

JANET AINLEY, LIZ BILLS and KIRSTY WILSON

DESIGNING SPREADSHEET-BASED TASKS FOR PURPOSEFUL ALGEBRA

ABSTRACT. We describe the design of a sequence of spreadsheet-based pedagogic tasks for the introduction of algebra in the early years of secondary schooling within the Purposeful Algebraic Activity project. This design combines two relatively novel features to bring a different perspective to research in the use of spreadsheets for the learning and teaching of algebra: tasks which are purposeful for pupils and contain opportunities to appreciate the utility of algebraic ideas, and careful matching of the affordances of the spreadsheet to the algebraic ideas which are being introduced. Examples from two tasks are used to illustrate the design process. We then present data from a teaching programme using these tasks to highlight connections between aspects of the task design and the construction of meanings for variable.

KEY WORDS: algebra, spreadsheets, task design, variable

1. INTRODUCTION

In this paper, we report on aspects of the *Purposeful Algebraic Activity* project, which is a longitudinal study of pupils' construction of meaning for some algebraic ideas as they move from primary school and into the early part of secondary education. The project takes up the challenge set by Sutherland (1991) to create “*a school algebra culture in which pupils find a need for algebraic symbolism*” by incorporating a particular understanding of ‘purpose’ into the design of pedagogic tasks which exploit the algebraic potential of spreadsheets. The tasks we have designed are purposeful in the sense that they have meaningful outcomes for pupils. Further, the purposeful nature of the tasks provides opportunities for pupils to appreciate the ‘utility’ of the algebraic ideas they are using, that is, why and how these ideas are useful. In this paper, we discuss the design of these tasks, which formed a teaching programme for 11–12 year-old pupils in the first year of secondary school. We then present two themes which have emerged in the analysis of data from the teaching programme.

2. SCHOOL ALGEBRA: A MEANINGLESS ACTIVITY?

In a typical lower secondary school algebra lesson, there is little opportunity for pupils to develop any understanding of the purposes of algebra. Two approaches are frequently used to provide *contexts* for the introduction of algebraic notation. Investigations of number patterns are designed to lead into the use of a general algebraic expression, but although this link may seem to the teacher to arise naturally, for pupils the purpose may be unclear. If you can already describe a rule for finding any term in the sequence, what is the point of expressing it again algebraically? Pupils are likely to see the use of algebraic notation as something imposed by the teacher, and fail to appreciate the purposes for which the notation may be used, or the power which it can provide.

A second approach to providing meaningful contexts is the use of word problems (for example, Stacey and MacGregor, 1997). Such problems can often be solved by arithmetic approaches, including trial-and-improvement. More importantly, providing *contexts* for the use of algebra does not necessarily imply that the *purpose* is apparent. Supposedly 'real world' contexts may prove uninteresting or confusing for pupils, and their contrived nature may encourage pupils to ignore the context as a distraction (Ainley, 2000), further emphasising the purposeless nature of school algebra.

Much of the existing research into the use of spreadsheets for the learning and teaching of algebra has used tasks taken from one or other of these well established approaches, and explored how spreadsheets may offer new ways of working on them. For example Sutherland and Rojano (1993) and Dettori et al. (2001) have carried out studies of pupils solving word problems with the support of spreadsheets. Despite being set in 'real life' contexts, these problems are often far from realistic, both in the actual contexts and in the ways in which mathematics is used within them. This tends to rob them of any sense of purpose, either in the solution to the problem set, or for the use of algebra. Also, the way in which spreadsheets may be used for such tasks is relatively limited: 'problem solutions based on them are essentially numerical computations performed according to a trial-and-error strategy' (Dettori et al., 2001). We argue that such use of spreadsheets, although it may have some benefits in terms of pupils' learning, fails to exploit the full potential of the technology for supporting the construction of meaning for algebra.

The Compu-math project (for example Tabach and Friedlander, 2004) has created spreadsheet tasks based on number patterns, which lend themselves more naturally to the use of spreadsheet facilities to model functional relationships. However, the tasks themselves are frequently lacking in a sense of purpose for pupils, so that the opportunities for pupils to see algebra as useful are limited.

Within the Purposeful Algebraic Activity project the design of tasks combines two relatively novel features to bring a different perspective to research in the use of spreadsheets for the learning and teaching of algebra: the creation of algebraic tasks which are purposeful for pupils and contain opportunities to appreciate the utility of algebraic ideas, and careful matching of the affordances of the spreadsheet to the algebraic ideas which are being introduced.

3. PRINCIPLES UNDERLYING THE TASK DESIGN

Our project is concerned with aspects of the introduction of algebra in the first year of secondary school. The teaching programme which we developed was used as part of the algebra strand in the curriculum for pupils across the ability range. It was not intended to cover all of the algebra content for the year, but focussed on expressing generality, the notion of a variable, and the use of a symbolic notation to express relationships.

As indicated in the introductory remarks, we have taken the aim of providing *purposeful* contexts for the use of algebra as the starting point for our design, and specifically used the design framework of ‘purpose and utility’ proposed by Ainley and Pratt (Ainley and Pratt, 2002; Ainley et al., forthcoming).

3.1. *Purpose*

Research on mathematics in out-of-school contexts (e.g. Nunes et al., 1993) has highlighted the contrast between levels of engagement of learners in mathematical activities in and out of school. However, setting school tasks in the context of ‘real world’ situations, for example through the use of word problems, is not sufficient to provide this level of engagement. Indeed, there is considerable evidence of the problematic nature of pedagogic materials which contextualise mathematics in supposedly real-world settings, but fail to provide purpose (see for example Ainley, 2000;

Cooper and Dunne, 2000). Schliemann (1995) concludes a discussion of the problems of bringing everyday mathematics into the classroom with the statement that 'we need school situations that are as challenging and relevant for school children as getting the correct amount of change is for the street seller and his customers. And such situations may be very different from everyday situations' (p. 57). Ainley and Pratt identify the *purposeful* nature of the learners' activity as a feature which may provide this challenge and relevance. They define a purposeful task as one which has a meaningful outcome for the learner, in terms of an actual or virtual product, the solution of an engaging problem, or an argument or justification for a point of view (Ainley and Pratt, 2002; Ainley et al., forthcoming). This feature of purpose for the learner, *within the classroom environment*, is a key principle informing our pedagogic task design.

Thus, we use the term *purpose* in a very specific way. The purpose of a task, as perceived by the learner, may be quite distinct from any objectives identified by the teacher, and does not depend on any apparent connection to a 'real world' context. This may, of course, be true in a trivial sense: learners may construct the purpose of any task in ways other than those intended by the teacher. However we consider *purpose* is a distinct overarching element that needs to be considered separately in pedagogic task design.

The purpose of a task is not the 'target knowledge' within a didactical situation in Brousseau's (1997) sense. Indeed it may have no apparent connection with the target knowledge. However, the purpose creates the necessity for the learner to use the target knowledge in order to complete the task, whether this involves using existing knowledge in a particular way, or constructing new meanings through working on the task. Movement towards satisfactory completion of the task provides feedback about the learner's progress, rather than this being judged solely by the teacher (Ainley et al., forthcoming). Harel (1998) proposes the 'necessity principle', which addresses learners' recognition of need in a different way. In Harel's terms an 'intellectual need' for a mathematical concept should be created before embarking on the teaching of the concept. However, intellectual need and purpose clearly differ, since intellectual need is related specifically to a mathematical concept, while the purpose of a task is not explicitly mathematical, but relates to the outcome. The 'necessity' principle perhaps relates more closely to the second concept for Ainley & Pratt's framework: utility.

3.2. *Utility*

Understanding the *utility* of a mathematical idea is defined as knowing how, when and why that idea is useful. Whilst engaged in a purposeful task, learners may learn to use a particular mathematical idea in ways that allow them to appreciate its utility by applying it in that purposeful context. This parallels closely the way in which mathematical ideas are learnt in out-of-school settings. In contrast, within school mathematics ideas are frequently learnt in contexts where they are divorced from aspects of utility, which may lead to significantly impoverished learning. By creating the need to use mathematical ideas to produce meaningful outcomes, purposeful tasks provide opportunities for learners to construct meanings for the utility of those ideas. Utility thus has some similarity to Harel's 'intellectual need'. However, Harel sees intellectual need as providing the motivation for learning a concept, whereas utility, why and how the concept is useful, is seen as an intrinsic, but frequently unacknowledged, facet of the concept itself.

3.3. *Applying the Purpose and Utility Framework to Algebraic Spreadsheet Tasks*

While purpose and utility provide the underlying principles for the design of our tasks, the detail of the content was guided by three key ideas which relate more specifically to the learning of algebra: a balance between types of algebraic activity, the continuum between arithmetic and algebra, and the algebraic potential of the spreadsheet. We comment briefly on each of these before giving two examples of tasks from our teaching programme.

3.4. *Types of Algebraic Activity*

The algebra curriculum in the early stages of secondary schooling is often characterised by an imbalance between three kinds of activities described by Kieran (1996) as

- *generational activities*: generating expressions and equations, expressing general relationships arising from a variety of sources;
- *transformational activities*: rule-based activities such as simplifying expressions and solving equations; and
- *global, meta-level activities*: such as problem solving, justifying and finding structure.

In the early stages of the secondary school curriculum, attention tends to focus on *transformational activities*, often in settings devoid of context, which aim to develop pupils' skills in simplifying expressions and solving equations. Where *generational activities* are provided, this is often through number pattern tasks where the utility of generalising in a formal notation may not be apparent. The kinds of problems which are set can often be solved by non-algebraic means, and so the opportunities for *global, meta-level activity* are limited. Within our task design we have attempted to redress the imbalance which we see within the existing curriculum by designing tasks in which generational activity is emphasised, and opportunities for meta-level activity are created.

3.5. *The Continuum from Arithmetic to Algebra*

Although one focus of the extensive body of research into the learning and teaching of algebra has been to identify a boundary between algebra and arithmetic by clearly defining what algebra is, (for example, Filloy and Rojano, 1989; Herscovics and Linchevski, 1994) this view of a separation between arithmetic and algebra which many pupils fail to negotiate successfully is not unchallenged. With other researchers we argue that the relationship between arithmetic and algebra may more usefully be seen as a continuum (for example Carraher et al., 2001) and that new technologies may play an important role in supporting the development from arithmetic to algebraic thinking (Sutherland, 1993). In our task design we aim explicitly to exploit pupils' fluency and familiarity with arithmetic structures to support the expression of generality through algebraic notation.

3.6. *The Spreadsheet as an Algebraic Environment*

Since spreadsheets are designed as commercial tools, rather than pedagogic tools for the teaching of algebra, there are significant differences between what *can* be done (and what it is natural to do) on a spreadsheet, and what might be done in traditional, pencil-and-paper algebra. Sutherland (1993) suggests that spreadsheet notation provides a 'language' which can mediate between pupils' natural language and formal algebraic notation. Radford's semiotic approach (Radford, 2000, 2002, 2003) offers us a powerful lens through which to examine such 'mediation'. Radford sees learning

(the construction of meaning) as a social process accomplished through the use of a variety of semiotic systems, including words, symbols, gestures, artefacts. In the learning and teaching of algebra, such semiotic means are used to *objectify* or make apparent particular (generalised) ways of seeing. Radford (2003) calls the signs and artifacts *semiotic means of objectification*.

Radford's extensive study of students' use of semiotic means of objectification in the early stages of the learning of algebra provides valuable insights into how meanings may be constructed which connect students' attempts to articulate generalised relationships in natural language, and their use of written notation. Within this model, we see the use of the spreadsheet adding an additional semiotic system which brings both new possibilities and new constraints.

The constraint presented by the spreadsheet is that, in order to access its power, a formal, algebra-like symbol system must be used. We have discussed elsewhere (Ainley et al., 2003) the transition we observed between pupils generalising the context of a problem, and generalising the calculation required to solve it. Using the spreadsheet gives an impetus for pupils to make this transition (Wilson et al., 2004). Further the spreadsheet requires the variable to be made explicit, in contrast to expressions in natural language which may refer only implicitly to the variable in a generalised description (Ainley et al., 2003).

However, in using the spreadsheet the use of notation to express relationships becomes important in its own right, since it has an immediate purpose in producing some kind of result (for example, a table of data which can be graphed) which contributes to the solution of a problem. In traditional school algebra it is often the case that generational activity is seen simply as a forerunner to transformational activity. Furthermore, these results produced by the spreadsheet provide immediate feedback for the user, in sharp contrast to the use of algebraic notation to express relationships in more traditional pedagogic approaches, where the only feedback accessible to the pupil may be the teacher's approval.

The algebra-like notation of the spreadsheet environment contains two ambiguities which we see as potentially powerful in supporting the construction of meaning for variables. The cell reference is used ambiguously to name both the physical location of a cell in a column and row, and the information that the cell may contain. The spreadsheet thus offers a strong visual image of the cell as a *number container* whose contents can be changed. This image seems likely to support the

development of the idea of variable as a “*placeholder for general number*” (Ursini and Trigueros, 2001), implying that pupils are able to:

“interpret a symbol as representing a general, indeterminate entity that can assume any value, and symbolise general statements, rules or methods” (p. 336).

The image offered by the spreadsheet is ambiguous in another powerful way, which may support the development of a different meaning for variable. When a formula is entered in a column, it can be ‘filled down’ to operate not just on a single cell, but on a range of cells in a column. The cell reference within a formula thus may be seen as both specific (referring to this cell) and general (referring to the whole column). This is likely to support the idea of variable as “*a range of numbers in functional relationships*”. Ursini and Trigueros (2001) associate this with (amongst others) the abilities to:

“determine the values of the dependent variable given the value of the independent one, and symbolise a functional relationship based on the analysis of the data of a problem” (pp. 336–337).

Although spreadsheets are a natural environment for *generational activities*, they do not so easily support opportunities for *transformational activities*. We have therefore incorporated the need to move away from the spreadsheet, and to use formal algebraic notation which requires *transformational activity*, in the design of the later tasks in our teaching sequence (see Ainley et al., 2005; Bills et al., in preparation).

4. TWO TASKS FROM THE TEACHING PROGRAMME

Limitations of space make it inappropriate to describe the whole of the teaching programme we developed, but we focus on two of the six tasks which illustrate a range of ways in which ideas underpinning the design have been applied. The tasks were used as three pairs; while each task presents a separate problem solving context, the tasks within each pair have similarities in both the algebraic ideas used and the technical demands of spreadsheet use. The tasks we describe here are the second and fourth in the teaching programme. All of the tasks were designed to be introduced through teacher-led discussion. Pupils

then worked in pairs, but teachers brought the whole class back together to share their ideas as they felt appropriate at points throughout the lessons.

4.1. *Hundred Square*

Hundred Square, which is the second task in the sequence, involves exploring patterns with a hundred square (created on the spreadsheet) by taking 3×3 cross-shapes from within the square, and comparing the sums of values on the two arms of the cross (see example in Figure 1).

After some discussion to recall the structure of the square, the task begins with pupils making their own hundred square on the spreadsheet. They would try some examples of ‘crosses’ as a whole class activity, calculating the totals mentally or with pencil and paper. Then, to make the exploration easier, they are asked to set up a ‘testing cross’ on their spreadsheet (shown in Figure 1 as a shaded cross shape below the hundred square). To make the testing cross, they must use formulae in the four ‘arms’ of the cross, so that all the numbers in the cross appear when the middle number is entered. They also use formulae to calculate the column and row totals. This means that pupils can try out a large number of examples quickly.

Pupils are asked to discuss and explain the patterns that they find in their results. They are then told about a primary school teacher

	A	B	C	D	E	F	G	H	I	J
1	1	2	3	4	5	6	7	8	9	10
2	11	12	13	14	15	16	17	18	19	20
3	21	22	23	24	25	26	27	28	29	30
4	31	32	33	34	35	36	37	38	39	40
5	41	42	43	44	45	46	47	48	49	50
6	51	52	53	54	55	56	57	58	59	60
7	61	62	63	64	65	66	67	68	69	70
8	71	72	73	74	75	76	77	78	79	80
9	81	82	83	84	85	86	87	88	89	90
10	91	92	93	94	95	96	97	98	99	100
11										
12										
13		6								
14	15	16	17							
15		26								
16										
17	Column	48								
18	Row	48								

Figure 1. The Hundred Square with a ‘testing cross’.

Tariff	Monthly	Calls		
Tariff A	£12.95	20p per minute	November	15 minutes
Tariff B	£14.50	15p per minute	December	48 minutes
			January	80 minutes
			February	44 minutes
			March	113 minutes
			April	63 minutes

Figure 2. The starting information for *Mobile Phones*.

who has been using this activity to explore number patterns for several years, and is rather bored with the cross shape. The final challenge is to design some other shapes that the teacher might use with her pupils which also show interesting patterns. Pupils produce their own designs as ‘testing’ shapes, that is shapes created on the spreadsheet away from the hundred square, using formulae to generate all the numbers within the shape when a single starting number is entered.

This task builds on a representation of number patterns which is familiar to the pupils, and then takes an approach used throughout the teaching programme: after working numerically on specific examples pupils are encouraged to ‘teach the spreadsheet’ to do some of the work for them. Making the ‘testing cross’ requires pupils to generalise their knowledge of the structure of cross within the hundred square. Expressing these relationships in the formal notation of the spreadsheet supports their understanding of why the totals for the horizontal and vertical arms of the cross are equal, and the relationship of this total to the middle number of the cross. The challenge of designing a new ‘interesting’ shape provides a purpose for this generalisation, and for the use of notation to express it, as pupils utilise the same ideas of structure in the design of their own shapes, both to create the shapes themselves on the spreadsheet, and to justify and explain the numbers patterns that are produced. This offers the opportunity to appreciate the utility of algebraic notation for showing structure.

4.2. *Mobile Phones*

In *Mobile Phones*, pupils are presented with information about two different tariffs offered by a mobile phone company, together with the monthly calltime of one user for a six month period. Each tariff includes a monthly line rental and a charge per minute for the calls made. Pupils are asked to set up a spreadsheet so that they can

investigate which tariff offers the best value. Since the answer is not clear-cut, discussion about different kinds of users is generated.

As in *Hundred Square* pupils need to ‘teach the spreadsheet’ a formula to calculate the total cost for each month, based on their experience of working with specific numerical examples. However, in this task these formulae will be ‘filled down’, exploiting the use of columns and formulae to generate data from a functional relationship. The data which is produced can be graphed to support the comparison of the two tariffs.

In the last part of the task, pupils are asked to investigate three more possible tariffs, and write a brief guide for customers to say which tariffs would be most suitable for different types of users. This provides a purposeful context for developing an understanding of variable in a setting which draws on the immediate feedback offered by spreadsheets, since it involves the interpretation of a table and of a graph. The opportunity to appreciate the utility of using generalised notation to generate such data emerges from this activity.

5. THE RESEARCH SETTING

The teaching programme, consisting of a sequence of six tasks, was developed in collaboration with a group of primary and secondary teachers. The tasks were developed for use in the classroom, with outline worksheets for pupils, and detailed teachers’ notes. The primary teachers trialled initial versions of the tasks with their pupils (aged 10–11 years), and this experience was fed into the development of a more polished set of materials to be used by the secondary teachers, and into discussion of teaching styles. The teaching programme was used as part of the normal curriculum with five classes in the first year of two secondary schools (aged 11–12 years), covering the range of attainment. The tasks were used in three pairs, spread through the school year.

6. DATA COLLECTION

The secondary school classes worked on each task for 2–3 one-hour lessons. Unfortunately, the time available for these lessons was limited by timetabling and curriculum constraints, and many classes did not have enough time to complete the tasks as they were originally

designed. All lessons were observed by a researcher, and four sources of data were collected:

- researcher's field notes, which included observations of pupils' working;
- audio-recordings from a radio microphone worn by the teacher;
- video and screen recordings from a targeted pair of pupils in each lesson; and
- examples of pupils' written work and spreadsheet files.

The audio and video recordings were transcribed. The transcripts of the pairs of pupils were annotated with observations from the video and screen recordings, and examples of the pupils' files saved during the lessons.

All sources of data were coded using NVivo software to identify examples of different kinds of generational, transformational and meta-level activity (Kieran, 1996). The coding was cross-checked amongst the project team, and sub-themes emerged and developed during the coding process. We coded examples of pupils:

- using (i) natural language, (ii) spreadsheet language, and (iii) standard notation in ways that suggest they are using variables;
- knowing that a letter, symbol or cell reference can be used to stand for a unknown number or variable;
- knowing that rules and relationships can be expressed in a general way by using a letter, symbol or cell reference to stand for a variable, and appreciating that it is useful to do so;
- referring to the purpose or context of the task; and
- knowing that expressing rules and relationships in a general way can be useful for (i) showing structure, (ii) explaining why something happens, (iii) generating many examples, and (iv) finding the value of an unknown number.

Having coded the large body of data from the teaching programme, we were able to construct various matrices to identify, amongst other things, patterns in the responses of particular pairs to different tasks, and patterns in the responses of a range of pairs to particular tasks.

Alongside the data collection from the teaching programme a second strand of the project involved tracking the algebraic thinking of groups of pupils over a 2–3 year period using semi-structured

interviews designed to explore the development of all three types of algebraic activity, and also their understanding of the purposes and utilities of algebra.

7. DIFFERENT UTILITIES AND DIFFERENT MEANINGS FOR VARIABLE

In the remaining sections of this paper, we discuss two sub-themes within our initial data analysis. The themes we consider are:

- the development of pupils' construction of meaning for variable from arithmetic-based activity, through use of spreadsheets, and
- the ways in which pupils construct utilities for algebraic activity.

We focus upon two utilities for algebraic activity: generating many examples (so that patterns can be seen more clearly) and showing structure. We make links between these two utilities and pupils' construction of meaning for variable. We illustrate these links using data from targeted pairs working on *Hundred Square* (task 2 in the programme) and *Mobile Phones* (task 4). These tasks have a number of superficial similarities, in that they require pupils to generate formulae and use these to solve problems. However, they are chosen here on two grounds: because they illustrate clearly how the different tasks offer opportunities to construct two different utilities for formal notation; and because the particular extracts of data from targeted pairs working on these tasks offer some insight into their construction of meaning for variable.

The data from the targeted pairs is very rich. However, it was often difficult to make interpretations about pupils' construction of meaning for variable in the course of their activity, since their speech is fragmented and inter-related with gestures both with their hands and with the mouse. The extracts that we draw upon here do offer some insight, often because the pupils were verbalising their thoughts in an attempt to explain something to their partner or because they were 'thinking aloud' in their attempts to make a spreadsheet formula work. The extracts were selected using searches for the codes listed above, and hence are examples of targeted pairs who did construct meaning for variable and utility for notation. Drawing upon these extracts, we discuss how different tasks offer different opportunities

for constructing meaning for variable and different opportunities for constructing utilities.

8. ACTIVITY IN THE *HUNDRED SQUARE* TASK

In Ann's class, Elizabeth and Shannon worked confidently on this task, producing appropriate formulae to make their 'testing cross' and exploring a range of examples. They noticed that the row and column totals were equal, and that they were three times the middle number of the cross. They explained their findings in general terms by describing how they could balance the bottom and top numbers by subtracting or adding ten.

Elizabeth: We found out that the formula was seventy-six times three equals the column and the row number (looks at Researcher)(.). Well not seventy-six, the middle number times three equals the column and the row number ...And that is because if you take ten from the, one of the column numbers (points to the bottom cell) and put it on the other column number (points to the top cell) then they both equal seventy six, so that's seventy six plus seventy six plus seventy six (pointing to each cell as she says 'seventy-six'). And if you take one from this one (pointing to right hand cell) and add it to the other side (pointing to left hand cell) that equals seventy six plus seventy six plus seventy six (pointing to each cell as she says 'seventy six') (*Pair transcript*).

Elizabeth's explanation suggests that her attention is focussed on a way of seeing the structure of the cross that makes apparent the reason for the column total being three times the middle number. Although she and Shannon had used spreadsheet formulae to make their own testing cross which subtracted and added 10 to the (variable) middle number (i.e. $=B14-10$ and $=B14+10$ for the cross shown in Figure 1), the calculation Elizabeth describes here is a different one. Taking 10 from the number at the bottom of the column and 'putting' it on the number at the top is a description of actions she imagines making and, despite her use of a numerical example, it seems that these actions are not tied to the particular case. In fact she repeated the same sequence of explanation and gestures with a further example immediately afterwards. Although Elizabeth's image is a powerful one for explaining one aspect of the structure of the problem, it may not be one which translates easily into further work with the spreadsheet to explore the design of other shapes.

In Graham's class, after the initial exploration, pupils were invited to offer explanations of why the row and column totals are the same, for example:

Pupil: Basically they cancel each other out because

Graham: What cancels each other out?

Pupil: Well the top, the number above would be minus ten and the number below would be plus ten so they'd cancel each other out and then that one would be minus one and that would be plus one (pointing to the left and right of the middle number) (*Teacher transcript*).

The combination of language and gesture here suggests that the pupil is describing the structure of a general pattern, although the variable (the middle number) is only referred to implicitly as the object to which 'plus' and 'minus' apply.

In the design of this task the exploration of number patterns is not the main 'target knowledge'. Such an exploration could be carried out productively without the use of the spreadsheet. The overarching intention in the task design is to emphasise the utility of generalised notation to show structure. Although creating the 'testing cross' focuses attention on expressing the structure of the hundred square in a formal way, it is the final challenge of designing a new shape which shows interesting patterns which is designed to encourage the construction of meaning for this utility.

In Judith's class, pupils (with a lower range of attainment than those in Ann and Graham's classes) spent the first lesson creating their hundred square, and exploring examples of cross-shapes. Most managed to find some patterns in their results when adding the numbers on the horizontal and vertical arms of the cross, noticing that each gave the same total, and some recognised that this total was three times the number at the centre of the cross. Judith began the second lesson with a class discussion, encouraging pupils to share their findings and to try to explain the patterns they had seen. Most pupils found it difficult to articulate reasons for these patterns. Louise was the only pupil prepared to offer a partial explanation.

Louise said that $16 + 10 = 26$ and $16 - 10 = 6$, explaining why the column total was three times the middle number. (*Field notes*)

With her partner Harriet, Louise was then comfortable with writing formulae to express this relationship to make a 'testing cross', using $= B14 - 10$ and $= B14 + 10$ as the formulae above and below the

middle cell (B14). Later they went on to make a new shape, a cross covering five rows and columns. They confidently used the cell reference of the centre cell (now M13) as the starting point for their formulae, but had difficulty in deciding how this related to the cells at the far ends of their cross.

Harriet: M13 minus a hundred

Louise: No ten

Harriet: No minus, minus a hundred

Louise: Minus, no minus two and then minus ten (*Pair transcript*).

They continued in this way for some time before producing the cross shown in Figure 3. Their formulae were correct on the horizontal arm, but the numbers which would appear in the vertical arm of the cross (4, 12, 14, 16, 24) clearly could not appear in those positions in the Hundred Square. At this stage in their work it seems that the girls were focussing on the task of producing interesting number patterns in their shape, but had lost sight of the connection to the hundred square. Harriet suggested many other possibilities, including changing the formula on the horizontal arm to plus five 'to make it harder', but Louise dominated the discussion. She explained to Harriet with words and gestures that the horizontal arm was correct, on the basis of the symmetry of the formulae.

Louise: That one is plus two (points to O13), that one is plus one (points to N13), that's minus two (points to K13).

We see this as a clear indication that she has appreciated the utility of the spreadsheet notation for showing structure. Their cross does, indeed, 'work' in so far as the sums for the two arms are equal, and these are five times the central number.

Louise and Harriet discussed their work in terms of operations on the unknown middle number in the cross, and they seemed to be

	K	L	M	N	O
11			=M13-10		
12			=M13-2		
13	=M13-2	=M13-1	14	=M13+1	=M13+2
14			=M13+2		
15			=M13+10		

Figure 3. Harriet and Louise's new cross.

comfortable with using the cell reference as a placeholder for a general number. However, in focussing on the calculations, and on ‘balancing’ these they seem to have lost touch with the original image of the hundred square. It may be that having added and subtracted 2 in the horizontal row, Louise felt that they should also use plus and minus 2 in the vertical column. Harriet’s suggestion of using one hundred may arise from a sense of the need to use 10 twice.

Other pupils in this class used similar arguments to explain the structure of their shapes, based on ideas of structure and balance but tended to talk about particular values and were less confident about expressing the structure in terms of a variable.

Across all five classes pupils created their own shapes on the hundred square and explored the number patterns that they generated. Many, like Harriet and Louise, made larger crosses; others produced L or T shapes, or diagonal crosses, and a few other more complex shapes were explored less successfully. Some of these shapes ‘worked’ better than others, in that they generated number patterns which could be easily identified.

9. ACTIVITY IN THE *MOBILE PHONES* TASK

The first stage of *Mobile Phones* is for pupils to understand how the total cost for each month is calculated by combining the rental charge with the cost of the calltime for the appropriate number of minutes. Like most of her colleagues, Judith began the lesson by working through examples with the whole class before pupils worked on ‘teaching’ their spreadsheets to perform the calculations. Harriet and Louise had set up their spreadsheet as shown in Figure 4, and realised quite quickly what they needed to do to calculate the cost for November (the first month) for tariff A.

	A	B	C
1	number of minutes	Tariff A	Tariff B
2	15		
3	48		
4	80		
5	44		
6	113		
7	63		

Figure 4. Harriet and Louise’s spreadsheet layout.

- Harriet:* No 'cause that's "monthly (points to £12.95 on the table on the worksheet), you have to pay that anyway and then ...that, times, where's the times (enters '= A2')
- Louise:* Why do you need "times"?
- Harriet:* You need times 'cause you need to that (points to 15 minutes) "times twenty" (*Pair transcript*).

After some difficulties with syntax, Harriet typed in the formula correctly in cell B2. Although she typed A2 for the calltime for November, she talked about the calculation as 'fifteen times nought point 2'. Louise heard the verbalisation, but apparently did not notice that Harriet had typed the cell reference, and she realised that 'fifteen times nought point 2' would not achieve what they wanted.

- Louise:* No, no, Harriet listen. If you do fifteen times twenty (she means 0.20) then it'll only work for fifteen, it won't work for forty-eight (the calltime for December)
- Harriet:* No, no I put A, A''2 (points to cell A2 then looks directly at Louise as if checking that she understands) times twenty
- Louise:* It will only work for A2
- Harriet:* No it "won't, it will work for A3 as well" (points to A3) (*Pair transcript*).

In this exchange both girls struggle to use language and gestures to make apparent the need that they see to give a general expression for the total cost, which would work for every month. We see their difficulty in understanding each other arising from the way in which Harriet was controlling the keyboard and mixing natural language and spreadsheet language in her talk. Throughout their discussion, Harriet seemed to be using the specific number of minutes (15) and the cell reference (A2) interchangeably, suggesting that she was thinking of this quantity as a variable. Louise also seemed to have a sense of the need for a generalised expression which would work for all the cases and knew that using 15 would not give them this. Her gestures at this point are indicative of her recognition of this need. When she said '*then it'll only work for fifteen*' she pointed to a specific place on the spreadsheet, but when she said '*it won't work for forty-eight*' she made a sharp downward movement with her finger, suggesting not just the next month, but all of the remaining months (see Figure 5) After Harriet's explanation she did not appear to immediately recognise the power of using the cell reference to produce such a generalised expression. However, she was quickly convinced by the feedback from the spreadsheet when they filled the formula down and the costs for each month were calculated.

In Ann's class, Max and Peter also struggled with the syntax involved in entering amounts of money, and despite being able to



Figure 5. Louise explains her point.

articulate a sense of the structure of the calculation, made several errors in working out the cost for November. When they began on the calculation for December (48 minutes) Peter was clearly getting more confused.

Peter: I don't get you

Max: (faces Peter and points to the worksheet throughout) That's like your monthly line rental right, so you have to do, you have to pay that every month ... And then say you used forty-eight minutes, that's twenty p per minute so you have to times them to there plus twelve pounds ninety-five

Peter: Go on then tell me (sits back in chair)

Max: ↑Do a ↑"formula [loudly] (palms face upwards then holds hands against forehead) (*Pair transcript*).

Max is able to articulate the structure of the calculation which they need to carry out. Although he describes a particular example, there are indications in both his gestures and his language ('you have to pay *that* every month', '*say* you used forty-eight minutes ... you have to times them to there') that he is trying to express a generalised pattern. Max's frustration continued for some time as they struggled with the syntax, getting answers which they recognised were incorrect. At this stage, their formulae were constructed with numbers rather than with cell references. Even though they appear to understand the general structure of the calculation, and had used cell references and 'filled down' formulae confidently in previous tasks, at this point they have become focussed on calculating the individual cases. This may have been because the initial class discussion had focussed on individual calculations.

Eventually they tried to attract the teacher's attention, but before she arrived, Peter looked across at the screen of the pupils sitting next to them.

Peter: (looking at the computer of pupils sitting next to them) Cool (.) 'ere we didn't get that answer, actually we didn't get that answer. ... What formula did you get for that (points) 'cause we couldn't get a formula (..) A2

Max: Oh yeah it's supposed to be A2 times

Peter: (looks at Max) You "stupid" [thing] you

Max: Yes I am stupid, we put twenty times fifteen (*Pair transcript*).

Max's explanation and his immediate recognition of where he had been 'stupid' suggests that his attempts to enter a formula were based on an image of the general structure of the calculation. Once he recognised the issue, Max insisted that he could quickly complete the rest of the task, saying '*I can get all these answers down in two seconds*'.

Again in this task we see pupils moving from arithmetic calculations, to generalised calculations of the dependent variable (the total monthly cost), which are often expressed in informal language. These functional relationships are then formally expressed as spreadsheet formulae which are replicated to generate sets of data, suggesting that pupils are constructing meanings for variable as a range of values.

Towards the end of the lesson, Peter and Max discussed their progress with the researcher.

Researcher: How are you getting on?

Peter: Okay

Max: We've now realised what we're doing (laughs)

Researcher: Why did you decide to write a formula?

Peter: 'Cause it's easier

Max: 'Cause it's quicker (*Pair transcript*).

Like Harriet and Louise, Peter and Max now seemed to clearly see the utility of expressing the calculation of monthly costs in a general way, so that they could generate a lot of data for each tariff quickly.

Pupils across all five classes went on to use line graphs to compare Tariffs A and B, and most were able to make some links between the way in which these costs changed as calltime increased, and the cross-over point on the graph. Comparing the costs of further tariffs proved to be more challenging, and for all but a few pupils, time ran out before they were able to complete their users' guide to all the tariffs. However, many did make some recommendations like the following in their reports, indicating that they had appreciated the value of comparing sets of data rather than individual values.

We have found out that if you spend a lot of time on your phone (1 and a half hours to 2 hours) you should go with Tariff C ... If you spend about (30 minutes to 1 and a half

hours) on your phone then you should go with Tariff B ... if you spend a little amount of time on your phone (0 to 30 minutes) you should go with Tariff A.

Tariff A is cheaper (*sic*) up to 30 minutes but when you get to 31 minutes Tariff A and B are equal, after that Tariff B is cheaper. (*Written reports*).

10. DISCUSSION

The snapshots of data presented here give an indication of the complex interactions between elements of task design and the affordances of the spreadsheet environment which shape the ways in which meanings are constructed and the semiotic means of objectification available within what may superficially appear to be similar activity on the part of pupils. From this data we examine two themes:

- the development of pupils' construction of meaning for variable, and the way in which specific affordances of the spreadsheet offer semiotic mechanisms which may be mobilised in making sense of generality (Radford et al., 2005); and
- the ways in which the purposeful nature of tasks supports pupils' construction of utilities for algebraic activity.

Both the tasks on which we have focussed involve generational activity, which takes the form of expressing calculations as spreadsheet formulae, and meta-level activity in solving meaningful problems. The contexts from which these generalised calculations arise are, however, quite different, as are the ways in which formulae are used within the tasks, both in the pupils' activity associated with them, and in the effects they produce.

In *Hundred Square*, the formulae in the testing cross, and later in the other shapes which pupils design, are constructed and entered individually. Pupils use the testing shapes by replacing the number in the cell on which the formulae are based. As each number is entered, a particular numerical example is created for that shape. Pupils need to record the outcome (i.e. the row and column totals) separately in order to explore patterns in the results. The use of the testing cross is designed to give experience of variable as placeholder through exploiting the ambiguity of the cell reference as representing both the specific contents of a cell, and any number which can be entered into the cell, building on pupils' existing familiarity with the structure of the hundred square.

In our data we observe pupils using a range of semiotic means of objectification as they construct meanings for aspects of the number patterns that they are exploring. These include gesture and verbal description, and also statements of calculations which make implicit or explicit reference to a generalised number. If the task were merely to describe and explain the patterns that they had found from their explorations, many pupils could, as we have seen, do this without reference to algebra, by means of describing the symmetry of the physical structure, or by using specific calculations as generic examples. The overall purpose of the task in creating a product for a teacher to use with another class, and the use of the spreadsheet play a key role in focussing pupils' attention both on the role of a variable within their descriptions of structure, and on the utility of notation to express relationships to the variable.

Pupils' activity in *Mobile Phones* has some similarities with *Hundred Square*, in particular generating a formula to explore many examples, based on experience of an arithmetic calculation. However, there are a number of significant differences. The formulae for calculating the total monthly cost for each tariff are based on pupils' understanding of the structure of the numerical examples which arise from a problem situation, rather than from an existing number pattern. On the spreadsheet these formulae fulfil a different function as they are 'filled down' to apply to a range of values for the calltime which are already in the spreadsheet. The value in any individual cell in the 'minutes' column will not be changed but the effect of the formula is seen applying to this range of values, and indeed to values which may later be entered as the tariffs are explored further. The task has been designed to exploit the ambiguity of the cell reference in a formula as representing a single value or any values which may be entered in that column, thus giving experience of variable as a range of numbers in a functional relationship. Harriet and Louise's discussion about how to create the formula to calculate the total monthly cost indicates that they have seen the need to express a relationship in a way which will work for a whole set of values, even though they have some confusion about how to express this. Similarly Max and Peter show a clear recognition of the power of the general formula which they have been struggling to create.

The purpose of the task, to produce a users' guide to the different tariffs, creates the need to produce sets of data which can be compared. The differences between the tariffs are not always apparent within the examples of calltime given in the initial problem, and so

pupils need to extend their data by adding more extreme values. In our data from this task it is easier to find evidence of pupils explicitly recognising the utility of formulae to quickly generate the data they need to decide on the best consumer advice.

In summary, we argue that aspects of our task design have given pupils opportunities to construct meaning for particular algebraic concepts.

- The purposeful nature of the tasks sets up possibilities to appreciate the utility of algebraic notation. In *Hundred Square* the purpose of producing more shapes offers the opportunity to appreciate the utility of spreadsheet notation to predict which shapes will ‘work’. In *Mobile Phones* the purpose of comparing tariffs across a range of calltimes offers the opportunity to appreciate the utility of spreadsheet notation to generate examples.
- The exploitation of the affordances of the spreadsheet sets up possibilities to construct meanings for variables. In *Hundred Square* the individual cell offers a metaphor for a variable as placeholder for a general number and pupils find a number of linguistic means of referring to this. In *Mobile Phones*, filling down a formula is understood as setting up a relationship between two sets of numbers.

Beyond this, we see ways in which the purpose of the tasks and utility of the spreadsheet notation are intimately linked with the spreadsheet affordances to create the opportunities for construction of meaning. The purpose of the task drives the use of the spreadsheet notation in a way which makes the utility visible.

ACKNOWLEDGMENTS

This project was funded by the Economic and Social Research Council.

REFERENCES

- Ainley, J. (2000). Constructing purposeful mathematical activity in primary classrooms. In C. Tikly and A. Wolf (Eds), *The Maths We Need Now*. London: Bedford Way Papers.

- Ainley, J. and Pratt, D. (2002). Purpose and utility in pedagogic task design. In A. Cockburn and E. Nardi (Eds), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 (pp. 17–24). UK.
- Ainley, J., Pratt, D. and Hansen, A. (forthcoming). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*.
- Ainley, J., Wilson, K. and Bills, L. (2003). Generalising the context and generalising the calculation. In N.A. Pateman, B.J. Dougherty and J. Zilliox (Eds), *Proceedings of the 27th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 (pp. 9–16). Honolulu.
- Ainley, J., Bills, L. and Wilson, K. (2005). Purposeful task design and the emergence of transparency. In H.L. Chick and J.L. Vincent (Eds) *Proceedings of the 29th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 (17–24) Australia.
- Brousseau, G. (1997). *Theory of Didactical Situations*. (Edited and translated by N. Balacheff, M. Cooper, R. Sutherland and V. Warfield) Dordrecht: Kluwer Academic Publishers.
- Carraher, D., Schliemann, A. and Brinzuela, B. (2001). Can young students operate on unknowns? In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1 (pp. 130–140). Netherlands.
- Cooper, C. and Dunne, M. (2000). *Assessing Children's Mathematical Knowledge*. Buckingham: Open University Press.
- Dettoni, G., Garuti, R. and Lemut, E. (2001). From arithmetic to algebraic thinking by using a spreadsheet. In R. Sutherland, T. Rojano, A. Bell and R. Lins (Eds), *Perspectives on School Algebra*. Dordrecht: Kluwer.
- Filloy, E. and Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics* 9(2): 19–25.
- Harel, G. (1998). Two dual assertions: The first on learning and the second on teaching (or vice versa). *American Mathematical Monthly* 105: 497–507.
- Herscovics, N. and Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics* 27: 59–78.
- Kieran, C. (1996). The changing face of school algebra. In C. Alsina, J.M. Alvares, B. Hodgson, C. Laborde and A. Pérez (Eds), *Proceedings of the Eighth International Congress on Mathematics Education: Selected Lectures* (pp. 271–290). Seville: S.A.E.M. 'Thales'.
- Nunes, T., Schliemann, A.D. and Carraher, D.W. (1993). *Street Mathematics and School Mathematics*. Cambridge: Cambridge University Press.
- Radford, L. (2000). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis. *Educational Studies in Mathematics* 42(3): 237–268.
- Radford, L. (2002). The seen, the spoken and the written. A semiotic approach to the problem of objectification of mathematical knowledge. *For the Learning of Mathematics* 22(2): 14–23.
- Radford, L. (2003). Gestures, speech and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning* 5(1): 37–70.

- Radford, L., Bardini, C. and Sabena, C. (2005). Perceptual semiosis and the microgenesis of algebraic generalizations. *Proceedings of the 4th congress of the European society for Research in Mathematics Education*. Barcelona, Spain.
- Schliemann, A. (1995). Some concerns about bringing everyday mathematics to mathematics education. In L. Meira and D. Carraher (Eds), *Proceedings of the 19th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1 (pp. 45–60). Brazil.
- Stacey, K. and MacGregor, M. (1997). Multiple referents and the shifting meanings of unknowns in students' use of algebra. In E. Pehkonen (Ed.), *Proceedings of the 21st Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 190–197). Finland.
- Sutherland, R. (1991). Some unanswered research questions on the teaching and learning of algebra. *For the Learning of Mathematics* 11(3): 40–46.
- Sutherland, R. (1993). Project AnA—the gap between arithmetic and algebra (R000232132). Final report to ESRC. Institute of Education, University of London.
- Sutherland, R. and Rojano, T. (1993). A spreadsheet approach to solving algebra problems. *Journal of Mathematical Behaviour* 12(4): 353–383.
- Tabach, M. and Friedlander, A. (2004). Levels of student responses in a spreadsheet-based environment. In M. Johnsen Høines and A.B. Fuglestad (Eds), *Proceedings of the Twenty Eighth Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 (pp. 423–430). Bergen University College.
- Ursini, S. and Trigueros, M. (2001). A model for the uses of variable in elementary algebra. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 335–342). Netherlands.
- Wilson, K., Ainley, J. and Bills, L. (2004). Spreadsheet generalising and paper and pencil generalising. In M. Johnsen Høines and A. B. Fuglestad (Eds), *Proceedings of the 28th Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 441–448). Norway.

JANET ANILEY, KIRSTY WILSON

Institute of Education
University of Warwick
Coventry, CV4 7AL
UK

E-mail: janet.ainley@warwick.ac.uk

LIZ BILLS

School of Education and Lifelong Learning
University of East Anglia
Norwich, UK