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INTEGRATION OF TECHNOLOGY IN THE DESIGN OF GEOMETRY TASKS WITH CABRI-GEOMETRY

ABSTRACT. Beginning with the gap in France between the institutional support for the use of technology in mathematics teaching and its weak integration into teacher practice, this paper claims that integrating technology into teaching is a long process. The aim of the paper is to identify and analyse the steps in this integration using as an example the evolution over time (3 years) in the design of teaching scenarios based on Cabri-géomètre for high school students. The analysis indicates that the role played by the technology moved from being a visual amplifier or provider of data towards being an essential constituent of the meaning of tasks and as a consequence affected the conceptions of the mathematical objects that the students might construct.

KEY WORDS: dynamic geometry, teacher beliefs, integration of technology, teaching scenarios, design of tables

INTEGRATION OF NEW TECHNOLOGY INTO THE FRENCH MATHEMATICS TEACHING

France is well known as a country in which technology for everyday life is widespread. Minitel, a forerunner of the Internet was designed more than twenty years ago and used by everybody for a large range of activities, from looking for a phone number or booking a flight or a train to performing banking operations. The number of cell phones with access to the Internet has been increasing dramatically over the past three years.

Concerning school technology the overall situation can be briefly described by saying that there is a gap between, on the one hand the availability of hand held technology for students and the intentions of the Ministry of Education, and on the other hand, the real integration of technology into the practice of teachers. All students from grade 10 have a graphing calculator bought by the family and according to the last TIMSS study (<http://timss.bc.edu/>), almost 60% of French students in the scientific track in the final year of secondary school have a computer at home. 89% of those students claim that they use a calculator weekly or daily (same source).

Since 1997, there has been a very strong incentive from the French Ministry of Education to promote the integration of information technolo-



gies into education at all levels. The Ministry provided financial support for organising pre- and in-service teacher education (IUFM) at national and local levels: 1/3 of the national sessions for teacher trainers is devoted to the integration of IT into teaching. Since 1996, the national curricula in mathematics have been renewed (one grade level per year). The new curricula offer a real integration of technology into the teaching of mathematics and claim the necessity of this integration as expressed in the following excerpt:

All students now have access to calculators and mathematics teaching must take it into account. [...] The new curricula show the necessity of working with calculators by ensuring the acquisition by everybody of abilities in mental and written computation (Middle school curriculum)

The curricula go further by arguing that introducing computers or calculators affects the very nature of mathematical activity and therefore the learning processes:

Computers allow a quasi-experimental approach in the field of numbers and figures of the plane and of space. They favour thus a more active approach and more involvement from students. Computers enlarge dramatically the possibilities of observation and manipulation [...] Computer environments allow various experiments and the study of same notion under a higher diversity of aspects; they contribute to the process of abstraction specific to mathematics and lead to deeper reflection and better understanding (High school curriculum, Grade 10).

A specific epistemological approach and learning hypothesis underlies the national curriculum, as evident in the preceding quotation: mathematics is by its essence abstract, and to acquire mathematical concepts, students must themselves achieve these abstraction processes by manipulating, experimenting and observing. Mathematical objects will be constructed by students as emerging from these experiments in a process of abstraction, eliminating all irrelevant aspects linked to the context of emergence. Technologies like computers favour abstraction processes in that they allow multiple experiments and thus may help distinguish the common relevant aspects from aspects attached to the context.

The demands of the national curricula are certainly high regarding this epistemological view of mathematics and its associated learning hypothesis. One suspects that it would be difficult for the majority of teachers to put what is recommended into practice. Although it is difficult to report the exact proportion of teachers making any real use of technology, it is estimated to be around 20% (Guin and Trouche, 1999). The gap between the institutional situation promoting technology and the actual situation in the classroom may appear surprising, and the effort underway in pre-service teacher education will no doubt improve the situation in the future.

Nonetheless, one must seek reasons for this reluctance to integrate technology into teaching practice. As Chevallard wrote almost ten years ago (1992), “the introduction of new tools in the didactic system must not be taken for granted . . . It is necessary to take into account didactic permanences, problems specific to teaching and learning that even new technologies cannot avoid. Quite often, computer-based strategies take rather little account of the logistic problems left to the teacher in his/her class.”¹

What follows is an attempt to analyse some of the problems encountered by teachers when trying to integrate technology into their practice. The aim of this paper is to show how and why the process of integration is long. It will be illustrated by a case study of the design of senior high school tasks with Cabri-geometry by a team of mathematics teachers and researchers.

THE PROCESS OF INTEGRATION

The expressions ‘integration of technology’ and ‘teaching systems’ are both very general. In particular, whereas the expression ‘integration of technology’ is used extensively in recommendations, curricula and reports of experimental teaching, the characterisation of this integration is left unelaborated.

The teaching system is complex, made up of several elements mutually interacting around three poles: the teacher, the students and knowledge. It is subject to several constraints (time, societal choices regarding curriculum, the inner structure of the mathematical domain of knowledge, the conceptions and ideas of students), and it evolves from one equilibrium state to another by choices made within this system of constraints. When a new element such as technology is introduced, the system is perturbed and has to make choices to ensure a new equilibrium is attained, choices that may be related to the various interrelated elements of the teaching system mentioned above. For example:

- The domain of knowledge: how are mathematical objects and relations affected by technology? What aspects are preserved? What aspects are changed?
- The interaction between teacher and students: by whom is technology used and for what purposes? For example, is it to be used to support the discourse of the teacher and to illustrate some points of the curriculum; or is it to support the learning of the students?

- The place and role of technology with respect to curriculum and to the course of teaching. To what extent is the use of the technology linked to the official curriculum? When is technology used: under exceptional circumstance to introduce a new idea, to apply an idea after its introduction or at any phase of teaching?

THE TEACHING SEQUENCES

In order to illustrate the process of integration, we report some teaching sequences involving a set of classroom scenarios, that were designed for the first year of French senior high school (15–16 year old students), and carried out in the classroom over three years, as part of an innovation project that aimed to develop the use of advanced technology in schools.² The teaching sequences were designed by four mathematics teachers, and discussed in the larger team involved in the project, which included a physics teacher, researchers in mathematics education and computer scientists involved in the development of software for mathematics teaching. All members of the team volunteered for the project. In terms of Noss and Hoyles (1996, p. 186), the aim of the project was to “design innovation *with*, rather than *for*, the teachers”. It is important to say that this project took place before the implementation of new curricula (and even before their design).

A complete “scenario” consisted of a combination of interrelated tasks, as well as collective phases in which the teacher made definitions and theorems explicit. For each scenario, an introduction presenting the aims of the scenario was written by its author. Observations of the class working on the scenario, the students’ behaviours and the classroom management gave rise to written comments (Clarou, Laborde and Capponi, 2001)³ and led to modification of the scenarios during the three years of the project, with two or three versions being produced for each sequence. The scenarios were based on the use of Cabri-geometry as available on the TI 92. Each student was given a TI 92 for the whole academic year with the hope that they would use it at home and not only for geometry. They also worked on some computer activities in a computer room, made possible by the availability of essentially the same dynamic geometry program in both environments (computer and calculator). The availability of the geometry application on a hand-held device certainly contributed to its integration in several aspects: the students could decide on their own to use it, and the teacher could give homework or class assignments to be done on the TI 92 (the files were easily transferred and collected on the teacher’s calculator).

Topics addressed in the scenarios were configurations, areas, vectors, geometrical transformations and, in particular, dilation and translation. The scenarios were designed for the whole attainment range since students do not make their choices of specialisation until aged 16 or 17, the year following this project.

The four teachers in the team had rather different profiles:

- two experienced teachers very familiar with the use of technology in mathematics teaching and with research in mathematics education; even though they were familiar with using technology, it was the first time they had integrated it into their teaching of geometry to this extent, probably because of the availability of the TI 92 for each student;
- a novice teacher but one who was experienced in computer science (he was a former engineer);
- an experienced teacher but one who had little familiarity in using advanced technology in mathematics teaching.

The first version of each scenario was designed by a single teacher who would try it in his/her own class. In some cases, it was discussed in the team before being tried out, but this was not always done. The team meetings took place once a month and it occasionally happened that between two meetings a teacher found an unexpected opportunity to use Cabri for a specific part of the curriculum, which could not be postponed in the course of the teaching. Since the scenarios were not significantly modified if their first version was discussed, we consider any version written before being tried out as a first version. It is interesting to note that a real discussion took place only after experimentation in class, perhaps because the team was more heterogeneous in terms of experience at that point, prior to which the experienced teachers in both domains did not want to impose their opinions *a priori*. After a scenario was tried out, all the teachers were able to argue from the basis of their observations of the classroom.

We call the 'second version' a version modified by the teacher (or another teacher) after being tried out in class. This second version was also tried out in class and subsequent versions might also be developed, as in the case of the scenario, 'Dilation.' Although the project lasted three years, some scenarios are still in use with ongoing modifications by the teachers, and members of the project. The fact that scenarios are continuously modified is part of the integration process of technology.

The first and second versions of the scenarios will be the focus of the discussion here. More precisely, we will analyse the choices made by the teachers and their evolution over time with respect to the following aspects:

- The content of a scenario, its place and role in the official curriculum and in the teaching progression of the class.
- The role of Cabri in the tasks.
- The creation of new tasks linked to technology.

In a second step, we will attempt to sketch the epistemological beliefs underlying their choices and to identify why the demands of a curriculum integrating technology require time before any effective integration can take place in class.

THE CONTENT AND PLACE OF SCENARIOS IN THE CURRICULUM

At the beginning of the project, the two teachers who were inexperienced (one in using technology and the other in teaching), both proposed activities that were not part of the normal curriculum.

Series of Disconnected Tasks

The novice teacher who was an expert in computer science designed several small activities based on various facilities of the TI 92. It was as if he wanted to show to his students the range of possibilities of the calculator rather than go deeply into the use of any one of them. He introduced programming activities, plotting functions and equations of straight lines, with each activity independent of the others.

He also used Cabri-geometry in the computer room within independent activities, peripheral to the curriculum or (more usually) as exercises. In these tasks, the role of technology was mainly to facilitate observation or collection of data. For example, the teacher asked the students to construct with Cabri the centroid, the orthocentre and the centre of the circumscribed circle of a triangle and to observe that the three points are collinear (The Euler line), and then to prove it but independently of the software. Or he asked the students to collect the measures of the angles of various quadrilaterals generated in Cabri through the drag mode. Two of the tasks designed by this teacher will be analysed in a following section.

A Long Problem

The experienced teacher (but a novice in using technology) chose a long problem she had found in a book, which aimed at gathering together uses of several geometrical properties and objects (loci, transformations, ellipse). The problem was not especially designed for a computer-based

environment and was conceived by the teacher as a way to check whether the students were able to coordinate and use various notions in a long problem. The teacher did modify the problem by introducing the task of constructing a rhombus by imposing

- in one question, the degrees of freedom of vertices (she mentioned that given vertices must be constructed so that they could be dragged anywhere),
- in another question, the trajectory of one vertex of the rhombus in the drag mode.

She kept the questions about determining the loci but asked the students, in an explicit way, first to obtain the locus with Cabri, second to conjecture the nature of the locus and third to prove it on paper. Even if the construction tasks were specifically designed for Cabri, it appears that in the locus problem, the teacher saw the role of technology as a way to facilitate the making of the conjecture but not as part of the solution of the task.

The Role of the Teacher Experience

At the beginning of the project, neither novice teacher used technology for teaching, but limited the use of technology to some sessions independently of the course of their teaching. In both cases the visual power of technology was used, but mainly for seeing and conjecturing and not for experimenting in order better to understand the mathematical situation.⁴ Cabri was not really used as a tool for solving a task. In the words of Pea (1985), we would say that in this kind of use for visualisation, technology was simply an *amplifier* and *not a reorganiser*.

However it is worth noting a difference between the novice teachers. The teacher who was a novice in technology, but experienced in teaching, designed tasks which could not be done with paper and pencil: construction tasks of dynamic diagrams with trajectories imposed and how to drag particular elements of the figure (for example, constructing a rhombus with a vertex moving in a straight line). She was very explicit about this aim: she wrote as a comment in the booklet, that the problem aimed to show to the students that a geometrical object is not static but can change while preserving its internal geometrical relations (Laborde, 1995). She stated that this is a good way to make students aware of the functional aspects of geometrical relations.

We interpret this behaviour as a change subsequent to the perturbation caused by technology, which affected the conception she had of the nature of a geometrical object. Paradoxically, the teacher who was expert

in technology did not change the nature of tasks he gave, in contrast to the novice in technology, who did, at least partly. Maybe this was because, as an experienced teacher, she had become aware that students experience difficulties in viewing geometrical objects as functions of other objects.

The teacher who was expert in technology but new to teaching, did not really change the scenarios he had written over time. The scenarios were all mainly short and dealt with topics independent of any progression in the class. This was not the case for the experienced teacher novice in technology. After some time, she decided to write a scenario for introducing students at tenth grade (15–16 year olds), to the new notion of dilation as a point transformation defined by using vectors.

'Real' Scenarios and Their Structure

From the very beginning of the project, the experienced teachers in technology and teaching started writing complete scenarios aimed at introducing students to new notions at the core of the curriculum, such as vector or translation, and to provide a complete teaching of these notions. Rather than being disconnected tasks, such scenarios required several teaching hours (between 6 and 10 hours). What must be stressed is that Cabri was used at several critical moments in the course of teaching:

- for introducing a notion and giving a meaning to it;
- for investigating properties of notions through Cabri;
- for applying the notion and solving problems with it.

In terms of object/tool dialectics (Douady, 1986), Cabri was used at various phases of the teaching process, as an object of teaching and as an explicit tool in the solution of problems. In contrast with Cabri tasks, paper-and-pencil tasks were mainly used to provide a change of context for the application of notions or properties that had already been introduced.

As illustration, we describe below the structure of Version 1 of the scenario 'Translation', and the first part of scenario 'Vector', both of which were written by one of the experienced teachers.

a) Structure of Version 1 of the scenario 'Translation'

The scenario was planned to require 6–7 hours. It comprised three phases.

The first phase was an introductory exploratory phase:

- Introduction of translation as the composition of two central symmetries (Cabri)
- Exploration of the effects of a translation on a polygon and a circle (Cabri)

- Identification of the translated image of a polygon among several polygons (paper and pencil)

After this phase of exploration there was an institutionalisation phase of the properties and invariants of a translation. The properties were to be applied immediately in an easy task:

- Determining and constructing the vector of a translation when an object and its image are given (Cabri)

Then, more open-ended tasks requiring the use of the definition or properties of a translation were given:

- Determining the nature of the composition of point symmetries as a function of the number of symmetries (Cabri, open problem)
- Determining the transformation that is the composition of two translations (Cabri)
- Constructing the image of a triangle and of a circle through a translation, without the Translation tool (Cabri)
- Introduction of the algebraic relations between the coordinates of a point and its image (Cabri)

Finally, there was an institutionalisation phase of the expression between the coordinates of a point and its translated image:

- Invariant points in the composition of point symmetries (Cabri, open problem).

b) Structure of Part I of version 1 of the scenario 'Vector'

This was planned to last 3–4 hours in distinct phases. First, an exploration phase:

- Exploration of a polygon and its image through a translation with a given vector, relations between the vector and the effect of the translation (Cabri)
- Construction of a vector equal to a given vector and passing through a given vector (Cabri)

Institutionalisation of the notion of vector as a congruence class:

- Coordinates of a vector (Cabri)
- Exercises of application in paper and pencil

Institutionalisation of the independence of the coordinates of a vector, of the position of its endpoints, and about change of coordinates when changing the vectors length or direction:

- Construction of a vector with given coordinates (Cabri)

- Sum of vectors, difference of vectors (Cabri)

Institutionalisation of the definition of the sum and of its properties:

- Exercises of application (paper and pencil).

It is clear from these two examples that, on the one hand, the scenarios dealt with the content to be taught by the curriculum, but on the other hand, Cabri was used in most parts of the scenario and gave rise to activities that were especially designed for a dynamic geometry environment. For example, it was possible to ask the students to investigate the effect of the composition of two central symmetries because the environment allows the user to obtain immediately the image of any figure through a central symmetry. Without this facility and the drag mode, such an investigation would be impossible. In the same way, it would be outside the students' competencies to explore the effect of changes of the translation vector on the image of a figure. Institutionalisation of new notions and properties followed a sequence of activities carried out only in the Cabri environment.

The fact that technology was used for meeting the demands of the curriculum leads us to consider that the level of integration of technology was higher in these scenarios than in a series of disconnected tasks or in the long problem. Cabri was used to introduce a new notion and its properties and the paper-and-pencil environment were used for reinvesting already-introduced notions in traditional exercises that form part of the school culture. It seems that in these scenarios the technology was meant to favour the construction of meanings by the learner, while paper-and-pencil exercises were intended to be a link with the dominant school culture and to present students with familiar forms of school problems.

The role of Cabri was different in the computer tasks. It offered possibilities to explore and to discover a notion and its properties. In many tasks, an additional role was to provide validation (or verification). And finally, some construction tasks were used that required students to use, in an explicit way, properties that had been studied in previous tasks. Cabri was not reduced to the role of data collector, but rather played a variety of roles.

The following section will analyse the various roles Cabri played in the tasks of the scenarios.

Various Roles of Cabri in the Tasks

Several research paradigms consider that the context (and in particular a technological context) deeply affects the task carried out by the student. The psychological approach of instrumentation of Rabardel and Vérillon (1995), inherited from a Vygotskian perspective, considers that tools are

not neutral; they not only change the actions of the users but deeply affect their conceptualisation of reality. This is consistent with the approach of Noss and Hoyles (1996) who claim that computer environments shape students' solution strategies when they are faced with a problem and forge their understandings.

The focus in this paper is not on the users but on the teachers designing the tasks and on their intentions. To what extent did they play on these changes introduced by Cabri? What type of role did they want to confer on Cabri? We propose to classify the tasks according to the role that the designer of a task attributes to Cabri and the degree of change that is anticipated. We distinguish *a priori* four types of roles:

- Cabri is used mainly as facilitating material aspects of the task while not changing it conceptually;
- Cabri is supposed to facilitate the mathematical task that is considered as unchanged: this is the case where Cabri is used as a visual amplifier (Pea, 1985) in the task of identifying properties; it is assumed that it is easier to observe that three lines always intersect in one point during the deformation of the diagram by the drag mode than in a static paper-and-pencil diagram;
- Cabri is supposed to modify the solving strategies of the task due to the use of some of its tools and to the possibility that the task might be rendered more difficult;
- The task itself takes its meaning or its 'raison d'être' from Cabri.

Let us present each type of role in a little more detail through an example.

Tasks in which Cabri facilitates the material aspects of the task

Constructing a triangle in Cabri, the midpoints of its sides, and then its medians does not really differ in terms of knowledge from the construction of the same diagram in a paper-and-pencil environment. The difference lies in the drawing facilities offered by Cabri. The solution strategies of both tasks do not differ deeply.

Cabri as facilitating the mathematical task

A polygon and its translated image are given in Cabri. Students are asked to conjecture relations between the sides of the initial polygon and its image. In this kind of task, a visual recognition of the relations should be relatively easy. The role of Cabri is to help students make conjectures about the relations using the drag mode.

Tasks modified when given in Cabri

In constructing a square with a given side (Laborde and Straesser, 1990), the solving strategy is deeply affected by the environment. With paper and pencil (squared paper) the task consists essentially of drawing four sides of equal lengths along the lines of the paper. This is achieved by following the lines of the paper and counting the squares. Solving the task is controlled by perception, since the shape produced by the drawing activity should be a square.

The same task in Cabri must be done by satisfying both the perpendicularity of sides and the congruence of two sides. The congruence cannot be obtained by eye and counting, but uses a circle as a tool for transferring a given distance. The task in Cabri requires more mathematical knowledge about the properties of a square and the characteristic property of a circle which students find difficult to put into action (see for example Hölzl et al., 1994).

Tasks only existing in Cabri

A task like reconstructing a dynamic diagram given in Cabri (so called 'black box' tasks) takes its meaning from the Cabri environment itself, in particular from the drag mode which preserves geometrical relations. Such tasks require identifying geometrical properties as spatial invariants in the drag mode and possibly performing experiments with the tools of Cabri on the diagram. These tasks differ from observation tasks in which Cabri supports the conjecturing activity in that the identification of underlying properties is not easy and constitutes the question. In particular, as shown below in the section 'New kind of tasks', it requires reasoning and knowledge.

Evolution of the tasks of the scenarios over time

We tried to assign the tasks of the scenarios to the proposed categories. This is of course a reconstruction informed by the comments of the teachers accompanying the scenarios.

Tasks of the first category were found only in the scenarios written by the novice teacher expert in the use of technology: they were tasks in which Cabri was facilitating the collection of numerical data. We also observed an evolution over time in the type of tasks proposed by the experienced teachers. The first and second versions of their scenarios did not mainly comprise tasks taking their meaning from the environment. It is easy to understand that the design of such tasks represents a conceptual break with the usual tasks performed in a paper-and-pencil environment. It is easier to discover new efficient strategies available in Cabri in an existing task

than to invent new tasks. We could observe an evolution over time in the frequency of occurrence of the types of tasks. At the beginning, most were observation tasks for conjecturing, whereas more diverse tasks appeared in later versions.

From the very beginning of the writing of scenarios, the experienced teachers in both domains often used the observation tasks in which Cabri facilitated the production of conjectures. Here are some excerpts from comments written by one of the teachers about the role of technology in these tasks.

About the task of conjecturing the nature of the composition of n point symmetries, n being any number, he wrote:

It is just about grasping at an intuitive level a possible generalisation of results just observed in particular cases. It is necessary to remain modest at the level of Seconde (Grade 10) and to take into account the real possibilities of students. The computer environment turns out to be useful here as a means of exploration and help to conjecture: If the number of the polygon is even, it is the image of MNPQ through a translation. If it is odd, it is the image through a point symmetry.

In the task of conjecturing properties of dilations:

The animation of the screen (which results from the use of the tool 'Dilation' in conjunction with dragging) implies that the students acquire without too much investment a global visualisation of a dilation: the effect of Dilation on usual figures and their main properties which are conjectured after multiple trials that they can be achieved in a short period of time. [...] In paper-and-pencil environment, the properties are immediately required to construct the image of a figure.

From these excerpts it is clear that the role of the software is to save time, to avoid complex constructions requiring the use of properties that are exactly the properties to be discovered, and to favour visualisation. The declared intention of the teacher was to keep the demands of the task at a modest level. The role of Cabri was mainly to facilitate conjecturing and not to cause a problem, as in construction tasks, where the solving strategies have to be constructed with the Cabri tools.

At the beginning of the project, the experienced teachers also used construction tasks but less frequently. It is interesting to note that it is only in Version 2 of the scenario 'Vectors' that they introduced black box and prediction tasks. And finally, influenced by the researchers, they introduced in Version 3 of the scenario 'Transformations', a geometrical transformation as an unknown macro-construction. These tasks will be discussed later in the section entitled 'New kinds of tasks'.

The experienced teacher novice in technology also introduced observation tasks leading to conjectures and rather fewer construction tasks in the first real scenario that she wrote. When she gave a construction in Cabri,

she also gave the same task in paper-and-pencil. She explained in one of our meetings that she was sure that students understood that the image by a dilation of a straight line is a straight line if they drew it with a ruler. The paper-and-pencil environment was for her the guarantee of theoretical knowledge.

The following sections are devoted to an analysis of the integration of Cabri in some of the tasks described above.

TECHNOLOGY AS A PROVIDER OF DATA

Two activities designed by the teacher novice in teaching illustrate the role assigned to technology of simply facilitating the collection of data and not changing the mathematical task. This type of task belongs to the first category (facilitation of material actions). They are tasks about the sum of the angles of a quadrilateral and the sum of distances of a variable point to fixed points.

The sum of the angles of a quadrilateral

This activity dealt with the sum of the angles of a convex quadrilateral. Because of the type of computers available in his school, Cabri-geometry I was used. The teacher gave the following task to the students:

Construct any convex quadrilateral ABCD. Mark and measure the interior angles of this quadrilateral by means of the menu items 'Mark an angle' and 'Measure' of the menu 'Miscellaneous'. Note the measures in the table below. Calculate the sum of the four angles and note the result in the table. Move A, B, C or D and note the new measures, then calculate the sum. Repeat it several times. What do you notice? Make a conjecture and prove it

In this activity, the computer is only used for measuring and not for performing calculations on the measures. The students had to add the measures on a sheet of paper as soon as they moved one of the vertices of the quadrilateral because there was no calculating facility on Cabri I. Two environments, Cabri-geometry I and paper and pencil, were involved in the task. Coordinating both might be difficult for students and this could lead students to focus only on the invariant sum visible in the table and to conjecture this invariance. They might then try to find a proof in the same paper-and-pencil environment.

The features of Cabri I might lead them to separate the conjecture and the proof (and this was supported by the statement of the task). In the conjecture phase, the features of the software may reduce its role to a provider of quadrilaterals and measures. It plays the role of a table of values of angle measures of quadrilaterals.

If the teacher wanted students to observe variations of the diagram for establishing the proof, he/she probably should have made this explicit in the task. After asking students to formulate a conjecture, he should have asked them to prove it by observing simultaneously geometrical variations and invariants when a vertex is dragged. Students could then have observed not only that the sum remains constant but also that three vertices remain invariant when one of them moves. This is the starting point for establishing a relation between the invariance of the sum and a geometrical analysis of the quadrilateral. Recognising an invariant triangle in the quadrilateral may prompt the use of the property of the sum of the angles of a triangle. By the way, one can notice how the conjunction of the drag mode and of the calculating facility (available in Cabri II) makes the linking of a numerical and a geometrical invariant easier and does not require the teacher to ask students to focus on this link.

To summarise, the design of the task did not involve software in the process of elaborating a proof.

The sum of the distance of a variable point to fixed points

Students were asked to construct the figure displayed below (Figure 1), in which M is a variable point of segment $[AB]$.

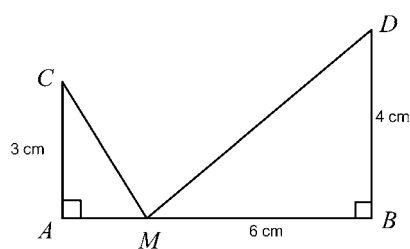


Figure 1. Using Cabri to provide data.

Students were asked to measure MC and MD for several values of AM and then to study, in the paper-and-pencil environment as normal, the function $MC+MD$ by making a table of values and drawing the graph. The teacher did not mention the possibility of obtaining this graph with Cabri as a locus which might have been a good reason to connect geometry with calculus.

In both tasks, the software was mainly used as a provider of data. It was *neither the source of the tasks* given to the students, nor *part of the solving process* of these tasks that were intended to be solved without the computer.

INTEGRATION OF TECHNOLOGY INTO AN OBSERVATION TASK

This point will be discussed by contrasting the first task of the scenario 'Dilation', as written by the teacher novice in the use of technology, and a revision of this task by one of the experienced teachers.

A first version of an observation task in scenario 'Dilation'

This first version of the scenario frequently called for immediate visual observations and generalisation by inductive reasoning. It gave a great prominent role to measuring. The very first activity shown in Figure 2 is representative of the whole scenario. Its outcome is shown in Figure 3.

1 - Definition of a new transformation
 In Cabri mark a point I; create by means of the tool Polygon any quadrilateral ABCD as you want.
 1 – Select the tool Number and type 3.
 Construct the image of ABCD by using the tool Dilation in the following way: designate successively the quadrilateral, point I and number 3.
 Label A', B', C' and D' the corresponding vertices of the new quadrilateral which has been obtained.
 Compare vectors IA and IA';
 vectors IB and IB';
 vectors AB and A'B';
 vectors BC and B'C';
 the area of ABCD and this one of A'B'C'D'.

Which equality is valid for vectors IA and IA'? IB and IB'?

2 – Modify number 3 into -0.5 and answer again questions of activity 1.

Do several trials by modifying the position of I and then of point A.

Figure 2. Version 1 of Activity 1 of scenario 'Dilation'.

The activity called only for discrete measuring and did not use the animation facility of the numerical display of the ratio. From Question 1 to Question 2, students had only to change the ratio and repeat the same comparisons. It was only after these two trials that they were asked to drag a vertex of the quadrilateral and the dilation centre.

Cabri was used more as a *provider of several static diagrams than as a provider of a variable diagram*. The constant ratio between the length of IA and IA' (or AB and A'B') was not presented as invariant in the drag mode but as the same from one diagram to another one. The focus of the activity was on the result more than on the variation, exactly as in the task of the sum of the angles of a quadrilateral discussed above.

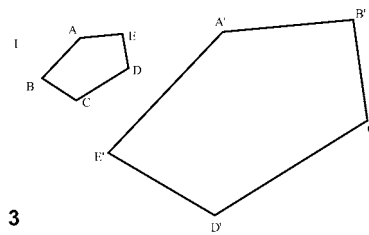


Figure 3. The outcome of the activity.

After this activity the definition of a dilation with centre I and ratio k was given: vector $IM' = k$ vector IM . Some exercises about writing vector equalities defining the image of a point through a given dilation were given independently of any use of the software.

Then in Part II, again students were asked to draw a polygon and to observe the images of the polygon through dilations with centre a fixed point and various ratios, changed in a discrete way. Then they were asked to generalise by giving a condition on the ratio for obtaining a smaller (or larger) image, an image with same (resp. with or different) orientation. Again students were not asked to investigate the variation of the size of the image with respect to the variation of the ratio.

Obviously the drag mode was used very little in these activities. The software was mainly used to provide a great number of dilations and to facilitate a conjecture about a general rule that students were asked to generalise by induction, without proving it. Then a proof was almost entirely given to students who had only to fill in some blanks in vector equalities of the proof.

What emerged from those activities is:

- a *reduced and static use of the possibilities of the software*: only measuring and no use of continuous drag of points or animation of ratio; activities were mostly inspired by paper-and-pencil activities;
- questions were restricted to numerical relations between elements, no qualitative questions about shapes, dynamic behaviours of the geometric elements;
- *strong guidance* to the students, *the absence of autonomous experimentation* by students: the elements to be compared and to be changed were given to the students.

The revision of the scenario 'Dilation' by another teacher

The following year, one of the experienced teachers modified the first version of the scenario Dilation. We use activity 1 (Figure 4) as an example of the modifications.

1. Study of a new transformation, Dilation In the toolbox Transformation of Cabri, in addition to reflection and point symmetry, there is the tool Dilation. You will study this transformation. Create a point I; edit a number k by using as starting value 2.5; create by means of the tool Polygon, a quadrilateral ABCD and construct its image through the dilation with centre I and ratio k (tool Dilation, designate the quadrilateral, then centre I and number k). Characterise the obtained image. (Do not hesitate to drag polygon ABCD, points A, B, C and D, centre I, and to modify number k; do not forget that you can display measures with tool distance and length and that a calculator is available within Cabri). Give to k a negative value (choose in a first step -0.5) and complete the previous characterisation (do not hesitate to vary k again).

Figure 4. Version 2 of activity 1 of scenario 'Dilation'.

Version 2 condensed the two activities of version 1 into just one activity, which in contrast to version 1, made a central place for dragging and did not ask students to perform discrete measures. Version 2 also gave more autonomy to students in that the text only mentioned a range of possible tools to be used. Version 2 was more open-ended while simultaneously prompting students to make more use of available tools. This is confirmed by the other activities of the revision of the scenario: use of tool Locus to explore the set of images of a point belonging to a polygon, use of Point symmetry to determine the centre and the ratio of a dilation, vectors, grid, system of axes, equation of a line . . .

Thus this new scenario exhibited both an increased use of the tools offered by Cabri and a greater autonomy of students, in particular in exploration methods. The increased number of tools to be used requires of course a high command of these tools by students and thus more familiarity with Cabri. In terms of perturbations, it is as if the novice teacher did not dare change her teaching too much and gave activities with little use of software. Her choices minimised the changes implied by the use of technology, whereas the revision took into account the perturbations and changed the tasks in a deeper way.

Contrasting the mediation of mathematical objects in both versions

In the novice teacher's version, the students' work dealt with a static diagram representing a theoretical object. The fact that the diagram was changed in a discrete way supported the conception that the modified diagram referred to a different mathematical object. In the remake, the diagram was continuously changed and it may support the idea that the diagram was representing a variable object. This conception is very

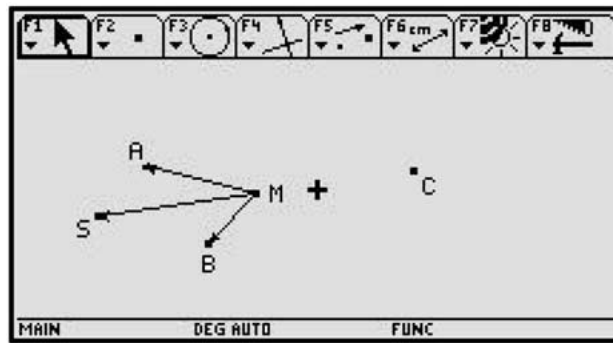


Figure 5. Constructing the third vertex of a triangle given two vertices and the centroid.

different from the usual one underlying schoolbooks and it may explain why it was not present in the novice teacher's version.

NEW KIND OF TASKS

The later versions (Versions 2 and 3) of the scenarios introduced two new kinds of tasks:

- tasks in which the environment allows efficient strategies which are not possible in a paper-and-pencil environment;
- tasks raised by the computer context, i.e. tasks which can be carried out only in the computer environment.

An example of the first kind of tasks is in the scenario 'Vectors': 'Construct a triangle ABC from the given points A, B and M, the centroid of triangle ABC'. Point C is theoretically determined by the relation:

$$(1) \quad \text{vector MA} + \text{vector MB} + \text{vector MC} = \text{zero vector.}$$

In a paper-and-pencil environment, it is impossible to make direct use of this relation. The sum MS of vectors MA and MB must be constructed through the parallelogram construction and point C is then constructed so that M is midpoint of CS. This strategy is no simpler than a purely geometric strategy consisting of considering M as a point two thirds along a median. Thus the vector relation is not efficient for the task in a paper-and-pencil environment. In Cabri, C can be constructed by just two operations; as the symmetrical point with respect to M of the endpoint, S, of the vector sum of the two vectors MA and MB (Figure 5).

This solution requires the change of relation (1) into

$$(2) \quad \text{vector MC} = -(\text{vector MA} + \text{vector MB})$$

and the identification of a point symmetry in relation vector $\vec{MC} = -\text{vector } \vec{MS}$.

Cabri thus contributes to linking the algebraic aspects of vectors to their geometrical aspects. Relation (1) is restricted in paper-and-pencil environment to algebraic calculations, whilst in Cabri it also receives a geometrical meaning since it is a tool of construction. It offers a new connection in the conceptual field of vectors (Vergnaud, 1991) or in the *web* of vectors (Noss and Hoyles, 1996).

In the paper-and-pencil environment, vectors and transformations are only used in reasoning and in proofs on theoretical objects and it has often been observed that it is quite difficult for students to have recourse to them in their proofs (Bittar, 1998). They prefer to use Euclidean arguments. We assume that Cabri allows tasks in which theoretical objects like vectors receive a kind of reification by being tools of construction.

The teacher introduced the task in the scenario because he was aware of the change of strategy and of the efficiency of vectors and transformations as tools of construction. He commented on it in one of the research meetings, stressing that it was a completely new kind of task belonging to a new culture in which theoretically complex objects become construction tools. He supported the idea that this culture needs time to be developed and must be introduced early in the school year. It was clear from his comments, that this teacher was aware of the conceptual change involved in this kind of task. It is interesting to note the evolution of this teacher, who at the beginning was mainly using Cabri as provider of dynamic imagery for facilitating formulations on the theoretical objects evoked by the diagrams. In this case, the theoretical objects are tools *operating* on the diagrams.

The second kind of tasks can be divided into two categories:

- ‘black box’ situations;
- prediction tasks.

In black box situations, students are given a diagram on the screen of the computer and are asked questions about it. Such a situation had been introduced by the experienced teachers in Version 2 of the scenario ‘Vectors’. It was only later used under the influence of the researchers of the group for introducing transformations. The first introduction of an unknown transformation in the scenarios was designed by Jahn (1998) and tried out by one the teachers several times. Then, only after discussion, the same teacher decided to introduce the same idea in Version 3 of scenarios ‘Translation’ and ‘Dilation’.

Instead of starting with the observation situation (presented above) about the effects of a dilation, Version 3 of scenario ‘Dilation’ started with a black box situation in which a point P and its image P' through

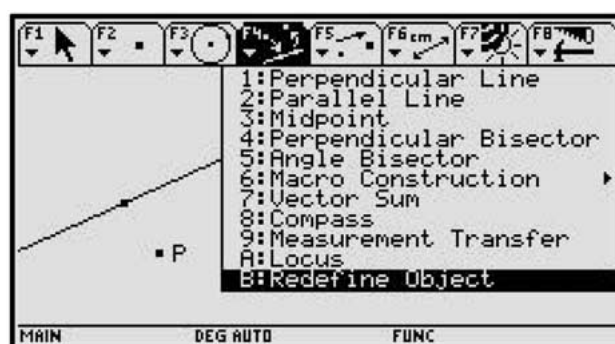


Figure 6. Redefining the given point P ...

an unknown transformation were given to the students. They could move P and observe the subsequent effect on P' . Students were asked to find the properties of the unknown transformation by means of this black box. In such a task, students must ask themselves questions about the transformation: Does it preserve collinearity? Does it preserve distance? Does it have invariant points?

Cabri can be used to design experiments and obtain empirical answers. For example, one may redefine P as belonging to any given straight line and obtain the image of this line as the locus of P' depending on the variable point P (Figures 6, 7, 8, 9). One may then redefine P as point of a circle and obtain then image of a circle through the unknown transformation (Figure 10). Two specific tools of Cabri are used 'Redefinition' and 'Locus'. It presupposes that the students not only master their use but also decide to use them. This decision actually involves mathematical knowledge, that is that the fact that the image of a figure is the set of images of points of a figure. This knowledge is often completely implicit in our curricula, even though it presents a conceptual cut (even an obstacle probably with both cognitive and didactical origins) between viewing a figure as an entity and as a set of points (Jahn, 2000).

Such a task offers a very different point of view on the notion of geometrical transformation, since instead of studying the effects of a known transformation, students are asked to characterise the transformation by means of its properties. This may be an attractive task, but only if some exotic transformations rather than just the usual ones are given to students. Theorems of invariance receive a new meaning in this kind of task: they are tools for identifying the category to which the unknown transformation belongs. An effect of this kind of task is that students may understand the reasons for studying all the theorems about invariant

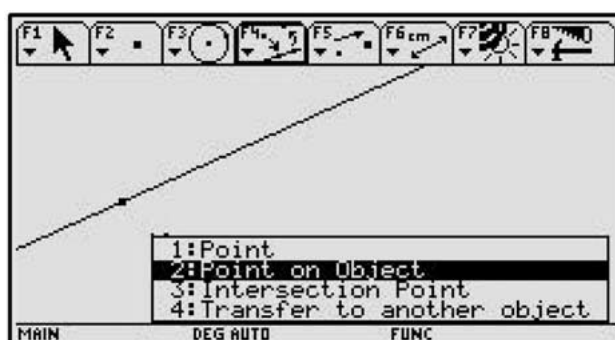


Figure 7. as a point on ...

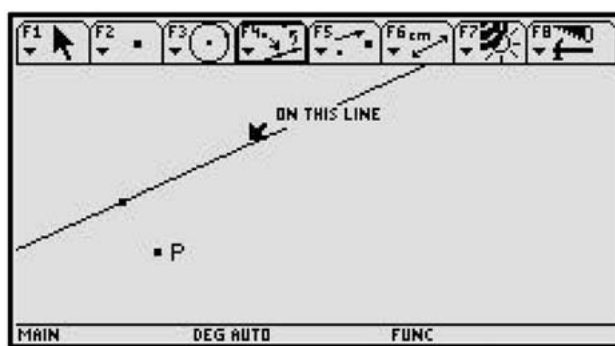


Figure 8. ... a line

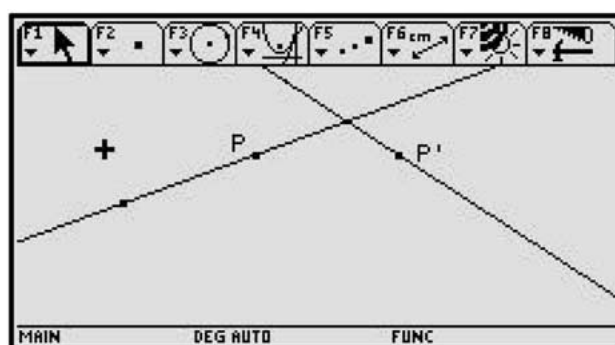


Figure 9. Locus of P' when P is moving on the line.

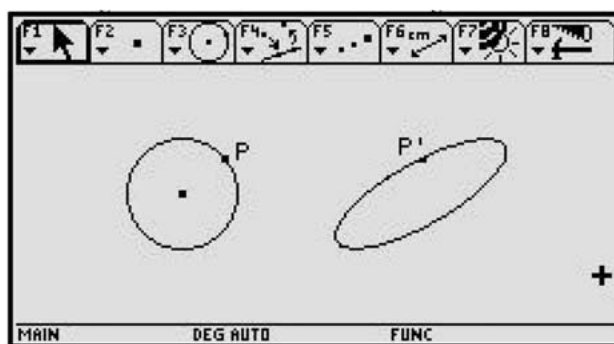


Figure 10. Locus of P' when P is moving on a circle.

elements of transformations: the invariance properties become remarkable rather than routine phenomena.

As already mentioned, black box tasks were not part of the first scenarios and appeared only gradually: at first for introducing the coordinates of a vector and much later for transformations. This shows that it takes time to integrate familiar kinds of tasks into new topics. It is not just a question of applying a given task, but of reconsidering the mathematics involved in the notion to be taught using the perspective of a black box task. Transformation is a second order object in that it is a process operating on objects, and transferring the idea of black box from objects to process requires a conceptual change of point of view.

Prediction tasks are also specific to the computer environment. As with black box situations, they appeared first in Version 2 of scenario "Vector" in the study of the coordinates of a vector. A vector was drawn on the screen and its coordinates displayed and updated when the vector was changed by dragging or rotating one of its endpoints. Before dragging, students were asked to predict the change of the coordinates when the vector was dragged, or when one of its endpoints was rotated by the pointer 'Rotate'.

Cabri allows a confrontation between what is predicted and what is observed. When students' predictions turn out to be wrong, this is a good opportunity for asking 'Why is it so?' and calling for an explanation or even proof. Teachers did not exploit all possible opportunities for such situations in the project; for example they could have used them more often in the scenarios about 'Transformations'. They preferred to propose observation rather than prediction situations for conjecturing and proving.

Learning Hypotheses – Epistemological Beliefs

The design of tasks is based on implicit assumptions about the way students learn mathematics and about the mathematical content itself. The reactions of the teachers when integrating technology could be used as a window on their own epistemology (Noss and Hoyles, 1996, ch. 8). From these reactions some hypotheses could be made about their beliefs about mathematics and their conceptions of learning. Three types of reactions will be commented on below in terms of possible beliefs:

- dichotomy between conjecture and explanation or proof;
- repetition of the same tasks in Cabri and in paper-and-pencil environment;
- complexification of tasks.

Dichotomy between conjecture and explanation or proof

The most obvious contribution of Cabri is the possibility of dynamic visualisation of geometrical relations preserved by the drag mode. Teachers (even the novice in using technology) immediately exploited this possibility by asking students to conjecture properties from what they could see. However, when the students were asked to justify, the teachers did not mention the possibility of using Cabri to find a reason or to elaborate a proof. It is as if there was no interaction between visualisation and proving. Technology was used in these tasks, as facilitating the formulation of conjectures but its role did not go beyond that. Quoting the formulation of Hölzl (2001, p. 65), we would say that the drag mode was “used only in a verifying manner” and that “learners are just supposed to vary geometric configurations and confirm empirically more or less explicitly stated facts”.

It was as if the process of elaborating a proof should deal with theoretical objects unrelated to their representations, not modified by actions on these representations. Bosch and Chevallard (1998) argue that mathematicians have always considered their work as dealing with non-ostensive objects and that the treatment of ostensive objects (expressions, diagrams, formulas, graphical representations) only plays an auxiliary role. This conception, according to which mathematical concepts exist independently of their representations, and which does not take into account interactions and mutual controls between non-ostensive and ostensive objects, seems to underpin this dichotomy. This is not unrelated to another conception: the intrinsic link of geometry with paper and pencil that is presented below.

Repetition of the same tasks in Cabri and in paper-and-pencil environment

The teacher novice in technology did not rely on technology-based activities for learning geometry and, in addition to technology-based activities, proposed similar paper-and-pencil tasks seemingly unaware that a paper-and-pencil task may be less demanding in terms of knowledge, by allowing perceptive strategies instead of strategies based on theoretical properties. It seems that she had an epistemological view of geometry as intrinsically linked to paper and pencil. This belief of the canonical form of mathematics linked to paper-and-pencil environment is widely shared. Povey and Ransom (2000) report an inquiry carried out among undergraduate students in mathematics in UK. Each of them seemed to refer to a single specific mode of understanding mathematics, the paper-and-pencil mode (pp. 52–53). “Technology can help if you have a paper-and-pencil understanding” stated one of the students; and this formulation could be taken as summarising the philosophy of the scenarios written by this teacher. As expressed by Povey and Ransom, the underlying learning assumption is that “doing maths by hand indicates that one understands it”. This is exactly the type of claim made by the teacher when she explained to us that, without the material action of drawing the image of a straight line through dilation with a straight edge, students would not appropriate this invariant of dilation.

This point of view is often linked with the conception of a paper-and-pencil environment as ‘not a context’. Knowing how to carry out a construction in paper-and-pencil environment would be the warrant of decontextualised knowledge. Noss and Hoyles (1996, p. 48) propose an alternative view of abstraction as not necessarily linked to decontextualisation and “as a process of connection rather than ascension”. They add that the “situated, the activity based, the experiential can contain within it the seeds for something more general” (p. 49). In the interaction with the computer, learners may construct what Noss and Hoyles call *situated abstractions*. Situated abstractions are invariants that are shaped by the specific situation in which they are forged by the learner. Although those invariants are situated, they simultaneously contain the seed of the general that could be valid in other contexts:

Within a computational environment, some at least of these objects and relationships become real for the learner (we are using ‘real’ here to mean something other than simply ontologically existent—perhaps meaningful or broadly connected are better descriptions): learners web their own knowledge and understandings by action within the microworld, and simultaneously articulate fragments of that knowledge encapsulated in computational objects and relationships—abstracting *within*, not *away from*, the situation. In computational environments, there can be an explicit appreciation of the form of generalised relations

within them (the relational invariants) while the functionality and semantics of these invariants – their meanings – is preserved and extended by the learner (p. 125).

Such linkage between understanding and paper and pencil may also be explained by the institutional context. Even if all kinds of calculators are allowed in our French national examination, all examination tasks are given in a paper-and-pencil environment. The teacher thus prioritises this context to be sure that students are able to perform the tasks in the examination environment.

Complexification of tasks

Experienced teachers involved in teacher education, such as the teachers we worked with, very often have a constructivist view of learning based on two assumptions:

- Students learn when they are faced with tasks for which mathematics notions are efficient tools of solutions;
- Feedback coming from the situation may favour an evolution of solution strategies more than a judgement coming from the teacher.

Feedback coming from dynamic geometry software may, from this point of view, be very rich in that it allows an interaction between the visual and the theoretical aspects of geometry. If a constructed diagram in the drag mode does not keep the shape that was expected, it means that the construction process must be wrong. The drag mode can also invalidate a conjectured property and thus lead the students to abandon it.

The teacher may rely too much on feedback from the calculator/computer and propose tasks of a greater complexity than corresponding paper-and-pencil tasks. The teacher underestimates the complexity of the task, and the time needed for the student to solve the task because he has little reference in his experience. He or she overestimates the possibility of interpretations by the student of feedback given by the software.

We observe this in Version 1 of the scenario ‘Vectors’ in which students were asked to construct all diagrams for the tasks in Cabri. Instead of teaching vectors for two weeks, it took two months! It is also a common phenomenon that any kind of teaching innovation provokes time inflation. Schneider (1999) reported on teaching about logarithms and exponentials based on the use of the TI 92 which took 40 hours of teaching instead of the usual nine hours.

Returning to the project, after the first year, the teachers attempted to find an optimal balance

- between what is prepared and demonstrated by the teacher on the LCD display and what is done by students,

- between what is ready made and given on the calculators to students and what has to be done by the students with the software.

For example, after one year the teachers preferred to give the macro-construction of the multiplication of a vector by a number for students to explore and interpret rather than for the students to construct themselves. Even apparently minor aspects may slow down the construction of a diagram. In the scenario ‘Vectors’, the first task was about polygons that students had to draw. To this end, they had to designate the successive vertices of the polygon and, at the end of the sequence, again the first vertex. Actually it turned out that students tried to do polygons with a large number of sides, and sometimes had difficulties in designating at the end exactly the first vertex and not a close point. If they had used a double click on the last vertex of the polygon, it would have avoided difficulties. But the teacher did not anticipate the long time spent on drawing the polygons and mentioned this shortcut only orally during the activity. This meant that only some students paid attention to his remark.

Evaluating the complexity of a task with technology requires taking into account not only the conceptual difficulties but also the use of the technology by the students. This is not easy and the wrong *a priori* evaluation by the teacher of the complexity of the task came from an absence of reference about students’ behaviour in the tasks. The complexification of tasks may also come from the uncertainty of the teacher about what students would learn from the tasks. They tended to presuppose that technology would facilitate the solving process, so that in order to be sure that students learned something from the new kind of tasks, the teachers increased the level of complexity. In all cases, it is clear that a deep and precise knowledge of students’ behaviour and strategies in the Cabri environment is essential for evaluating *a priori* the degree of difficulty of a task.

CONCLUSION

In our introduction we claimed that the process of integrating technology into mathematics teaching is a long and complex process. In analysing the types of tasks developed by teachers over the three years of the project and their evolution, we can formulate tentative explanations for the length of this process.

In terms of our initial framework about the didactic system as a complex system, technology is not just an additional element in the system since it interacts with all the components of the system, which are subject to change. As already mentioned above (see section ‘Various roles of Cabri’),

this point of view is based on two theoretical approaches, the notion of instrument as developed by Rabardel and Vérillon and the mediating function of a computational learning environment (Noss and Hoyles, 1996). Vérillon and Rabardel (1995) stressed how an artefact is not taken as such by the learner but reconstructed by him/her. The learner constructs both a representation of the artefact (the instrument) and the structures that allow him/her to perform activities with the artefact (schemes of utilisation of the artefact). Both types of constructs depend on previous knowledge of the learner and affect this knowledge. According to a Vygotskian perspective, Rabardel claims that the 'instruments' constructed by the learner constitute forms which structure the relationships with situations and knowledge and thus may have a considerable influence on the construction of knowledge. Noss and Hoyles (1996) investigated for many years how learners construct situated abstractions dependent on the means of action and expression offered by the environment. In particular, the influence of Cabri on students' strategies has been analysed in depth by Jones (1999) and Hölzl (1996, 2001). Hölzl (2001, p. 83) distinguishes two ways of using the mediating functions of the drag mode, as a test mode on the one hand and as a search mode on the other. All his observations led him to conclude that the second use of the drag mode is not "a short term affair" but results from "a learning process that is characterised by different layers of conceptions".

We interpret the behaviour of both novice teachers as resulting from their perspective that technology is an additional component of the teaching system but external to the learning processes. Technology facilitated material aspects of the actions of the students (teacher novice in teaching), technology was used in observation and construction tasks but activities in paper-and-pencil environment were given in addition by the teacher who was a novice in the use of technology. It is interesting to note that this latter teacher planned what might be interpreted as a more verifying or test way of using the drag mode than search way (in Hölzl's terms). In the observation tasks that she gave, all steps of the conjectures were given explicitly.

A second interesting feature of the design process of the scenarios by the experienced teachers can also be interpreted in terms of instrumentation and mediating function of the environment. These teachers offered more open exploration activities involving more of a search use of the drag mode; they did it in two kinds of circumstances: at the beginning of sessions in observation tasks to introduce new properties and at the end of sessions in open problems to be solved. But the comments they added expressed clearly that the drag mode was for them more to facilitate

visualisation than to act in the solution process, even for the open-ended problem. It took one or two years for them to accept that investigating the invariants of an unknown transformation under the form of a black box situation through the drag mode and the tool 'Redefinition' could be part of a scenario. The difference between a black box situation and an observation situation for conjecturing must be stressed here. A black box situation is a problem situation and the invariants are the tools of solution of this problem. In an observation situation, in which students are asked to conjecture properties, the question is more to satisfy a contract of finding properties relevant from the perspective of the teacher.

We assume that really integrating technology into teaching takes time for teachers because it takes time for them to accept that learning might occur in computer-based situations without reference to a paper-and-pencil environment and to be able to create appropriate learning situations. But it also takes time for them to accept that they might lose part of their control over what students do. Povey and Ransom (*op. cit.*) concluded from their inquiry among undergraduate students (already cited above) that the plea for learning by doing 'by hand' could be related to a "desire to feel in control" (p. 56). As they mentioned, speaking about technology as 'taking over' and depriving the human of control is usual in a wider social context. The situation is far more complex for a teacher who must not only understand what the computer does but also what the students do with the computer.

Cabri covers a broad domain of knowledge and action. It is a micro-world allowing multiple ways of exploring, experimenting and solving a problem. If the basic use of Cabri can be learned rapidly because of its friendly interface, constructing a global and structured representation of all of its possibilities requires time. It requires even more time to understand on the spot in the classroom the strategy of students faced with a task with Cabri that they are not able to describe. Teachers may be uneasy if they do not control everything happening in the class.

If we reconsider the evolution of the choices of the teachers, we could interpret the early behaviour of both novice teachers as an unconscious way of trying

- to avoid interaction of Cabri with the core of the didactic system (anecdotal tasks whose content was not essential for the curriculum),
- and to limit the scope of the schemes of utilisation developed by students only to dragging, observing and measuring.

When the teacher novice in the use of Cabri decided to write a real scenario, the activities she proposed to students made use of a small range of tools available in Cabri. This might have been due to:

- her partial knowledge of the software: she may have not yet developed a large set of schemes of use, for example she did not propose to vary a number by dragging its display on the screen
- and/or to her willingness to control what the students could do with Cabri.

This resistance of teachers to introduce in their classrooms computational ideas over which they were not in command was also found by Noss and Hoyles (1996, 1993, ch. 8) in a University course for teachers on the use of microworlds in mathematics. They developed caricatures summing up a cluster of features shared by several teachers, and two of these caricatures, Dennis and Bob, were very concerned with the idea of understanding completely an idea before using it. In particular, the microworld approach and its openness were at first rejected by Bob. Bob was also careful not to offer an over-demanding curriculum to low-attaining students and this could have also been one of the reasons of the strong guidance of the teacher of our team in the scenarios that she wrote.

One of the experienced teachers had a different way of formulating the activities that the novice teacher wrote, by opening up both the task and the range of possible tools used by students. The consequence is that class management would be more difficult since the range of possible answers would be larger, as well as the range of possible questions and difficulties of students. This is why managing the integration of technology in tasks given to students requires the teachers to know enough about possible strategies and uses of the technology by the students. This can be learned from research but also from practice in class.

Another step is reached when the teachers consider the changes brought by technology on the mathematical concepts, as the experienced teachers did when proposing new kinds of tasks in which vectors or transformations became construction tools or black boxes with unknown transformations. It requires a change of teacher perspectives on the mathematical objects, which is not easy to achieve, or to be accepted if it is proposed by somebody else. The second difficulty lies in the distance between the new perspective on mathematical concepts and their normal presentation. This difficulty arises in countries like France in which there is a national curriculum and a national examination at the end of the secondary school. Teachers make the decision to introduce these new tasks, only if they are sure that the learning expected by the institution will benefit from this. This requires teachers to conceive possible interrelations between the new conceptual aspects introduced by technology and the actual curriculum. This depends also on their views about the curriculum. Although the introduction of vectors as construction tools of points may be easily accepted,

since the equality ‘vector $GA + \text{vector } GB + \text{vector } GC = 0$ ’ (where G is the centroid of triangle ABC) is an explicit part of the French curriculum, working on exotic transformations (which are not part of the French curriculum) with black boxes may not be so easily accepted by teachers unless they are convinced that students need to understand why invariance theorems are of interest in geometry.

Generally, these activities allowing rich interactions with the students require time and the development of specific schemes of instrumentation. They bring a new kind of perturbation with respect to the legitimacy of knowledge specific of technology and of constraints of time. Whilst in more classical tasks as observation tasks, the perturbation affected the relationship between the students and the mathematical content of the tasks, in the second kind of tasks, it affects the entire didactic system: the choice of the official content to be taught and the management of time.

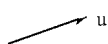
This is why the experienced teachers in our project had two reactions after one or two years:

- they brought changes beyond the class itself, they gave homework and part of the usual assessment in class on Cabri;
- they institutionalised mathematical contents in relation to technology.

The introduction of homework was a means of giving time to students for developing their schemes of instrumentation at their own pace at home on exercises of various types, simple or more open-ended. For example, some open-ended problems of the scenario ‘Translation’ were given as homework since they required time for exploration and generalisation. The possible ways of solution were then discussed in the classroom. But homework could also be devoted to carrying out instrumented construction procedures as in the case of Version 2 of scenario Dilation. Students had to construct at home images of figures without using the ready made tool Dilation. This supported both the use of invariants of dilation as well as the use of the tools of Cabri, related to the observation of Guin and Trouche (1999) in the case of an integrated use of Derive on the TI 92: they claim that the instrumentation processes cannot take place without a complete reorganisation of study time, giving room to students’ own manipulations.

Including assessment on Cabri in the usual assessment in addition to paper-and-pencil assessment was a way of giving legitimacy to the instrumented solutions. It was also a means for the teacher to judge the degree of appropriation of instrumented techniques by the students.

The assessment in Cabri comprised simple but critical exercises in which technology could reveal better than paper and pencil how a mathematical notion was acquired, such as in the task of constructing a straight



A

Figure 11. The vector task.

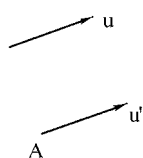


Figure 12. One solution.

line passing through a given point A and whose direction vector is a given vector u (Figure 11).

For most students a direction vector should be on the line until they understand that a vector is just giving the direction and not the location of the line in the plane. Students with a conception of a direction vector attached to the line construct a vector u' equal to u with origin A (Figure 12) and then a line passing through point A and the endpoint of u' . Students with the second conception simply construct a line passing through A and parallel to vector u (Figure 13). This strategy is far more efficient than the preceding one (requires only use of the tool Parallel Line instead of several tools).

In the first year of the project, the institutionalisation of mathematical content was always done 'as usual' without any reference to Cabri. Institutionalisation is the process by which the teacher formulates what is to be learned, to be known by the students as the official knowledge (Brousseau, 1997, p. 236). The underlying philosophy is that the institutional formulation must be the cultural form of knowledge, general enough and context free to allow a shared use among all users of mathematics. In the case of students in school, the formulation should be shared (or understandable) by all students and teachers of the country. The general form is also supposed to facilitate a reinvestment of knowledge in other contexts because it is the part of knowledge that is detachable from the context. At the beginning of the project, properties, theorems or definitions were formulated in the

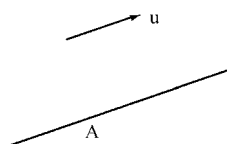


Figure 13. A more efficient solution.

same way they are in the schoolbooks, without reference to a context.⁵ But after some time, the experienced teachers of our team discovered that some ideas or notions difficult to conceptualise could give rise to ‘tangible’ phenomena in Cabri, whilst it was quite impossible to formulate them jointly in a de-contextualised manner and satisfy mathematical rigour.

Let us comment on an example coming from the scenario ‘Vectors’. The notion of direction of a vector is not easily understood by students. There is a paradox that this notion is understood as soon as the notion of vector is acquired, but to acquire the notion of vector, the students need to understand the notion of direction. Usually it is explained that the coordinates of a vector depend on its direction and magnitude. But as students do not understand the notion of direction, it can be orally replaced by the ‘equivocal’ formulation that the coordinates of a vector do not depend on the origin and endpoint of their representatives.

The use of the two kinds of pointers in Cabri showed the students that when the vector is moved by the usual pointer, its coordinates remain unchanged whilst they are changed when the vector is rotated by the pointer ‘Rotate’ although its norm is unchanged. This is a tangible phenomenon not relying on a prior conceptualisation of direction. So the teacher introduced the Cabri property: coordinates of a vector are modified by the Rotate pointer which lead to the usual property ‘coordinates of a vector depend on the direction of the vector’. This could be interpreted in Vygotskian terms. The pointers played a mediation role of the notion of direction (Bartolini Busi and Mariotti, 1999). The pointer Rotate was used at the beginning as an external tool displaying the change of coordinates with the rotation of the vector with fixed magnitude, in opposition to the usual pointer that did not change the coordinates. The teacher assumed implicitly that after it was experienced by the students in prediction situations, pointers became a psychological tool and were interiorised. The direction of a vector is the invariant attached to this internal construction. The institutional formulation evoking the pointers should refer to this construct of direction.

In the same way, the degrees of freedom of points was introduced by the teachers as linked to their geometrical status. The fact that a translation is determined by two points was made explicit through the use of a macro construction with two points as initial objects.

This reference to the environment in the institutionalised content must be stressed since it is not part of the norm in the teaching. It implies that Cabri became a reference for official knowledge and we consider that this is a sign of high integration of technology into the teaching. Guin and Trouche (*op. cit.*, p. 225) claim that teaching that supports an instrumental

genesis must provide institutionalisation of efficient instrumented techniques. We consider that here the institutionalisation process went further by dealing with content knowledge and not only with techniques.

Finally, in order to meet the demands of the didactic system, the reaction of the participating teachers was to reinforce the links between mathematics and technology: technology gives a meaning to mathematics and mathematics justifies the use of technology. This could be our final proposal for the definition of integration.

NOTES

¹ Original text “L’introduction d’outils nouveaux dans le système didactique ne va pas de soi. [...] il convient de tenir compte des ‘permanences didactiques’, des problèmes propres à l’enseignement et à l’apprentissage, que même les technologies nouvelles ne peuvent contourner. Bien souvent, les stratégies utilisant l’ordinateur prennent peu en compte ‘l’intendance’, qui est laissée à l’enseignant dans sa classe.”

² This project was funded by the Region Rhône-Alpes and the department “New Technologies and Education” of the National Institute for Pedagogical Research.

³ Some scenarios in mathematics, their presentation, and their comments are published in a book (Clarou, Laborde and Capponi, 2001). The whole set of scenarios is available from the author of the paper.

⁴ This shows how the new curricula are ambitious when recommending the use of technology for experimenting to reach a deeper understanding of a mathematical idea.

⁵ In French we say “connaissance décontextualisée” (de-contextualized knowledge).

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