on the act of thought underlying the process and their scientific concepts were mediated through other concepts. In the course of constructing the task container, both the plumber and the environment changed. The plumber started with perceptual action, responded to what has been executed by continuously reviewing the model container, executed more actions, and so on. The goals for container construction changed as the container evolved and its actuality became possible. This interaction elicited critical skills such as "recognizing opportunities or problem finding, knowing when and how to apply skills that have been learned in other contexts, and exploiting properties of the present situation"

(Lave et al., 1990, p. 63). On the other hand, the students' interaction with their physical environment was minimal and they approached the problem of constructing the task container by implementing the procedure that was derived directly from the classroom practice. Except for the interventions, they relied exclusively on mnemonic and cognitive tools by reading the problem, selecting the formula, calculating the unknown, and writing the answer.

The complexity of the activity structure is an indicator of the degree of control and self-monitoring during the problem solving process. Figure 1 shows that the activity structure of the plumber is more complex than that of the students. The plumber had proper means of 'gauging' his own skills. He knew (or he thought he knew) whether he can complete the task of constructing the container or not and whether he had made mistakes. Being free to generate problems for himself, the plumber was also at ease and free to change a problem, resolve it, transform it as well as solve it. This was obvious when the plumber resorted to changing the height of water in the container by simply hitting on the base of the cylindrical container. The procedures he used were invented on the spot as part of situated ongoing activity. The students showed little control over the problem solving process, they strongly believed in the power of formulas and algorithms and hence did not feel the need for self-monitoring other than checking the correctness of their calculations. They followed standard school procedures and did not go beyond them until they were prompted to do that.

5. DISCUSSION

The results of this study ascertain the power of the activity theory and its methodology in explaining and identifying the structural differences of the same activity done by the plumber and the students in two different cultural settings. Thus, despite the fact that both the students' and plumber's actions had a common intentional aspect i.e. the construction of the task container, they significantly differed in the operational aspect (actions) or the means and concrete conditions (operations) under which such a goal was carried out. It is clear that activity theory has the potential to explain the differences between the two activities. First, there is a difference in the motive of the two activities: The production of a concrete object in the course of normal job in the case of the plumber and doing a school task at the request of the teacher in the case of students. Second, the plumbing workshop and the school are two different social-cultural settings. Third, the tools that were available and accessible at the time the task was ex-

ecuted, were different and resulted in different actions. The tools used by the plumber were mostly concrete (hand tools, machines and basic equipment, measuring scales). The students used symbols as 'mnemonic-technical' devices and de-contextualized mediation means to calculate the unknown in the formula. Only after the second intervention, the students shifted to utilizing concrete construction tools that were available in their immediate environment. Fourth, there was an obvious difference in the constraints (operations) under which the task was executed. Whereas the plumber was constrained by the properties of the material he was working with and making the material meet the required specifications, the students were mainly constrained by translating the symbols into physical reality.

The findings of this study are in general, but not full, agreement with those of relevant studies in the literature. First, the findings of this study support those of other studies in that the problem acquires different meanings from the point of view of a practitioner in the workplace than from the perspective of an observer (Pozzi et al., 1998; Jurdak and Shahin, 1999; Masingila, 1996). For a practitioner, the problem is not to decontextualise the situation to arrive at a mathematical model but rather to build a connection between the situation (with its concrete resources and tools) and mathematical relationships in order to 'bring down' relevant mathematical meanings to the situation. Second, the findings of the present study add to the accumulating literature that had consistently reported that the problem solving strategies used in the workplace are more meaningful and idiosyncratic than those used in the school (Reed and Lave, 1981; Carraher et al., 1985; Nunes et al., 1993; Jurdak and Shahin, 1999). However, the findings of this study do not support the conclusions of those studies in that the solutions were almost error-free in the workplace unlike the school. Though he was able to deal with the problem in a few cases, the plumber was not able to use his method effectively when we gave him unfamiliar and extreme versions of the same situation. On the other hand the students were able to find a correct solution for all cases, though they were not able to actually construct the container without prompting. One would imagine that the students would be able to use their mathematical tools to deal with the same problem but replacing, say, the cylinder with a regular right pyramid, whereas, the plumber would be helpless in this case. It seems that the relative superiority of practitioners, compared to school settings, in having almost error-free solutions to situations calling for elementary arithmetic, does not carry to more advanced mathematical spatial concepts and relationships.

The pedagogical implications of the findings of this study and similar studies are paradoxical and problematic. According to Resnick (1986), this

paradox is due to the dual role of formal mathematical language as both signifier and signified. Thus, a mathematical expression takes its meaning from the situations it refers to; whereas, the same expression derives its mathematical power from being detached from the situations which give it meaning. This paradox has become problematic to mathematics education, because historically school mathematics and the workplace have developed an almost non-overlapping division of labor in which the school deals with mathematics as a signifier (conceptual tools) to accomplish mathematical tasks, and as signified objects (concrete tools) in the workplace to solve problems that may arise there.

The results of this study support the hypothesis that working with mathematics in the workplace is more meaningful (connection to a referent) than in the school. However, school mathematics has more power, is more generalizable, than work mathematics. On one hand it is more economical, in time and effort, to learn mathematics, as conceptual tools hoping that meaning will come later. Both experience and research do not support this claim. On the other hand, basing the mathematics curriculum on experiential learning only, in addition to being unrealistic, deprives the learner of the power of mathematics. What is needed is to strike a reasonable balance between the power and meaningfulness of mathematics. The challenge of working out the modalities for reorienting the mathematics curriculum in this direction remains open for investigation. One possible approach is for the mathematics curriculum to build bridges between conceptual tools and concrete tools. Constructing solids from plane sheets is a case in question. Normally, dealing with solids at the conceptual level comes rather late in the school curriculum. There is no reason why goal-oriented activities, that involve actual constructions of solids from plane sheets of different materials, cannot be included in earlier grades. The key words here are 'goal-oriented' and 'actual'. Perhaps, the paradox may turn out to be not only a challenge but also a window of opportunity.

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