

BLACK BOXES IN WORKPLACE MATHEMATICS

ABSTRACT. We ground Cultural-Historical Activity Theory (CHAT) in studies of workplace practices from a mathematical point of view. We draw on multiple case study visits by college students and teacher-researchers to workplaces. By asking questions that ‘open boxes’, we ‘outsiders and boundary-crossers’ sought to expose contradictions between College and work, induce breakdowns and identify salient mathematics. Typically, we find that mathematical processes have been historically crystallised in ‘black boxes’ shaped by workplace cultures: its instruments, rules and divisions of labour tending to disguise or hide mathematics. These black boxes are of two kinds, signalling two key processes by which mathematics is put to work. The first involves automation, when the work of mathematics is crystallised in instruments, tools and routines: this process tends to distribute and hide mathematical work, but also evolves a distinct workplace ‘genre’ of mathematical practice. The second process involves sub-units of the community being protected from mathematics by a division of labour supported by communal rules, norms and expectations. These are often regulated by boundary objects that are the object of activity on one side of the boundary but serve as instruments of activity on the other side. We explain contradictions between workplace and College practices in analyses of the contrasting functions of the activity systems that structure them and that consequently provide for different genres and distributions of mathematics, and finally draw inferences for better alignment of College programmes with the needs of students.

KEY WORDS: black boxes, case studies, cultural – historical activity theory, maths-in-work, situated cognition, transfer

1. INTRODUCTION

We draw on a multiple case study enquiry into the ‘mathematics of workplaces’, with each case involving a field study of workplace practices from the College mathematics point of view in the UK.¹ Our motivation was to gain insight into mathematics in vocational settings and hence infer how College students might be better prepared mathematically to understand the workplaces in which they might later become employed (Wake and Williams, 2003).

The focus of our attention here is the mathematics *identified by* College students and teacher-researchers on workplace visits and placements, i.e. by outsiders crossing the ‘boundary’ of College and workplace systems. By joining with students and workers in examining and discussing practices of the workplace, we aimed to break out of previous research lacunae in which

the two systems of College and workplace were separately investigated and interpreted. Our view is that traditional ethnographic approaches describe differences between the two systems without resolving them, and that this may lead to unhelpful over-simplifications or misleading inferences for College curricula. Instead we sought to engage with differences as contradictions in practice, to expose and explain the 'gap', develop a dialectic literally through dialogue, and hence eventually try to understand how to build bridges between College mathematics and workplace practice. Each case study therefore draws on observations and records of the workplace activity seen as potentially 'mathematical', but also relies essentially on teacher-researcher mediated discourses with workers and students about workplace practices.

The focus of this paper is the result of cross-case analysis of how mathematics is shaped and black-boxed by the local work activity system: we try to answer the questions:

- how is mathematics shaped, indeed often hidden, by workplaces?
- what processes serve to shape mathematics in workplaces differently from that in Colleges, and hence cause a 'gap' for students when presented with mathematics from workplaces?

From the answers to these questions, we try to draw inferences for mathematics education.

In the paper that immediately follows this (Williams and Wake, this issue) we address the next obvious question: how can outsiders sometimes come to make sense of workplace practices using their College mathematics? The paper following this therefore explicitly addresses the problems that arise when outsiders enter into dialogue in which they explore (workplace) mathematics, whereas in this paper we examine in detail how workplace mathematics is formed.

We draw on our original Leverhulme study (see Wake and Williams, 2001) involving a dozen workplace field studies, and several others conducted since. Each of the original field studies involved a number of events that included usually about five days of field work, sometimes more, (i) visiting, observing and discussing with workers and so identifying at least one (usually several) practices of some mathematical interest to us, (ii) discussing the curriculum with teachers in College in relation to these preliminary findings and selecting some students to visit, though in some cases the students were already engaged in a work-placement, (iii) conducting a visit with the student(s) to the workplace, discussing again with workers there the practices they were involved with, and (iv) discussing what had been seen with the student(s) separately after the main focus visit.

In this paper our intention is to use these studies to attempt to ground Activity Theory as an analytic lens: we mainly draw on our findings from (i) to (iii), but in the next paper data from (iv) are also particularly significant. Case study is the method of choice when the phenomenon under investigation, in this case workplace mathematical practices, is believed to be not readily distinguishable from its context (Yin, 2002). In such studies, data sources are expected to be multiple but the limited number of units of analysis does not allow for traditional 'sample to population' generalisation; rather it allows 'analytic generalisation', i.e. generalisation to theory. The analogy is with experimentation rather than survey logic: each case is a replication. Thus, theory becomes more convincing and grounded to the degree to which its concepts and frameworks provide more credible insights and explanations for events in a more diverse set of cases.

In our study, we initially hypothesised that while traditional cognitivist-type frameworks (in particular we used our theory of general mathematical competences, Williams et al., 1999) might explain how some mathematics in the workplace is recognised by outsiders; situated theories, in particular Cultural Historical Activity Theory (CHAT), would be more useful in accounting for difficulties in understanding and 'gaps'. Indeed we did in each case study identify and describe some mathematics that could be named by a general mathematical competence (Wake and Williams, 2001). But we also found that CHAT served to provide insight into how mathematics tended to be black-boxed and so embedded and sometimes hidden in the workplace, causing gaps and difficulties for outsiders.

Our methodology is informed by Yin (2002) for 'explanatory' cross-case study analysis: he claims that this method is particularly powerful in serving to ground theory, and even more so when there are competing theories. The main idea is that the selected cases should be as diverse as possible, thus subjecting theoretical propositions or generalisations to diversity of conditions and hence stress. In our case, 'theory' is the CHAT view of practice as being mediated by its Activity System versus a traditional, essentially cognitivist view of 'general mathematical competences'. Indeed we originally identified a very broad range of such mathematical competences, from handling experimental data graphically, to using mathematical diagrams, interpreting large data sets and so on, but that does not concern us here. Our strategy in this paper is to show how CHAT can help us understand the phenomenon of workplace mathematics and 'black-boxing' by applying it to the case data.

Each selected case referred to in this paper is given in greater detail in conference papers and reports elsewhere (Wake and Williams, 2000; Wake et al., 2000, 2001, 2003; Williams and Wake, 2000, 2002; Williams et al., 2001). The focus here is to bring together our understanding of how these

workplaces shaped and crystallised mathematics, and to draw inferences for College curriculum and practice. Being aware of the noise involved in so many diverse contexts, we select only a few case studies to ground theoretical insights, and refer briefly to the other cases only insofar as they offer variations of theoretical significance. In fact in every case we analysed we gained insights from a CHAT analysis of mathematical practice, i.e. we saw that instruments and artifacts mediated the mathematics and shaped a local mathematical genre, and the social norms and division of labour were significant in understanding the work-process and therefore the 'who does what' mathematically: i.e. how mathematics is socio-culturally distributed.

1.1. *Mathematics and black boxes*

We note how workers, students and even teachers (for further evidence, see Nicol, 2002) find it difficult to identify mathematics in workplaces. Some research has concluded that workplaces demand minimal mathematics (Riall and Burghes, 2000) while others say rather that significant mathematical demands arise during particular 'breakdown' moments when routines are challenged by new or unusual situations (Pozzi et al., 1998), or that mathematics may be difficult to recognise because it is bound up or hidden in black boxes by technology (Hall et al., 2002). Some in mathematics education have used the term black box for the result of this technological process, and have sought to clarify how technology can also serve as a means to illuminate what is hidden in these black boxes. This approach has been applied by Noss, Hoyles et al. (1996, 2002) to workplace mathematics, where they also refer to Activity Theory and to Leont'ev's notion of crystallisation.

Furthermore, Straesser (2000) suggests that the hiding of mathematics in black boxes signals an increasingly 'mathematical' social life, even while workers and professionals continue to report that mathematics is progressively becoming invisible, 'disappearing' from the workplace. We agree with him that this contradiction is at the heart of the problem for vocational mathematics education. Our theoretical work will build on and add to these concepts of breakdown and black box: in particular we seek to show two processes by which black boxes are formed and how these processes shape as well as hide the mathematics involved in distinct ways.

We take Latour's very broad view that historical and cultural processes hide mathematical and scientific work, inter alia, in 'black boxes', which include machines, but also ideas and concepts, and 'scientific facts' (Latour, 1987). The term black box is drawn from cybernetics, and is used

‘Whenever a piece of machinery or a set of commands is too complex. In its place they draw a little black box about which they need to know nothing but its input and output.’ (ibid, p. 2, 3).

However, Latour’s notion is broader than this: he notes the role of objects, people and ideas in networks that can become black boxed. Thus he describes the Kodak camera as a distributed complex ‘network’ of actants—persons, materials and organisations – without which the machine would not function, i.e. would not be what it is. For Latour, this network becomes a black box for the user, because ‘it acts as one piece’ and he says ‘when many elements are made to act as one, that is what I will now call a black box.’ (p. 131). Later, Latour (1999) defines black-boxing as referring to:

‘the way scientific and technical work is made invisible by its own success. When a machine runs efficiently, when a matter of fact is settled, one need focus only on its inputs and outputs and not on its internal complexity. Thus, paradoxically, the more science and technology succeed, the more opaque and obscure they become.’ (p. 304).

2. UNDERSTANDING PRACTICE THROUGH ACTIVITY THEORY

Our theoretical starting point is to view mathematical activity and discourse as part of, and enmeshed in social practice (Bourdieu, 1990; Chaiklin and Lave, 1993), which suggests mathematical cognition is situated (c.f. Lave and Wenger, 1991). Thus doing College mathematics is inseparable from the social life of the classroom, assessment and pedagogy (Cobb et al., 2000; Kirshner and Whitson, 1997). The same is, of course, true of workers’ activity. Their mathematics is situated in *working* practices and is lived as such: indeed it may not be recognised as ‘mathematics’ if it is sufficiently embedded in, or fused with the workplace culture (Roth and Bowen, 2001)

We found mathematics in black boxes of many different kinds: in workplace instruments, procedures and routines, of course, but also in local mathematical languages or language genres and in the workplace division of labour. Cultural Historical Activity Theory (CHAT) provided us with analytical foci that helped us understand the processes by which these black boxes arise in significantly different ways, with consequences for how workplaces shaped mathematics. A key concept from CHAT is ‘crystallisation’, which metaphorically describes how processes historically come to be reified within artefacts of all kinds: tools, routines and jargon or language. Mathematics can thereby become subconscious for the workers involved, and ‘outsiders’ who seek to understand how mathematics is pertinent may be presented with substantial difficulties. Thus, we will draw on the corpus of work in Activity Theory, which Latour himself has rarely

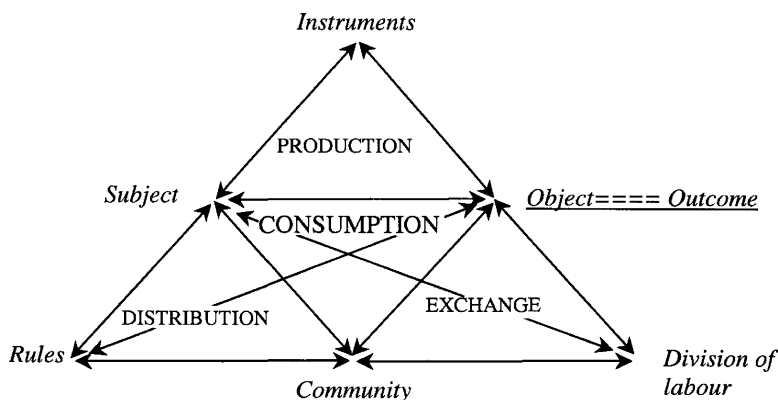


Figure 1. Schema of activity system.

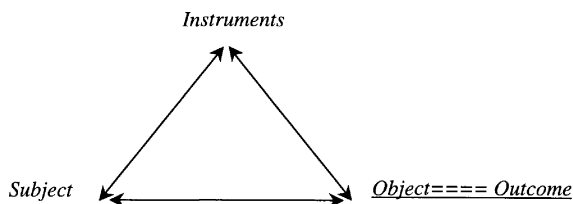


Figure 2. Vygotsky's schema of mediated action.

referenced or sourced, to analyse the workplace and College 'networks' as Activity Systems with particular structures and properties. CHAT helps us to understand differences and expose contradictions between these systems and this allows us to draw inferences for how they might be better aligned.

Engestrom and Cole (1997) sum up the corpus of work in activity theory due to Vygotsky, Luria, Leont'ev and their followers in the notion of Cultural Historical Activity Theory, in which a principal unit of analysis is the culturally-mediated and historically-evolved Activity System, represented in the schematic diagram (see Figure 1).

The origin of this figure is in Vygotsky's notion of *mediated action* of the subject in dialectical interaction with an object, shown in the top triangle (see Figure 2): just as labour is mediated by artefacts, tools and 'instruments', so also communicative action is mediated by cultural tools, concepts and language (Engestrom, 1995; Wertsch, 1991, after Bakhtin et al., 1986, 1981). Thus Vygotsky's theory of language (and sign mediated action in general) draws on Marx's theory of labour (for example, Ilyenkov, 1974; Vygotskii, 1987). Vygotsky draws the distinction between physical tools and semiotic tools, however: while physical tools operate on or control

natural objects, semiotic tools operate on ideas and concepts. They are initially used in dialogue interpersonally but later used intrapersonally, i.e. they become psychologically, inwardly directed, facilitating planning activity in particular.

So it is, CHAT suggests, with mathematics in use: a mathematical formula can only make practical sense in its context of activity. As an abstract expression only, a mathematical sign has a general, abstract meaning (i) associated with a network of other mathematical signs, and (ii) its many potential uses, but as yet no practical meaning. But the ideal form that is represented by mathematical signs is pregnant with many different particular potential meanings and uses in different activities. The meaning of a sign for Pierce is in the interpretant, and hence arises within an interpretative field (see Whitson, 1997). But because there are many different possible fields of interpretation that the reader can orient a sign to, signs afford an infinity of specific interpretations. Thus, for instance, *the linear model* is abstract and general, but within the context of a particular action and activity, it can come to have practical meaning (i) as a mediating tool of activity, say as a model for extrapolating from past data to predict future gas consumption or (ii) as a pure mathematical function signed as ' $y = mx + c$ ', which is the subject of algebraic and graphical transformations and manipulations. We note, therefore, that a mathematical sign may have an infinite variety of different meanings in different settings, and in particular that the same sign might be a tool for one activity but the object of action in another.

In Vygotsky's mediational triangle in Figure 2, symbolic instruments of action are abstract, and action is made practically meaningful by virtue of its purpose. Leont'ev adds to this a specific mediation of activity as communal: he suggests that activity draws its meaning from a community, without which any particular action is devoid of motivation. This community motivation may be hidden from some individuals in the community, but it must be there nonetheless. Thus CHAT takes from Leont'ev the other nodes in Figure 1 to describe the subject's action in relation to a community: this includes the ways in which the *division of labour* and associated *norms and expectations* mediate the subject's activity in relation to the community.

We will here follow Leont'ev's account of this in the example of communal activity of the primeval hunt (Leont'ev, 1981). A beater (the subject) frightens the prey (the object of action) in order to make it flee (the action's goal). But the *motivation* of this act is to fulfil a need (arising from hunger, or cold). There is an *idea* of the prey as a potential object of consumption to satisfy needs (food, clothing). But this would be a meaningless, motivation-less act without the others in the community who will ambush

the frightened prey (according to the *division of labour* in the hunt) and a set of *rules*, including the norms and expectations about how the prey will be shared. We will use these concepts in analysing how mathematical actions sometimes become socially black-boxed, and hence hidden by the division of labour in the workplace.

Leont'ev (1978) develops Vygotskian theory further and argues that analysis and understanding of any subject's action should involve a distinction between (i) the *action* and its *operation*, on the one hand, and (ii) the *action* and the communal *activity*, on the other hand. The hierarchy "activity – action – operation" is associated with the parallel hierarchies of "community – subject – instruments" and "motivation – goal – methods". By way of example he painstakingly distinguishes the motivation for the communal hunting activity (catching and sharing the prey) from the particular goal of the individual subject's action (frightening the prey). The goal of an action thereby only acquires meaning in the context of understanding the motivation of communal activity (and also how the work is shared and the rewards divided). We will return to this distinction later, as it raises all the salient issues of rules and division of labour (e.g. the rules about the division of labour between the 'beaters' and the 'shooters').

The *operation* however is the method by which the action is instrumentally accomplished e.g. the frightening of prey by the loud noise may be accomplished by beating or by shouting. With other instruments available the operation would change but the conscious goal (frightening prey) may not. Leontev comments that when driving a car, the conscious goal of action might be to 'speed up', whereas the subconscious practical operation might be to change gear; he notes that it is the fate of such operations to become automated. The automatic gear changes the required operation corresponding to the same action. So it is with practice, actions tend to become automated as operations, and they then become sub-conscious. This process, we argue, underlies the formation of technical black boxes as instruments, and we will show how these on the one hand hide and on the other hand shape new mathematics in workplaces.

As soon as we consider psychological activity we notice that the effect of different instruments and operations is highly significant. The action of calculation is operationally very different with an electronic calculator from that with a paper and pencil or abacus. Leont'ev notes that operations are available because of the experience of actions in previous activities: it matters not for the action at hand what operation is used as long as it is available, but this operation is only available if it has been mastered earlier. So we should expect to find, and do find that the use of idiosyncratic instruments and procedures in the workplace causes cognitive difficulties for the unpractised outsider.

On the other hand, we notice that when driving a car, if the clutch starts to malfunction, to accomplish a gear shift we may have to re-orient, 'double de-clutch' etc. Thus at 'breakdown' moments what was merely routine and operational now becomes the object of attention and hence conscious action. Hoyles et al. (2001) used this notion in the context of nurse's mathematical practices, but it is a very general one according to Leontev. Thus, in our methodology we design interactions deliberately to try to induce 'breakdown': that is, by organising situations where we, as outsiders, find the routine incomprehensible, and demand explanations, sometimes of a mathematical nature. Thus workers are confronted with a 'controversy', and so a need to justify and explain the routines in terms we might understand. Although this use of the term is similar to communication 'breakdown', it is in fact not very different from the way Pozzi et al. (1998) have used it: when a nurse who is new to a practice on a ward questions whether or how this routine works, a very similar breakdown moment occurs in their study.

Latour (1993, 1999) refers to the crystallisation process as one of 'black box' formation, but he also uses the term breakdown in the context of opening up these boxes:

"Take for instance an overhead projector, it is a point in a sequence of action (in a lecture say), a silent and mute intermediary, taken for granted, completely determined by its function. Now suppose the projector breaks down. The crisis reminds us of the projector's existence. As the repairmen swarm round it . . . we remember that the projector is made of several parts, each with its role and function and its relatively independent goals. Whereas a moment before the 'projector' scarcely existed, now even its parts have individual existence, each its own 'black box'. In an instant, our projector grew from being composed of zero parts to one to many." (p. 183)

Recall that Latour's black boxes include not just physical objects and tools, but also scientific 'facts' and theories, and indeed the networks of actors and actants that allow such objects to 'act as one thing'. He describes the extensive, distributed and coordinated labours required by networks of tools, practices, papers, and scientific communities to produce and maintain them. We include the whole of mathematics and all the tools and instruments, hardware and software that are built with mathematics in this category: in our data examples include machine-produced graphs, programmes to control machinery, spreadsheet formulae, rules for mixing cement or calculating financial measures, police performance indicators and so on.

In summary, Cultural Historical Activity Theory offers a powerful conceptualisation of systems which takes us beyond what most 'situated' social practice theorisations provide in at least two ways. First, mediating artefacts

place any activity in a socio-cultural and historical context: these objects fill the social space around us, connecting us with the rest of humanity historically and contemporaneously. Second, these systems are dynamically evolving in response to efforts to manage contradictions within and between systems (Engestrom, 1987). Thus in CHAT, social practice is conceived as being structured by multiple, dynamic systems of activity which are interlinked (by boundary crossers and objects) in ways which provide conflicts explained by contradictions. To take a well-known example, the development of powerful new technological tools for mathematical calculation (responding to needs from contradictory industrial, business and scientific-technological activity systems) potentially introduces contradictions between mathematics assessment and the intended curriculum in schooling.

In our analyses we find CHAT fruitful in drawing our attention to the workplace structuring of mathematics, i.e. to the way black boxes are formed. CHAT analysis also assists in explaining significant differences between workplace and College mathematical activity, and contradictions between them. Our methodology is designed to ensure that such differences are manifested in contradictions between work practice and outsiders' mathematical discourse. These contradictions might sometimes make it difficult for workers and outsiders to recognise College mathematics as relevant to the work practice and communication 'gaps' are indicative of such contradictions.

3. INSTRUMENTS USED AS A MEANS OF MEDIATING MATHEMATICAL ACTIVITY

At the core of each case study (for example, see Wake and Williams, 2001) is an activity and some actions of an individual worker within a workplace system that structures the community activity. This CHAT perspective leads to an initial focus on how their action is mediated by *instruments*. We can illustrate mediation of mathematical activity in the workplace by reference to any one of our case studies and other studies besides. The most obvious instruments are machines which either input or output mathematical signs. We initially draw on one case study here to illustrate this.

A student said by her mathematics teacher to be mathematically very competent was on placement in an industrial chemistry laboratory and carried out a typical experiment for the laboratory in which a substance was heated so that it became thermally unstable. Data was logged in real-time, and a number of graphs were produced (for a fuller description see Williams et al., 2001). One of the resulting graphs is shown

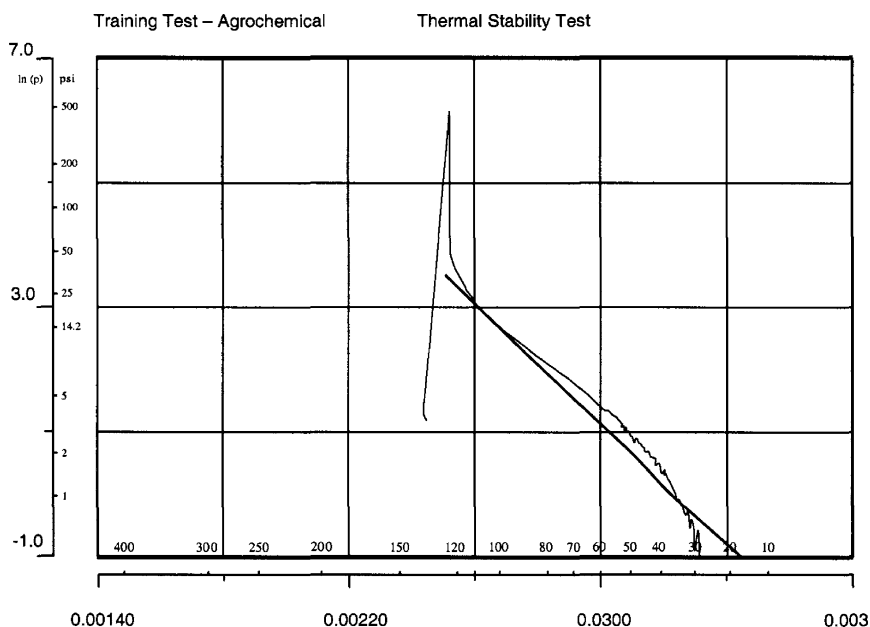


Figure 3. Industrial chemistry laboratory graph demonstrating thermal instability.

in Figure 3: this graph proved highly problematic for the student when she was asked to interpret it in the context of the experiment she had performed.

As we have remarked elsewhere (Williams et al., 2001) the ‘College’ graphs students typically meet obey very different conventions to many of the graphs produced by machines at work; and here is a contrasting pair of examples (Figure 4). Although familiar with graphical work from her studies in both mathematics and chemistry, the graphs from the workplace (Figures 3 and 4a) had features for which the student was unprepared.

In discussion with the industrial chemist who uses these graphs every day we identified the reason for these features: he was able to instantly ‘read’ from these graphs the ‘spike’ (Figure 3) or lack of one (Figure 4a) which to the trained eye reveals temperature and pressure changes: thus the spike in Figure 3 reveals conditions at which the material being studied exploded and hence the chemist drew conclusions about the safety of the material, which is the motivation for the laboratory experiment. This laboratory activity shapes the mathematics of graphing in particular ways: it requires a graph to output a wide range of values of pressure for instance, so a logarithmic scale is conventionally used: this is a rare feature in academic graphing practices in College mathematics and makes it difficult for the student to make the required connection with what she knows.

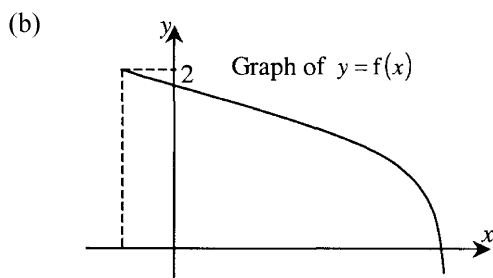
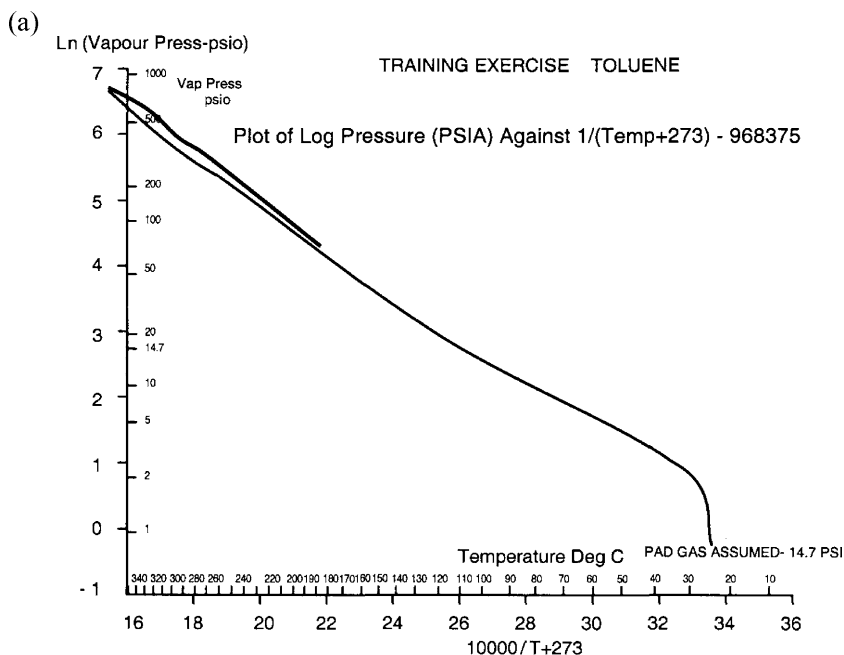


Figure 4. (a) An example of a graph produced in an industrial chemistry laboratory contrasted with (b) an example of a graph met by students studying Advanced Mathematics.

Perhaps the most problematic feature of the graph of Figure 4a for the student was that in effect it should be 'read' from right to left, i.e. the starting point of the experiment is at the right-hand end of the trace. The industrial chemist uses a coefficient that has particular importance in gas laws and hence in the analysis of the situation and which is defined by the inversion:

$$\frac{10000}{\text{temperature} + 273}.$$

This new variable is motivated by the relation of absolute temperature to

pressure, and increases from left to right along the horizontal axis, i.e. as temperature decreases. But this led to confusion as the new variable is low when temperature is high and is high when temperature is low. This has the effect that the familiar quantity (temperature) has high values at the left end and low values at the right end of the horizontal axis. This is the reverse of the student's common experience of conventional graphs in both mathematics and chemistry; so much so that her reading of the graph was from left to right. To be consistent with this she was willing to suggest that during the experiment, as time increases (reading from left to right), the materials cooled down. At the same time she certainly 'knew' that during the experiment the sample was being heated at 2 K/min and hence throughout the experiment temperature must be rising: a manifest conflict!

On the other hand the industrial chemist was able to interpret the resulting graph with ease: the workplace artefact allowed him to quickly make sense of the heating process. The graph serves a very special function in helping the expert to diagnose the situation, and its properties have been accordingly designed. For the initiated, the reading of the graph is fluent, fused with its practical use (Roth, 2002). The spike in Figure 3 "is" for the expert what we outsiders say it "represents". The outsider, on the other hand, has to learn to read these graphs like a second language, associating the elements of this weird product with meanings until we begin to notice its key features. Eventually, with practice, we may learn and become experts, too, and the sense of effortful association of parts of graphs with the explosive properties of substances will disappear. The experience of interpreting these graphs will sink to the operational level and become unconscious.

Graphing in College mathematics generally is designed for the study of algebraic functions: a typical syllabus lays down the rules as, "Sketch graphs of polynomials of such and such a type, and functions of the form $A/Bx + C$ " etc. The syllabus takes this form historically because the system of assessment and examination holds that such statements and exemplars should be clear, equitably interpreted in different Colleges across the nation, and so on. Local idiosyncracies and conventions are regarded therefore as dysfunctional. In the College context too the new student has to expend effort to begin to interpret particular features of these graphs (possibly with the aid of an instrument like a graph-plotter) such as zeros, turning points and asymptotes. The expert College graph-reader comes to interpret these things with ease: it may become automated, an 'operation' in the performance of other actions, like finding a maximum value of a function. When this happens the operation is often not regarded by the student as really 'mathematical': typically we found the student will then say "that's just basic stuff". This is somewhat the same as how the worker may feel when reading the graph as an explosion without being conscious of doing any real

mathematical interpretation as such: the complex work of interpretation has been automated and become similarly invisible.

In both cases then expert graph reading has similar features and a similar genetic process, but their conventions are different and in some significant ways contradictory, confusing the newcomer. The contrasting communal purposes of the two different activities of 'explosions lab' and 'doing College' have historically evolved the functions of the two different graphing practices and hence their rules and conventions in contrasting ways. The particular features and conventions involved are functions of the instruments used, and these were in turn developed historically by the object of activity: to diagnose explosions on the one hand or to represent functions on the other. Hence the gap between the two graphing practices.

When the outsider solves the problem, noticing that the graphing conventions are a problem in need of special attention and re-interpretation, a new synthesis becomes possible. The newcomer may reflect that their own practice of graphing relies on certain implicit conventions, and that these are in a sense arbitrary. Perhaps better, they are not arbitrary as such, but shaped to the activity context, that is, to the Activity System in which the mathematical instruments have developed or historically evolved. We found that students were in many cases ill-prepared to see mathematics as conventional in this social sense. By exposing social differences between College and work to a contradiction in practice, we learnt that mathematical conventions and rules are a product of activity. This insight seems to us salient to helping students who want to learn how to make sense of workplace mathematics: we discuss this sense-making in more detail in the paper that follows (Williams and Wake, this issue).

We now turn more briefly to a second example in which the historical process of crystallisation of mathematics in language is more stark. The case involved a small metal workshop in which Steve devises programs for punching out metal plates with a CNC device. He receives specifications by fax for metal plates which have to be costed for customers. The cost depends critically on the machine time required and hence is a function of the efficiency of the program. If he wins the order, his workers will usually do the actual work using the program he has devised on the machines to cut the plates, but the smart work is in the program. Figure 5 shows a program developed by Steve that controls a press to punch small components, often with complex shapes, from large metal sheets. This program controls the movement of a metal sheet below a turret armed with a variety of differently shaped tools that punch holes out of the metal.

This program was also initially incomprehensible to the uninitiated outsider due to the particular and special conventions and notation used. It makes use of, but also hides, mathematical ideas: vectors and coordinates,

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(SIZE, 125, 185)
(CL, 1, 50.00)
(CL, 2, 150.00)
(T7, RE, 5x30mm)
(T12, RE, 30x5mm)
X7.5 Y85. T7
G91 G61 Y70. R25. X105.
G61 Y-70. R25.
X 25. Y 172.5 T12 G90
G91 G61 X 70. R25 Y -105.
G61 X -70. R25.
M30 M50

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Figure 5. Punch press program for punching out a plate.

repeat-loops, subroutines and number conventions have all been used by programmers in their development of a useful programming language. For example the commands G90 and G91 call subroutines that switch coordinate frames for movement between absolute and relative referencing. Perhaps the most easily recognisable mathematical references are the X and Y signs which refer to coordinates in absolute referencing mode, but to vector movements when in relative referencing mode. However, even these are not necessarily unproblematic for the academic outsider, as X denotes a movement from left to right whilst Y denotes movement from top to bottom, and their whole number arguments must have a decimal point at the end. The historical reason for this was lost to the worker as well as to us as outsiders: sometimes it is not easy to breakdown the crystallisations of ossified, forgotten historical work.

Both these examples arise in the context of workers communicating with machines designed to automate particular mathematical actions, and hence make the labour process more efficient. Most of our case studies provided similar examples: spreadsheet formulae were used by office workers to calculate indices of customer indebtedness, police-force performance indicators, and a gardener used a scale-ruler for reading from a map, and so on. Each involves a genre of mathematics with its own, particular idiosyncratic conventions which are often influenced by the technology, and might require considerable technical research or knowledge at the operational

level to understand, e.g. the gas laws that underpin the temperature conversion for degrees Celsius to Kelvin in the case above. But the mathematics can also be seen to have been structured by the purpose of the action in the context of activity, which required sometimes considerable work-process knowledge to understand: for instance the understanding of the critical role of the chemistry lab explosion experiments for fuel safety reports, or the significance of efficient programming to the pricing and winning of contracts.

The two 'mathematical' instruments we described are very particular to their specific workplace activities, essentially structured by a productive 'object'. Note that the operational form of the action is shaped by the instrument: the mathematical (or non-mathematical) signs involved in each action are mediated by the mathematical language embedded in each instrument. Thus, we may legitimately speak of different genres of mathematical language (Bakhtin et al., 1986; Gee, 1996), in the two examples here we might call these: 'Data-logging graphical mathematics' (addressing the explosions lab community) and 'Machine tool programming mathematics' (addressing the machine tool community).

Action	== (mediating instruments)	== object	=> outcome
Interpret graph	== (data-logging machine)	== graph	=> diagnosis of explosive properties
Write/design program	== (program language)	== program	=> to cut shapes from metal plate

By analogy with the 'voice' of persons communicating in genres with particular social groups (Wertsch, 1991), we conceive of these voices as means for speaking to or reading from particular machines (data-logger or machine tool respectively) in genres that were constructed by others, elsewhere. In the case of the chemistry lab the scientist 'reads' the machine's output; while the metal-worker 'writes' programs to the machine. In other cases we have workers who both 'read and write' with their machines' spreadsheet, and one case (more of this later) where the reading and writing is shared between the work-unit in a particular, vital division of labour. This notion of mathematical 'communication' requires us to be democratic about these machines. Latour elects to call them communicative actants in their own right, suggesting they sometimes even have agency when coordinated in networks, when they become collective subjects (in Engeström's, 1987 sense). We think this notion that a community consisting of more than just a collectivity of humans, but extending to a distributed network including tools etc, is one that is particularly significant for mathematics, science and

technics/technology: the language of mathematics and the genres of which we speak are especially significant in such communities.

These particular mathematical genres are historical formations that presumably were influenced by other, earlier mathematical genres developed or used by their designers. We speculate that these were historically related to past 'academic mathematics', among other genres, and hence (as distant cousins) to the 'school mathematics' which our College students learn, and we can see the traces of these signs in the use of axes, coordinates and expressions involving variables. In addition we assume that the particular process of creation of these instruments and machines were conditioned by the particular scientific and technological problems their industries posed. They are more or less idiosyncratic, and the conventions of the resulting contextualised mathematics are hence idiosyncratic to the case: e.g. the use of 'G90', or the logarithmic scales for the axes above. But the precise genesis and evolution of the relationship is lost to us: we see only its resulting crystallisation. Our inquiries serve to reopen the mathematical problems that have already been solved in the making of the black boxes, so to speak, and perhaps construct or re-construct the mathematical processes that created them.

We next attempt to shine some light into another kind of black box, constructed within Activity Systems by the division of labour at work.

4. ACTION WITHIN THE WORKPLACE COMMUNITY

Up to this point we have focused on how mathematical processes become blackboxed in or crystallised within *instruments*. We turn now to illustrate how the mathematical actions of individuals should be understood by their function within the activity of a *community* and its *division of labour*. For the workplace, with the overriding objective of the production of goods or services, we expect to find its *rules*, and *divisions of labour* structuring actions accordingly. We will argue that these involve black boxes, too, but of a different kind, consisting of social units rather than instruments (though often this social division is mediated by an instrument such as a schedule or program).

First let us note that in the two cases we have drawn upon here there have already been indications of a significant division of labour between operators and managers/engineers. The operatives who use Steve's program to cut out metal plates had no understanding of the way the program was written. The program for the operator is purely instrumental, in much the same way that Steve's understanding of the mathematics of 'G90' is instrumental and different from that of the programmer who wrote the code for the rou-

time, G90. Similarly, the trainee operator in the explosions lab knew to pick out the graph's 'spike' as meaning that there had been an explosion, and to locate the temperature and pressure at which this occurred. But she could not explain in mathematical terms why the temperature transformation involves +273 in the way that the scientist could, or presumably in the way the machine's designer could have. The operator's understanding serves to perform the action, not to explain the layers of crystallised mathematics to an outsider.

We therefore sought to explore the significance and role of this division of labour in our collection of cases as a whole. As an illustration of how the action of the individual subject is mediated by the *community*, i.e. by its *division of labour* and *rules*, we consider the case of the engineer, Dan, who is responsible for overseeing the consumption of gas and the production and distribution of electricity and steam to sub-units of a large industrial chemical plant. He collects data to estimate gas consumption each day: essential to the functioning of the plant. Dan has been responsible for devising spreadsheet models to allow him to do this from data collected and supplied by his operatives. One set of data is required at 6 o'clock each morning. To assist with this Dan has devised a "daily loading sheet" which controls the data collection of his operatives: this is a complicated proforma with many labelled boxes used by operatives to record data from meters. Dan explained to the visiting researcher and students how this "daily loading sheet" is completed by a technician walking round the control room reading dials and filling in the sheet. Dan then takes these readings from the sheet, enters them into his spreadsheet to calculate the information required by the gas supplier and the plant's finance office.

Analysis of the transcript of Dan's description of filling in a blank Daily Loading Sheet shows the obscurity of the procedures the technician uses and of which Dan has 'control'. In one part of the sheet a value has to be entered into a 'box' labelled "*CellsAC*". Dan says: "*Cells AC is J, K, L, KOH*" i.e. the sum of the values in four boxes on another part of the sheet. Although in other parts the sheet uses lines and arrows to make clear how the power is distributed, this particular addition and transfer is opaque. Dan and his technicians know how to use it but there is little attempt to ensure that it is accessible to others. Thus local idiosyncracies are here unconstrained by want of clarity or elegance.

The technicians and manager told us that Dan was the 'keeper of the spreadsheets', the one who did the maths. The work seems not to demand that the technicians know how the data they supply is used in Dan's spreadsheets: they are completely 'black-boxed' from the calculation process. This struck us as being particularly stark in this case study, with Dan's 'ownership' of the mathematics seeming important in establishing his iden-

tity in the community. As Dan explained, “I built these because I’m one of a team of seven people, and I’m the only one that knows how to do this. I try and make it idiot proof . . .” It seemed to us there were issues of power, control and professionalism involved in the engineer’s construction of the loading sheet and its maintenance of the division of labour in the plant.

This case showed particularly clearly how the division of labour between Dan and the operatives in his team is managed by the sheet itself: the operatives fill in the sheet and the engineer uses it as a record to enter data into his calculations. As such the sheet forms a boundary between two activities, and reveals two sets of meanings ‘on either side’ of it. In the case of the metal workshop a similar boundary is marked by the program: the program holds different meanings for the programmer (it is the object of his action, and the goal is to save production time and metal. . .) and the operative (it is the instrument of cutting actions on the machine). Though the program was not consciously written for the purpose of separating and regulating the work of the programmer and his operatives, it does in fact serve this function. In a sense also, one can see how the difference in understandings between the chemist and the lab operative is related to the idiosyncratic workplace structuring of the graph. The operator can interpret the spike, which is fused with the notion of explosion; the scientist however can be made conscious of and can explain the graphical representation and transformations of its axes. This knowledge was used in the design of the graphing machine but is made unnecessary at the operative level, while the scientist may be expected to have that depth of knowledge. Thus the historical disappearance of mathematics in the machine is also reflected in the social black boxing of knowledge in the workplace division of labour.

Once we started to look, every case study revealed that mathematics is highly sensitive to the distribution of knowledge connected to the workplace division of labour, and hence to its power relations. These are often mediated by objects at the boundary of two or more subunits of activity that coordinate their activities. These objects are acted on by different people in different ways, and may involve different mathematical meanings. Mathematical knowledge and activity is in this way black-boxed, with groups of people excluded or insulated from mathematical knowledge and power.

We have highlighted in this section that workplace activity is mediated not only directly by workplace instruments and machines, but also by the community rules and division of labour. These rules and expectations are often made visible in tangible artefacts and instruments such as the ‘daily loading sheet’, the spreadsheet formulae, the timetable, the program or the work-schedule that crystallise them. Similarly College rules and norms are sometimes codified in documents, syllabuses and job descriptions. But we

found that they may also be tacit. There are unwritten rules, implicit norms and cultural models in play that are less easy to identify. For instance, we found that there was an important 'hidden curriculum' at Dan's workplace: there was a hierarchical workplace language in use in which Dan's authority was vested in his role as the expert keeper of the mathematical formulae, while the operators' work was made 'idiot proof'. This language served to position him in a powerful way in relation to his operatives (and even his manager).

We suggest that we have here a different kind of black box, then, which serves to contain the operator's work, isolating them from consciousness of the whole activity. On one side is the operative network (the technician notes down readings, the clerk calculates financial measures, the workshop technician operates the metal cutting machine), while on the other side is the engineer (or scientist, or manager) who uses their work to perform apparently 'higher level' work. The consciousness of the activity and hence the mathematical meaning therein is very different on either side of the boundary: the boundary serves to isolate one side of the system from the other, the 'inside' from which (or in which) mathematical meaning is hidden. The boundary thereby distributes the mathematics (Hutchins, 1995). We suggest that this aspect is as important as the process of automation traditionally thought of as black-boxing.

In conclusion, therefore, we find that there are two different black-boxing processes; the first crystallises mathematical work from earlier activity (the machine's designer, the software programmer etc.) in instruments and tools. These serve as boundary objects between mathematics performed historically in their design and their current use by operatives: there is a distribution of mathematics in *time*. Consequently, tool-mediated work engages a workplace mathematical genre which is mediated by idiosyncratic conventional forms: the mathematical origin and significance of these may even be quite lost in time. The second kind of black box involves workers in a subsystem more or less isolated within the wider system by a division of labour held in place by rules: there may be boundary objects maintaining and controlling/regulating these, as in the loading sheet. These seem to distribute and crystallise work 'horizontally', in a division of labour across social *space*, i.e. across boundaries within the present system. We have noted that the division of labour is often regulated by new artefacts and instruments, but can be maintained also by hidden forces.

Thus, while it is true that some mathematics can readily be identified by academics in workplace practices, we find that workplace mathematics has its own distinct genre, inflected by the local practice and its activity system, its instruments and division of labour and power, as well as the productive goal of the whole activity. We speculate that there are genetic relationships

between these two historical blackboxing processes. The disappearance of mathematics may involve a historical progression of automation related to the division of labour, power and knowledge: first a social division of labour/knowledge may emerge which is asserted through power relations in the present community, and then this is increasingly regulated by rules that become crystallised in artefacts that control operations, and finally operations are automated in machines. On the other hand the historical development of efficient machines necessarily engages a division of labour between those who design, make and repair machines and those who use them: so the historical development of instrumentation reinforces the 'spatial' growth of the division of labour.

5. IMPLICATIONS FOR EXPANDING COLLEGE MATHEMATICS EDUCATION

We have so far only implicitly contrasted the cultural-historical processes at work in the formation of mathematical practices in College with that of the workplace. The differences between work and College systems, i.e. their objects, motivations, tools, division of labour and expectations, seem stark. We now focus on, and extend, our analysis into the contradiction of the two, in an attempt to infer possibilities inherent in this contradiction for curriculum development. What lessons might we learn from our analysis of workplaces? How might we better structure the mathematical experiences of young people in schools and Colleges?

The practice of teaching and learning mathematics in College is evidently structured by an academic system: it is defined by its academic objects, texts and tasks usually set by the teacher, curriculum and examinations, with the intended outcome of individual 'learning' of the mathematics documented in a syllabus, and mediated by assessment and examination rules. There are traditional norms which have historically developed and are maintained by various professional and training organisations and monitored by agencies and documents, but they are also locally shaped by the particular College institution with its particular communities, divisions of labour and rules of cooperation. The teaching profession is itself shaped by certain audit cultures of accountability, historically slowly evolving but broadly 'universal' at least in the English educational culture. The profession is itself the object of action by national institutions and committees which define how mathematics should be written, what tasks should be performed and so on. These government committees are to some extent influenced by voices from workplaces, though academic voices from universities have historically been generally relatively rather stronger.

We have now merely scratched the surface here of the complexity of activity systems which interact with, constrain and impact on the central activity site in the College classroom. The result is that College mathematical activity comes to be a part of our national academic history and culture, obedient to its own conventions, though many of these conventions may be invisible as such to its practitioners. We pointed above for instance to the conventions of graphical interpretation predominant in mathematics learning in advanced mathematics courses. We suggest we can view College mathematics as just 'another genre' of mathematical semiotic activity: it interacts with, but also contrasts with those of university academic mathematics, engineering mathematics, and so on. This contrast becomes a contradiction when the two are brought into contact, through boundary objects and crossers (Engestrom, 2003) as in our research programme.

Thus the academic genre of mathematics values elegance, simplicity and certain conventions which afford the perception of mathematical structure and mathematical relations. It is a particularly privileged genre, to the extent that it is aligned with 'professional mathematics' which in our culture has come to be regarded as the 'proper' mathematics of a universal community of scientists. It is this genre which has influenced the College genre of mathematics our teachers and students try to speak and practise, though it is by no means the only or most powerful factor influencing what actually goes on in classroom life. It is also privileged by virtue of providing a national (and somewhat international) common culture: everyone goes to school and learns to some extent to speak this genre: it is the commonly accepted cultural model of what "mathematics" is for most people. This in turn inflects our interactions with workplaces, wherein workers say they 'don't do mathematics'.

We argue that the key to the invisibility of workplace mathematics, both for workers and for visiting students, is in College mathematics itself. From our experiences of working with students attempting to make sense of the mathematics of others we are led to suggest that it demands dispositions, skills and awarenesses that would be useful to workers themselves. To make sense of workplace mathematics, outsiders need to develop flexible attitudes to the way mathematics looks, to the way it is 'black boxed' by instruments and divisions of labour. We found that students were not well prepared for this challenge. For instance, students had little awareness that the College mathematics they had learnt was itself idiosyncratically 'conventional', and that mathematical conventions might vary in time and place. In general, they had little awareness of the extent and limit of their own mathematical knowledge, and were not well prepared with the inquiry skills needed to probe the mysteries of the workplace situation.

This, we argue, is crucial meta-cognitive knowledge for a pre-vocational mathematics curriculum. As a result, we argue, curriculum design should include learning outcomes such as “making sense of the mathematics of others” and tasks should be included in the curriculum that involve students in inquiry into mathematical artefacts. We also note the centrality of technology, often new technology, in all our cases, and it seems critical to us that any pre-vocational mathematics deals with this as a concern. In other words, the mathematics of ‘others’ should include a diversity of unfamiliar technical media, whose mathematical genre has to become the object of study before its use can be understood. Clearly spreadsheets are commonplace at present, but the point is not only to become proficient at using a spreadsheet, but also to learn to decode a particular spreadsheet, taking into account its contextualisation, its place within the work process and division of labour as well as its embedded rules.

In general we suggest that mathematics curricula be specified to ensure that students meet a diversity of mathematical conventions and methods, have experience of developing mathematical thinking in contexts that reflect realistic, complex workplace situations, e.g. incorporating appropriate artefacts, real group problem-solving tasks in which the object is to make sense of how mathematics ‘works’. New mathematics qualifications (Free-standing Mathematics Qualifications and AS Use of Mathematics) that might allow development of such an approach to the curriculum have been developed in the UK (for a brief description see Wake et al., 2004), although there is as yet little evidence that such innovation is taking place in College classrooms.

We also need to draw inferences from our observations of how social aspects of workplaces may mediate the mathematical experiences of workers and consider how this might inform curriculum design. In College, learning mathematics is often a solitary experience in contrast to the community endeavour of workplaces. Students do not usually have experiences, for example, of having to decide what (mathematical) information they need to gather from others to be able to proceed with solving of a problem. In some instances we have observed workers subcontracting to ‘mathematicians’ when the going gets tough, and then at a later stage operating with the supplied results, in much the way that a student might delegate the calculation of a regression equation to a software package or calculator. It seems therefore that students might benefit from sometimes experiencing mathematics as a communal activity and becoming aware that this is common practice in workplace situations.

These suggestions should challenge the ‘normal’ passivity of UK College students’ mathematical experiences. They seek to help students become more actively engaged with challenging tasks which require them

to critically 'inquire', to work out how others use mathematics (e.g. in work contexts) and to develop social and problem solving skills as well as mathematical resources and confidence.

We now reflect on our research method and the nature of the boundary crossings we have engaged in. By inquiring into the workplace we made the workplace practices problematic, both for us in coming to understand them and the workers who felt obliged to explain them. Thus, opening black boxes where necessary, explicit and implicit mathematical models were exposed and examined. Naturally, we who speak the academic genre of mathematics problematise local idiosyncrasies, its conventions, jargon, processes, instruments and routines that are new to us. On reflection, we therefore come to see our own mathematical genre as one which has its own conventions and rules.

We argue that this should be a significant aim in a pre-vocational mathematics education: coming to understand how the school mathematics learnt is a particular, historically and culturally situated one. Others have made efforts to effect such an aim in their curricula (especially at university and high school levels, e.g. the French IREM, multicultural mathematics etc). We would argue that a pre-vocational curriculum should teach about workplace diversity as such a multi-cultural mathematics. On the other hand we noticed sometimes that the process has a reflexive impact on the workplace: workers who did not perhaps see their activity as mathematical were sometimes brought to see things our way, and thus look at their practice with a new, more mathematical perspective, e.g. the policeman who came to see the 'error' of using an average of the averages in performance management, from a mathematical point of view (Wake and Williams, 2000).

We see our research methodology then as a way of living the contradictions between the two systems of College mathematics and working practices, with potential consequences for both. Engestrom (1987) has postulated this kind of activity in the R&D unit of the work context as a possible example of a new system of activity which interfaces between a system and a more advanced system of activity. Thus, we suggest, the development of teams of students, teachers and researchers in studying workplace mathematical practices (such as those in our case studies) may serve as an R&D activity for educational systems. By developing accounts of inquiries, we suggest that such R&D activity may provide new insights into pedagogy and curricula (for examples of the implications of this work for vocational curricula see Wake and Williams, 2001, 2003). Already the examples of workplace mathematics reported here have informed curricula by providing examples of situated mathematics for students to study, but more importantly implications from the difficulties posed for students by the research has informed curriculum and assessment policy.

Finally, we suggest that such inquiry activity might constitute a microcosm of a new more advanced pre-vocational educational activity, i.e. as a College programme. We are currently investigating this possibility.

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