

# **MATHEMATICS LEARNING WITH THE USE OF THE BALANCE, A COMPUTER PROGRAMME FROM ENCICLOMEDIA**

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*This study has as its main purpose the analysis of primary school students' mathematical activity in a sequence of lessons on operations with fractions and equivalence of fractions. In these lessons, an interactive programme called The Balance designed for Enciclomedia, a national project in Mexico, was used. Using enactivism as theoretical framework the researches observed the lessons, discussed with the teachers and talked to students. From the analysis of the observations it was found that The Balance was an instrument which invited students to act mathematically in a number of ways, and helped them change their actions from trial and error to the use of systematic procedures such as arithmetic operations and comparison of fractions using different representations.*

## **INTRODUCTION**

The introduction of digital technologies in the teaching of mathematics has been considered by some as an answer to the problems students present when they learn mathematics, allowing for the development of conceptual understanding. This is not always the case, as tools often introduce different problems and their use generates new sets of questions about student's learning (Lagrange, et. al., 2001; Laborde, 2004). In this paper, we investigate and analyse the relationships between the use of a computer programme, specifically designed for a large-scale Mexican project called Enciclomedia, and students' mathematical learning.

Enciclomedia was devised with the purpose of enriching primary school teaching and learning by working with computers in the classrooms. An electronic version of the mandatory textbooks that are used in all primary schools in Mexico is being enhanced with links to computer tools designed to help teachers with the teaching of all subjects. As members of the Mathematics group in Enciclomedia, we create resources and strategies which can help teachers and students in their teaching and learning of mathematical concepts. An additional part of our work is to investigate how students learn mathematics as they use the computer tools that Enciclomedia provides them with.

To begin with, we consider some theoretical ideas about the learning of mathematics; in particular, with the use of computer interactive programmes. We also give a brief description of the interactive programme that is being used in mathematics lessons and that we want to consider in our analysis. We discuss the way in which the learning of mathematics with these programmes is being investigated. We talk about

the approach we have taken in Enciclomedia and the methods we are developing for our project, **and discuss some of the results obtained.**

## **THEORETICAL FRAMEWORK**

Our ideas about learning are based on enactivism, a theory of knowing which considers learning as effective or adequate action (Maturana and Varela, 1992). In enactivism, our minds are seen as ‘embodied’ and cognition as ‘embodied action’. These ideas of ‘embodiment’ entail two fundamental senses: on the one hand cognition is seen as ‘dependent upon the kinds of experience that come from having a body with various sensorimotor capacities’ and on the other, individual sensorimotor capacities are considered to be ‘themselves embedded in a more encompassing biological and cultural context’ (Varela, 1999, p. 12). The first meaning of embodiment locates cognition in our bodies, and prevents us from thinking about it as an abstract notion that is detached from our everyday experience. The second situates learning in a wider social and cultural context.

In enactivism learning occurs when, as a result of interactions with each other and with the world, individuals act in a way that is effective in a certain domain. When faced with a problematic situation, individuals act by making use of what they already know and are prepared to do within a certain context, according to their biological and social history. Their actions can be effective in some situations and ineffective in others.

In a particular context or location, the participants create together the conditions that allow actions to be adequate or effective. Learning outcomes cannot be predetermined or predicted, but the criteria for the adequateness of actions are, at least in part, specified by teachers and students. In mathematics classrooms, certain kinds of behaviour are promoted while others are discouraged. For example, in a particular classroom, following specific procedures and giving right answers might be an expected behaviour, while in others, exploring and looking for multiple ways of solving a problem might be common activities. In a given context, the behaviour of all the participants will be modified in a similar way, according to the conditions that make actions effective in that environment.

### **Learning mathematics with computer tools**

From an enactivist perspective, the use of computer tools is part of human living experience since ‘such technologies are entwined in the practices used by humans to represent and negotiate cultural experience’ (Davis et. al., 2000, p. 170). Tools, as material devices and/or symbolic systems, are considered to be mediators of human activity. They constitute an important part of learning, because their use shapes the processes of knowledge construction and of conceptualization (Rabardel, 1999). When tools are incorporated into students’ activities they become instruments, which are mixed entities that include both tools and the ways these are used. They are not

merely auxiliary components in the teaching of mathematics; they shape students' actions and therefore their learning (ibid, 1999).

Every tool generates a space for action, and at the same time it poses on users certain restrictions. This makes possible the emergence of new kinds of actions. The influence that tools exercise on learning is not immediate. Actions are shaped gradually, in a complex process of interaction. In the classrooms, students construct meanings through the use computer tools, in a process of social interaction and with the guide of the teacher (Mariotti, 2001).

The purpose in Enciclomedia is to develop initiatives that can help teachers to create contexts in which certain actions, related to the learning of different mathematical concepts, can be fostered through the use of computer tools. Because we believe learning occurs in the process of interactions, when students gradually modify their behaviour, we create digital resources and teaching guides that promote the joint exploration of mathematical ideas and concepts. Computer programmes in Enciclomedia are intended to broaden users' experiences with mathematics by providing spaces where the need for using mathematical procedures arises naturally. Our activities are meant to challenge and develop students' (and teachers') intuitive (and sometimes inadequate) thinking and to trigger, in them, ways of acting that we consider mathematical.

## RESEARCH QUESTIONS

We are interested in investigating those mathematical activities that we find to be effective as students work in mathematics problems using *Enciclomedia*. In particular, in this study, we address students' mathematical learning with the use of a computer interactive programme called *The Balance*. This is the second part of a previously reported investigation in which we analysed how three teachers used the programme to teach fractions (García and Trigueros, 2005). Here we address the questions: 'How does the use of *The Balance* contribute in shaping students' learning?' 'What are the mathematical actions of students during lessons where the interactive programme is being used?'

## ANTECEDENTS

The computer programme *The Balance* has been devised with the purpose of helping teachers and students in the teaching and learning of rational numbers. A general overview of the research literature in this area indicates that although there is agreement on the inherent complexities of the teaching and learning of rational numbers (Hunting and Davis, 1997; Mack, 1998; Hecht, 1998; Hunting et al., 1998; Moss and Case, 1999; Tzur, 1999; Cramer et al., 2002; Litwiller, 2002), there is no consensus about how to facilitate the understanding of the concepts related to these numbers and their operations (Behr, et al, 1997; Taube, 1997). A great number of research reports are focused on identifying the experiences children need to develop

meanings for rational numbers (Taube, 1997) and several research studies have built upon students' and teachers' difficulties with concept of fraction (Davis and Thipkong, 1991; Behr et al, 1997; Cramer et al., 2002). For example, The Rational Number Project Curriculum (Cramer et al., 1997, Cramer et al., 2002 ), includes activities to work with rational numbers that emphasize translations within modes of representation. Results showed that when these materials were used, students approached mathematical tasks that involved fractions in a conceptual way. Other studies concerned with the same project concluded that children's learning about fractions can be enhanced with the use of multiple concrete models. It was also shown that students benefit from opportunities of talking to one another and to their teachers about their ideas about fractions. Finally, authors conclude that teaching materials should focus on the development of conceptual knowledge prior to formal work with symbols and algorithms (Cramer et al., 1997).

*The Balance* was created as a space in which students and teachers could explore their ideas about rational numbers by working with activities involving equivalent fractions. It reproduces a problem in the students' mandatory textbook in which there is a representation of a mobile toy which needs to be balanced on different levels by using fractions. The computer programme provides automatic feedback that helps users in identifying which parts of the toy are balanced and which are not. In enactivist terms, the programme is meant to trigger in students mathematical effective actions such as comparing rational numbers, and finding equivalent fractions.

In a previous study, it was shown that many teachers have difficulties with the concept of equivalence of fractions, but that the use of *The Balance* can help them to reconsider their strategies and to understand the purpose of the activities in the textbook (García and Trigueros, 2005). It was also found that *The Balance* helped teachers reflect on the concept of equivalence of fractions and that, when the programme was used in the classroom, it enhanced the use of several representation registers (*ibid*, 2005). The evidence in that study, however, was insufficient to conclude that the tool helped students in their learning of the concepts related to fractions. We address students' learning with *The Balance* more directly in this study, using an enactivist framework.

## METHODOLOGY

In enactivism, research is considered to be a way of learning, and therefore researchers are seen as individuals developing their learning in a particular context. From this perspective, researchers interpret the world in a particular way, influenced by their previous experiences. In addition, in the process of doing research, researchers influence and shape the context in which they are immersed (Reid, 1996, p. 206). The interdependence of context and researchers makes the research process a flexible and dynamic one. Research does not occur in a linear fashion; rather, it is seen as a recursive process of asking questions. The work reported in this paper is part of a complex process of interaction and development of ideas. Because of the

nature of our work we consider it to be not only research but ‘action research’. The methods we use to investigate mathematics learning will change in the future according to what we observe in the classrooms and to the feedback we receive from colleagues.

In order to research the learning of mathematics with *The Balance*, we contacted a school in Mexico City where we worked with two Year 5 and two Year 6 groups of about 25 students each (aged 11-13). Two of us visited the classrooms at a time and our role was that of participant observers. We helped the teacher in giving general directions on how to use the computer programmes and we walked around the room, making comments or asking questions about students’ work. As we entered the classrooms we contributed in creating certain kinds of classroom cultures – that is, patterns of actions and interactions. When digital technologies are used, these change the way students and teachers interact with each other and therefore particular classroom cultures emerge.

In order to analyse students’ learning with *The Balance*, we carried out detailed observations of students’ actions in the classrooms. We used multiple methods for the collection of data. We used audio recording during the lessons. We recorded whole group discussions as well as interactions that occurred between two or three students and/or between students and teachers or researchers. We also used a video camera, with the purpose of recording, for each lesson the actions of particular pairs of students. So far we have videoed different students on every session.

Additionally, for each lesson, we filled in two observation sheets developed using the enactivism framework. We used the first to obtain a sense of the culture in the classroom through the following aspects of students’ effective behaviour: *Active/Passive*, *Attentive/Inattentive*, *Working with others/Working individually*, *Freedom/Constraint*, *Giving correct answers/ Formulating explanations*, *Understanding/Remembering*. These aspects had emerged in a previous study in which they had been helpful in analysing students’ mathematical actions (Lozano, 2004). The second observation sheet refers more specifically to mathematical behaviour, and includes the following headings: *Initial mathematical behaviour* (which refers to students’ actions related to mathematics during the whole group introductory discussion at the beginning of the lesson), *Mathematical actions* (those observed during the rest of the lesson, which are related to the mathematical concept(s) in the textbooks’ chapter) and *Other mathematical actions* (they do not explicitly address concepts in that chapter). Particular incidents, where mathematical behaviour is observed, were written at length under each heading. In addition, we have kept records of students’ work with paper and pencil.

Acting mathematically does not necessarily mean, to us, solving a problem in a conventional ‘correct’ manner. We collectively decide on what is mathematical by having discussions in which we talk about our notes, our transcripts from the audio tapes, and about what we observe on the videos. To support our interpretations about

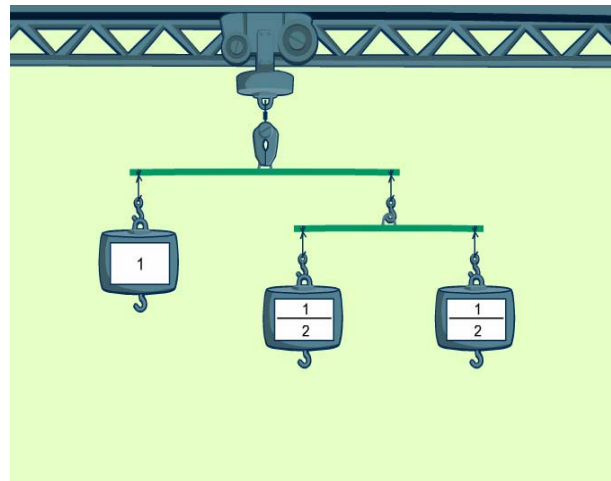
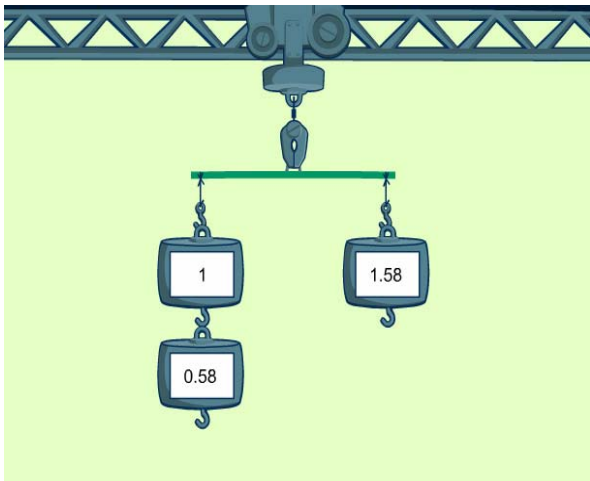
mathematical actions, we also read the literature on the teaching and learning of the concepts related to fractions, that we already mentioned as the antecedents to this study to identify the experiences and actions of students with *The Balance* that facilitate the conceptual understanding of the concepts related to fractions, and the use of different representations. We use the textbooks to identify these concepts, and to learn about the purpose of the chapters in them.

## RESULTS

We observed six sessions in which students from two different school-groups worked with *The Balance*. During these sessions we identified mathematical actions at the beginning, during and at the end of every lesson. The purpose of the sequence of the lessons, as found in teachers' lesson plans was 'to solve problems by adding or subtracting fractions using the concept of equivalence of fractions'. At the beginning of the sequence of lessons, teachers conducted an exploratory discussion which included comparing fractions and decimal numbers. We observed that most students were not able to compare decimal numbers and fractions adequately. Some would think that, for example, .4 is equivalent to .04, or that the latter is greater than the former. Other students would also say that  $\frac{1}{3}$  is greater than  $\frac{1}{2}$  because 3 is greater than 2. During following sessions, students were asked, in whole-group discussions, about addition and subtraction of fractions. Our records indicate that some students often added numerators and denominators when adding fractions and that they applied procedures algorithmically, without being able to explain what they were doing.

After these initial explorations, *The Balance* was introduced to the groups. In the beginning, teachers closely guided students' actions but later students were left to explore and to work on their own. At first, *The Balance* was used to compare weights with pairs of numbers such as the following: 2 and 1, .04 and .4, .04 and .040, .4 and  $\frac{4}{10}$ . The computer programme indicates, visually and with sounds, whether the weight of two arms of the balance are equivalent or not and we repeatedly observed students having to modify their answers due to the feedback they received.

Later on, students were asked to build more complicated models and to reproduce the problems posed in their textbooks, which often involve addition, subtraction and even multiplication or division of fractions:



As students used *The Balance*, they often encountered situations in which they did not know, immediately, how to balance the mobile. In this case, it was observed that students started out by answering with trial and error. However, they gradually refined their strategies. In small groups they started discussing the use of better strategies, and they produced more efficient methods for obtaining fractions that equilibrated the balance. We observed them using different kinds of drawings and testing some hypotheses with the programme. Some of them found systematic ways of solving problems with the balance, finding out when it was appropriate to add or to subtract fractions. Using ‘random’ numbers proved to be an inadequate manner of addressing the problem. Effective behaviour included looking for systematic ways of balancing the mobile.

In every session, whole-group discussions were organised, during which students were asked to explain their strategies for balancing the scales. Explanations differed in their degree of mathematical adequacy and in the use of representations of fractions, but some students could clearly verbalize their strategies and exhibited conceptual understanding related to comparison and operations with fractions. Teachers and students, together with *The Balance*, determined the adequacy of the answers and explanations.

When the students worked with *The Balance*, mathematical actions such as the following were registered:

- Students ask questions such as ‘Why is this heavier than this?’
- Students give explanations about how to equilibrate the balance: ‘fractions have to get smaller’ ‘this and these two have to be the same; you need one that is the same here’.
- Students perform arithmetic operations with fractions and decimal numbers.  
(28/11/05)

The data show that the use of *The Balance* invites students to act mathematically in a number of ways: comparison of numbers, verification of hypothesis based on the use

of other representations, use of operations with fractions. They work with mathematical concepts such as equivalent fractions and they operate on mathematical symbols. When students make mistakes, they often reconsider their answers after using *The Balance*:

Student1: You have to add this and this, see,  $\frac{1}{3}$  add  $\frac{1}{3}$ , that is  $\frac{2}{6}$  (They try their answers in *The Balance*)..... No, it doesn't work...

Student2: No, it is  $\frac{2}{3}$ !!  $\frac{1}{3}$  add  $\frac{1}{3}$  is  $\frac{2}{3}$ , not  $\frac{2}{6}$ . (06/12/05)

Different representations were used by students when dealing with fractions. While working with *The Balance* they often used paper and pencil and resorted to numerical and pictorial representations of fractions. Mathematical questions such as 'why is not  $\frac{1}{3}$  heavier than  $\frac{1}{2}$ ? Mathematical explanations were given by students within small groups and in whole-group discussions. Justifications were usually incomplete although we have also recorded sophisticated explanations, such as using graphic representations (drawings of pizzas) to show how a fraction with odd denominator could be divided in two.

## **CONCLUSIONS: SOME REFLECTIONS ON METHODOLOGY AND DIRECTIONS FOR FUTURE RESEARCH**

Learning mathematics is a complex process. The introduction of digital technologies does not automatically generate mathematical understanding. In fact, different computer programmes trigger in students different kinds of behaviour, as we have observed during our analysis of mathematics learning with the programmes from Enciclomedia (see Lozano *et al.*, 2006).

When investigating the learning of mathematics with *The Balance*, we found that the programme invited students to act mathematically; using concepts from the textbooks in a variety of ways. We noticed that students started looking for explanations which could help them interact with their peers and teachers as they talked about their work with *The Balance*. Over time, patterns of effective behaviour were shaped and students acted mathematically; that is, in terms of an enactivist perspective, mathematical learning occurred. Students gradually modified their actions from trial and error to finding systematic methods to solve the problems which included the use of operations with fractions and comparison of fractions using equivalence of fractions. Changes were also observed in the explanations of their procedures. Explanations included several mathematical concepts and the use of different representations of fractions.

From our observations we can say that the use of *The Balance* triggered changes in students' behaviour. Feedback provided by the tool contributed in shaping students' learning. *The Balance* became an instrument in their activity, and as such, it became part of their mathematical actions and explanations. This was only possible with teachers' support and intervention. Even when *The Balance* proved to be useful as a means of enriching children's experiences with fractions, teachers' organisation and

strategies were crucial.

The theory used and methodology followed during this part of the study proved to be effective for recording, with detail, the activities and behaviours during the class. This can be considered a contribution of this study. The use of the computer programme as a tool, facilitated students' conceptualization of fractions, and promoted the use of different representations, activities that are closely linked to the learning of fractions. Students felt confident with the presence of the researchers and talked freely with them. More research is needed to study how students use what they learned while working with *The Balance* when they face other lessons or problems where operations with fractions and equivalence of fractions are needed.

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