

Mathematics teacher education advanced methods: an example in dynamic geometry

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Abstract We present a research work about an innovative national teacher training program in France: the Pairform@nce program, designed to sustain ICT integration. We study here training for secondary school teachers, whose objective is to foster the development of an inquiry-based approach in the teaching of mathematics, using investigative potentialities of dynamic geometry environments. We adopt the theoretical background of the *documentational approach to didactics*. We focus on the interactions between teachers and *resources*: teachers' professional knowledge influences these interactions, which at the same time yield knowledge evolutions, a twofold process that we conceptualise as a *documentational genesis*. We followed in particular the work of a team of trainees; drawing on the data collected, we analyse their professional development, related with the training. We observe intertwined evolutions and stabilities, consistent with ongoing geneses.

Keywords Community of practice · Documentational approach · Dynamic geometry environment (DGE) · Inquiry-based learning and teaching · Mathematics teacher education · Training path

1 Introduction

Dynamic geometry environments (DGEs) have been developed in the mid of the 1980s, in the thread of the powerful

idea of “direct manipulation” (Laborde & Laborde 2008). Their interest for the learning of geometry appeared right away, as they offered new possibilities for visualisation and for checking properties by dragging geometrical objects. This interest has been recognised by many educational systems, which have integrated DGE into their curricula (Kortenkamp et al. 2010). Nevertheless, 25 years afterwards—a quarter of a century—the use of DGE in mathematics classrooms is not so spread; this use appears quite complex to a number of teachers, and the “ordinary” teacher training program seems helpless to change this situation.

We want here to address this issue by putting forward an example of mathematics teacher education advanced method, aiming to foster the use of technology—in this case DGE in mathematics—in classrooms.

In the next section, we present our theoretical perspective, situating it in the dynamics of the field of research: “technology in mathematics teacher education”. In Sect. 3, we describe Pairform@nce, an innovative teacher training program, focusing on the case of a DGE training path. In Sect. 4, we analyse the effects of this training path on teachers' knowledge and practices. Finally, we draw, in the conclusion, some perspectives and raise some new questions, about both the training program at stake, and the theoretical framework supporting our research.

2 Teacher resources and teacher education

The reflection on technology¹ in mathematics teacher education is active since 20 years; we enlighten, in this

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¹ We use in this article the word “technology” to point out both hardware and software likely to be integrated in mathematics teaching. We also use the acronym ICT (Information and Communication Technology) with the same meaning.

section, its dynamics, and propose our theoretical approach as a means for understanding deep evolutions and analysing new phenomena in this field.

2.1 Technology and mathematics teacher education

We are today far from the naïve idea that “technology makes mathematics teaching easier”. A number of studies have put in evidence the *complexity* of integrating technology in mathematics teaching. For example, Hennessy et al. (2005, p. 162) note that “In addition to the new interpersonal and pedagogic skills which teachers require to use ICT in their classrooms, other contextual factors which can act as barriers to using ICT include lack of confidence, experience, motivation, and training; access to resources and timetabled use of dedicated ICT class-rooms; unreliability of equipment; classroom practices which clash with the culture of student exploration, collaboration, debate, and interactivity within which much technology-based activity is said to be situated”.

In the case of a DGE, Laborde (2001, p. 92) pointed out that, when it was used, “the software was mainly used as an amplifier for visualizing properties, but not really the source of the task given to the students, nor as a tool for solving the task”. Going further would lead to exploit the “powerful idea of direct manipulation” (Sect. 1): integrating DGE for conjecturing and supporting a search for evidence would require, for the teacher, to rethink her way of teaching.

Renewing teaching, including DGE, supposes to have access to relevant *innovative situations of use*, what is certainly not a sufficient condition: “There was a tendency to focus on *teachers’ development* and an implicit assumption that the transfer of *innovative situations of use*, possibly supported by outcomes of research, would provide the teacher with sufficient material for an easy integration. Aware of the complexity of teaching and learning situations with ICT, researchers are now more cautious” (Lagrange et al. 2003, p. 259).

There is a gap between *having access* to resources and *appropriating* them for usage in the classroom. The European project Intergeo (Kortenkamp et al. 2010, Trgalova et al. 2010) is tackling these issues of designing, sharing and appropriating resources devoted to the use of DGE in the classroom.

The question of resource appropriation meets necessarily *teacher education*, appearing indeed as a *key issue* (Artigue 1998) for the integration of computer technologies. Regarding in-service teacher education on ICT, Trouche & Guin (2005) underlined that usual training strategies in France are essentially based on the transmission of “expert resources”. Mostly organised in a short period (about 3 days), isolated from school practice, they do not allow a *continuous* support to be provided during the

necessary adaptation of resources to each teacher’s usage context. To overcome this inadequacy, innovative programs had been developed, from the 1990s, in the US (Allen et al. 1996), relying on teachers’ *networks*, designing, with the help of experts, situations of use for DGE; this perspective of “teachers empowering teachers” appeared powerful, for both integrating DGE and promoting new inquiry-based teaching practices. Trouche & Guin (2005) have taken up this idea, conceptualising *collaborative work on resources* as a way of teacher education on technology in developing an innovative teacher training program named SFoDEM.² More generally, a recent survey (Kraimer & Wood 2008) enlightened the importance of *teams, networks and communities* for teacher education. Joubert et al. (2010) drew the same lesson, from analysing several models for teachers *continuing professional development*, and added to this picture the importance, for a teacher, to experiment in her own class, and to have time away from school for thinking and discussing.

Conceptualising resources as *a theme* for teacher education, conceptualising collaborative work on resources as *a way* of teacher education, led us to rethink our theoretical background on teachers’ resources, practices and knowledge.

2.2 From the instrumental to the documental approach

The awareness of the complexity of ICT integration in mathematics teaching went, in the 1990s, with the emergence of a new theoretical perspective situating the artefacts as *active partners* of the learning processes. The understanding of the crucial role of *mediation* for human development, of course, does not constitute in itself a novelty: Vygotski (1978) already enlightened it. Nevertheless, Vérillon and Rabardel (1995) developed a seminal approach by distinguishing *artefacts*, as products of passed human activity, available for further activity, and *instruments*, as individual constructs, mixed entities made of artefacts and *schemes*. A scheme is defined by Vergnaud (1998) as an *invariant organisation of finalised activity*, encapsulating knowledge, structured by *operational invariants*, both result and guides of activity. The development of an instrument is described as an *instrumental genesis*. Vérillon and Rabardel proposed to distinguish, within this genesis, two processes, *instrumentation* (the artefact shaping user’s activity) and *instrumentalisation* (the user shaping the artefact), the interaction between these two processes fostering the instrumental geneses.

² SFoDEM: Suivi de Formation à Distance pour les Enseignants de Mathématiques (“Continuous Distance Support for Mathematics Teachers Training”).

This new perspective on instruments has been integrated in the field of mathematics education (Guin & Trouche 1999), giving birth to the *instrumental approach*, which, articulated with other theoretical approaches in the field (Artigue 2002), gave new means for understanding learning processes. For example, in DGE environment, Restrepo (2007) identified various schemes of dragging geometrical objects, some operational invariants involved (for example: “if a conjecture is verified when dragging a diagram, therefore it is always true”) and analysed their consequences in terms of students’ knowledge. The dynamics of this approach leads to reconsider the teacher’s responsibility for monitoring students’ instrumental geneses: the notion of *instrumental orchestration* (Guin & Trouche 2002, Trouche & Drijvers 2010), defined as a didactical management, by the teacher, of the artefacts available in a given environment, for tackling a given mathematical activity, thus appeared as a necessity for analysing teacher’s work with technology, both out-of-class (preparing her teaching) and in class (adjusting it ‘on the spot’).

This new sensibility to teacher’s work raised new questions: what does a teacher work with? With whom? When? The digitalisation of information and communication in society has strong consequences for teaching: vanishing of the frontiers between hardware, software and situations of use,³ abundance of ‘resources’ (Sect. 2.1). The notion of *resource* thus appears more relevant than ‘technology’ or ‘artefacts’ to describe ‘what a teacher deals with?’, to understand what ‘re-resources’ teacher’s activity (Adler 2000). This conducts to a more holistic view on teacher’s work, considering it both in and out-of-class, wrestling with resources for performing her teaching. We have therefore proposed (Gueudet & Trouche 2009) to substitute to the dialectic artefact/instrument a new dialectic between *resources* and *document*.⁴ We call *documentational approach* this new perspective on teachers’ work: teacher’s work consists (Fig. 1) in developing documents, along *documentational geneses*, from a set of resources, in a given institution, for a given class of situations.

The *documentational genesis* jointly develops a new resource (made up of a set of resources selected, modified, recombined) and a scheme of utilisation of this resource (for examples, see Sect. 4.4). We can represent, in a very simplified way, this process by the equation: *Document = Resources + Scheme of utilisation*. A scheme of utilisation integrates *practice* (how to use selected

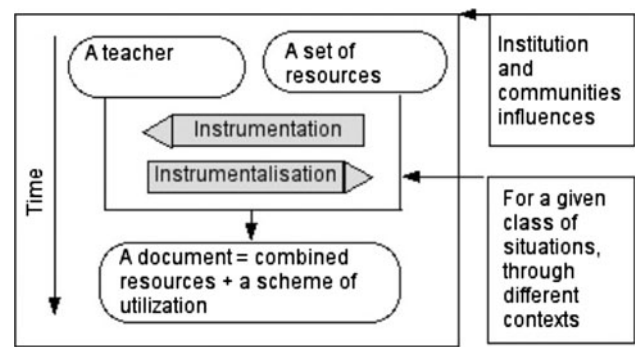


Fig. 1 Schematic representation of a documentational genesis

resources for teaching a given subject) and *knowledge* (on mathematics, on mathematics teaching, on students, on technology). A scheme combines elements of stability (‘for teaching this subject, I use to...’) and of evolutions (for adapting to new context, new resources, new curriculum...).

A *documentational genesis* develops in a field of interactions: interactions between teachers and resources (instrumentation vs. instrumentalisation), interaction between teacher and students, interactions between teacher and colleagues. This last kind of interactions is certainly to be nowadays carefully considered: digitalisation gives new occasions (e-mails lists, blogs, websites, etc.) for communicating and sharing resources with colleagues. The frame of *community of practice* (Wenger 1998) appears thus relevant to describe new forms of teachers’ engagement in common projects, resulting in shared resources.

Finally, the *documentational approach* gives new means for thinking teachers’ development through their documentation work, individual as well as collective (Gueudet & Trouche 2010). It allows reformulating the question at stake in this article: What are the effects of a collective documentation work, organised in the frame of a DGE in-service training device, on the practice and knowledge of the trainees?

2.3 Needs for rethinking research methodology

Considering teachers’ development through the magnifying glass of their *documentational geneses* has deep methodological consequences that we summarise around three principles:

- The *reflexive investigation* principle: understanding long-term processes as *documentational geneses* implies long-term follow-up, in and out-of-class. This necessary long⁵ and continuous follow-up led us to

³ For example, it is possible to have access from any connected computer to free DGE online, to websites offering examples of use, mathematical problems, suggestions of solutions, etc. The hardware seems transparent; software and situations of use appear as merged.

⁴ We have chosen the word ‘document’ to match the vocabulary of the document engineering (Pédaque 2006), a document including usages (effective or intended).

⁵ The duration of the follow-up depends on the precise research question. For a given academic year, we follow the teacher during at least 3 weeks, to capture different aspects of her mathematical work, corresponding to different objectives (introduction of a new notion, assessment, etc.).

design tools for *reflexive investigation* (Gueudet & Trouche, 2011), calling up the teacher's look on her own work: a logbook fulfilled by the teacher herself (describing her documentation work), the gathering of her resources, interviews and classrooms observations;

- The *design-in-use* principle (Rabardel & Bourmaud 2003): any resource evolves through its use by a teacher, along instrumentalisation processes (Sect. 2.2);
- The *collective* principle: a documentation work is never isolated. Understanding this documentation work requires to exploit the interrelations between peers within different collectives that a teacher is part of.

These principles frame our research methodology. They can also be considered as principles for the design of a teacher training program, organised around the documentation work of the trainees: such a training should support a reflexive stance; it should propose the design of resources by teachers, their use and their continuous revision; it would also gain of exploiting the potential of collective documentation.

In the next section, we present an innovative teacher training program, which follows these principles.

3 An innovative teacher training program supporting ICT integration

3.1 The French national program Pairform@nce and the associated research

Pairform@nce, aiming to develop in-service teachers' skills in using ICT in class, follows the principles listed above. It proposes *training paths* for all topics, from primary to secondary school levels.

These training paths are available on an online platform (<http://national.pairformance.education.fr/>). Each path is structured in seven stages: (1) introduction to the training session, (2) selection of teaching contents and organisation of teacher teams, (3) collaborative and self-training, (4) collaborative design of a lesson, (5) test of this lesson in each trainee's class, (6) shared reflection about feedback of class tests, and (7) evaluation of the training session.

Our team⁶ has followed, since its beginning, the development of Pairform@nce, providing this program with tools, methods and recommendations, and analysing documentation work of designers, trainers and trainees. We focus here on trainees,⁷ analysing their collective

involvement in a training path dedicated to DGE and its effects on practice and knowledge.

3.2 Inquiry-based approach and the use of DGE in mathematics

The exact title of the training path is (Fig. 2, our translation): "Design and implementation, *in a computer lab*, of *practical work* with a DGE". This path has been elaborated in 2007–2008 by a design group gathering teacher trainers, experimented both in training about DGE and in distant training (they have all been members of the SFoDEM, Sect. 2.1). Its objective is introduced from the first lines: "Mathematics curriculum more and more often emphasizes the experimental dimension of mathematical activity and the helpful potentialities of DGE for the effective implementation of *inquiry* practices into the classrooms [...] The objective of this path is to support you, face-to-face, and at distance, in the collaborative design of practical work in a DGE". This objective justifies that we designate this path under the label "Inquiry with dynamic geometry" (IDG).

What is inquiry in mathematics has been extensively discussed. We retain here the definition of inquiry given by Fuglestad (2007, p. 1410): "Inquiry means to ask questions, make investigations, acquire information or search for knowledge". In the context of mathematics teaching with a DGE, it can be interpreted as giving responsibility to students, towards both the mathematics content and the DGE.

We will not discuss here all the links between DGE and investigation in mathematics, but only mention some possibilities which have been considered by research works. The *direct manipulation*, crucial feature of a DGE (Sect. 1), means simultaneity between students' action and DGE feedback. For Leung (2003), this simultaneity is a promising agent for bridging the gap between experimental and theoretical mathematics, or the transition between the processes of conjecturing and formalising. Laborde (2001, p. 99) illustrated this power, for the teaching of geometry, with the example of the "black-box" situations:

In the black box situations, the students are given a diagram on the screen of the computer and they are asked questions about it. This kind of situation was used in our scenarios for introducing new transformations. A point P and its image P' through the unknown transformation were given to the students. They could move P and observe the subsequent effect on P' . Students were asked to find the properties of the unknown transformation by means of this black box. In such a task, students must ask themselves questions about the transformation

⁶ Several institutions were involved in the research, which was coordinated by the INRP (French National Institute for Pedagogical Research).

⁷ For results about designers and trainers, see Gueudet et al. (2009).

Fig. 2 Extract of the IDG path (stage 3) used in 2009–2010



This description is consistent with the definition of inquiry we retained; it clearly emphasises the responsibility of the students, who formulate questions about a transformation instead of following a presentation of its properties.

The potential of DGEs for inquiry is not limited to the field of geometry. In precalculus, DGE can also be used to introduce the notion of function (Falcade et al. 2007): the dragging tool, e.g., can support the development of a feeling of functional dependency. Geometrical situations and functions can also be associated in specific problems, where DGE can be used for the visualisation of the dynamic aspect of functional dependencies simultaneously in different representations (Hoffkamp 2010). We consider such uses in Sect. 4.

For teachers, using a DGE in an inquiry perspective implies a double difficulty: they have to master both the mathematics at stake and the DGE, and face the DGE instantaneous answers (Sect. 2.1). The objective of the IDG path is to help teachers in overcoming this double difficulty.

3.3 The IDG path: presentation and implementation

We present here a version of the IDG path used by trainers (who were not the initial designers) in 2009–2010.⁸

⁸ The initial designers of the path are: B. Clerc, J. Haraki, N. Moreau & J.-M. Ravier; the “designers-in-use” are F. Loric, H. Hili & G. Gueudet.

The trainees are lower secondary school teachers, teaching from grade 6 to 9. The DGE used in the path are GeoGebra⁹ and Tracenpoche¹⁰ (for legal reasons, all software used in the Pairform@nce paths must be under free licence). The training takes place during 13 weeks (outside holidays); it comprises three face-to-face workshops of 1 day each. Between these workshops, a continuous work is done, using the e-mail, and the training distant platform. Table 1 presents the agenda of the training. In this table and in the rest of this article, when we refer to a mathematical text, without indications of use (an exercise in a textbook, a problem elaborated by the teachers, etc.), we call it a problem text. When the problem text comes with elements of orchestration (Sect. 2.2), we call it a *scenario*: a scenario is the description of a planned teaching, which does not necessarily lead to a classroom implementation. The actual classroom implementation is called a *lesson*.

During the first workshop, the trainers present two different examples of inquiry-based lessons using a DGE (triangle area, introduction of the formula; identification of the circumcentre of a triangle with a black-box type dynamic diagram). The presentation permits to start the discussion about the use of a DGE for inquiry, and to emphasise some features of an inquiry-based lesson with a DGE: presentation of the task; responsibility left to the

⁹ <http://www.geogebra.org/cms/>.

¹⁰ <http://tracenpoche.sesamath.net/>.

Table 1 Agenda of training as planned by the IDG path

Week 1	Week 2	Weeks 3 and 4	Week 5	Weeks 6–12	Week 13
Questionnaire sent via e-mail by the trainers, filled by the trainees: equipment, experience with DGE, expectations	<i>Workshop 1</i> Presentation of the training (principles, agenda, platform). Constitution of the teams (4 teachers in 2 different schools). Presentation by the trainers and discussion of two examples of lessons. Presentation of three grids: description of scenario, observation of a lesson, final report. Proposition of two problem texts by the trainers	Design, by the teams of trainees, of a scenario for one of the problems texts proposed	<i>Workshop 2</i> Presentation by the teams of trainees and discussion of the scenarios. Choice of a lesson theme. Beginning of the lesson design	Design and experimentation of the lessons by the teams of trainees. At least one implementation in class by a member of the team, observed by an other member. The three grids (scenario, observation, report) are filled and uploaded on the platform	<i>Workshop 3</i> Presentation by the teams of trainees and discussion of the lessons

students, both for looking for a mathematical solution and for using the DGE; debate and argumentation; articulation between experimentation and proof. The three grids which trainees are required to use (scenario presentation, observation and final report) are also introduced at this moment. Between workshops 1 and 2, the teams design a scenario for a given problem text (since only two problems are proposed, in order to have comparative discussions in workshop 2, the trainees are not required to test their scenario in class). During workshop 2, the trainees choose the theme of their lesson, and begin its design. The scenario designed by the team has to be implemented in class and observed at least once; if possible, it is tried several times, with modifications resulting from the observation. During week 12, the three grids are filled and uploaded on the platform. Each team is responsible for reading the three grids of an other team, in preparation for the last discussion during workshop 3.

According to our theoretical approach (Sect. 2.2), such a work is likely to yield documental geneses, thus, in particular, evolutions of the teachers' practices. We followed in 2009–2010 two trainees, in order to investigate precisely such geneses.

4 IDG and documental geneses: a case study

In this section, we focus on a case study, about two teachers who participated in 2009–2010 to a training corresponding to the IDG path. They retained, as a starting point, an optimisation problem for grade 9, to be studied

with GeoGebra. We present first our methodology; we then analyse the potential use of GeoGebra for studying this optimisation problem with the students. We give at last a short account of the lesson designed, before studying the corresponding documental geneses.

4.1 Methodology

The Pairform@nce trainings are organised in the frame of the official regional training program. It means that the training is announced in an official training sessions list around June; the interested teachers apply for it, the participants are selected by the regional mathematics inspectors. Two weeks before the training, the trainers receive the list of selected teachers.

In 2009–2010, 16 teachers were selected for the IDG training. When receiving the list of trainees, we contacted two teachers of the same school, Lauren and Gilda, who accepted to participate in the reflective investigation follow-up (Sect. 2.3) we proposed. This choice of two trainees was more or less constrained: we had a short time to convince two teachers to participate, and we already knew Lauren, who intervenes sometimes as a trainer. Nevertheless, their background and experience with DGE is quite similar to those of the other trainees. Both of them know quite well the functionalities of a DGE; they used Geoplan during several years and started to use GeoGebra in 2008–2009, after a specific training. Nevertheless, they mostly use the DGE to illustrate their courses with a video projector. Lauren sometimes organises sessions in the computer lab for her grade 6 students, about the

functionalities of a DGE. None of them tried inquiry with a DGE, or even a use by the students of a DGE as a tool to solve a mathematics problem, before the training.

All the Pairform@nce trainings that we organised were associated with data collection, drawing on the very features of Pairform@nce. The trainees fill two questionnaires, at the beginning and at the end of the training. We also collected their discussions on the forums, and the files they uploaded and the platform.

Lauren and Gilda filled moreover, during all the training, a logbook, noting their activity related with the training: not only the workshops, the lesson in class, but also all the preparation activities. We observed and videotaped one of the lessons, made by Gilda in her grade 9 class. We met afterwards Lauren and Gilda for an interview about the training and about their lesson. We also collected all the resources they used in their preparation.

For organisational reasons, it was not possible to observe the trainees before the training (their names being known only at the last minute). Thus, we partly rest on their declarations, to identify the changes brought by the training.

In this article, we focus on geneses related to exact uses of GeoGebra. In order to identify such geneses, we start with an analysis of the possible choices for the scenario (see Sect. 4.2), in order to situate the actual choices within these possibilities. We look for regularities and for evolutions, in the trainees' choices, by confronting several kinds of data: not only the lesson designed by the team, the scenario they elaborated between workshops 1 and 2, but also files collected on their computers and corresponding to other lessons, designed before and after the training. We confront these data with the notes of the trainees in their logbooks, and their declarations during the interview; we especially retain in this interview the evolutions identified by the trainees.

Our approach shares similarities with the one developed by Laborde (2001, p. 90) to study the integration of DGE,

who proposes to “reconstruct the choices of the teachers underlying the design of the teaching sequences based on the use of technology and their evolutions; from these choices [...] infer their conceptions about the nature of mathematical activities and about learning mathematics.” The reconstruction of choices, in our case, is grounded in a large variety of sources: questionnaire, logbook, classroom videos, interview, files and discussions on the platform, etc. Moreover, the inference following this reconstruction takes the particular form of inference of schemes (Sect. 2.2).

4.2 Optimisation with GeoGebra: analysis of potential uses

The optimisation problems we consider here, in the context of the French national curriculum for grade 9, are presented as “real life” problems, where a maximum or minimum must be determined under given constraints.

The “real life” situation can be modelled in geometry; the value to maximise or minimise is a geometrical measure: area, perimeter, and angle, depending on another measure. The situation can also be modelled with a function, which is most of the time a polynomial of degree two. At grade 9 in France, the students discover the notion of function; they do not learn techniques to determine the extreme value of a degree two polynomial, but can solve a guided exercise about this topic. The exercise retained by Lauren and Gilda (Fig. 3) is of this kind.

We do not intend here to make an exhaustive analysis of all possible uses of a DGE for such a problem; we only want to point relevant dimensions of analysis, in an inquiry-based teaching (Sect. 3.2), focusing on the respective students' and teacher's roles, regarding the mathematical content and the DGE.

As mentioned above, two main possibilities of modelling appear: one with a geometric construction, the other

Fig. 3 Diagram of the ‘swimming zone’, problem chosen by Lauren and Gilda’s team



A lifeguard uses a rope and two buoys (B and C) to form the boundary of a swimming zone. In this way, he forms a rectangular zone. The length of the rope is 160 m = 16 dam. He wonders where to place the buoys B and C to obtain a swimming zone with the largest possible area. The point A is fixed.

with a function. In both cases, the modelling can be associated with the use of a DGE.

For the geometric construction, the students have to construct a rectangle ABCD, respecting the length constraint: $AB + BC + CD = 16$ (we consider here the decametre as the length unit). This can be done with coordinates, or only using perpendiculars, for the right angles, and a circle to respect the length constraint. A is fixed; as soon as another point is chosen (B for example), the rectangle and its area are determined. The area of the rectangle can be displayed on the DGE; the free point can be dragged, the students can observe the area variations, and notice that 32 seems to be the maximum value, observed for a 4 over 8 rectangle. Either with coordinates, or with a circle, the students have to notice that $AB = CD$ (property of the rectangle), and thus $BC = 16 - 2AB$. This can be associated with an algebraic modelling: AB can be labelled as x , which gives $BC = 16 - 2x$. GeoGebra, labelling all the objects with a single letter, influences such an algebraic formulation of the geometric modelling.

The modelling with a function is somehow a second step, because it requires first to use the rectangle (but this can be limited to a paper-and-pencil stage). Extending the reasoning evoked above, the students can write the area as a function a of the length $AB = x$, defined by $a(x) = x(16 - 2x)$. The graph of this function can be constructed with GeoGebra, and the coordinates of the maximum identified by a graphical reading.

The geometrical and functional modelling can also be associated (Fig. 4): once the rectangle is constructed, a point M, whose coordinates are the length AB and the rectangle area, can be created. Activating the trace of point M, and dragging the free point B, creates a set of points, belonging to the graph of the function a (which can stay

implicit or not). Then, the same graphical reading can be done to find the maximum.

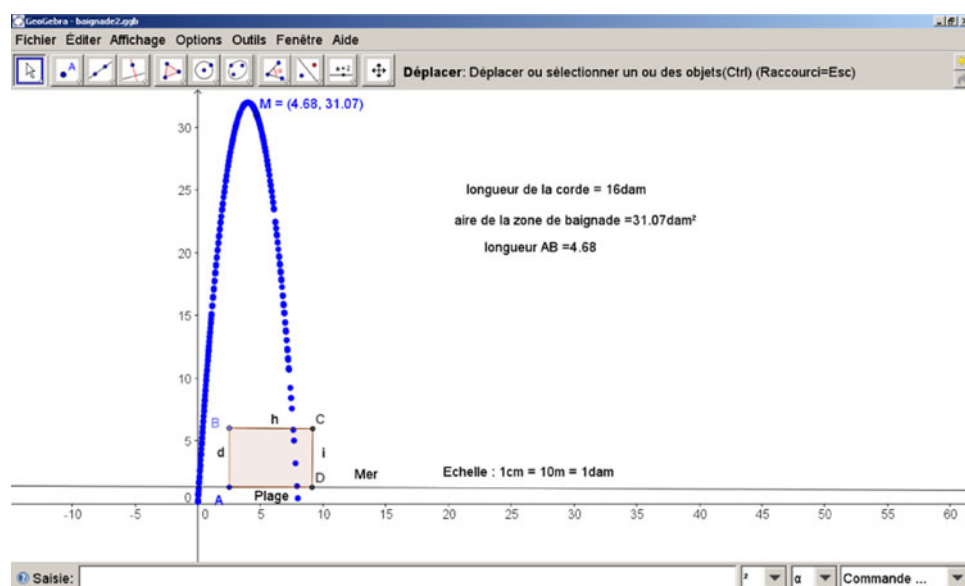
In both cases, the dynamic construction, the dragging, the observation can be completely under the responsibility of the students, or under the teacher's responsibility, with all the intermediate steps between these two extreme possibilities.

Many other choices of the teacher are important, for the orchestration (Sect. 2.2) of this situation: the presence, or not, of a final work about the associated proof; the articulation between the software and paper-and-pencil work. We can notice that, for this problem, the conjecture (the value of the maximum is 32, and it is obtained for a rectangle of size 4 over 8) involving 'simple' numbers is easily found on paper and pencil. The interest of the other procedures is more attached to the geometrical, algebraic or functional modelling. The role of the DGE can also be to induce these modellings, and to evidence the different possible modellings of the situation.

4.3 Scenario and lesson

During the training, Lauren and Gilda were in a team with two colleagues from another school. The team chose the problem text mentioned in Fig. 3, found in a textbook, during the second workshop. During this workshop, the team started to transform the text, and to think to a scenario involving GeoGebra and a spreadsheet. Unfortunately, the two other colleagues were not familiar with the use of the platform and distant communication. After the second workshop, the scenario was designed by Lauren and Gilda without any collaboration with their colleagues. Lauren and Gilda worked together by e-mail. Gilda started by writing the first draft for a sheet to be distributed to the students

Fig. 4 Use of GeoGebra to represent the 'swimming zone' situation, with a rectangle and the trace of a point



and filled by them in class; and a first draft of the scenario. Lauren suggested adaptations.

The scenario started with a homework: drawing a graph of the function f defined by $f(x) = 16x - 2x^2$, for x belonging to the interval $[0,8]$. This homework was proposed to save time for the work in class. It was followed by a work in five parts. The first part is a paper-and-pencil work (each student is given a paper with a line figuring the sea limit, and draws a line figuring the rope), the area of the corresponding rectangles is compared, leading to a first hypothesis. Then, the hypothesis is formulated again with three other strategies:

- *With GeoGebra and a geometrical modelling*: a file including a line (the limit of the sea) with point A is given, they construct point B with a perpendicular, and create the segment [AB]. This segment is labelled d ; then point C is created with a perpendicular and a circle of centre B and radius $16 - 2d$, and the diagram is completed with point D, and the creation of the rectangle. The area is displayed; students drag point B and observe the maximum value obtained for the area. All the steps of the construction are indicated on a student's sheet (Table 2);
- With GeoGebra, the geometrical modelling, and the trace of the point M whose coordinates are d ($=AB$) and the area. In this case the file (Fig. 4) is given to the students who do not construct but manipulate and observe the maximum of M y-coordinate. A link is made with the graph proposed for homework;
- *With Open Office Calc and an algebraic modelling*: the students fill a sheet with two columns: one with values between 0 and 8, the other with the corresponding values of the area.

Finally, a guided proof is proposed, the students must look for the proof and write it on a paper.

Gilda and Lauren plan two hours for the whole lesson in Gilda's class. It is set up twice, with two subgroups: Gilda's grade 9 class has 30 students, and the computer lab is a small room, with only 10 computers. The first subgroup

comprises 12 students, encountering no difficulties in mathematics. The students go through the whole activity, but, according to Gilda (interview) and to Lauren (observation notes and logbook), they have difficulties to connect the initial situation and the successive steps. The lessons of the two subgroups are separated by 2 days, during which Gilda changes the student sheet, to evidence more the fact that "when B is chosen, the whole figure is determined", essential according to her to understand the situation and connect the different parts of the lesson. With the second subgroup (some of them very low-achieving in mathematics), difficulties arise early because several students do not master GeoGebra. These students worked on GeoGebra the previous year; this year, Gilda used GeoGebra to illustrate her course, but the students are for the first time using it themselves. The students remain motivated (even though they already found the conjecture after the work on paper, which perhaps confirms that they do not connect the different representations), but Gilda has to recall the functionalities of the software and the related icons. She is finally obliged to cancel the spreadsheet part, which is proposed as homework.

4.4 Lauren and Gilda documentation work and geneses

Lauren and Gilda, as trainees in the Pairform@nce program, developed a documentation work involving many resources: a textbook (not the usual class textbook), GeoGebra, a spreadsheet, e-mail to communicate, etc. This documentation work yielded practice evolutions, general evolutions corresponding to the aims of the training path, and more specific geneses.

4.4.1 General evolutions: collective work, DGE and inquiry

Gilda and Lauren declare in their interview and their questionnaires that they have chosen the training for two main reasons: the collective work, and the inquiry with software (which corresponds to institutional expectations).

Table 2 Extract of the student's sheet, beginning of the work with GeoGebra (our translation)

Open the file 'swimming.ggb', and save it in your folder with your names. Follow the construction program:

1. Create the perpendicular through A to the water limit,
2. Create a free point B on this perpendicular, display its label,
3. Create the perpendicular to (AB) through B,
4. Create the segment [AB], rename it and display its label as d , if necessary. d corresponds to the distance AB.
5. Create the circle with centre B and radius $16 - 2d$, explain the link with point C position:
6. *Move the point B and observe what happens:*
7. Create the intersection point between this circle and the perpendicular to (AB) through B, rename it C and display its label.

...

Question 6 in italics has been added between the first and the second realisations

At the end of the training, they express their satisfaction both in the final questionnaire and in the interview. It was their first opportunity to design a lesson together; they appreciate it very much, and declare that they praise in particular the observation (with the observation grid).

Gilda designed and tested during the training her first scenario for a lesson in the computer lab; she declares in the interview that the collective work helped her to be confident enough for this first attempt. Lauren, who sometimes works with her grade 6 students in the computer lab, declared in her initial questionnaire that she was interested in evolving towards lessons where the DGE is a tool to practise mathematics. She also considers that the training contributed to the development of her practice with a DGE, meeting her expectations.

About inquiry-based teaching, they seem to have retained some of the aspects emphasised in the training (according to the features of their first scenario, and of their lesson). They carefully prepare, and set up, the presentation of the task. They organise a work of the students on the DGE; the students have to drag a dynamic diagram, and formulate a conjecture. Their lesson does not meet all the criteria of an inquiry-based teaching: in particular, the use of the DGE is tightly guided, the students do not build their own experiment. We consider that they only started an evolution towards inquiry. Beyond these general considerations, we analyse more precise geneses linked with the objectives of the path.

4.4.2 *Inquiring the evolution of measures, in a geometrical situation*

The students' responsibility with the DGE, in Gilda and Lauren's lesson, concerns the observation of the dynamic diagram. Dragging point B has several consequences; the students have to select which consequence to focus on, and to formulate their observation. This kind of observation of a numerical value variations, when dragging a point in a dynamic diagram, followed by the formulation of the observation, and leading in some cases to the formulation of a conjecture is present in several productions of Gilda and Lauren. It appears in the problem text they prepared during the training between workshops 1 and 2 (looking for a minimal distance), the lesson discussed here, and a lesson that Gilda set up afterwards about the 'angle at the centre' theorem. These different observations lead us to consider that Gilda and Lauren developed a document, for the class of situations "designing and implementing a problem about measure properties". This document includes recombined resources: a DGE, a mathematical text; we infer that it also includes operational invariants like 'a DGE can be used to help the students to observe properties of measures'; 'Numerical values evolving with the dynamic values on a

DGE permit to observe properties of measures of a geometrical figure'. The development of this document probably started before the training. At the beginning of this training, Gilda and Lauren already developed knowledge about the use of a DGE for the formulation of hypotheses by the students. This knowledge guided their choices in the design of this lesson, in an instrumentalisation process. Their documentation work during the training also contributed to a further step in this long-term process, a further evolution of this knowledge, which will certainly go on after the training.

4.4.3 *Articulating different dynamic representations to support the learning of functions*

Another important point for Gilda and Lauren in this lesson is proposing to the students to work with different representations, evidencing functional relations. In fact the use of GeoGebra seems mostly directed towards evidencing functions as processes, enhancing dynamic aspects of functions through the different representations allowed by the software. In her work about the use of DGE to support the learning of calculus concepts, Hoffkamp (2010) demonstrates the potential of such dynamic visualisations to explore the functional dependency; she also evidences obstacles, attached in particular to the connection of different representations. Lauren has observed such difficulties here. She writes, in the observation grid (Sect. 3.3) and in her logbook, that the students do not seem to connect the different representations proposed in the lesson. Discussing via e-mail with Gilda, they identify that a central issue, for the problem and for establishing connections between the different representations, is that the students notice that when B is fixed, there is no choice left for the rest of the figure. Gilda introduces question 6 (Table 2), expecting the students to answer that when B is dragged, the radius of the circle evolves. Indeed with the label d used for the distance AB, the students can miss the connection between B and the radius of the circle (naturally, when B is dragged the centre of the circle moves as well, but Gilda does not expect this answer). She also asks the students to observe how the rectangle, and then its area, changes. In the following section, she asks to observe the 'numbers placed in the parentheses' next to point M and their variation; then she asks the students to connect the trace of M and the graph of the function f proposed for homework. The trace of the point M (Fig. 4) is a dynamic representation, changing each time that B is dragged. It is connected on the one hand with the dynamic diagram of the rectangle, each position of the point M corresponding to one rectangle; on the other hand with the static graph of the function that the students drew on a paper. This last connection is linked with a discretisation of the graph, as a set of points; it

confers to the graph a dynamic aspect, as being progressively constructed as a set of points. In this case, we consider that Gilda started to develop a document, for the class of situations: “Designing and setting up the introduction of the notion of function”. We infer from the different elements described that this document entails, in particular, operational invariants like ‘students need to connect several representations to discover the concept of function’; ‘dynamic representations help to construct the idea of function as a process’; ‘a DGE permits to elaborate different dynamic representations and to connect these representations’. One of the problems proposed for designing scenarios between the first and the second workshops was quite similar to the ‘swimming zone’ situation (maximising the area of a rectangle inscribed in a right-angled triangle). The trainers proposed during the second workshop, for this problem, a scenario using a DGE with the geometric figure, and the trace of an associated point. Gilda and Lauren did not download this example on the platform, but they appreciated it during the workshop. An instrumentation process, consequence of the use of this resource during the training, probably contributed to the emergence of this document. It is certainly also a consequence of the features of the software used, an instrumentation process linked with the possibilities of representation offered by GeoGebra.

4.4.4 Avoiding the use of coordinates

Confronting Gilda’s and Lauren’s choices to the analysis of potential uses presented above evidences the facts that we already mentioned: the inquiry is limited to observation of measures variations; the different dynamic representations are articulated, to support the learning of functions. It also evidences that Gilda and Lauren seem to avoid the use of coordinates. All the points, in the first GeoGebra file, must be constructed using geometrical objects: perpendiculars, and a circle. The following excerpt of the lesson about the construction of point C confirms this hypothesis:

Gilda: Which is the geometric tool permitting to construct a point at the right distance [from point B]?

Student: The input field?

Gilda: No, not the input field. It will not be simple with the input field.

The student intervening here was at ease with the use of GeoGebra. In fact he was representing the rectangle without reading the instruction sheet, which imposes using the circle to construct C. It would have been possible to construct C with its coordinates, using the input field. But Gilda does not want to exploit this possibility. Moreover, in the two files she prepared for the students, the algebra

window is hidden. Gilda only started to use GeoGebra the year before; she was accustomed to use Geoplan before. According to GeoGebra designers: “The most remarkable feature of GeoGebra is the dual view of objects: every expression in the algebra window corresponds to an object in the geometry window and vice versa”.¹¹ Gilda has not yet started developing documents including this feature. This might be related with the class level: in grade 9, students are not yet very familiar with coordinates. However, we hypothesise that it is more a consequence of her experience with Geoplan. She has developed operational invariants attached to the use of Geoplan, which has no algebra window or input field (points can be constructed from their coordinates in Geoplan, but it requires to open a specific window). These operational invariants (outcomes of the instrumentation attached to her use of Geoplan) intervene in her documentation work, even with a new DGE, yielding a form of inertia; it appears as a consequence of the *curriculum script*, a loosely ordered model of relevant goals and actions, a key feature for technology integration, according to Ruthven (2010).

5 Conclusion

The IDG path of Pairform@nce shares with the Intergeo project the aim of supporting the integration of dynamic geometry, and the use of a platform offering resources and networking possibilities. In the IDG path, the collective documentation work is the key feature retained in an objective of professional development. The resources proposed, in particular: the scenario, observation and report grids, and the examples of lessons aim at supporting the documentation work of the teams of trainees. The implementation of the scenario elaborated by each team as lesson in class, and the revision of this scenario, following a design-in-use principle, are also essential features of the path. All these could be organised in a distance training, using a platform. The workshops, gathering the trainees and trainers in the same room, were nevertheless also essential. They permitted indeed to discuss about important features of an inquiry-based lesson using dynamic geometry, in particular: presentation of the task, appropriation of the problem by the students; design of an experiment by the students using the DGE, formulation and test of conjectures; argumentation, organisation of a debate; articulation between experimentation and proof. These elements were discussed during each one of the three workshops: about the examples of lessons presented by the trainers (workshop 1), about the scenarios built by the teams (workshop

¹¹ GeoGebra quickstart guide, <http://www.geogebra.org/source/translation/quickstart/>.

2), about the lessons designed by the teams (workshop 3). We consider that the intertwining of such discussions and of scenarios design by the teams of trainees contributed to the development of the trainees' practices.

As mentioned above regarding Gilda and Lauren, all the trainees appreciated the collective work, and used the resources provided to support it (the observation grid in particular). Regarding the use of a DGE for inquiry, the conclusions on the impact of the IDG training are more balanced. Along the documentation work of the teams of trainees, documental geneses certainly occurred. Evolutions and stabilities are interwoven in such geneses. In the case of Gilda and Lauren, we observed stabilities: they designed a lesson where the inquiry by students was curtailed to the observation of measures evolutions; their use of GeoGebra seemed to be held back by their previous use of Geoplan. However, at the same time, Gilda organised her first lesson in the computer lab (thanks to the collective work, according to her declarations); Gilda and Lauren tried for the first time to use the coordination of multiple representations for the learning of the concept of function. Moreover, they carefully prepared the presentation of the task, its appropriation by the students; we consider that this aspect is (at least partly) a consequence of the training. The trainees' uses of a DGE, before the training, were far from inquiry-based teaching. A single training, even lasting 13 weeks, is probably not enough to reach this objective. However, we identified geneses, the beginning of the development of documents, which indicate evolutions in these teachers professional knowledge about DGE and inquiry. These geneses are long-term processes, they should not stop at the end of the training. The resources mobilised during the training, or designed by the teachers themselves, remain available on their computers, interactions with these resources can go on the following year, all the more if pairs of teachers in the same school followed the training.

Following the teachers after the year of the training is naturally a perspective for our further work. The methodology presented here can be adapted to this new context, with a focus on the use of the resources coming from the training, in the frame of the methodological principles attached to the documental approach. More generally, this reflexive investigation methodology and the related theoretical approach are still being developed, in particular about the issues raised by collective dimensions. What is the 'common part' of the individual documents generated by a collective work? To what extent is it possible to speak of a common knowledge coming from a community documentation genesis? Which methodological tools can contribute to answer these questions: a common logbook, use of videos for auto-analysis or cross-auto-analysis? These issues must still be studied,

and probably require to draw on several approaches (Gueudet & Trouche 2011).

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