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The 'Resources' Approach to Mathematics Education

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Chapter 1

Studying teachers' documentation work:

Emergence of a theoretical approach⁷

Ghislaine Gueudet, CREAD, ESPE de Bretagne, University of Brest, France.

Abstract

The documentational approach to didactics is a young theory – less than ten years old. In this chapter I look back at the process of development of this approach. I recall the initial context, motivating the elaboration of a specific frame for studying teachers' documentation work. Several kinds of evolutions are emphasized: theoretical, methodological, but also evolutions of the questions studied in the research works referring to the documentational approach. I conclude by evidencing the main evolutions so far, and evoking some perspectives.

Keywords

Documentational approach to didactics; Reflective investigation method; Resources systems; Teachers' resources

⁷ This chapter originates from a lecture given to the Re(s)ources 2018 International Conference. Video in English, with French subtitles at http://video.ens-lyon.fr/ife/2018/2018-05-28_003_Ressources2018_Ghislaine_Gueudet_v1.fra.mp4

Introduction

This text proposes the analysis of an historical process: the development of the documental approach to didactics (DAD in what follows). I do not claim to present a scientific analysis: I have been actively involved in this process; what I describe here is my own view on it. I consider in particular that this short history (from 2007 to 2018) can be divided into four important periods, and this chapter is organised according to these successive periods. The first period corresponds to the initial proposition of the theoretical approach; section 1 presents the sources used for this theoretical construction, and this early version of the approach. In section 2 I present the second period (2008-2010), which has seen the development of methods, and the investigation of secondary school mathematics teachers' documentation work. Then the (2011-2013) period (section 3) marked the enlargement of the scope of the approach, in different directions. In section 4 I present works done since 2014, and discuss the major ongoing evolutions. Naturally these dates must not be considered as very precise borders between different periods: the work in the projects and the corresponding publications can be separated from several years; the development of DAD was a continuous process. Nevertheless this somehow artificial splitting in periods is helpful to evidence the main steps of this short history.

1.1 Origins and first theoretical propositions (2007 and before)

The development of DAD took place in a context of generalized availability of online resources for teachers and students. The first version of the theory has been presented at the French Mathematics Didactic Summer School (an event taking place every two years) in 2007 by Ghislaine Gueudet and Luc Trouche (Gueudet and Trouche 2009a, Figure 1.1).

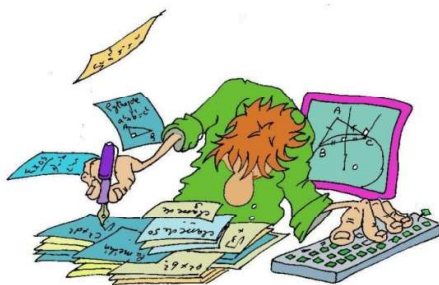


Figure 1.1. A teacher at work. An illustration by Serge Cecconi (Cecconi 2007), used in Gueudet and Trouche (2009a, p. 109)

The work in the Mathematics Didactic Summer School is organised according to themes, each theme proposing courses and associated tutorials. In the 14th Summer School, we were co-responsible for a theme entitled “Mathematical situations and teachers’ documents” and decided to give in it a course about teachers’ work in this evolving context. This decision has been driven by our previous work in related fields (section 1.1.1). We had from the beginning the aim to propose theoretical constructs, and we drew for this objective on diverse sources (section 1.1.2). We also collected some data on teachers’ work, presented in section 1.1.3. Most of the concepts now used in DAD were already present in the course, but also other propositions, which disappeared in further works. We discuss these aspects in section 1.1.4.

1.1.1 Previous works by Luc Trouche and Ghislaine Gueudet

Our previous works related with the development of DAD mainly concerned educational technologies.

Luc Trouche has been one of the early contributors of the introduction of the instrumental approach in mathematics education research. While his first works mainly concerned the use of the calculator by students (Guin et al. 2005), he also introduced the notion of instrumental orchestration (Trouche 2004), which contributed to a shift of the focus with a new attention to the use of technologies by teachers. Another dimension of Luc Trouche's work that has been strongly influential was his interest for teachers' collective work, strongly linked with the SFODEM project (Guin and Trouche 2005). The SFODEM (2000-2006) was a project of in-service teacher training, mainly at distance, and aiming to support the integration of ICT in the teachers' practices by a rich offer of resources on a platform. Studying the SFODEM's case led to raise the issue of the collective design of resources. Finally, Luc Trouche also contributed to a collective book entitled "Technological environments and digital resources for learning" ("Environnements informatisés et ressources numériques pour l'apprentissage", the book published in French has not been translated). This book initiated a coordination between several research fields: mathematics education, computer science, cognitive ergonomics, but also what is called in France "document engineering", which played a determining role as we will see below.

On my side, after several years of research (including my PhD) on the teaching and learning of linear algebra at university, I started around 2000 to work on learning processes with online exercises. This led me to use the instrumental approach (see e.g. Cazes et al. 2007) and to develop an interest for all the kinds of online resources for the teaching and learning of mathematics. The focus of my research went beyond university to include all school levels, and shifted to teachers' work, considering online exercises as resources for teachers. Following a work by Haspekian (2014), I considered with my colleague Laetitia Bueno-Ravel teachers' instrumental geneses (Bueno-Ravel and Gueudet 2007), and we tried to build connections between these instrumental geneses and the structuring features of the classroom practice framework as introduced by Kenneth Ruthven, in particular in his plenary at the CERME5 conference (Ruthven 2007).

In a context where the Internet became a major tool in teachers' work, extending these previous works by the collective construction of a relevant frame to study the consequences of the ongoing evolutions was natural. For this aim we drew on a variety of theoretical sources, presented in the next section.

1.1.2 Theoretical sources

The first theoretical source for DAD was naturally the instrumental approach (Rabardel 1995). The instrumental approach distinguishes between an artefact, product of the human activity, designed for a goal-directed human activity, and an instrument developed by a subject using the artefact. The instrument is a mixed entity, comprising the artefact and a scheme of use (Vergnaud 1998) of this artefact. A scheme of use is a stable organisation of the activity, for a given class of activity situations: a set of situations corresponding to the same aim of the activity. The scheme comprises several components: the aim of the activity; rules of action, of control and rules for taking information; operational invariants and possibilities of inferences. The operational invariants are of two kinds: theorems-in-action and concepts-in-action. A theorem-in-action is a proposition considered as true by the subject; a concept-in-action is a concept considered as relevant. The development of an instrument is called an instrumental genesis. The process has two strongly interrelated components: instrumentation (the features of the artefact shape the schemes developed by the subject) and instrumentalisation (the subject modifies the artefact according to his/her existing schemes).

Along these first concepts, which directly inspired DAD, other aspects of the instrumental approach nurtured our first theoretical constructs. I mentioned above the instrumental orchestration (Trouche 2004), leading to a focus on the teacher. Another important idea was the “double instrumental genesis” (Haspekian 2014). This concept has been introduced by Haspekian to emphasize the fact that a teacher working with a software develops two instruments. A personal instrument, when he/she learns the functionalities of the software: how a spreadsheet works, for example. And a professional instrument, developed for teaching objectives: how to use a spreadsheet to support the learning of algebra by grade 9 students, for example. Other concepts came more directly from the work of Rabardel and his team. The activity families are sets of classes of situations with the same kind of aim. The subjects develop instrument systems (Rabardel and Bourmaud 2005), structured according to the activity families. Hence we have used these families in our attempts to identify, from the beginning, the structure of the document systems; we discuss it below (§ 12.1.4). We also referred to the principle of design-in-use and design-for-use (Folcher 2005): the artefacts are designed for a given, initial use; but their design continues during their use. Researchers working within the instrumental approach have used this statement as a principle, guiding an efficient design: the users must be associated to the design from the beginning.

The second source came from the research field of document management, which was extensively represented in a book coordinated by Luc Trouche and colleagues (Baron et al. 2007). In this field the consequences of the use of digital resources was at that time a major issue. Researchers of this field introduced a distinction between resource and document. « The notion of resource is used as resource to design documents ... The document bears an intention depending of the context of use. » (Crozat 2007, p.260). This distinction is linked with the study of digital resources. Pédaque (2006, 2007) stressed that with the digital means, each reading was not only associated with re-interpreting, but even possibly led to re-writing, hence producing something different from the initial resources. For Pédaque, a document is “a contract between humans” (Pédaque 2006 p.12). The intentions, in the documents, can be interpreted as possible aims and hence as components of schemes.

Another important source was the theory of communities of practice (Wenger 1998) which inspired from the beginning the work on teacher collaboration within DAD. A community of practice in a group characterized by a shared objective, a shared commitment, and a shared repertoire. This repertoire can be considered as a set of resources shared by the members of the community. Moreover, Wenger emphasizes the dialectical link between participation (in the common practice) and reification (design of new resources in the repertoire). A community of practice always designs resources, whatever its common project is. This link between the collective practice (or activity) and the resources used and produced by this activity coincided with some of the observations made about collective work in the case of the SFODEM (Guin and Trouche 2005).

While our sources, except for Wenger, were mostly French, an international opening came from the work of Ruthven, presented as a plenary conference at the CERME5 congress in 2005 (Ruthven 2007). The frame of the structuring features of the classroom practice (SFCP), presented at this conference, comprised five components: working environment; resource system; activity format; curriculum script; and time economy. Several of these features appeared as linked with the instrumental approach: the idea of resource system, connected with instrument systems; but also the curriculum script. This concept was indeed defined by Ruthven as: “a loosely ordered model of relevant goals and actions which serves to guide [the teacher's] teaching of the topic” (Ruthven 2007, p.61). In this definition, the goal suggests a possible connection with schemes. The curriculum script could be interpreted as a set of schemes developed by the teacher. The interested reader can find in the chapter 2 of this book

a presentation of the SFCP frame, its origins, its evolutions, and its links with DAD, in particular concerning the concept of resource system.

Along these concepts, the CERME5 plenary by Ruthven also played another central role in the early stage of DAD. In this plenary indeed, Ruthven referred to the development of new conceptualizations of how teachers use curriculum material, and in particular to the work of Janine Remillard (2005). The field of research on curriculum material was not known in France at that time; it became very influential in the development of DAD. The perspective developed by Remillard was indeed very close from the instrumental approach, considering that teachers shape and are shaped by the curriculum material they use. It confirmed for us the relevance of an extension of the instrumental approach (as it was used in mathematics education research), to encompass not only educational technologies but all the kinds of resources used by the teachers.

1.1.3 The early version of DAD

In the summer school course on DAD, where the first version of the theory was presented, finally a single question was studied, corresponding to the aim of theoretical development:

“Which concepts are needed to analyse the activity and professional development of secondary school mathematics teachers?”

Along the theoretical work, drawing on the sources presented above, we also used empirical data. We met nine secondary school teachers for interviews, at their homes where their documentation work took place. We asked them about their documentation work, about its evolutions (during the last ten years) and its expected evolutions (during the next ten years). This methodology was clearly limited, and in particular did not include observations in class. Nevertheless the empirical data usefully complemented the theoretical work.

We presented in Gueudet and Trouche (2009a) a first version of DAD. It already introduced the notions of: documentation work, resources, documents (defined as a set of recombined resources and a scheme of use of these resources), documentational genesis (defined as the process of development of a document, Figure 1.2).

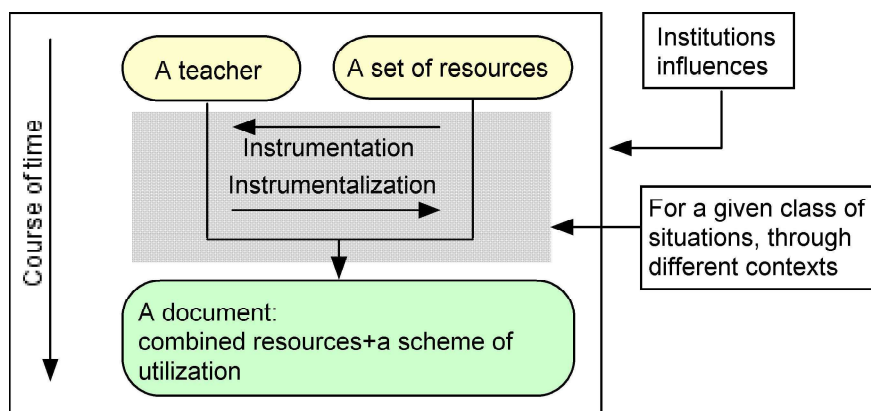


Figure 1.2. A documentational genesis

The methods we used only allowed us to formulate hypotheses on possible schemes (since we only relied on teachers’ declarations). For example, Benoîte declared that with her grade 6 class, she started every course by ten minutes of mental calculation. She used for this slides that she found online on the Sésamath⁸ website. We considered that Benoîte developed a

⁸ <http://www.sesamath.net/>

document, for the aim “practice mental calculation”. This document comprised the original resources (slides), modified by Benoîte who introduced her own calculations; a rule of action: “I start every course with ten minutes of mental calculation using a slideshow” and operational invariants. “Mental calculation” is a concept-in-action; “the grade 6 students need to practice every day mental calculation” is a theorem-in-action.

The course considered individual documentation work, but also documentation work in communities of practice, drawing for example on the case of the SFODEM mentioned above (Guin and Trouche 2005). It also introduced document systems, defined as: “the system of all documents developed by the teacher”, structured by the situation classes (sets of situations with the same aim of the activity) and activity families (sets of situation classes with the same type of aim). The resource systems were at that time defined as the “resource part” of the document systems.

We made in this course some first attempts to investigate the structure of document systems. We suggested that teachers’ work was organized according to three activity families:

1. Design and organise the teaching;
2. Participate to the school’s organisation;
3. Reflect on his/her practice.

We also formulated a hypothesis, concerning the existence of “pivotal documents”, defined as documents that:

- Have a central place in the document system;
- Contribute to articulate other documents;
- Play a privileged role on the time axis. Concentrate the memory of previous resources, and intervene for the integration of new resources.

Moreover, we hypothesized that one of these pivotal documents was central, articulating the other documents and playing a role of integration and memory of new resources. We called it “le recueil” in French, which can be translated in English as “the compendium”.

As we will see in the next sections, these first attempts to investigate the structure of the document systems were not satisfactory. They have been followed by other propositions, and the issue of the structure of document system and resource system remains unresolved.

1.2 Theoretical and methodological developments (2008-2010)

From 2008 to 2010, starting from the first version, we deepened the work on DAD (Gueudet and Trouche 2008, Gueudet and Trouche 2009b, Gueudet and Trouche 2010). Probably the most important evolutions concerned the methods, with the introduction of the reflective investigation method (§1.2.2), but the sources and the theory also evolved.

1.2.1 Evolutions of the sources

At the very beginning of 2008, we found out the work by Jill Adler (2000) and her proposed conceptualization of resources for teachers. It became, and remained, a major source for DAD. Jill Adler proposes indeed to focus on resources-in-use. According to her, “it is possible to think about resource as the verb re-source, to source again or differently” (Adler 2000, p. 207). This perspective invites to consider material, but also socio-cultural and human resources. This definition of resource was especially relevant for DAD. Indeed, instead of focusing on some digital artefacts (the instrumental approach would be enough for this) our aim was to consider all the resources intervening in the teacher’s activity. Hence the definition proposed by Adler became the definition of resource retained by DAD, with some

modifications/differences. We did not indeed consider human resources as such. While networks and collective work are central in DAD, we do not consider humans themselves as resources. The resource is for us a discussion with a colleague, in presence or by e-mail; a student's production, or even a puzzled expression on the face of a student.

Gueudet and Trouche (2010) is a collective book in French which accounts for all these sources. It starts with a translation in French of Adler's (2000) paper on resources. It also contains a chapter by Ruthven (2010) and by Remillard (2010) (translations in French of original papers). The French sources are also present, in particular the field of document management is represented by a chapter by Bachimont (2010): "Digital medium for knowledge: between materialization and interpretation". Moreover this book intended to gather a variety of works from the French community of mathematics education research, directly addressing the issue of teachers' resources (e.g. Margolinas and Wozniak 2010) or relevant for it (e.g. Chevallard and Cirade 2010; Sensevy 2010).

One direction of our theoretical work at that time was also to better situate DAD within the landscape of French theories in mathematics education (these links are summarized in Gueudet and Trouche 2008, see also chapter 4 by Artigue in this volume). The Anthropological Theory of the Didactics (ATD, Chevallard 2006) has been present from the beginning as a reference used by DAD: the work of the teacher is indeed situated within institutions, which clearly influence the available resources and their use. In Gueudet and Trouche (2008), we developed it further by using the concept of didactical moments to refine the structure of the documents systems (see § 2.3 below). The link with the theory of didactical situations (Brousseau 1998) has been explained in Gueudet and Trouche (2008) by saying that the set of all the resources available for the teacher can be considered as the *milieu* of the teacher. Moreover, DAD is linked with the Joint Action Theory in Didactics (JATD, Sensevy 2012), since it considers the students' productions and utterances as central resources for the teacher, acknowledging a joint action of the teacher and the students in class but also out-of-class. For example JATD has extensively studied the design of lessons by groups associating teachers and researchers, and has evidenced that students' productions can be essential resources in this design process (Sensevy 2012). The same processes could be studied by DAD – but the focus would be on the documents developed by teachers, producing different research results.

1.2.2 Evolution of the methods: introducing the reflective investigation

The interviews used in the early stages of development of the approach were clearly not sufficient to obtain results in terms of documental geneses. The elements of schemes are not all conscious, hence the declarations of the teachers have to be confronted with the observation of their actual activity. These statements led us to the development of a specific method, that we called *the reflective investigation method*. The data collection with this method follows four principles:

- The teacher is actively involved in the collection of resources in a reflective stance;
- The activity of the teacher is observed in class and out-of-class;
- The resources used and produced by the teacher are collected;
- The follow-up is organized over a "long" period of time, in order to observe stabilities and evolutions.

These principles can lead to different kinds of data collection. Figure 1.3 presents one example (extracted from Gueudet et al. 2012a, p. 29).

First encounter: - presentation of the methodology, its spirit and its tools	First visit: - about the resources in general; - about the chapter followed.	Second visit: - about the lesson observed.	Classroom observation	Third visit: - About the lesson observed; - Complements about the chapter and resources.
Tools: - Schedule; - Questionnaire; - Logbook.	Tools: - Interview guidelines; - SRRS	Tools: - Interview guidelines	Tools: - Observation guidelines	Tools: - Interview guidelines; - SRRS; - Collection of resources
Week 1		Week 2		Week 3
Filling the logbook				

Figure 1.3. Data collection following the reflective investigation principles.

In this case, the follow-up was organized during three weeks – with the intention to follow the same chapter the following year. The researcher first encountered the teacher to present the data collection organization. A first visit was organized at the teacher's home. Indeed at that stage the work still only concerned the secondary school teachers in France, who prepare their courses at home. During this visit, the teacher is interviewed about his/her resources in general, and about a specific chapter and a lesson in this chapter. The lesson in class was observed and video recorded, then discussed during a new interview. Two specific data collection tools were used during this follow-up. Firstly, the teacher noted in a logbook all his/her activity, the resources he/she used, with whom he/she worked etc. Secondly, the teacher was asked to produce a schematic representation of his/her resource system (SRRS, see figure 1.4). This representation was progressively produced during the three weeks.

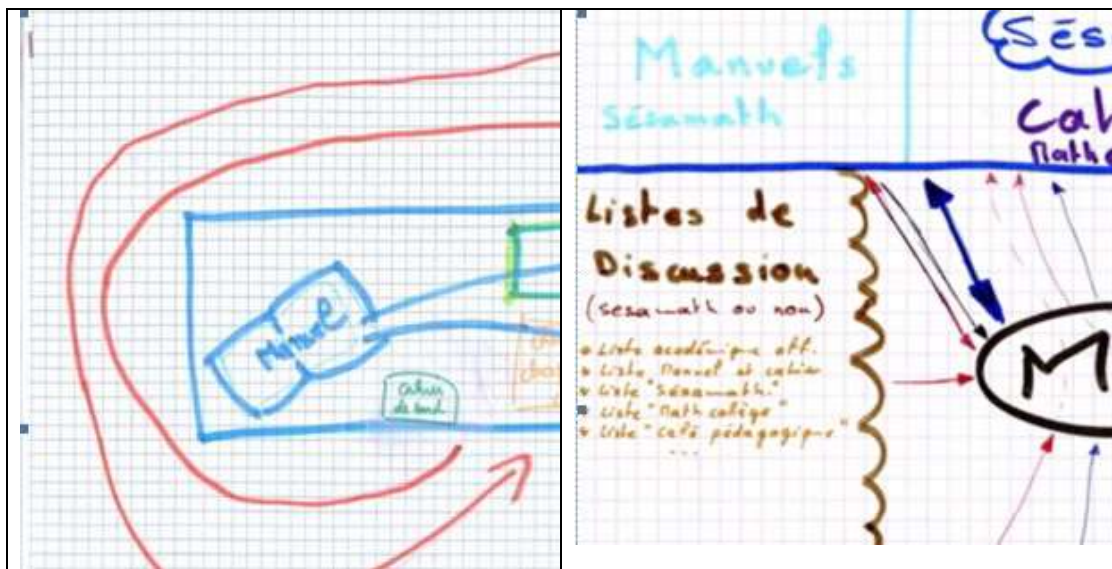


Figure 1.4. Examples of Schematic Representations of the Resource System by two lower secondary school mathematics teachers. On the left Corinne, on the right Pierre.

The reflective investigation method can also take other forms, as long as it follows the four principles presented above, we will discuss some of its evolutions in section 3. It is naturally associated with case studies. For a single teacher, or group of teachers, it provides an important quantity of data. The different kinds of data have to be confronted, and discussed

with the teacher. In particular, the classroom observations, the resources used and produced are confronted with the teachers' declarations, in order to infer operational invariants. Indeed the operational invariants are propositions considered as true (theorems-in-action) or concepts considered as relevant (concepts-in-action) which guide the teacher activity. This link between the convictions expressed by the teachers and their actual activity is thus central.

1.2.3 Evolutions of the theory

Using the reflective investigation method, during this period our aim was to study the documentation work of teachers, hence to answer questions like:

- Which are the documents developed by secondary school mathematics teachers along their professional activity?
- How do the schemes of use of resources evolve, which are the operational invariants in such schemes?
- Which are the consequences for the documents developed by mathematics teachers of the generalized availability of digital resources?
- How are the resources systems and the document systems of an individual teacher structured?
- How does the resources system of a community evolve, along individual and collective contributions?
- ...

While our central aim was to use DAD in order to understand the transactions between teachers and resources, and their consequences in terms of professional development, studying these questions also led us to further theoretical attempts.

Concerning the structure of the documentation system, we were still trying to elucidate the relevant activity families for the teacher. Instead of three families (see §1.3 above), we suggested that the teachers' activity was structured by 9 activity families:

1. Reflecting on his/her practice
2. Planning
3. Preparing and setting up introductory activities
4. Preparing and setting up syntheses
5. Preparing and setting up drill and practice
6. Preparing and setting up assessment
7. Manage the class and follow the students
8. Participate to the school life
9. Participate to collective work out-of-class

The activity families numbered from 3 to 7 came from the work of Chevallard (2002) who introduced the notion of didactic moments. These nine activity families were coherent with our case studies. Nevertheless, they have not been used since by other authors or even by ourselves in recent works.

We also replaced the notion of “pivotal document”, proposing to use instead “pivotal resources”, defined as resources engaged in several activity families. Indeed, it appeared clearly in our case studies that a given resource could be used by the teachers for different aims, corresponding to different activity families. In particular the textbook was used by several teachers for “Preparing and setting up introductory activities” and for “Preparing and setting up drill and practice”; sometimes also for “Planning”. This resource led to the development of a different document for each different aim. Indeed a single document cannot correspond to several activity families, since an activity family is defined by an aim, and this aim is one of the component of the scheme, hence of the document. Thus the concept of “pivotal resource” is more relevant, it has been used in several works after its introduction.

Another aspect to be retained from this period is that some authors very early appropriated the terms of the documental approach, and introduced personal theoretical constructs. In particular, Sánchez (2010) introduced the concept of “documentational orchestration” as a natural extension of instrumental orchestration, for the study of teacher education programs.

1.3 Extension of the scope and new contributions (2011-2013)

The determining collaboration with Birgit Pepin started in 2011, once again thanks to Kenneth Ruthven who introduced us to Birgit Pepin and to her work on textbooks. We first worked together on the book "Lived resources" (Gueudet et al. 2012). Far from being a translation of the French book “Ressources vives” (Gueudet and Trouche 2010), this work constituted a determining step in the insertion of DAD within a landscape of international works. While 12 of the 18 chapters in the French book were written by French authors, the book in English comprised only 5 chapters from French authors (for a total of 17 chapters). New authors have been involved, coming from various countries from Europe (Germany, Norway, The Netherlands) and outside Europe (Canada, Mexico, Australia, USA). Soon after this book, we edited together a special issue of ZDM entitled: “Resourcing teacher work and interaction: new perspectives on resources design, use, and teacher collaboration” (Pepin et al. 2013), associating again new authors. More generally, this 2011-2013 period was a period of extension: of the international links, but also of the issues studied with the approach, of the school levels considered, and even of the disciplines, with the first works using the approach outside of mathematics.

1.3.1 New issues studied and extension of the scope

From the study of teacher education programs to perspectives on collective design

An important issue studied during this period concerned professional development programs for in-service teachers (it was already present in the work of other researchers, like Sánchez 2010, but new in our work). The ministry of education in France opened indeed at that time a national platform called “Pairform@nce”. This platform offered “training paths”, which are structured sets of resources for the organisation of blended teacher education courses. The ministry wanted researchers to be associated with this innovative project: for designing training paths, and for assessing their use by teacher educators and by trainees.

This context opened for us different new research directions. The first one concerned the kind of teacher in-service education programs likely to lead to evolutions in the teachers’ practice. DAD suggested that collective documentation work was likely to contribute to professional development. This guided our choices for the training paths and for the associated teacher education programs (Figure 1.5).



Figure 1.5. First page of the training path “Using online exercises to individualize teaching”.

Studying these teacher education programs permitted to confirm the relevance of this hypothesis. We observed indeed a rich collective documentation work by teams of trainees, and changes in their classroom practices, linked with the professional development aims of the training path (Gueudet and Trouche 2011). A second direction was the study of the documentation work of teacher educators. Indeed the training paths can be considered as resources for the teacher educators who decided to use it for setting up their own training. We organised an experiment where two teams of “training paths” designers became teacher educators using the path designed by another team. We observed this way documental geneses of the teacher educators (Gueudet et al. 2012b). We also incorporated the improvements suggested by the users in a new version of the training paths. Beyond teacher education issues, this work led us to a new and more general research direction: the design of curriculum resources, in particular collective design. According to DAD, this design is linked with the documentation work and its outcomes. It is a continuous process, incorporating the contribution of users (teachers or teacher educators for example) in “living resources”. First developments of this perspective on design can be found for example in Pepin *et al.* (2013); it is still central in our present work (see chapter 5 and chapter 12 for more details about teacher design).

Documentation work at Kindergarten and primary school

The design of curriculum resources became also central in another research project, called “TREMA-1” for Technologies and Resources in Mathematics at Primary School. In this project primary school teachers, teacher educators and researchers worked together to study the use of technologies at primary school, and at the same time to design teaching resources. The research question guiding the beginning of the project was “Which are the factors shaping the integration or non-integration of technologies by primary school teachers in their mathematics teaching practices?” We studied this question in terms of integration of technologies in the resource system of the primary school teachers (Poisard et al. 2011), and documents developed by the teachers incorporating technologies. Following the reflective investigation methodology, we identified documents developed by the teachers, and in particular operational invariants. Some new operational invariants were developed; the already existing operational invariants played a central role for the integration of a new resource. The compatibility with other resources already present in the teacher’s resources system was also an important factor. These results guided a further stage of the work within

this project: the design of curriculum resources supporting the integration of technologies by primary school teachers. This work on design issues was only initiated at this stage; in fact it is still a major issue in 2018 as I will discuss it in the next sections.

I want to emphasize here another aspect of this project: it led us indeed to investigate for the first time the documentation work of primary school teachers. We observed important differences, compared with secondary school mathematics teachers. In France the primary school teachers mostly work in the schools, while the secondary school teachers prepare their lessons at home. Hence the resources are present in the classroom. The kind of resources used is also different: manipulatives are very important resources (in particular at Kindergarten, see e.g. Besnier 2016). Moreover primary school teachers in France teach all the subjects: mathematics, but also French, science, sports, arts etc. It means that their document system and resource system concern all these subjects, which makes even more complex the issue of the document system structure. We considered that these teachers develop a sub-system of documents for their teaching of mathematics. Nevertheless some resources are clearly used for different subjects, for example to make syntheses at the end of a work in groups (much more frequent at primary school). Hence the structure of the resources system, in the case of primary school teachers, is especially complex.

Documentation work at university

This period also marked the beginning of works concerning the documentation work of university teachers (Gueudet 2013). Here the complexity does not come from the teaching of several subjects, but from the links between teaching and research. Interviews with lecturers evidenced that they developed documents not only for their teaching activity, but also for their research activity. This suggests a new development of the approach: it can be used for different aims of the professional activity.

Lecturers have a document system for teaching, and a document system for research. The links between both systems are complex. Moreover we observed that, in a context where lecturers do not receive any teacher education (for most countries), the participation in communities engaged in a collective documentation work plays a central role for professional development. For this reason, the teaching practices at university are probably even more stable than the practices at secondary school for example. At secondary school, changes in the curriculum and in teaching approaches can be supported by in-service or preservice teacher education. At university, beginning teachers, in an instrumentation movement, develop operational invariants influenced by the resources they use. These resources have been designed by their colleagues in previous years. Aligning with these resources, the beginning teachers contribute to the stability of the teaching practices (and even sometimes to the stability of the curriculum, while changes in the secondary school curriculum would require modifications). These issues require specific investigations, which are still ongoing now.

1.3.2 Evolution of the methods and contribution of the first PhDs

During this period, the reflective investigation method was complemented in several ways. The first way was the introduction of what is called since “the documentation valise” (Figure 1.6). While the metaphor of the valise was chosen at the beginning to evoke the resources gathered by the teacher for a journey, the content of the valise goes far beyond such resources. It is more a valise for the researcher, gathering all the data of a teacher’s case study. This new organisation of the data was linked with the intention of sharing data with other researchers. Indeed the data collection with the reflective investigation method is a long process. Gathering many cases is an important aim, to observe regularities across cases and to

allow comparisons (international comparisons for example). This can only be achieved by researchers working together and sharing their data. This project had new developments and produced important results in recent years (see Chapter 12 by Luc Trouche).

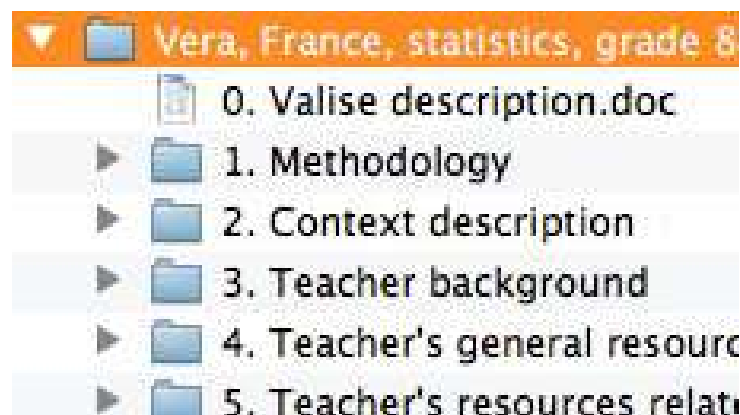


Figure 1.6. Example of a documentation valise, the case of Vera⁹.

Other methodological developments have been proposed in the two first PhDs defended using the documentational approach: Aldon (2011) and Sabra (2011). Aldon studied communities of teachers and researchers designing resources to support the integration of technologies by secondary school mathematics teachers. He introduced the notion of “incident”, defined as an unplanned event. He used it as a methodological tool: indeed when such an incident happened in class (linked in his study with the use of technology) the teacher had to draw on his/her professional knowledge to react on the spot (this could be connected with the aspect of instrumental orchestrations described by Drijvers et al. 2010 as on-the-spot decisions, part of the didactical performance). The circumstance made the knowledge more visible.

Sabra also used the notion of documentational incident. His work addressed especially collective documentation work, in the context of the French association Sésamath whose members produce free online resources, including e-textbooks. For this purpose Sabra introduced new methodological tools, in particular a collective logbook for the community of textbooks authors. Comparing the collective logbook and the individual logbooks can reveal tensions in the collective work or at least differences in the interpretations of the teachers. This work by Sabra has been the source for many future works concerning the documentation work in communities of practice. Indeed Sésamath remains a unique case in France, and probably at an international level: a community of teachers that produced digital resources, including e-textbooks and a complete virtual learning environment (LaboMEP) covering the whole secondary school curriculum, and widely used by teachers. While the documentation work of teachers using Sésamath resources has been researched with DAD from the beginning, the work by Sabra was the first one studying the design processes in communities of Sésamath authors.

These two early PhDs contributed to the development of methods for the identification of documents developed by teachers, individually or collectively.

⁹ http://educmath.ens-lyon.fr/Educmath/recherche/approche_documentaire/documentation-valise/documentation-valise-1

1.4 Recent works (2014 and after)

I will only mention here some recent works using the documentary approach, and evoke related evolutions of the theory. A more complete account can be found in chapter 12. I focus on the evolutions brought by a French national project: REVEA. I firstly present the REVEA project, and then describe methodological developments and theoretical evolutions resulting from works linked with this project.

1.4.1 Living resources for teaching and learning (REVEA): a national project in France.

“Living resources for teaching and learning” was a national project in France (financed by the National Research Agency, ANR) involving 5 different research teams, and piloted by Eric Bruillard. It took place between April 2014 and March 2018. The aim of this project was to investigate the documentation work of secondary school teachers and its consequences for 4 subjects: English; Mathematics; Science; Technology. The use of digital resources and its consequences received a specific attention.

The comparison of different subjects within the project evidenced that, even if the development of documents is an individual process, there are some regularities across teachers of the same subject. These regularities can naturally concern the kind of resources used: English teachers use more videos than mathematics teachers; in mathematics the textbook is central, but not in technology etc.. But other kinds of regularities exist: the alignment, or not, with the official curriculum; the more or less important involvement in collective work, for example. Concerning the use of digital resources, according to the subject specific software can be more or less used. Nevertheless all the teachers followed in the project declare that they spend more time (now, compared with 5 years before) searching for online resources. In the different secondary schools followed in the project, we observed the use of digital means to share resources with colleagues. Nevertheless this use does not seem to be stable yet: sometimes a group of colleagues only uses e-mail, sometimes they share a folder in the school virtual learning environment (or another institutional platform), and sometimes they use a Google Drive etc. We even observed in some groups of colleagues working together that some members of the group were not aware of the existence of a shared folder.

The REVEA project also examined the collective work of groups of teachers working out of schools with an objective of resources design. Drawing on the work of Sabra concerning the Sésamath association (2011), it studied the “life” of communities of practice designing resources, including three different communities concerning mathematics. Studying these communities led in particular to observe specific features of their resource systems. These systems comprised indeed meta-resources: resources whose aim is to support the design in the community. They also included pivotal resources, which articulate different other resources designed with different aims (Trouche et al. 2018a). More details on the theoretical and methodological developments linked with collective documentation work can be found in chapter 12.

The REVEA project also evidenced the need to combine studies of the documentation work at different scales: from processes developing on several years, and concerning groups of teachers to evolutions of schemes of a single teacher, during a few weeks. We will illustrate this reflection on macro-scale / micro-scale studies in the next sections, concerning respectively methodology and theory.

1.4.2 Methodological developments

The REVEA project also opened opportunities for several methodological developments. We present here two of them, chosen to illustrate the different time scales that the documentational approach has to take into account.

Investigating schemes, and, in particular, operational invariants (Vergnaud 1998) is a complex issue. As coined by Vergnaud, only a minor part of operational invariants corresponds to explicit knowledge. Some of them are not conscious. Thus, it is impossible to reach them only by asking the teacher: “why did you act this way?”. Most of the time the answer to this question is not accessible to the teacher.

For this reason, after gathering all the data mentioned above with the reflective investigation method, the researcher(s) builds from them a “documents table” (Table 1, see e.g. Gruson et al. 2018). This table comprises elements of the document: the goal of the activity, the resources used, the rules of action and potentially corresponding operational invariants. The goal and operational invariants are inferred from the declarations of the teacher in the interviews. The resources used and the rules of action are observed in the activity.

Goal of the activity: Prepare and implement...	Resources used	Rules of action	Operational Invariants
Stabilization of previous knowledge	“LaboMEP” (online exercises)	Valeria chooses exercises in LaboMEP to test whether the students master previous knowledge and to supplement if necessary.	“Some of the grade 10 students need to practice on grade 9 knowledge”

Table 1. Example of a “documents table” in the case of Valeria (Gruson et al. 2018)

This table is then submitted to the teacher who complements or corrects if necessary. The “documents tables” are efficient methodological tools to support the identification of the documents developed by the teachers. For example, using Table 1, we can claim that Valeria has developed a document, for the aim: “prepare and implement the stabilization of previous knowledge” (at least with her grade 10 students). The document comprises online exercises chosen on LaboMEP (a virtual learning environment designed by the Sésamath association), rules of action like “before starting a new chapter, I assess the students previous knowledge with online exercises”; “I propose online exercises for the students who do not master the previous knowledge”, and an operational invariant like: “Some of the grade 10 students need to practice on grade 9 knowledge”.

At a completely different time scale, Rocha (2018) introduces the notion of “documentational experience” (defined as the accumulation of documentation work along the years) and “documentational trajectory” (defined as the set of collective and individual events that took place along this experience). It led her to propose a new methodological tool: Reflective Mapping of Documentational Trajectory (RMDT). It is a new kind of representation produced by the teacher during a reflective interview, concerning his/her experience since the beginning of his/her career (Figure 1.7).

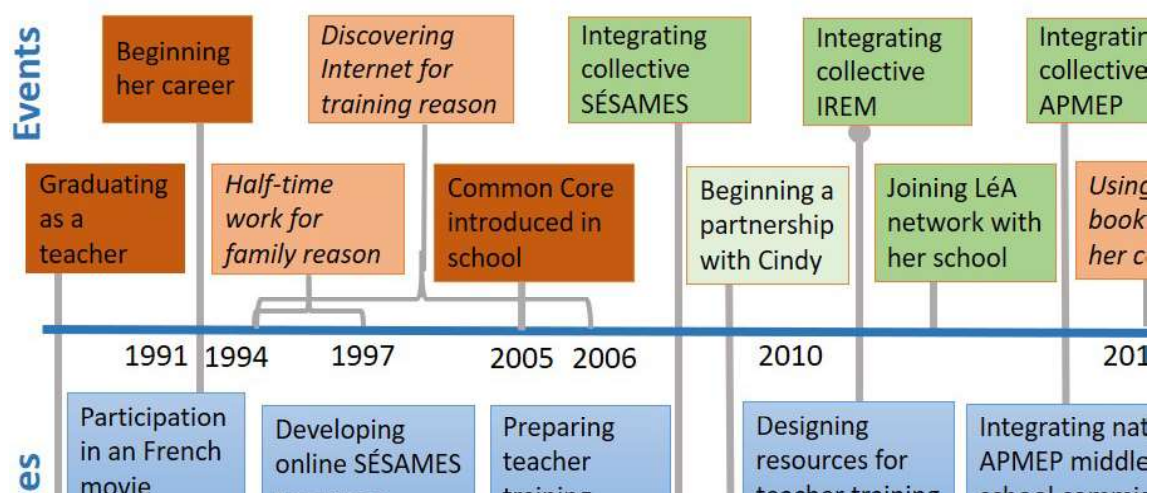


Figure 1.7. Example of Reflective Mapping of Documentational Trajectory (Rocha 2008)

It permits to identify crucial events, important collaborations etc., still influential in the teacher's work, even many years after their end.

1.4.3 *E-textbooks and theoretical evolutions*

The REVEA project concerned all kinds of teachers' resources. For mathematics teachers it confirmed the importance of textbooks in their resources systems. In France secondary school teachers use between 4 and 8 different textbooks to search for "introductory activities", to build the organization of the mathematical content over the year (their "yearly progression"). One of them is the classroom textbook, shared with the students. It is mainly used to give exercises: homework, or work in class. Nowadays, a pdf version of each textbook is provided to the teacher with the textbook on paper. The teachers use this pdf version for projection in class: even if a student has forgotten his/her textbook, he/she can read the text of the exercise. Moreover, some publishers developed e-textbooks: private publishers, and the Sésamath association (studied by Sabra in his PhD, 2011). The e-textbooks from private publishers are still quite expensive, hence not much used. The Sésamath e-textbook is free, associated with a complete environment called LaboMEP, and used by many teachers.

Within the REVEA project we worked on e-textbooks, in particular the Sésamath e-textbook. It led us to propose specific theoretical constructions and articulations.

Firstly we investigated the design of the Sésamath e-textbook. It involved a community of practice of authors, coordinators, developers. We consider this design as a collective documentation work. Its development during several years led us to use the Cultural-Historical Activity Theory (CHAT, Engeström 2001), and to combine it with DAD. In (Gueudet et al. 2016), we proposed to associate two scales, to investigate the activity of the e-textbook designers (Figure 1.8).

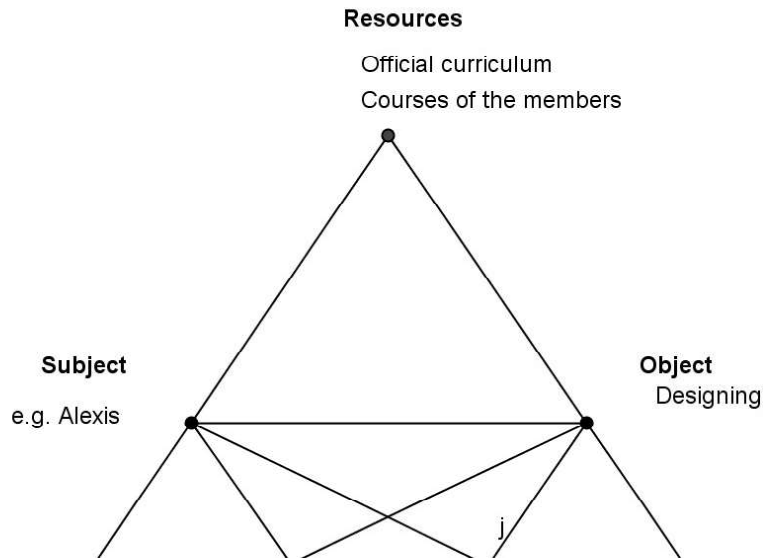


Figure 1.8. The activity system of an author of the Sésamath e-textbook

The macro scale considers the evolution of the different components of the activity system, resulting from tensions between different aspects of this system. The micro scale refers to DAD, and studies the development of documents by some of the community members. We focus on documents concerning particular mathematical topics; for this reason we refer to this aspect as “micro”.

After this study of the design of the Sésamath e-textbook, we worked more generally on the features of e-textbooks (including the Sésamath e-textbook). It led us to introduce the concept of “connectivity” (drawing in particular on “connectivism”, Siemens 2005). An e-textbook is a structured set of resources; many possibilities for building links exist within such a textbook, and with its environment. We defined connectivity (in Gueudet et al. 2018) as: *“connecting potential [of an e-textbook] for a given user (student or teacher) both practically as well as cognitively”*. We distinguished again a macro and a micro level for the analysis of an e-textbook connectivity:

- The macro-level takes into account the potential connections “outside” of the e-textbook (connections with other online resources, but also connections with the user’s resources system);
- The micro-level considers connections within the textbook, for a given mathematical theme (connections between representations, between different aspects of a given concepts etc.)

We designed an analysis grid for e-textbooks using these two categories. We argued that this grid was helpful to characterize different kinds of e-textbooks.

Linking DAD and the study of particular resources can seem surprising: DAD is originally devoted to the study of the teacher’s work. The criteria we used for defining the connectivity of e-textbooks incorporate the DAD perspective for the macro level. Indeed, this macro level includes the possibilities of connections between the e-textbook and the user’s resource system (for example, possibility for the user to download extracts of the book). Thus it takes into account the potential of the e-textbook for the documentation work of the user.

Naturally the period starting in 2014 has seen many other research works referring to DAD, that also contributed to the evolution of the questions, methods and theory: the reader of this book has many opportunities to observe it in other chapters.

1.5 Conclusions: towards solid findings in DAD?

In this chapter I have tried to synthesize the short history of the documentational approach. I did not present a literature review; I have not taken a scientific stance, I have only described my point of view on this history. The previous sections evidence many evolutions: in the theory, in the methods, in the questions studied and in the scope of the approach. In this conclusion I present what I consider to be the most striking evolutions and perspectives (for a more complete discussion on perspectives see chapter 12).

Firstly, I will look at the theoretical evolutions through the lens of “solid findings”. The concept of solid findings in mathematics education has been introduced by the European Mathematical Society educational committee in 2011 (EMS, 2011), and defined as findings that:

- “ 1) Result from trustworthy, disciplined inquiry, thus being sound and convincing in shedding light on the question(s) they set out to answer;
- 2) Are generally recognized as important contributions that have significantly influenced and/or may significantly influence the research field;
- 3) Can be applied to circumstances and/or domains beyond those involved in this particular research;
- 4) Can be summarized in a brief and comprehensible way to an interested but critical audience of non-specialists (especially mathematicians and mathematics teachers).” (EMS 2011, p. 46)

I will mainly use the criteria 2 and 3, and consider as solid the concepts that have been used since by other colleagues, and that have been applied to other contexts (meaning here, not only applied to secondary school mathematics teachers).

The notions of resources, documentation work (individual or collective), documentational genesis and document seem now to be “solid concepts”, since they are used by a large international community of researchers, in mathematics education and beyond. Moreover, people using these concepts share the same definitions.

While the notions of *resource system* and *documents system* are also used by several researchers (in particular, asking teachers to draw a SRRS relies on the assumption of a shared meaning for the term resource system, at least in this context), I argue that these concepts cannot yet be considered as solid. Concerning resource systems, a new definition has been introduced recently: “We call the set formed by all the resources used by the teacher his/her *resource system*” (Trouche et al. 2018b). With this perspective, the resource system can be considered independently of the document system. The consequences of the change of perspective still need to be investigated. Chapter 2 of this book contributes to this investigation, by identifying different possible meanings of the concept of resource system. Concerning document systems, I have described several attempts to approach their structure: first with three activity families (Gueudet and Trouche 2009a), then with nine such families (Gueudet and Trouche 2010). These two propositions have never been used by other authors afterwards. This is directly linked with the complexity of the issue of resource system and document system structure. Only the concept of “pivotal resource”, as resource used for several activity families seems to start becoming more stable and shared, while “pivotal document” has disappeared from recent papers.

More generally, in recent publications referring to DAD, I observe that the concept of “resource” appears much more frequently than the concept of “document” (the conference in

2018 was entitled “Re(s)ources 2018”, which is probably also significant about this matter). This can be linked with the methodological complexity for the identification of schemes. This evolution should not obscure two important issues. The move towards a focus on resources should not be limited to material resources, easier to observe. Evidencing that resources are not only material is a major contribution of DAD, coming from the work of Adler (2000). Moreover another major contribution from DAD is to offer theoretical tools to link teachers’ use of resources with their professional growth. The already existing schemes and operational invariants of a teacher shape his or her use of resources. The work with resources leads to the development of new schemes. Thus it is essential to continue to investigate documents (which means not only resources, but also schemes).

The theoretical evolutions also concern theoretical connections, whose development appeared in this chapter. The original connections with the instrumental approach (Rabardel 1995), with the communities of practice (Wenger 1998), with ATD (Chevallard 2006) etc. naturally remain. New connections have been developed, I have presented here connections with activity theory (Engeström 2001); other links are central in recent works, for example with teacher design capacity (Pepin et al. 2017, see also chapter 5 of this book) or curriculum ergonomics (Choppin et al. 2018). Chapter 4 of this book provides a frame to study this theoretical networking, and opens the path for a reflection on these theoretical links, which is also a promising direction for future research. Concerning the methods, in the recent evolutions I retain the apparent need to take into account a combination of different scales. These scales can concern the mathematical content (from a precise theme to a whole year for a given class) or the periods of time (from a few weeks to the whole career of the teachers). Some studies have also started to use quantitative methods, about the use of particular resources for example (see e.g. Gueudet & Lebaud 2016 about the choice and use of the textbook). Combining quantitative methods with case studies could also bring new results about teachers’ documentation work and its evolution.

Concerning the scope of DAD, I consider that its expansion is one of the most striking evolutions during these last years. While we started by a work limited to experienced secondary school mathematics teachers, DAD is now used from Kindergarten to University and in-service teacher education and for several subjects: chemistry, physics, biology and language education. Researchers still investigate the documentational geneses of experienced teachers, but also of preservice and novice teachers, of teacher educators and recently even of students (at university, see Gueudet and Pepin 2018). The proceedings of the Re(s)ources 2018 conference (Gitirana et al. 2018), and the chapters 8 to 11 (Section 3 of this book) provide abundant evidence of this expansion.

Maintain and extend collectively the reflection initiated in this chapter (and in the other chapters of this Section 1) could be useful in this context of expansion. It could contribute to a coherent development of the manifold research works, and to the production of more solid findings concerning teachers’ documentation work.

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